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Review article

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The randomized measurement toolbox

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Quantum gate noise characterization

The examples in the main text have demonstrated the surprising power of randomized measurements (RM), and the closely related idea of randomly applying unitary gates. Here, we dive a little deeper in the direction of applying random unitaries, and we consider what happens when random unitary gates are used throughout a quantum circuit.

Randomized dynamics were originally suggested as a way to decouple unwanted interactions from an environment and to put noise into a standard form known as a Pauli channel [1–3]. Moreover, all that is required to achieve this noise projection is the ability to insert random Pauli gates (or π-pulses) into a quantum circuit. A Pauli channel is any quantum channel whose Kraus operators are the Pauli matrices, and where the operator sum is weighted by a probability distribution. Thus, many common channels such as depolarizing noise, dephasing noise, and bit-flip noise are Pauli channels, as are some more complicated correlated noise channels across multiple qubits. Non-examples include amplitude damping or coherent over-rotation errors.

Pauli channels are a natural class of noise channels because their stochastic nature makes it easy to report a single figure of merit, an error rate, to a given noise process. They also enjoy a central role in the theory of quantum error correction because local Pauli noise can be efficiently simulated when surrounding quantum circuits are comprised solely of Clifford gates. Lastly, the Pauli channel error rates in a quantum device provide an important metric for progress towards fault tolerance.

These considerations have motivated a research effort to use random unitary dynamics to simplify the noise in a quantum gate and to enable efficient characterization of noise by reducing the problem to estimating Pauli error rates. The literature on noise characterization is already the subject of entire review articles [4, 5], so we necessarily limit the scope of our discussion.

The quintessential method for estimating average error rates in few-qubit quantum systems is called randomized benchmarking (RB) [6, 7]. In RB, sequences of random Clifford gates of varying length are applied to the initial state $|0\rangle^{\otimes n}$. At the end of the circuit, the inverse Clifford circuit is computed, compiled, and applied, then the state is measured in the computational basis. If the circuit had no noise, then one would always measure the 0 outcome, but owing to noise in the system the probability of 0 decays exponentially in the length of the circuit. Suppose that the noise on each Clifford gate is identical, Markovian, time-stationary noise. Then it can be shown [8] that the slope of this decay curve estimates the average error rate r_{avg} between the noise $\mathcal{E}$ and the ideal gate U, defined as

$$r_{\mathrm{avg}} = 1 - \int \mathrm{d}\psi \langle\psi|U^\dagger \mathcal{E}(|\psi\rangle\langle\psi|)U|\psi\rangle \,,$$

where the integral is taken over the uniform Haar measure. Fitting to an exponential decay by using sequences of varying lengths achieves two goals: first, it decouples the noise in the state preparations and measurements from the noise in the gates, improving the accuracy of gate error estimates; second, the long sequences also improve the precision of the estimates by amplifying small gate errors into a signal that is observable with a reasonable amount of sampling. These strengths have made the RB method the de facto standard for experimental estimation of error rates in one- or two-qubit experiments.

The success of RB has spawned numerous modifications to improve and extend the method. Two early ideas in this direction were interleaved RB (IRB) [9] and simultaneous RB (SRB) [10]. In IRB, standard RB is first performed to get a baseline average error rate estimate r_0. Then new random circuits are sampled by systematically appending to each random gate the same fixed Clifford gate U. This new experiment will generally give a worse average error rate r, and then the ratio r/r_0 provides an estimate of the average error rate of U. In this way, IRB allows one to estimate gate-specific average error rates. SRB works in a similar comparative manner. In SRB, the baseline error rate is estimated by doing RB on a composite system, and this baseline r is compared to RB done simultaneously on the constituent subsystems. This facilitates estimation of crosstalk error rates and correlated errors, which can be especially detrimental for fault tolerance. Finally, several variants have been proposed to extract just the average error rate r_{avg} (or related parameters) in larger-scale circuits than is possible with standard RB [11, 12].

As mentioned above, randomized dynamics can be used to ensure that the noise affecting a quantum computation is of the form of a Pauli channel [1–3]. This idea was further developed into a scheme called randomized compiling [13], which improves over naive schemes by reducing circuit depth slightly and comes with a perturbative error analysis. These ideas have been demonstrated experimentally in superconducting qubits [14, 15].

The experimental success of these methods justifies the efforts to estimate the Pauli noise on individual gates or rounds of gates in a quantum circuit [16–24]. Two notable experiments in this space are a trapped ions experiment [17] that estimated the average noise on a 10-qubit Mølmer-Sørensen gate, and an experiment [19] that estimated all of the locally Clifford-averaged Pauli error rates in a 14-qubit transmon device. These methods have recently been put into an overarching framework called ACES (for Averaged Circuit Eigenvalue Sampling) [24]. ACES has been shown numerically to scale to at least 100 qubits, and offers a promising avenue for scalable Pauli noise estimation in large-scale quantum devices. Similar in spirit are the shadow sequences introduced in Ref. [25], whereas Refs. [26] provide channel generalizations of classical shadows that can help to capture any type of quantum channel.

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