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Reviewers' comments:

Reviewer #1 (Remarks to the Author):

Dear authors,

I have found your manuscript to be a novel, well written and organized paper. The experimental results are well described and explained. The conclusions and recommendations are complete and informative. I have some recommendations:

1) You only have two resonators, so instead of calling them longest and shortest, I would change it to "the longer" and "the shorter".

2) In Fig. 1, instead of plotting the frequency vs temperature, people might be more interested to see a  $(f-f_0)/f_0$  plot. I hope you can update that.

3) Perhaps you can show a plot of the fit, error and data you used to extract your kinetic inductance portion.

4) The caption of Fig. 2 needs to add a bit more information. For instance, Fig. 2(a) has points I II III and IV and an arrow not mentioned in the caption. Although you did a good job describing the data and plot in the main text, the caption should reflect most of the features that readers can find. Probably a sentence like "details about those labels are in the text" will do the job.

5) I have to admit the magnetic field are not weak in your experiment. Can you maybe give some reference in your introduction or conclusion part about how much field a resonator chip might be exposed to for solid-state systems, phase-slip qubits and trapped electrons? Many readers might be curious about how terrible it might be for those types of systems in term of magnetic field.

Overall, I enjoy reading and reviewing the manuscript. Thanks!

Reviewer #2 (Remarks to the Author):

The authors of the paper "Catastrophic magnetic flux avalanches in NbTiN superconducting resonators" present complementary electromagnetic characterizations of superconducting microwave resonators to demonstrate the relevance of thermomagnetic instabilities.

From a technology perspective, the topic is important because microwave superconducting resonators are instrumental for quantum and high frequency electronics.

The paper is very clear and well written. The measurements are presented in detail, along with a large amount of complementary data.

I have only a few remarks:

- Pay attention to acronyms: e.g., "MOI" is defined in row 79, but not used in row 87 and in caption of Fig. 2;

- The dotted circle in Fig. 2 (d) can be enlarged;

- Colormaps/contrast of Fig. 2 (d) can be improved (like Fig. 3);

- A "local magnetization" is defined in row 237. However, by magneto-optical imaging with an indicator film you can measure only the perpendicular component of the magnetic induction field. Therefore, the difference between the local value of the perpendicular component of the magnetic field

and the applied field gives only the perpendicular component of the self-field induced by the supercurrent. I would suggest renaming this quantity, also in Fig. 2.

- Key factors for triggering thermomagnetic instabilities are the applied magnetic field ramp and the heat exchange. These conditions should be the same in the two experimental setups in order to observe a coincident onset of the instabilities. Some details on these aspects could be given at least in the Methods section.

Finally, I recommend the publication with minor revisions.

Below you can find our reply (black lined) to each of the reviewer’s comments (blue lined) and we also indicate the changes we made to the manuscript (green lined).

**Reviewer 1**

Dear authors, I have found your manuscript to be a novel, well written and organized paper. The experimental results are well described and explained. The conclusions and recommendations are complete and informative.

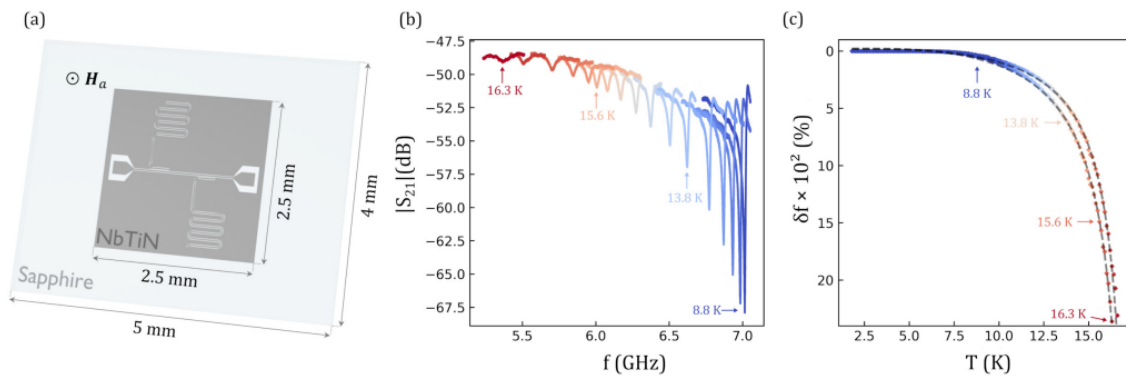
We are glad to read the positive assessment of the referee. In the following sections, we have taken the opportunity to address his/her remarks and answer his/her questions.

