Supporting information for the paper: "Spread of risk across financial markets: better to invest in the peripheries"

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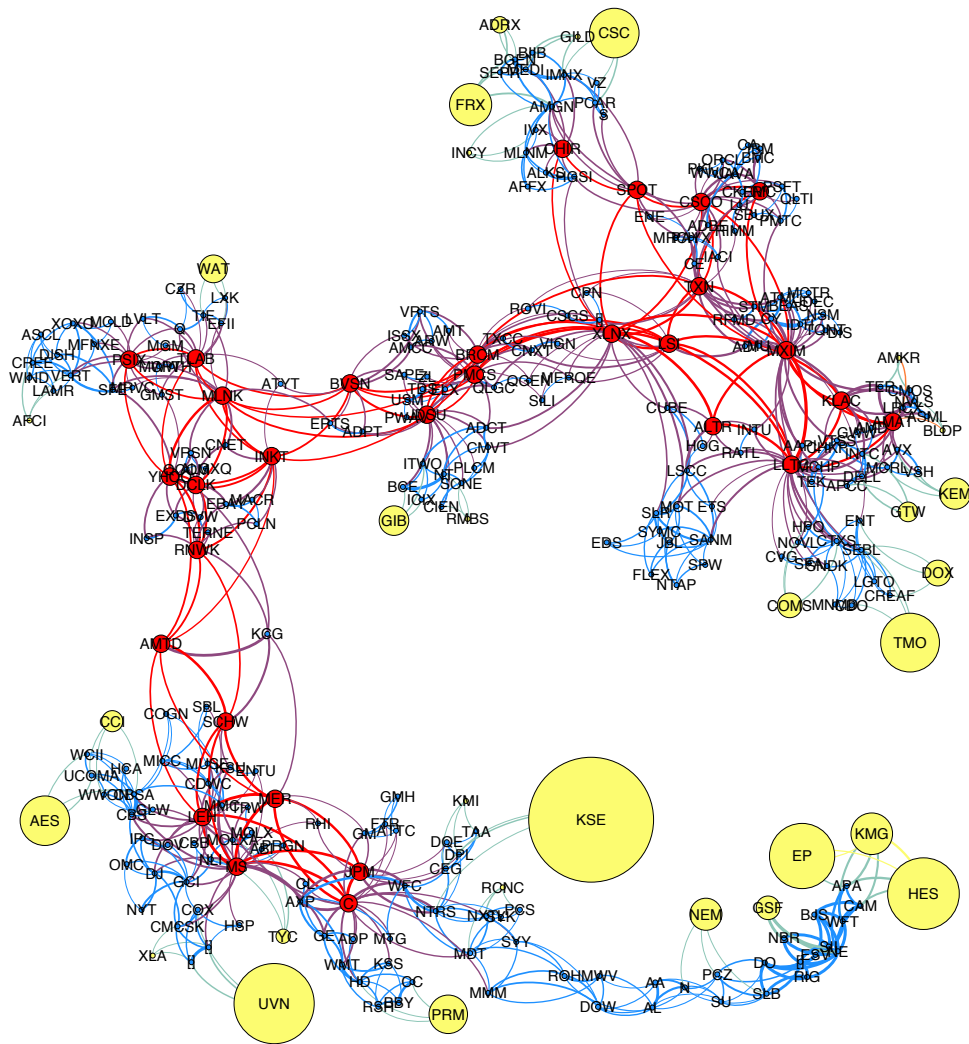
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Figure S.1: Same figure as Fig. 1 in the main paper but, in this case, with the ticker symbol of each stock reported.

S.1   Peripheral vs. central nodes: frequencies

In this section we show that the center of $PMFG$ is dominated by a small number of central stocks whereas the periphery exhibits larger variations.

For each date we select the most *central* (*peripheral*) stocks defined as the 5% companies characterized by smallest (largest) values of $X + Y$ - a total of 15 stocks for each date. By aggregating all the dates and counting the number of times a stock is selected as 'central' or 'peripheral', we assign a frequency to each of the 2286 stocks analyzed over the whole period. The resulting cumulative frequency is reported in Fig. S.2. We see that the two curves for central and peripheral stocks are rather different. For instance, for central stocks, the 229 largest frequencies (corresponding to 10% of all stocks) account for 83% of all "most central stocks"; conversely, for peripheral stocks, the 229 largest frequencies account for only 50% of the total. This means that central stocks are more stable in central positions than peripheral stocks in peripheral positions.
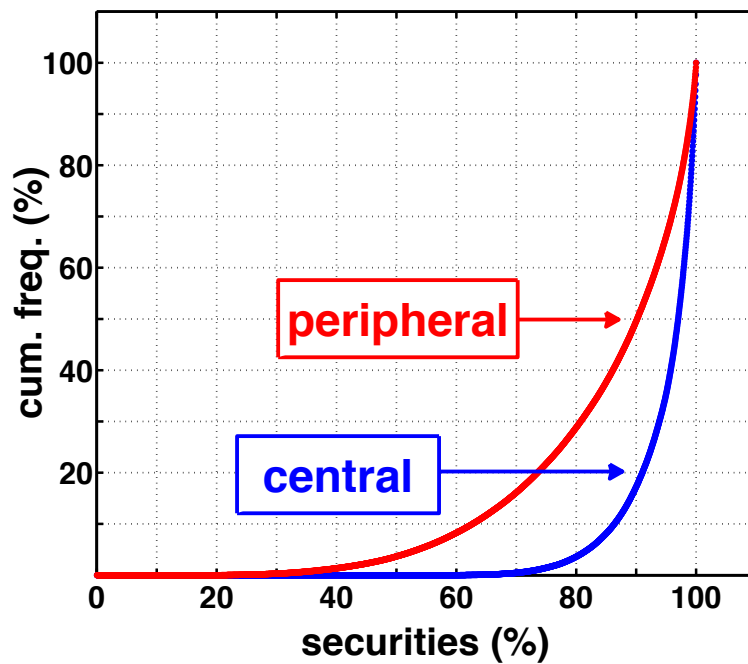

Figure S.2:   Cumulated frequencies, for each security, of centers and peripheries: 229 central stocks represent 83% of total frequencies for central stocks; the same number of peripheral stocks represents just 50% of all frequencies for peripheral stocks.

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Table S.1: Average yearly returns for portfolios of MKT, $RAND$, $BEST$, $PMFG$'s central nodes ($PMFG$-c) and $PMFG$'s peripheral nodes ($PMFG$-p). In round brackets the standard deviations are reported for all 7071 yearly returns.

