Peer Review File

Sign reversal diode effect in superconducting Dayem nanobridges

Reviewers' comments:

Reviewer #1 (Remarks to the Author):

Authors report on sign reversal of supercurrent rectification in a Niobium Dayem bridge structure. The possible origin of the sign reversal is discussed to be due to contribution from higher harmonics in the current-phase relation. The conclusions regarding the sign reversal of SDE are supported by the presented data. I have some questions and comments for the authors:

1. It has been shown in previous works that metallic bridges with high non-uniform critical currents and non-uniform critical currents can realize asymmetric critical currents as shown in Refs.
[1] Appl. Phys. Lett. 102, 072602 (2013)
[2] Phys. Rev. B 6, 855 (1972)
[3] J. Appl. Phys. 50, 8135 (1979)
simply due to self-field effects. This self-field due to the current flow is sufficient to introduce additional harmonics in the CPR and the vortex effects or spin-orbit effects that authors allude to are not necessary. Authors should explain why they are ignoring the self-field effects in construction of their theory. Authors should cite the above references and make distinctions clear.

2. I would like to point to the authors that in a recent works sign reversal of diode polarity without reversal of the sign of out-of-plane field was demonstrated in Refs:
[1] Nat Commun 14, 3078 (2023).
[2] Nano Lett. 2021, 21, 12, 5240–5246 (In this work the additional phase shift is generated by a superconducting vortex).
Along with this it was also demonstrated in [1] that the sign of diode polarity can be reversed by electrostatic gating in the same work. Given the focus on sign reversal of diode polarity in the introduction it seems appropriate to cite these works.

3. It would be useful for authors to provide comments on whether it is possible to control different terms that contribute to the CPR in their device architecture.

Given that the mechanism (Even if it is indeed not just self-field) and effects both have been observed before in similar Josephson structures, it would be helpful for readers if the authors can explain why this work is distinct.

Reviewer #2 (Remarks to the Author):

The paper by Margineda et al. reports the observation of the superconducting diode effect in Nb nanobridges. The manuscript also provides a plausible explanation based on the harmonicity of current- phase relation of the Dayem bridge. Superconducting diode effect has been extensively explored in various systems both experimentally and theoretically, by now, as is evident from the plethora of publications with similar titles. Although the particular observation made in the paper, to my knowledge, is unique, I doubt the novelty of the observation and the theoretical explanation warrants a publication in Communications Physics. Nevertheless, I have the following comments / questions regarding the manuscript.

1) I suggest providing more details on the measurement. How was the value of Isw chosen. Is it the mean of 5-10 iterations? What was the spread (or standard deviation) of the switching currents? What was the ramp rate of the bias current? The value of the switching current depends heavily on the ramp rate of the current, especially with only 5-10 iterations.
2) What is the critical magnetic field of the Nb film.? Is there a reason for making the 2 wires different in thickness? How do you justify ignoring the thickness difference while comparing them?

3) Given the simplicity of the fabrication and measurement techniques as the authors claim, I am wondering why the experiment was limited to only 3 nanowires. A dependence of the effect on the length of the nanowire would have provided a deeper insight into the physics.

4) What are the dimensions of the third device in the extended data?

5) Figures 2c and 3c are slightly confusing. I suggest using a better color code matching the color of the arrows.

6) Effect of the banks: The authors mentions that the screening currents do not contribute as the width is less than the penetration depth. This is not true for the banks. Do you have any comments on the effects of the banks especially at higher fields where the critical currents are much lower.

7) Is there any realistic experimental estimates of Rashba and Dresselhaus parameters in Nb? I am questioning the applicability of the model 2 for Nb realistically.

Daniel Margineda,PhD
NEST, Istituto Nanoscienze CNR & Scuola Normale Superiore
Piazza San Silvestro 12, 56127, Pisa, Italy

Phone: +34 696655083
Email: daniel.margineda@nano.cnr.it
Website: http://web.nano.cnr.it/sqel/

We wish to thank all the Referees for their careful comments on our paper. We believe that in the attached revised version of our manuscript, we have addressed all the issues that you have pointed out. In the following, we answer your comments and specific concerns. At the bottom, we shortlist the changes to the manuscript, indicating the corresponding line number for each of them.