**(Q1)** You only have two resonators, so instead of calling them longest and shortest, I would change it to “the longer” and “the shorter”.

**Reply :** We accommodated this suggestion throughout the text.

**(Q2)** In Fig. 1, instead of plotting the frequency vs temperature, people might be more interested to see a  $(f-f_0)/f_0$  plot. I hope you can update that.

**Reply :** We agree with the suggestion of the referee to introduce a normalized resonance frequency and Figure 1 has been adapted accordingly.



**Fig. 1** (a) A schematic representation of the investigated sample. In the superconducting NbTiN region indicated in dark grey, two  $\lambda/4$  resonators are capacitively coupled to a central feedline. (b) The  $S_{21}$  transmission parameter of the longer resonator plotted for different fixed temperatures between 8.8 K and 16.3 K. The color change from blue to red indicates the increase in temperature. (c) For both resonators indicated in panel a the relative frequency shift  $\delta f = [f(0) - f_T] / f(0)$  as a function of the temperature is indicated from blue to red with increasing temperatures. The dashed black line corresponds to a fit of both resonance frequencies as described in the text. The data points corresponding to the transmission measurement shown in panel b are indicated with the corresponding colored arrow.

(Q3) Perhaps you can show a plot of the fit, error and data you used to extract your kinetic inductance portion.

In order to clarify the fit we adapted equation 1 in the manuscript as follows:

“... based solely on a geometric inductance contribution of  $L'_g = 4.19 \cdot 10^{-7}$  H/m. However, it is essential to consider the contribution of the kinetic inductance, which is dependent on the superconducting energy gap. By utilizing an interpolation formula for the superconducting energy gap,  $\Delta(T) \approx \Delta(0) \tanh(1.74\sqrt{T_c/T - 1})$ , the following expression for the temperature-dependent kinetic inductance is obtained [31,32] :

$$L'_k(T) = L'_k(0) \frac{1}{\tanh(1.74\sqrt{\frac{T_c}{T} - 1})} \cdot \frac{1}{\tanh(\frac{\Delta(T)}{2k_B T})},$$

whereby  $L'_k(0) \propto \frac{l}{w} \frac{R_{\square} h}{2\pi^2 \Delta(0)}$  with  $l$  and  $w$  corresponding to the length and width of the resonator,  $R_{\square}$  the normal state sheet resistance,  $h$  and  $k_B$  are Planck's and Boltzmann's constants, and  $T$  is the temperature [31,32]. By combining the general expression for the resonance frequency and Equation 1, the temperature-dependent resonance frequency can be fitted with fitting parameters  $L'_k(0)$ ,  $T_c$  and  $L'_g$ . The fit for both...”

Combining the expression for  $L'_k(T)$  and the general expression for the resonance frequency of a  $\lambda/4$  resonator we used a least squares method with fitting parameters  $L_g$ ,  $T_c$  and  $L_k(0)$ . The remaining geometrical parameters are predetermined by the values known from our design and fabrication e.g.  $l = 4089 \mu\text{m}$ ,  $w = 20 \mu\text{m}$ . The main message we intend to convey is that the system follows the expected temperature dependence associated with the depletion of the superconducting condensate whereby the fitting parameters are fairly close to the expected values. For completeness, we included the standard deviation on all the fit parameters determined by the covariance matrix.

(Q4) The caption of Fig. 2 needs to add a bit more information. For instance, Fig. 2(a) has points I II III and IV and an arrow not mentioned in the caption. Although you did a good job describing the data and plot in the main text, the caption should reflect most of the features that readers can find. Probably a sentence like “details about those labels are in the text” will do the job.