# of stocks	MKT			RAND			BEST			PMFG-c			PMFG-p		
	u	ns	s	u	ns	s	u	ns	s	u	ns	s	u	ns	s
5 stocks	0.152 (0.195)	0.137 (0.148)	0.142 (0.156)	0.151 (0.250)	0.142 (0.211)	0.141 (0.211)	0.236 (0.359)	0.182 (0.285)	0.183 (0.286)	0.144 (0.272)	0.122 (0.235)	0.118 (0.234)	0.165 (0.219)	0.154 (0.196)	0.153 (0.196)
10 stocks	0.152 (0.195)	0.137 (0.148)	0.142 (0.156)	0.154 (0.223)	0.139 (0.189)	0.138 (0.190)	0.214 (0.318)	0.159 (0.221)	0.159 (0.222)	0.142 (0.251)	0.118 (0.212)	0.114 (0.207)	0.163 (0.196)	0.148 (0.174)	0.147 (0.174)
20 stocks	0.152 (0.195)	0.137 (0.148)	0.142 (0.156)	0.152 (0.209)	0.139 (0.176)	0.138 (0.176)	0.194 (0.286)	0.150 (0.204)	0.150 (0.205)	0.142 (0.236)	0.118 (0.197)	0.112 (0.192)	0.159 (0.185)	0.143 (0.161)	0.141 (0.161)
30 stocks	0.152 (0.195)	0.137 (0.148)	0.142 (0.156)	0.151 (0.202)	0.138 (0.170)	0.136 (0.171)	0.186 (0.274)	0.147 (0.196)	0.150 (0.198)	0.142 (0.230)	0.118 (0.187)	0.113 (0.186)	0.158 (0.183)	0.142 (0.157)	0.139 (0.158)
40 stocks	0.152 (0.195)	0.137 (0.148)	0.142 (0.156)	0.152 (0.201)	0.138 (0.166)	0.136 (0.168)	0.179 (0.263)	0.146 (0.188)	0.150 (0.192)	0.144 (0.226)	0.123 (0.179)	0.118 (0.181)	0.157 (0.182)	0.140 (0.156)	0.137 (0.157)

S.2   Measures of performance and risk

We report here a selection of measures of performance and risk for portfolios "MKT" (all 300 stocks), "$RAND$" (random stocks), "$BEST$" (of stocks achieving best performance on the year preceding the investment), "$PMFG$-c" ($PMFG$ central stocks) and "$PMFG$-p" ($PMFG$ peripheral stocks).

Table S.1 reports the average yearly returns and the associated standard deviation computed over the whole period of 7071 days. Analogously, Table S.2 reports the average yearly excess returns (defined as the difference between the portfolio returns and returns of the benchmark S&P 500 Composite index) over the whole period of 7071 days. In both tables it is evident how peripheral nodes perform systematically better than central nodes both in terms of averages and standard deviations; they perform equivalently or better than "MKT" and "$RAND$", both in terms of averages and standard deviations (except for standard deviations of excess returns, which are slightly worse although associated with usually higher averages); their averages are smaller than in the case of "$BEST$" but they have smaller standard deviations.

By looking at the generalized Hurst exponent [1, 2, 3, 4] of yearly returns and yearly excess returns, we observe differences in long-term memory for different portfolios. Results, for the generalized Hurst exponent $H(1)$, computed using [5], are reported in Tables S.3 and S.4. Let us recall that the generalized Hurst exponent must be equal to 0 for a white noise process and 0.5 for a random walk, and deviations from 0.5 reveal deviations from a Brownian motion. We find that returns obtained from "MKT" are characterized by highest $H(1)$ while those from "$RAND$" are characterized by very

Table S.2: Average yearly excess returns for portfolios of MKT, $RAND$, $BEST$, $PMFG$'s central nodes ($PMFG$-c) and $PMFG$'s peripheral nodes ($PMFG$-p). In round brackets the standard deviations are reported for all 7071 yearly excess returns. Excess returns have been here calculated as the difference between yearly portfolio returns and S&P 500 Composite index returns.

# of stocks	MKT			RAND			BEST			PMFG-c			PMFG-p		
	u	ns	s	u	ns	s	u	ns	s	u	ns	s	u	ns	s
5 stocks	0.053 (0.073)	0.038 (0.127)	0.043 (0.151)	0.052 (0.172)	0.043 (0.154)	0.042 (0.155)	0.137 (0.283)	0.083 (0.234)	0.084 (0.235)	0.045 (0.169)	0.023 (0.149)	0.019 (0.156)	0.066 (0.181)	0.055 (0.174)	0.054 (0.175)
10 stocks	0.053 (0.073)	0.038 (0.127)	0.043 (0.151)	0.055 (0.128)	0.040 (0.135)	0.040 (0.139)	0.115 (0.238)	0.060 (0.154)	0.060 (0.157)	0.043 (0.138)	0.019 (0.125)	0.015 (0.145)	0.064 (0.146)	0.049 (0.150)	0.048 (0.152)
20 stocks	0.053 (0.073)	0.038 (0.127)	0.043 (0.151)	0.053 (0.105)	0.040 (0.128)	0.039 (0.138)	0.095 (0.197)	0.051 (0.136)	0.051 (0.141)	0.043 (0.114)	0.019 (0.121)	0.013 (0.151)	0.060 (0.120)	0.044 (0.137)	0.042 (0.141)
30 stocks	0.053 (0.073)	0.038 (0.127)	0.043 (0.151)	0.052 (0.092)	0.039 (0.127)	0.037 (0.140)	0.087 (0.184)	0.048 (0.131)	0.051 (0.139)	0.043 (0.105)	0.019 (0.122)	0.014 (0.158)	0.059 (0.107)	0.043 (0.132)	0.040 (0.139)
40 stocks	0.053 (0.073)	0.038 (0.127)	0.043 (0.151)	0.053 (0.088)	0.039 (0.126)	0.037 (0.142)	0.081 (0.172)	0.047 (0.129)	0.051 (0.138)	0.045 (0.100)	0.024 (0.124)	0.019 (0.159)	0.058 (0.099)	0.041 (0.130)	0.038 (0.138)

Table S.3: Generalized Hurst exponents of yearly returns for portfolios of MKT, $RAND$, $BEST$, $PMFG$'s central nodes ($PMFG$-c) and $PMFG$'s peripheral nodes ($PMFG$-p). Standard deviations are small and therefore omitted.

# of stocks	MKT			RAND			BEST			PMFG-c			PMFG-p		
	u	ns	s	u	ns	s	u	ns	s	u	ns	s	u	ns	s
5 stocks	0.507	0.462	0.387	0.022	0.018	0.018	0.322	0.329	0.329	0.244	0.231	0.219	0.137	0.143	0.142
10 stocks	0.507	0.462	0.387	0.033	0.018	0.019	0.361	0.341	0.338	0.278	0.246	0.225	0.143	0.152	0.150
20 stocks	0.507	0.462	0.387	0.059	0.028	0.024	0.367	0.354	0.351	0.292	0.209	0.193	0.174	0.186	0.181
30 stocks	0.507	0.462	0.387	0.095	0.042	0.035	0.395	0.348	0.346	0.333	0.208	0.177	0.211	0.212	0.204
40 stocks	0.507	0.462	0.387	0.102	0.040	0.033	0.408	0.352	0.349	0.344	0.204	0.168	0.238	0.230	0.214

low values of $H(1)$; returns from "BEST" exhibit relatively high $H(1)$, but smaller than "MKT", while returns from $PMFG$s are characterized by relatively small $H(1)$, but larger than "$RAND$". It has been pointed out that the Hurst exponent can successfully detect the level of development/liquidity of a market [1, 2] and it has been argued that it could be used as a tool to detect market instabilities [4]. Generally speaking, we can affirm that small $H(1)$ should be associated with lower risk of large persistent deviations. Further studies will be devoted to understand the relation between scaling exponents and portfolio investment risk.