With best regards on behalf of all the authors,

D. Margineda

Reviewer 1 (Remarks to the Author)

Authors report on sign reversal of supercurrent rectification in a Niobium Dayem bridge structure. The possible origin of the sign reversal is discussed to be due to contribution from higher harmonics in the current-phase relation. The conclusions regarding the sign reversal of SDE are supported by the presented data. I have some questions and comments for the authors:

Q 1.1: *It has been shown in previous works that metallic bridges with high non-uniform critical currents and non-uniform critical currents can realize asymmetric critical currents as shown in Refs.*

[1] Appl. Phys. Lett. 102, 072602 (2013)
[2] Phys. Rev. B 6, 855 (1972)
[3] J. Appl. Phys. 50, 8135 (1979)

simply due to self-field effects. This self-field due to the current flow is sufficient to introduce additional harmonics in the CPR and the vortex effects or spin-orbit effects that authors allude to are not necessary. Authors should explain why they are ignoring the self-field effects in construction of their theory. Authors should cite the above references and make distinctions clear.

A 1.1: We thank the Referee for the useful remarks which help to clarify the main outcomes of our work. We agree with the Referee that spatial inhomogeneities of the critical currents and self-field effects can yield asymmetric critical currents and rectification. Regarding our work, we have neglected self-field effects because they are weak in amplitude. Such conclusion can be directly deduced from the stark contrast of the profile of the switching current vs B field data points as justified below.

Firstly, we would like to point out that self-field effects and asymmetric critical currents in dc SQUIDs geometries as reported in Phys. Rev. B 6, 855 (1972) or as recently investigated by our group in Appl. Phys. Lett. 123, 092601 (2023) requires asymmetric Josephson junctions $I_{c1} \neq I_{c2}$.

On the other hand, self-field effects in single-junction superconducting systems as reported in Appl. Phys. Lett. 102, 072602 (2013) (see Fig. 3b) and extensively discussed and analyzed in Ref. 28 feature a characteristic skewness in the $I_{sw}(B_z)$ profile. The additional contribution to the B-field due to self-field effects results in a polarity-dependent maximum I_{sw} at $B_z \neq 0$ responsible for the SDE reported. Instead $I_{sw}(B_z)$ in our devices decays monotonically at low fields with no signature of the characteristic skewness. Another experimental evidence that allows us to rule out contributions from self-field effects to the rectification is that, in the publications aforementioned, amplitude asymmetry does not change sign as a function of the magnetic field without reversing its orientation (within the first lobe of the Fraunhofer pattern). For this reason, we have considered the combination of high-harmonic content of the CPR with vortex and intrinsic anomalous phase shift.

In order to clarify the issue of self-field effects, we have revised the document. (See changes 4-6)

Q 1.2: *I would like to point to the authors that in a recent works sign reversal of diode polarity without reversal of the sign of out-of-plane field was demonstrated in Refs:*

[1] Nat Commun 14, 3078 (2023).

[2] Nano Lett. 2021, 21, 12, 5240–5246 (In this work the additional phase shift is generated by a superconducting vortex).

Along with this it was also demonstrated in [1] that the sign of diode polarity can be reversed by electrostatic gating in the same work. Given the focus on sign reversal of diode polarity in the introduction, it seems appropriate to cite these works.

A 1.2: We thank the Referee for bringing these articles to our attention. We would like to point out that Ref. [1] was published during the writing process and a manuscript very similar to Ref. [2] by the same author was included in the original version (i.e. Ref. 28). Notwithstanding, both references are now added to the revised version. (Ref. 29,30).

Q 1.3: *It would be useful for authors to provide comments on whether it is possible to control different terms that contribute to the CPR in their device architecture.*

Given that the mechanism (Even if it is indeed not just self-field) and effects both have been observed before in similar Josephson structures, it would be helpful for readers if the authors can explain why this work is distinct.