**Reply :** We agree with the referee that the reader will benefit from a more informative caption. Thus we modified the caption as follows :

“(a) Resonance frequency as a function of the applied magnetic field  $\mu_0 H_a$  obtained at  $T=5$  K. The magnetic field has been swept to complete a full loop  $0 \rightarrow 5 \rightarrow -5 \rightarrow 0$  mT. The inset shows a zoom-in of the framed region. The different features associated with the peculiarities of the magnetic field are indicated with the Roman numbers I to IV. These features are visualized in panel (d) and described in the text. Similarly, the region indicated by the red arrow is described in the text and visualized in the supplementary information. (b) ..”

(Q5) I have to admit the magnetic field are not weak in your experiment. Can you maybe give some reference in your introduction or conclusion part about how much field a resonator chip might be exposed to for solid-state systems, phase-slip qubits and trapped electrons? Many readers might be curious about how terrible it might be for those types of systems in term of magnetic field. Overall, I enjoy reading and reviewing the manuscript. Thanks!

**Reply :** Indeed, the applied magnetic fields are not negligible. Following the suggestion of the referee concerning the reference to other systems we included the following paragraph in the conclusion.

“...their performance at GHz seem to be less promising [58].

Even though the investigated maximum applied magnetic field of 5 mT may seem significant, the required field intensities for various applications can exceed the range explored in this work. For instance, in solid-state systems the required field typically falls within the range of 15-200 mT [3-5], for phase-slip qubits the range is around 0.3 mT [6,7], whereas it lies in the tesla range for trapped electrons [10,11]. It is worth noting that while the magnetic field is applied in-plane in some of these systems, a slight misalignment can introduce a significant perpendicular field component.

It is interesting to mention..... “

## **Reviewer 2**

The authors of the paper “Catastrophic magnetic flux avalanches in NbTiN superconducting resonators” present complementary electromagnetic characterizations of superconducting microwave resonators to demonstrate the relevance of thermomagnetic instabilities. From a technology perspective, the topic is important because microwave superconducting resonators are instrumental for quantum and high frequency electronics. The paper is very clear and well written. The measurements are presented in detail, along with a large amount of complementary data.

We thank the referee for his/her critical reading of our manuscript and for the overall positive judgment. In the next paragraphs, we address his/her comments.

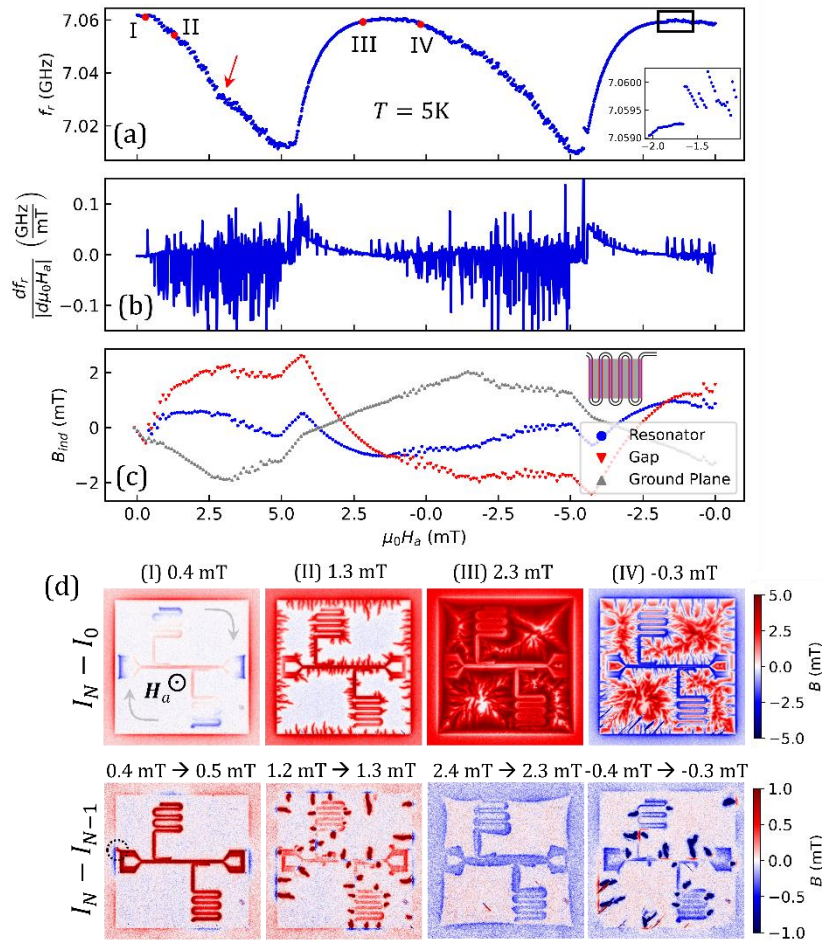
**(Remarks)** Pay attention to acronyms: e.g., “MOI” is defined in row 79, but not used in row 87 and in caption of Fig. 2