Tables S.5 and S.6 report the Information Ratio (IR) and the Sharpe (Information) Ratio (SIR). The Information Ratio is calculated as average divided by standard deviation of yearly returns; the Sharpe (Information) Ratio - also called revised Sharpe Ratio [6] - is the information ratio of the excess yearly returns (the benchmark being the S&P 500 Composite index). (We do not use a risk-free rate as benchmark because our portfolios contain only stocks and no risk-free alternative.)

Table S.4: Generalized Hurst exponents of yearly excess returns for portfolios of MKT, $RAND$, $BEST$, $PMFG$'s central nodes ($PMFG\text{-}c$) and $PMFG$'s peripheral nodes ($PMFG\text{-}p$). Standard deviations are small and therefore omitted.

# of stocks	MKT			RAND			BEST			PMFG-c			PMFG-p		
	u	ns	s	u	ns	s	u	ns	s	u	ns	s	u	ns	s
5 stocks	0.449	0.431	0.392	0.008	0.010	0.010	0.316	0.325	0.324	0.209	0.202	0.194	0.132	0.138	0.137
10 stocks	0.449	0.431	0.392	0.008	0.011	0.012	0.342	0.330	0.328	0.219	0.217	0.200	0.127	0.144	0.142
20 stocks	0.449	0.431	0.392	0.010	0.013	0.017	0.336	0.333	0.335	0.204	0.180	0.176	0.142	0.166	0.162
30 stocks	0.449	0.431	0.392	0.027	0.028	0.029	0.358	0.337	0.342	0.229	0.179	0.172	0.167	0.188	0.184
40 stocks	0.449	0.431	0.392	0.025	0.023	0.029	0.366	0.345	0.342	0.229	0.186	0.168	0.169	0.204	0.191

Table S.5: Information Ratio for portfolios of MKT, $RAND$, $BEST$, $PMFG$'s central nodes ($PMFG\text{-}c$) and $PMFG$'s peripheral nodes ($PMFG\text{-}p$). (**a**) Information Ratio over the whole period of 7071 days; (**b**) Average Information Ratio calculated over 6821 samples (corresponding to as many sub-periods each of 250 observations); (**c**) Standard deviations for the samples as in (**b**). The Information Ratio has been here calculated for the yearly returns without benchmark.

# of stocks	measure	MKT			RAND			BEST			PMFG-c			PMFG-p		
		u	ns	s	u	ns	s	u	ns	s	u	ns	s	u	ns	s
5 stocks	(a)	0.782	0.926	0.908	0.603	0.671	0.669	0.657	0.639	0.638	0.528	0.519	0.507	0.754	0.785	0.782
	(b)	1.955	2.023	2.018	0.872	0.959	0.954	1.093	1.073	1.073	1.002	0.912	0.869	1.063	1.129	1.127
	(c)	(1.893)	(1.745)	(2.055)	(0.898)	(0.929)	(0.927)	(1.419)	(1.281)	(1.282)	(1.382)	(1.238)	(1.211)	(0.883)	(0.965)	(0.966)
10 stocks	(a)	0.782	0.926	0.908	0.690	0.737	0.730	0.672	0.723	0.717	0.565	0.559	0.548	0.830	0.850	0.844
	(b)	1.955	2.023	2.018	1.132	1.143	1.130	1.229	1.329	1.322	1.200	1.014	0.915	1.345	1.374	1.362
	(c)	(1.893)	(1.745)	(2.055)	(1.123)	(1.127)	(1.126)	(1.489)	(1.510)	(1.513)	(1.479)	(1.282)	(1.166)	(1.139)	(1.166)	(1.167)
20 stocks	(a)	0.782	0.926	0.908	0.728	0.792	0.782	0.678	0.736	0.732	0.601	0.602	0.584	0.859	0.891	0.875
	(b)	1.955	2.023	2.018	1.356	1.342	1.300	1.366	1.519	1.539	1.337	1.093	0.984	1.617	1.628	1.584
	(c)	(1.893)	(1.745)	(2.055)	(1.317)	(1.299)	(1.286)	(1.556)	(1.695)	(1.710)	(1.529)	(1.296)	(1.155)	(1.426)	(1.391)	(1.388)
30 stocks	(a)	0.782	0.926	0.908	0.747	0.813	0.795	0.678	0.750	0.757	0.618	0.634	0.608	0.861	0.900	0.878
	(b)	1.955	2.023	2.018	1.521	1.451	1.378	1.460	1.572	1.624	1.403	1.194	1.062	1.761	1.778	1.696
	(c)	(1.893)	(1.745)	(2.055)	(1.491)	(1.399)	(1.370)	(1.604)	(1.653)	(1.708)	(1.555)	(1.379)	(1.256)	(1.615)	(1.556)	(1.524)
40 stocks	(a)	0.782	0.926	0.908	0.759	0.835	0.810	0.683	0.774	0.780	0.635	0.686	0.654	0.861	0.895	0.870
	(b)	1.955	2.023	2.018	1.585	1.513	1.428	1.520	1.644	1.722	1.450	1.292	1.138	1.853	1.816	1.712
	(c)	(1.893)	(1.745)	(2.055)	(1.523)	(1.438)	(1.411)	(1.614)	(1.641)	(1.765)	(1.572)	(1.413)	(1.296)	(1.715)	(1.606)	(1.568)

We report in (**a**) the measures computed over the whole period of 7071 days; the averages of all subperiods in (**b**) and the standard deviation of the measures observed over all subperiods in (**c**).

The IR of peripheral nodes performs better than that of central nodes, "$RAND$" and "$BEST$" and analogously to that of "MKT". The SIR of peripheral portfolios performs better than that of central nodes and "$RAND$" but worse than that of "$BEST$" and "MKT".

Table S.7 reports the beta coefficients calculated over the yearly returns, with S&P 500 Composite index as benchmark. Let us recall that 'beta' is a measure of systematic risk of a portfolio in comparison with the market. Values of beta smaller than one indicate that the portfolio's excess returns have an anti-cyclic behavior with respect to the benchmark market. In (**a**) the measures computed over 7071 periods (since the standard deviation of beta coefficients is always very small it is omitted); the averages of the beta

Table S.6: Sharpe Information Ratio for portfolios of MKT, $RAND$, $BEST$, $PMFG$'s central nodes ($PMFG$-c) and $PMFG$'s peripheral nodes ($PMFG$-p). (**a**) Sharpe Information Ratio over the whole period of 7071 days; (**b**) Average Sharpe Information Ratio calculated over 6821 samples (corresponding to as many sub-periods each of 250 observations); (**c**) Standard deviations for the samples as in (**b**). The Sharpe Information Ratio has been here calculated as the information ratio of the excess yearly returns (the benchmark being the S&P 500 Composite index).