A 1.3: As shown in our work, the harmonic content of the CPR and therefore the SDE can be modified by geometry and temperature. The high-order harmonic content is modified by the effective length $l_{eff} = l/\xi(T)$ tending to zero near T_c as suggested by the temperature dependence of the nonreciprocal transport we report. The strong dependence of the CPR lineshape with l_{eff} has been deeply studied in similar systems by our group in Phys. Rev. Appl. 6, (2016). However, such CPRs remain odd-in-ϕ and no SDE is expected. To have SDE, the rich harmonic content of the weak link CPR has to coexist with other sources of spatial symmetry breaking, which in our case are either a vortex or a spin-orbit coupling. Small variations of the vortex position in the bridge or variations in the actual dimensions of the nanowires will strongly affect the rectification lineshape at low fields as shown in Fig. 2 of this document.

In conclusion, we can claim that the major novelty of this work is the sign-reversal of the rectification without reversing the orientation of the magnetic field. Such behavior is inconsistent with physical scenarios related to Meissner currents or self-field effects and fairly described by our complementary theoretical models. Novelty of this work has been emphasized in the conclusion Section. (See change 9).

Reviewer 2 (Remarks to the Author)

The paper by Margineda et al. reports the observation of the superconducting diode effect in Nb nanobridges. The manuscript also provides a plausible explanation based on the harmonicity of current- phase relation of the Dayem bridge. Superconducting diode effect has been extensively explored in various systems both experimentally and theoretically, by now, as is evident from the plethora of publications with similar titles. Although the particular observation made in the paper, to my knowledge, is unique, I doubt the novelty of the observation and the theoretical explanation warrants a publication in Communications Physics. Nevertheless, I have the following comments / questions regarding the manuscript.

Before addressing the specific questions, we would like to emphasize that the major novelty of this work is the sign-reversal of the rectification without reversing the orientation of the magnetic field incompatible with mechanism based on self-field effects or screening currents. Instead, we demonstrate that the critical current-field profiles are fairly described by our complementary theoretical models. On them, vortex phase windings present in the bridge or an anomalous phase shift compatible with anisotropic spin-orbit interactions act as spatial symmetry breakers resulting in nonreciprocal superconducting transport.

Q 2.1: *I suggest providing more details on the measurement. How was the value of Isw chosen. Is it the mean of 5-10 iterations? What was the spread (or standard deviation) of the switching currents? What was the ramp rate of the bias current? The value of the switching current depends heavily on the ramp rate of the current, especially with only 5-10 iterations.*

A 2.1: The ramp rate ΔI was adapted depending on the switching current but always lower than $0.002 I_{sw}(B=0) \sim 1\mu\,$A and $100\,$nA for "short" and "long" devices, respectively. I_{sw} for each iteration is extracted from the maximum of the derivative dV/dI. Thus, I_{sw} presented in the manuscript is the mean value of all IV curves. Error bars account for the standard deviation and propagation of errors in I_{sw}, ΔI_{sw} and η. For the characteristic fields, the error is given by the magnetic field step $\Delta B_z < 10\,$mT. Although most of the error bars are masked by the size of the plot dots, they are clearly observed in ΔI_{sw} plots. Information about the measurements is now expanded in Methods.

Q 2.2: *What is the critical magnetic field of the Nb film.? Is there a reason for making the 2 wires different in thickness? How do you justify ignoring the thickness difference while comparing them?*

A 2.2: Measurements of the out-of-plane critical magnetic field as a function of temperature for the three devices presented in the manuscript are shown below in Fig.1. B_c at the experiment temperature T $< 0.1\ T_c$ is in the order of a few Tesla, while most of the relevant features in η take place for $B \leq 0.4\,$T. The zero-temperature, in-plane coherence length $\xi_{||0}$ is estimated from fittings