The dotted circle in Fig. 2 (d) can be enlarged;

Colormaps/contrast of Fig. 2 (d) can be improved (like Fig. 3);

## **Reply :**

We replaced all instances of the acronym MOI after its definition throughout the text, except in the conclusion. The contrast and the circle have been improved.



(Q1) A “local magnetization” is defined in row 237. However, by magneto-optical imaging with an indicator film you can measure only the perpendicular component of the magnetic induction field. Therefore, the difference between the local value of the perpendicular component of the magnetic field and the applied field gives only the perpendicular component of the self-field induced by the supercurrent. I would suggest renaming this quantity, also in Fig. 2.

Reply :

We agree with the referee, the MOI measurement with an indicator film allows us to solely obtain the perpendicular component of the SC-induced field. In order to clarify this in the manuscript we made the following changes :

“In order to identify the origin of this feature, we have measured the local magnetic field  $B$  in different regions of the device and computed the perpendicular component of the induced field  $B_{ind} = B - \mu_0 H_a$ . The resulting induced field  $B_{ind}$  as a function of the applied field is shown in Figure ...”

Similar for the caption for Figure 2 :

“(c) The perpendicular component of the induced field as a function of the applied magnetic field inside the region schematically represented in the inset...”

All other instances of “local magnetization” have been replaced, including the labels of the supplementary material.

(Q2) Key factors for triggering thermomagnetic instabilities are the applied magnetic field ramp and the heat exchange. These conditions should be the same in the two experimental setups in order to observe a coincident onset of the instabilities. Some details on these aspects could be given at least in the Methods section.

**Reply :**

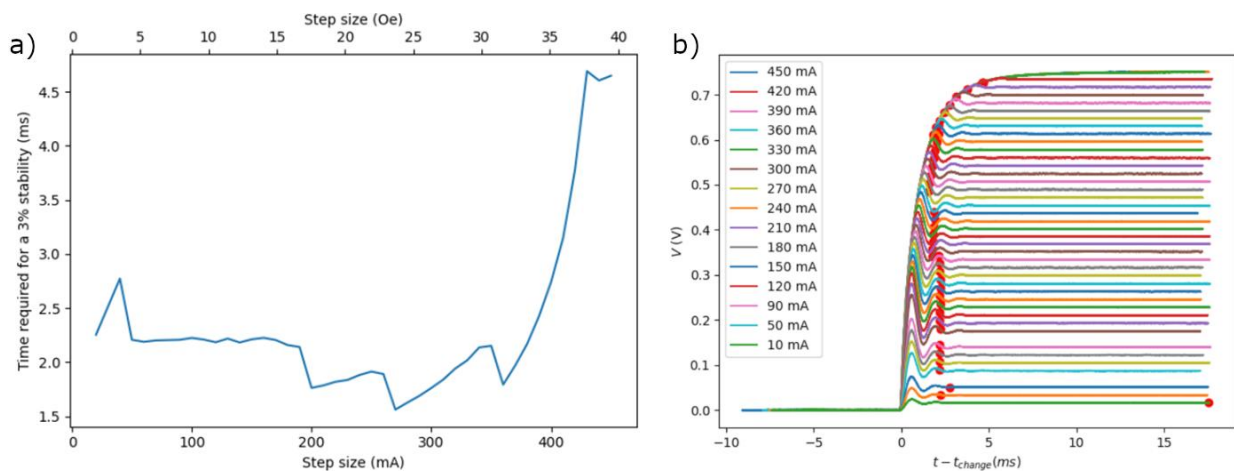
We agree with the referee that the magnetic field ramp and heat exchange are key factors for triggering thermomagnetic instabilities.