# of stocks	measure	MKT			RAND			BEST			PMFG-c			PMFG-p		
		u	ns	s	u	ns	s	u	ns	s	u	ns	s	u	ns	s
5 stocks	(a)	0.731	0.298	0.284	0.301	0.278	0.273	0.484	0.356	0.357	0.264	0.154	0.125	0.365	0.314	0.311
	(b)	1.596	0.691	0.707	0.342	0.339	0.335	0.705	0.562	0.561	0.406	0.260	0.226	0.481	0.444	0.443
	(c)	(1.814)	(1.794)	(1.705)	(0.340)	(0.460)	(0.461)	(0.873)	(0.822)	(0.821)	(0.967)	(0.750)	(0.736)	(0.598)	(0.741)	(0.743)
10 stocks	(a)	0.731	0.298	0.284	0.426	0.300	0.284	0.482	0.392	0.384	0.313	0.156	0.101	0.435	0.328	0.317
	(b)	1.596	0.691	0.707	0.512	0.406	0.397	0.752	0.631	0.625	0.582	0.242	0.159	0.605	0.512	0.503
	(c)	(1.814)	(1.794)	(1.705)	(0.482)	(0.703)	(0.705)	(0.962)	(0.882)	(0.880)	(1.052)	(0.798)	(0.770)	(0.732)	(0.942)	(0.943)
20 stocks	(a)	0.731	0.298	0.284	0.508	0.313	0.282	0.482	0.375	0.362	0.374	0.159	0.089	0.498	0.323	0.297
	(b)	1.596	0.691	0.707	0.680	0.505	0.471	0.836	0.695	0.672	0.732	0.272	0.147	0.744	0.556	0.532
	(c)	(1.814)	(1.794)	(1.705)	(0.683)	(0.921)	(0.930)	(1.070)	(1.081)	(1.038)	(1.033)	(0.935)	(0.954)	(0.908)	(1.113)	(1.111)
30 stocks	(a)	0.731	0.298	0.284	0.562	0.309	0.267	0.472	0.369	0.367	0.409	0.159	0.090	0.547	0.322	0.285
	(b)	1.596	0.691	0.707	0.799	0.517	0.463	0.873	0.706	0.702	0.828	0.291	0.150	0.850	0.584	0.538
	(c)	(1.814)	(1.794)	(1.705)	(0.828)	(1.042)	(1.035)	(1.068)	(1.161)	(1.107)	(1.042)	(1.020)	(1.128)	(1.032)	(1.195)	(1.179)
40 stocks	(a)	0.731	0.298	0.284	0.606	0.311	0.264	0.468	0.363	0.365	0.445	0.194	0.122	0.585	0.314	0.274
	(b)	1.596	0.691	0.707	0.899	0.558	0.490	0.906	0.707	0.737	0.913	0.337	0.209	0.941	0.587	0.529
	(c)	(1.814)	(1.794)	(1.705)	(0.891)	(1.115)	(1.098)	(1.094)	(1.170)	(1.180)	(1.063)	(1.047)	(1.184)	(1.144)	(1.232)	(1.209)

coefficients calculated over $7071 - 250 = 6821$ subperiods of length 250 days are reported in (**b**) and the standard deviation of the beta coefficients observed during all subperiods in (**c**). We observe that "$BEST$" and central nodes provide the worst performance, being unable to diversify market risk and often amplifying it. Peripheral nodes provide beta coefficients superior to "$RAND$" and comparable to "MKT". Beta coefficients of peripheral portfolios with uniform weights are much smaller than those obtained with any other portfolio with uniform weighting.

Tables S.8 and S.9 report the probability of respectively positive yearly returns and positive excess yearly returns. We report in (**a**) the measures computed over 7071 periods (since the standard deviation of the probabilities is always very small it is omitted); the averages of all subperiods in (**b**) and the standard deviation of the probabilities observed over all subperiods in (**c**). The probability of positive yearly returns of peripheral nodes is superior to that of "$RAND$", "$BEST$" and central nodes while it is similar to that of "MKT". The probability of positive excess yearly returns of peripheral nodes is superior to that of central nodes and comparable to that of "$RAND$", "$BEST$" and "MKT".

Overall we can say that from all previous analyses it emerges clearly that central nodes perform worse than any other alternative while peripheral nodes are often better than others, sometimes equivalent and seldom inferior.

Table S.7: Beta coefficients for portfolios of *MKT*, *RAND*, *BEST*, *PMFG*'s central nodes (*PMFG-c*) and *PMFG*'s peripheral nodes (*PMFG-p*). (**a**) Beta coefficients over 7071 periods; (**b**) Average Beta coefficients calculated over 6821 samples (corresponding to as many sub-periods each of 250 observations); (**c**) Standard deviations for the samples as in (**b**). The Beta coefficients have been here calculated for the yearly returns (the benchmark being the S&P 500 Composite index).

# of stocks	measure	*MKT*			*RAND*			*BEST*			*PMFG-c*			*PMFG-p*		
		u	ns	s	u	ns	s	u	ns	s	u	ns	s	u	ns	s
5 stocks	(a)	1.018	0.591	0.525	1.023	0.832	0.827	1.274	0.922	0.925	1.222	1.023	0.979	0.741	0.627	0.624
	(b)	1.164	0.605	0.471	1.127	0.850	0.847	1.637	1.062	1.070	1.318	1.134	1.112	0.773	0.624	0.622
	(c)	(0.431)	(0.490)	(0.485)	(0.452)	(0.363)	(0.365)	(1.380)	(0.969)	(0.978)	(0.870)	(0.672)	(0.697)	(0.688)	(0.595)	(0.596)
10 stocks	(a)	1.018	0.591	0.525	1.026	0.781	0.765	1.209	0.895	0.893	1.203	0.965	0.851	0.769	0.626	0.617
	(b)	1.164	0.605	0.471	1.143	0.794	0.773	1.588	0.922	0.925	1.292	1.011	0.946	0.782	0.630	0.622
	(c)	(0.431)	(0.490)	(0.485)	(0.454)	(0.372)	(0.376)	(1.223)	(0.742)	(0.739)	(0.762)	(0.518)	(0.571)	(0.669)	(0.552)	(0.554)
20 stocks	(a)	1.018	0.591	0.525	1.020	0.729	0.692	1.182	0.866	0.851	1.176	0.880	0.724	0.815	0.613	0.595
	(b)	1.164	0.605	0.471	1.173	0.749	0.699	1.526	0.853	0.831	1.302	0.956	0.834	0.853	0.635	0.616
	(c)	(0.431)	(0.490)	(0.485)	(0.435)	(0.412)	(0.421)	(1.052)	(0.685)	(0.692)	(0.663)	(0.504)	(0.590)	(0.562)	(0.497)	(0.504)
30 stocks	(a)	1.018	0.591	0.525	1.013	0.703	0.656	1.154	0.839	0.817	1.162	0.817	0.657	0.849	0.615	0.589
	(b)	1.164	0.605	0.471	1.154	0.738	0.669	1.472	0.832	0.790	1.315	0.894	0.751	0.903	0.626	0.596
	(c)	(0.431)	(0.490)	(0.485)	(0.424)	(0.428)	(0.438)	(1.026)	(0.621)	(0.618)	(0.608)	(0.527)	(0.611)	(0.519)	(0.488)	(0.502)
40 stocks	(a)	1.018	0.591	0.525	1.018	0.683	0.630	1.127	0.799	0.780	1.153	0.767	0.618	0.872	0.620	0.590
	(b)	1.164	0.605	0.471	1.151	0.714	0.637	1.411	0.770	0.718	1.322	0.823	0.688	0.951	0.623	0.584
	(c)	(0.431)	(0.490)	(0.485)	(0.436)	(0.438)	(0.459)	(0.984)	(0.573)	(0.576)	(0.569)	(0.536)	(0.613)	(0.465)	(0.485)	(0.502)