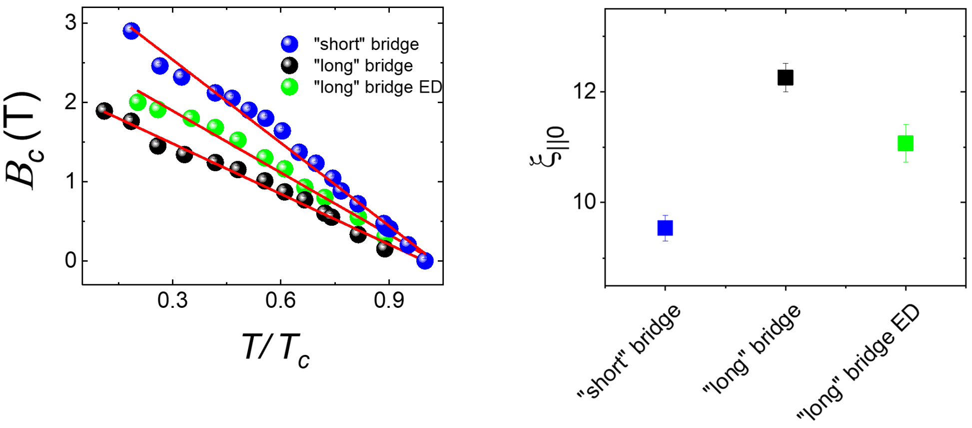

Figure 1: Temperature dependence of the upper critical field of the three devices presented in the manuscript and the estimated zero-temperature coherence length.

to (see e.g., Quaterman et al. Phys. Rev. Mat., 4, 74801 (2020)):

$$B_c(T) = \left(\frac{\Phi_0}{2\pi\xi_{||0}^2}\right)(1 - T/T_c)$$

$\xi_{||0}$ values fairly agree with those obtained from resistivity measurements reported in the manuscript. Although the effect of the nominal Nb film thickness t_{Nb} in the nonreciprocal transport has not been investigated in detail, several observations suggest that it plays a residual effect on the SDE: 1) Small variation of thin film critical temperature (see Fig. 1 of the manuscript) suggests minor impact of the film thickness on the superconducting state 2) $\xi_{||0}$ remains shorter than the sample thickness for both sets of samples and similar dispersion in $\xi_{||0}$ is observed among the three samples investigated regardless of the thickness. This issue is addressed by change 7.

Q 2.3: *Given the simplicity of the fabrication and measurement techniques as the authors claim, I am wondering why the experiment was limited to only 3 nanowires. A dependence of the effect on the length of the nanowire would have provided a deeper insight into the physics.*

A 2.3: As pointed out in Q1.3, the CPR of Dayem-like constrictions is highly anharmonic for lengths l of the order of ξ_0. However, the high-order harmonic content in the skewed CPR is not the only relevant parameter. On top of it, we have also either the vortex position or the strength of the spin-orbit. Thus, we have judged that a systematic study over many lengths would have not provided further insight into the effect. This can be observed in long wires with the same nominal length that shows sizeable differences. Below, in Fig. 2, we plot the rectification of three identical nanowires (two of them presented in the manuscript.) $\eta(B_z)$ differences at low fields are consistent with model I as discussed in the manuscript. Small variations of the vortex position in the bridge or variations in the actual dimensions of the nanowires will strongly affect the rectification lineshape at low fields. Furthermore, phase slips complicate the picture as suggested by the rectification profile on the third nanowire (green color). $\eta(B_z)$ exhibits a more noisy trend near the sign change than the other ones whose origin is now under investigation.

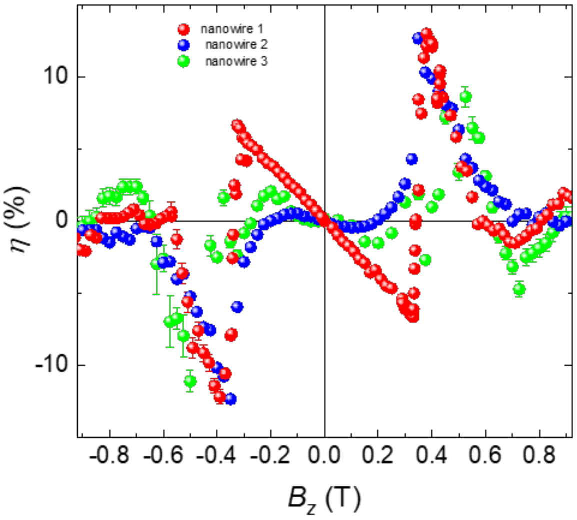

Figure 2: Rectification of three nanowires with same nominal dimensions at T = 50 mK

Q 2.4: *What are the dimensions of the third device in the extended data?*

A 2.4: The third device has the same nominal dimensions that the nanowire investigated in Fig. 3 of the manuscript. This is clarified in the revised manuscript by change 2.