Regarding the heat exchange, while the cooling powers of both setups are quite similar at the 4 K stage ( $\sim 100 \text{ mW}$ ), the heat exchange could differ. Most likely the limiting factor will be the quality of the thermal contact between the sample and the cold finger, or simply the thermal resistance of the sapphire. As such, it remains difficult to quantify. We included the cooling power in the methods section of both the MOI and RF measurements.

Concerning the magnetic field ramp rate, below we introduce some theoretical considerations followed by some additional measurements.

For the RF setup the magnetic field is provided by a superconducting magnet with an inductance  $L = 15.1 \text{ H}$  and in the investigated field regime a voltage of  $\sim 2 \text{ V}$ , resulting in an estimation of a sweep rate of  $\frac{V}{L} = \frac{2}{15.1} = 0.132 \text{ A/s}$  which corresponds to  $183.6 \text{ Oe/s}$ .

The external magnetic field in the MOI setup is applied through a copper coil with resistance  $R = 22.1 \Omega$  and inductance  $L = 26 \text{ mH}$ . The corresponding time constant is  $\tau = \frac{L}{R} \sim 1.18 \text{ ms}$ . Since the field step is set to  $1 \text{ Oe}$ , a maximum rate of about  $850 \text{ Oe/s}$  is obtained at the transition between two consecutive field steps. In addition, we experimentally determined the time constant by inserting a  $\sim 2 \Omega$  resistance in series and probing its voltage, which is proportional to the current and therefore also proportional to the field produced. Panel (a) of Figure R1 below shows the time necessary to reach 97% of the final voltage value as a function of the current (field) step. The full curve is shown in panel b for different current values, at time  $t = 0$  the current is switched on. The red dot indicates the time at which the voltage reaches to 97% of the final (stabilized) value. From these measurements one can extract a time constant  $\sim 2.25 \text{ ms}$ , corresponding to a maximum field ramp rate of  $\sim 444 \text{ Oe/s}$ .



**Figure R1 :** a) The time required for the system to stabilize to 97% of the stable value as a function of the current step size, therefore also proportional to the magnetic field step size. b) The voltage curves obtained after switching on the current at  $t=0$ , for different current values. The red dot indicates the 97% point shown in panel (a).



Although the sweep rates of both measurement setups are similar in magnitude, it is anticipated that the threshold field for triggering flux avalanches decreases with higher field ramp rates [1]. Previous studies conducted by Nowak et al. [2] demonstrated that avalanche activity in Nb rings remained unaffected within a wide range of rates, spanning four decades from 0.002 Oe/s to 20 Oe/s. Similarly, recent experimental investigations on bulkier Nb samples indicated that thermomagnetic instabilities remained insensitive to sweep rates between 500 – 20 000 Oe/s [3]. Even though our sweep rates fall below these values, and considering that in literature the impact of ramp rate has been investigated mainly in the range of T/s for different superconducting materials, one should remain cautious for the sweep rate. Nevertheless, it is worth mentioning that although our aim is to provide a qualitative analysis of the observed phenomena, the correspondence and reproducibility of the avalanche regime in both techniques allow us to make a comparison between the sweep rates employed for MOI and RF measurements. For the sake of completeness, we included both sweep rates in the method sections of the manuscript.

## References

- [1] Mints, R. G., & Brandt, E. H. (1996). Flux jumping in thin films. *Physical Review B*, **54**(17), 12421.
- [2] Nowak, E. R., Taylor, O. W., Liu, L., Jaeger, H. M., & Selinder, T. I. (1997). Magnetic flux instabilities in superconducting niobium rings: Tuning the avalanche behavior. *Physical Review B*, **55**(17), 11702.
- [3] Chabanenko, V. V., Rusakov, V. F., Piechota, S., Nabialek, A., Vasiliev, S., & Szymczak, H. (2002). The structure of vortex matter avalanches in a niobium plate. *Physica C: Superconductivity*, **369**(1-4), 82-86.

REVIEWERS' COMMENTS:

Reviewer #1 (Remarks to the Author):

Thank you for addressing my comments and made the corresponding changes in your manuscript. I am pleased to see this version.

Reviewer #2 (Remarks to the Author):

The authors improved the paper following the suggestions.

Only a small remark: on 394, 395 and 398 lines the acronym "MO" is used instead of "MOI", which was defined in the text.

I recommend this paper for publication.