Table S.8: Probability of positive yearly returns for portfolios of *MKT*, *RAND*, *BEST*, *PMFG*'s central nodes (*PMFG-c*) and *PMFG*'s peripheral nodes (*PMFG-p*). (**a**) Probabilities over 7071 periods; (**b**) Average probabilities calculated over 6821 samples (corresponding to as many sub-periods each of 250 observations); (**c**) Standard deviations for the samples as in (**b**).

# of stocks	measure	*MKT*			*RAND*			*BEST*			*PMFG-c*			*PMFG-p*		
		u	ns	s	u	ns	s	u	ns	s	u	ns	s	u	ns	s
5 stocks	(a)	0.782	0.840	0.816	0.760	0.770	0.769	0.760	0.763	0.763	0.747	0.742	0.737	0.794	0.802	0.802
	(b)	0.774	0.834	0.810	0.752	0.763	0.762	0.751	0.756	0.756	0.738	0.735	0.730	0.787	0.796	0.796
	(c)	(0.302)	(0.250)	(0.286)	(0.241)	(0.236)	(0.236)	(0.255)	(0.254)	(0.254)	(0.304)	(0.277)	(0.274)	(0.217)	(0.209)	(0.209)
10 stocks	(a)	0.782	0.840	0.816	0.781	0.784	0.780	0.774	0.791	0.791	0.768	0.746	0.725	0.814	0.813	0.811
	(b)	0.774	0.834	0.810	0.773	0.776	0.773	0.766	0.784	0.784	0.759	0.739	0.721	0.807	0.807	0.805
	(c)	(0.302)	(0.250)	(0.286)	(0.258)	(0.241)	(0.242)	(0.248)	(0.254)	(0.255)	(0.312)	(0.274)	(0.264)	(0.224)	(0.212)	(0.212)
20 stocks	(a)	0.782	0.840	0.816	0.788	0.791	0.787	0.782	0.794	0.799	0.774	0.750	0.732	0.823	0.827	0.819
	(b)	0.774	0.834	0.810	0.780	0.784	0.780	0.774	0.787	0.792	0.766	0.742	0.728	0.816	0.821	0.813
	(c)	(0.302)	(0.250)	(0.286)	(0.275)	(0.244)	(0.248)	(0.266)	(0.263)	(0.263)	(0.312)	(0.271)	(0.263)	(0.235)	(0.213)	(0.217)
30 stocks	(a)	0.782	0.840	0.816	0.788	0.795	0.792	0.787	0.800	0.804	0.772	0.758	0.743	0.820	0.825	0.813
	(b)	0.774	0.834	0.810	0.781	0.788	0.785	0.779	0.793	0.797	0.763	0.749	0.737	0.813	0.819	0.807
	(c)	(0.302)	(0.250)	(0.286)	(0.282)	(0.252)	(0.257)	(0.267)	(0.260)	(0.266)	(0.315)	(0.273)	(0.275)	(0.248)	(0.223)	(0.232)
40 stocks	(a)	0.782	0.840	0.816	0.790	0.802	0.797	0.791	0.805	0.807	0.770	0.770	0.756	0.816	0.827	0.813
	(b)	0.774	0.834	0.810	0.782	0.795	0.790	0.783	0.798	0.799	0.762	0.762	0.749	0.809	0.821	0.807
	(c)	(0.302)	(0.250)	(0.286)	(0.286)	(0.248)	(0.254)	(0.269)	(0.255)	(0.269)	(0.315)	(0.273)	(0.279)	(0.256)	(0.224)	(0.238)

Table S.9: Probability of positive excess returns for portfolios of *MKT*, *RAND*, *BEST*, *PMFG*'s central nodes (*PMFG-c*) and *PMFG*'s peripheral nodes (*PMFG-p*). (**a**) Probabilities over 7071 periods; (**b**) Average probabilities calculated over 6821 samples (corresponding to as many sub-periods each of 250 observations); (**c**) Standard deviations for the samples as in (**b**). The benchmark for yearly excess returns is the S&P 500 Composite index.

# of stocks	measure	MKT			RAND			BEST			PMFG-c			PMFG-p		
		u	ns	s	u	ns	s	u	ns	s	u	ns	s	u	ns	s
5 stocks	(a)	0.777	0.623	0.618	0.628	0.623	0.622	0.707	0.664	0.664	0.617	0.587	0.574	0.654	0.640	0.639
	(b)	0.770	0.616	0.614	0.622	0.619	0.619	0.703	0.660	0.660	0.608	0.585	0.573	0.653	0.640	0.639
	(c)	(0.255)	(0.325)	(0.302)	(0.132)	(0.170)	(0.171)	(0.233)	(0.221)	(0.221)	(0.274)	(0.234)	(0.235)	(0.189)	(0.224)	(0.224)
10 stocks	(a)	0.777	0.623	0.618	0.679	0.642	0.638	0.713	0.671	0.671	0.658	0.579	0.555	0.689	0.662	0.661
	(b)	0.770	0.616	0.614	0.673	0.638	0.635	0.707	0.670	0.669	0.648	0.580	0.559	0.684	0.660	0.658
	(c)	(0.255)	(0.325)	(0.302)	(0.166)	(0.217)	(0.218)	(0.223)	(0.239)	(0.238)	(0.262)	(0.234)	(0.235)	(0.211)	(0.255)	(0.257)
20 stocks	(a)	0.777	0.623	0.618	0.710	0.651	0.638	0.726	0.666	0.664	0.702	0.585	0.541	0.716	0.663	0.654
	(b)	0.770	0.616	0.614	0.703	0.648	0.635	0.720	0.665	0.663	0.692	0.582	0.545	0.709	0.658	0.649
	(c)	(0.255)	(0.325)	(0.302)	(0.197)	(0.240)	(0.243)	(0.231)	(0.246)	(0.244)	(0.254)	(0.245)	(0.252)	(0.230)	(0.280)	(0.283)
30 stocks	(a)	0.777	0.623	0.618	0.741	0.645	0.630	0.739	0.671	0.674	0.730	0.576	0.535	0.722	0.662	0.650
	(b)	0.770	0.616	0.614	0.733	0.641	0.628	0.734	0.668	0.672	0.720	0.573	0.539	0.715	0.656	0.644
	(c)	(0.255)	(0.325)	(0.302)	(0.202)	(0.261)	(0.260)	(0.227)	(0.250)	(0.245)	(0.252)	(0.244)	(0.271)	(0.240)	(0.288)	(0.288)
40 stocks	(a)	0.777	0.623	0.618	0.749	0.651	0.626	0.744	0.673	0.680	0.743	0.585	0.554	0.727	0.658	0.647
	(b)	0.770	0.616	0.614	0.740	0.647	0.623	0.739	0.668	0.677	0.734	0.581	0.556	0.719	0.652	0.642
	(c)	(0.255)	(0.325)	(0.302)	(0.210)	(0.268)	(0.267)	(0.225)	(0.251)	(0.244)	(0.258)	(0.247)	(0.281)	(0.254)	(0.287)	(0.285)