Q 2.5: *Figures 2c and 3c are slightly confusing. I suggest using a better color code matching the color of the arrows.*

A 2.5: We are very thankful to the Referee for the valuable suggestion and the color code has been changed accordingly.

Q 2.6: *Effect of the banks: The authors mention that the screening currents do not contribute as the width is less than the penetration depth. This is not true for the banks. Do you have any comments on the effects of the banks especially at higher fields where the critical currents are much lower.*

A 2.6: The profile of I_{sw} vs B_z and sign reversal diode effect are by themselves an indication that screening current does not drive the rectification in our nanometric constrictions. Other works in micrometric size Nb stripes (Ref. 31-34) exhibit a characteristic I_{sw} profile with no sign reversal rectification without reversing the magnetic field. SDE in those works is ascribed to the interplay of screening currents and asymmetric edges. In particular, nonreciprocal transport is mostly given by an asymmetric linear increase of $I_{sw}(B_z)$ at low B field values, which is not observed in any of our devices where the switching current-field relation monotonically decays. This issue is addressed by change 6.

Q 2.7: *Is there any realistic experimental estimates of Rashba and Dresselhaus parameters in Nb? I am questioning the applicability of the model 2 for Nb realistically.*

A 2.7: Model 2 proposes a scenario compatible with the experimental observations in Nb bridges. At the best of our knowledge, literature does not provide a direct measurement of the Rashba interaction or a univocal estimate of the Dresselhaus coupling in Nb thin films.

Still, Nb has a high atomic number, and thus possesses an atomistic spin-orbit of the order of 100 meV [J. Less Common Met. 19, 405–411 (1969)]. We extend such a concept to our metallic bridges where we call for a spin-orbit coupling of the order of very few meV, i.e. 2 orders of magnitude weaker than the atomic limit. In heavy metals, crystalline potentials break mirror and/or inversion symmetries by primarily coupling the orbitals contributing to the electronic structure and then the spin through the atomic spin-orbit coupling. This has been shown for instance in Nature 549, 492–496 (2017), Nature Materials 10, 521–526 (2011), Nature Communications 1, 17 (2010), and discussed in the review paper Nature Reviews Physics 4, 642–659 (2022). We model this effect as Rashba and Dresselhaus-type interactions ranging from 0.2 and 6 meV (i.e., from 1% to 30% of the effective electronic amplitude describing the low energy properties of the superconductor).

We conclude by pointing out that a similar working hypothesis has been considered to explain the signatures of spin-orbit effects in hybrid devices where graphene (where spin-orbit is notoriously very weak) is contacted by Nb leads (Physical Review X, 12(2), 021057). This issue is addressed by change 9 and 10.

Summary of changes:

1. Fig. 2c and Fig. 3c, Color of the IV curves are changed for better visualization.
2. Line 241. *devices with same nominal dimensions* has been added.
3. Line 653. Methods section has been modified to give more detailed information about the measurements as discussed in **Q 2.1** .
4. Line 58. *or self-fields* is added to introduce self-field effects as a source of SDE.
5. Line 87. *which is inconsistent with physical scenarios related to Meissner currents or self-field effects* is added to emphasize the novelty of this work.
6. Line 250. Differences between our results and SDE reported in other works on conventional superconductors are discussed before introducing the theoretical models. *Previous SDE reported in conventional superconductor are normally attributed to Abrikosov vortices and self-field effects [Ref.] or Meissner currents [Ref.]. We have neglected those mechanisms since they are weak in amplitude. Such conclusion can be directly deduced from the stark contrast of the profile of the switching current vs B-field data points. In both scenarios $I_{sw}(B_z)$ reaches a maximum at nonzero magnetic fields Moreover, amplitude asymmetry does not change sign as a function of the magnetic field without reversing its orientation before the critical current vanishes. Instead, the switching current-field profile in our devices decays monotonically at low fields with no signature of the characteristic skewness. For this reason. . . .*
7. Line 116. *suggesting minor impact of the film thickness on the superconducting state* is added referring to the effect of the thickness on the critical temperature of the Nb films.
8. Line 307 Comments on the spin-orbit coupling used are introduced before presenting model 2. *Nb is a heavy metal and thus possesses an atomistic spin-orbit of the order of 100 meV [J. Less*