S.3 Portfolio variance and remonetized quantities

In Fig.S.3 are reported the portfolio performances when weights are computed with the Markowitz method with short-selling. One can note that the results are almost undistinguishable from the one for the case with no short-selling, reported in Fig.3 of the main paper.

As further quantification of risk let us here report in Figs. S.4, S.5 and S.6 the variance of portfolio returns at various time lags from 1 to 250 days. One can observe that the variance of the portfolios made of peripheral stocks ($\square$) is always lower than that of portfolios made of central stocks (∇) and it is comparable or lower than that of portfolios made of all 300 stocks (thick line), with as little as $m = 10$ stocks.
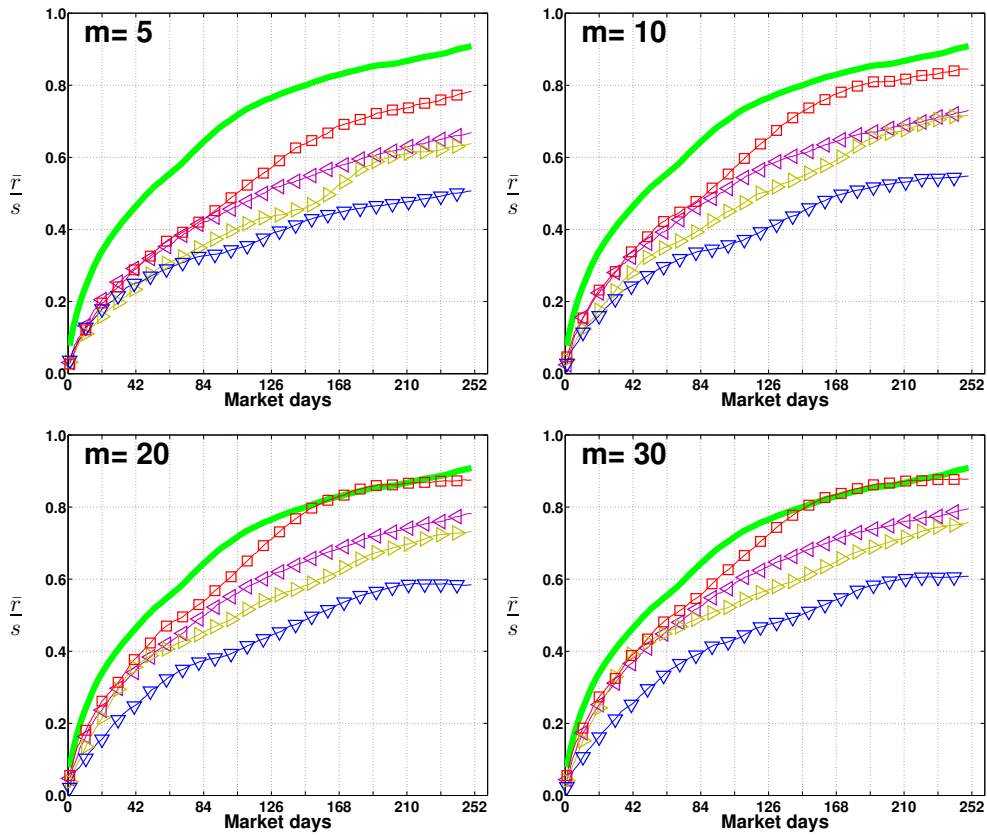
Figure S.3: Demonstration that portfolios made with peripheral stocks ($\square$) perform better than portfolios made with central stocks (∇) also in the case of weights obtained by solving the Markowitz problem with short-selling. Portfolio sizes are respectively $m = 5, 10, 20, 30$ stocks; weights are uniform. The plots report the 'signal-to-noise ratio' $\frac{\bar{r}(\tau)}{s(\tau)}$ (average return divided by its standard deviation) for $\tau = 1, .., 250$ days following the investment day. The performance is compared with: ($\triangleleft$) portfolios made of m randomly chosen stocks; ($\triangleright$) portfolios made with the m stocks that have achieved the best performance over the period preceding the investment date. The thick line is a 'market portfolio' made by taking all 300 stocks.

10
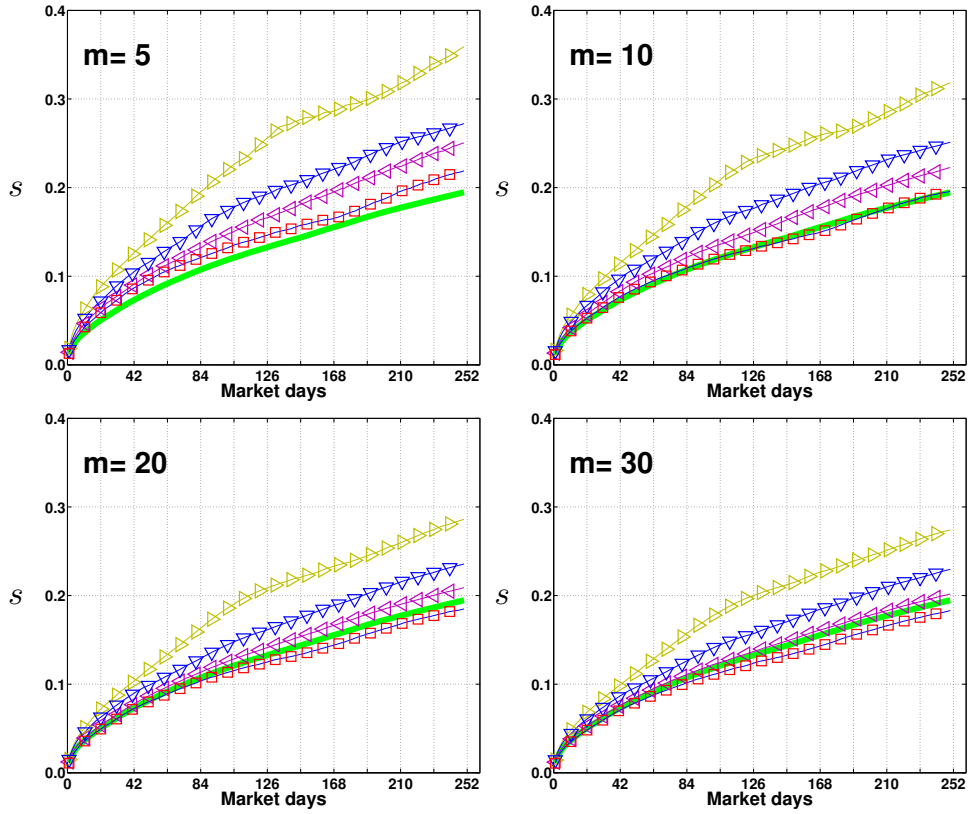
Figure S.4: Comparison between the variance of different portfolios with uniform weights. The symbol $\square$ indicates portfolios made of the $m = 5, 10, 20, 30$ most peripheral stocks (i.e. with largest $X + Y$). $\triangledown$ indicates portfolios made of the m most central stocks (i.e. with smallest $X + Y$). These are compared with: (thick line) 'market portfolios' made of all 300 stocks; ($\triangleleft$) portfolios made of m randomly chosen stocks; ($\triangleright$) portfolios made of the m stocks that have achieved the best performance over the period preceding the investment date.