Common Met. 19, 405–411 (1969)]. We extend such a concept to our weak links where we call for a spin-orbit coupling of the order of very few meV, i.e. 2 orders of magnitude weaker than the atomic limit [Physical Review X, 12(2), 021057].

9. Line 392. *The evolution of the critical current on B field decays monotonically without showing any skewness, which rules out self-field effects or Meissner currents.* is added to the conclusion to emphasize the novelty of this work.

10. Line 735. Details of the calculations of the spin-orbit coupling, estimation of the required coupling amplitudes, and related references are added to Methods.

References added:

[29] Nat Commun 14, 3078 (2023).
[30] Nano Lett. 2021, 21, 12, 5240–5246
[35] Appl. Phys. Lett. 102, 072602 (2013)
[57] J. Less Common Met. 19, 405–411 (1969)
[58] Physical Review X, 12(2), 021057 (2022).
[59] Nat. Rev. Phys. 4, 642-659 (2022)
[60] Nature 549, 492-496 (2017)
[61] Phys. Rev. B. 105, 125143 (2022)
[62] Phys. Rev. Lett. 107, 156803 (2011)
[63] Phys. Rev. Appl. 14, 034041 (2020)
[64] Nat. Mater. 10, 521-526 (2011)
[65] Nat. Comm. 1, 17 (2010)

REVIEWERS' COMMENTS:

Reviewer #1 (Remarks to the Author):

The authors have edited the manuscript and clarified all the points. I find the manuscript suitable for publication.

Reviewer #2 (Remarks to the Author):

I have a remark on A2.1. There seems to be a misunderstanding on the meaning of ramp rate. The ramp rate of the dc bias current is defined as dI/dt (uA/sec). The actual measured switching current depends on the rate at which the current is ramped up. I would recommend to include this number in the manuscript. It is also important to keep this rate same for all measurements unless there is a specific experimental reason for varying it. I am otherwise satisfied by the authors replies and recommend the manuscript for publication.

CNRNANO
ISTITUTO NANOSCIENZE CONSIGLIO NAZIONALE DELLE RICERCHE

Daniel Margineda,PhD
NEST, Istituto Nanoscienze CNR & Scuola Normale Superiore
Piazza San Silvestro 12, 56127, Pisa, Italy
Phone: +34 696655083
Email: daniel.margineda@nano.cnr.it
Website: http://web.nano.cnr.it/sqel/

Reviewer 2 (Remarks to the Author)

Q 2.1: *I have a remark on A2.1. There seems to be a misunderstanding on the meaning of ramp rate. The ramp rate of the dc bias current is defined as dI/dt (uA/sec). The actual measured switching current depends on the rate at which the current is ramped up. I would recommend to include this number in the manuscript. It is also important to keep this rate same for all measurements unless there is a specific experimental reason for varying it. I am otherwise satisfied by the authors replies and recommend the manuscript for publication.*

A 2.1: For DC current-voltage measurements, the excitation and acquisition have been conducted point by point, e.g. voltage drop is measured for each bias current value with an integration time of 20 ms, (number of power line cycles NPLC =1). Therefore, the ramp rate dI/dt is given by the current step size ΔI. This value was modified according to the maximum switching current at different temperatures.

Method section has been modified to clarify this issue as "DC current-voltage characteristics were measured by sweeping a low-noise current bias positively and negatively, and by measuring the voltage drop across the weak links with a room-temperature, low-noise pre-amplifier for each current value every ~ 20 ms. Current step size ΔI was adapted depending on the switching current keeping the values lower than $0.002 I_{sw}(B = 0) \sim 1 \mu$A and 100 nA for short and long devices, respectively."