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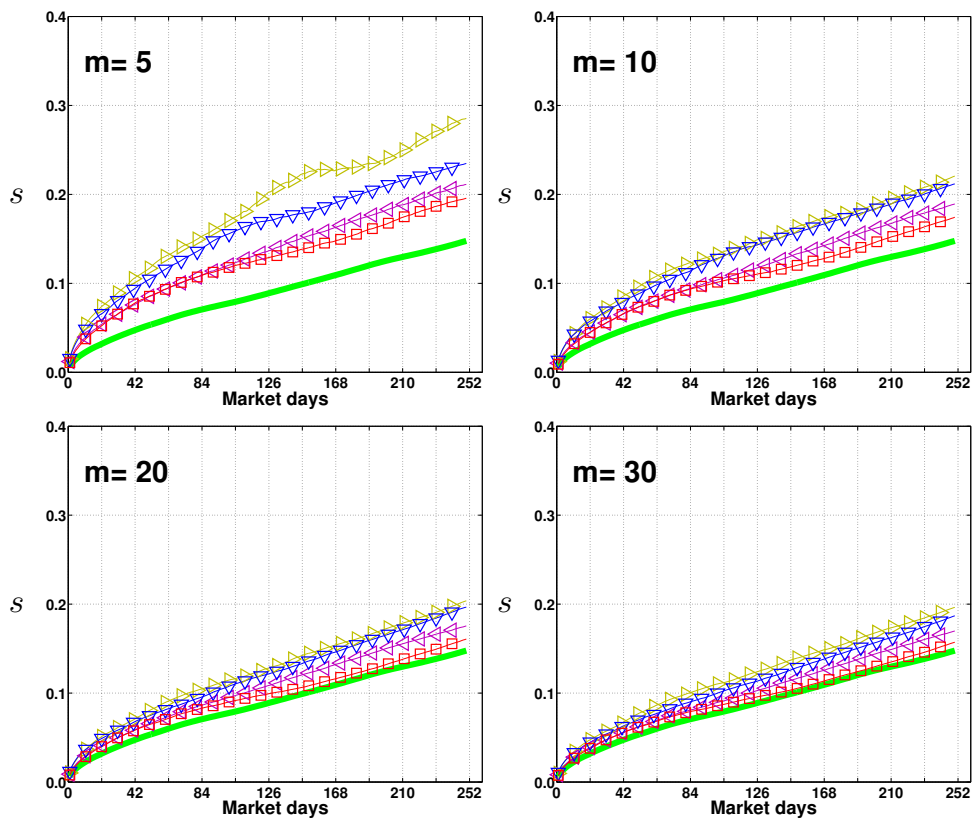
Figure S.5: Comparison between the variance of different portfolios obtained by solving the Markowitz problem with no short-selling. Labels are the same as in Fig. S.4: the composition of portfolios is the same while the weighting is different.
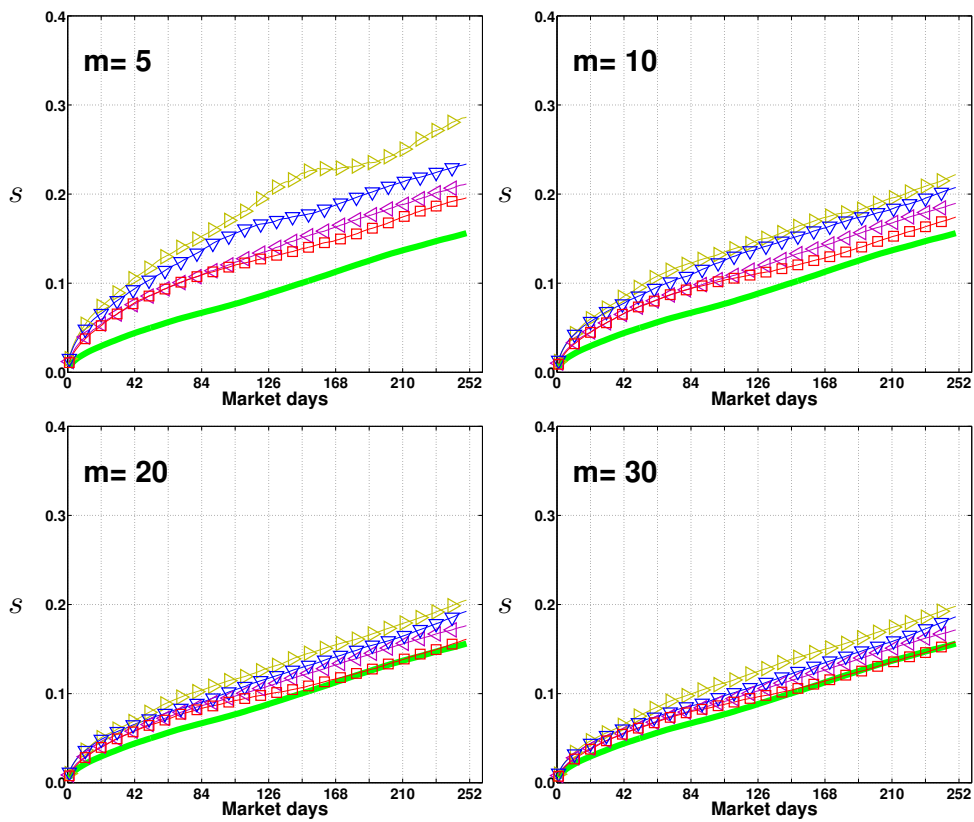
Figure S.6:   Comparison between the variance of different portfolios obtained by solving the Markowitz problem with short-selling. The labels are the same as in Fig. S.4.

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S.4 Comparison of performances by using different centrality measures

In the paper we introduced the hybrid centrality measure $X + Y$ to select stocks in the peripheral or central parts of the filtered graphs. We mentioned in the paper that the selection through this measure gives consistently better results than the use of the centrality measures in isolation. Let us here compare performances obtained with the hybrid measure with the ones obtained by using Betweenness Centrality on the weighted $PMFG$ graph (C_{BC}^w) and Eigenvector Centrality on the weighted $PMFG$ graph (C_{EC}^w). This is reported in Figure S.7 for the Betweenness Centrality measure with portfolios made by weighting stocks uniformly and in Figure S.8 for the case of Markowitz weights with no-short-selling. Figure S.9 reports the results for the Eigenvector Centrality measure with portfolios made by weighting stocks uniformly and Figure S.10 for the case of Markowitz weights with no-short-selling. As one can see, performances obtained by using the hybrid measure are consistently better than the ones obtained by using the centrality measures in isolation. Let us stress, that despite the different performances, the main result of the paper that portfolios made of peripheral stocks are less risky and more rewarding than portfolios made of central stocks is always retrieved for all centrality measures and their combination.
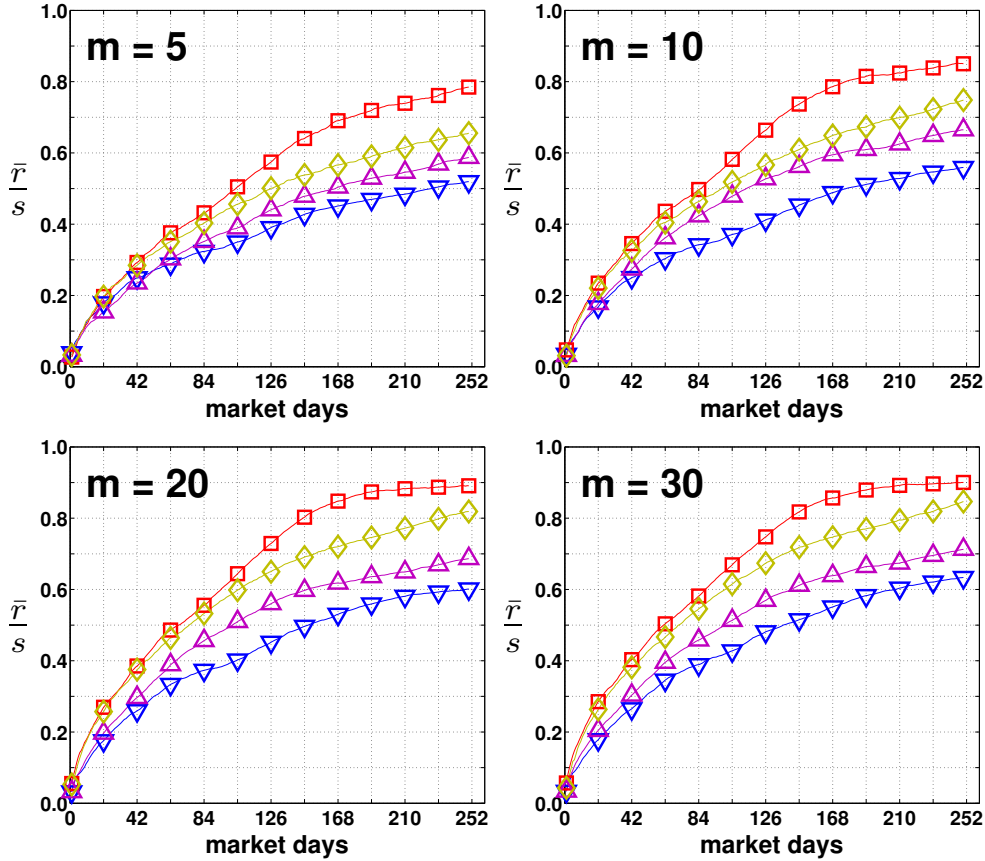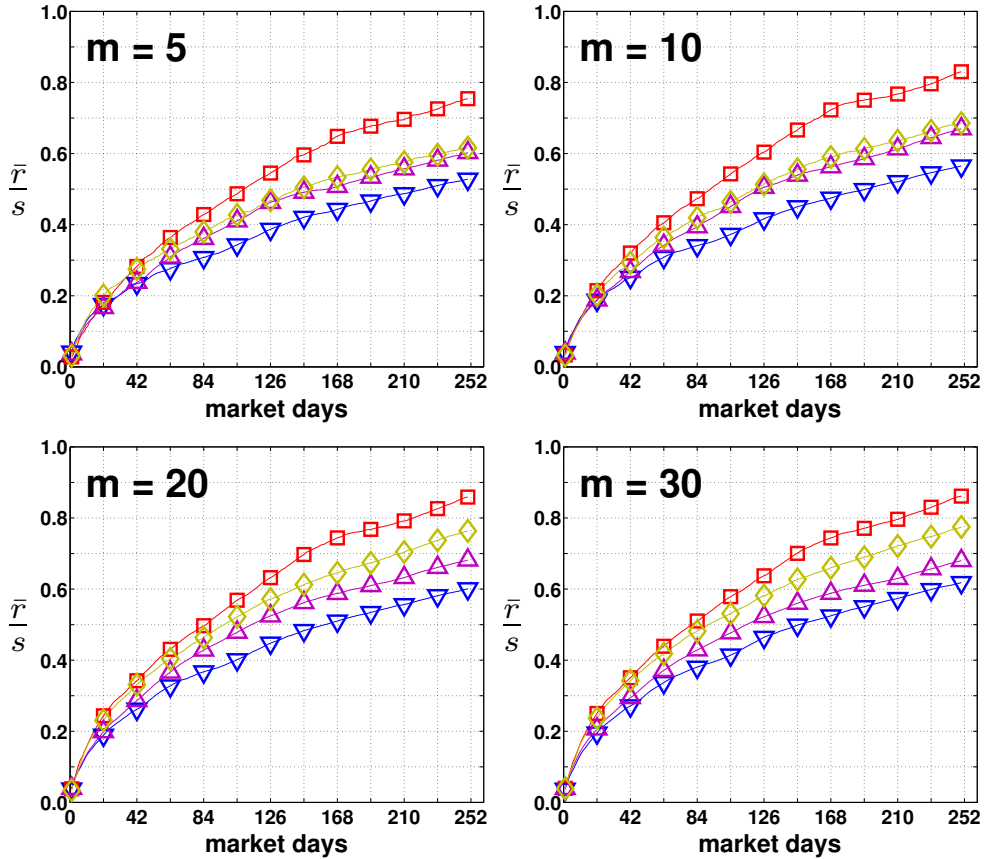
Figure S.7: Demonstration that Markowitz weighted portfolios, constructed by using Betweenness Centrality index (C_{BC}^w) on PMFG to select peripheral or central vertices are less effective than portfolios constructed by using the hybrid measure $X + Y$ introduced in the paper. The plots report the 'signal-to-noise ratio' $\frac{\bar{r}(\tau)}{s(\tau)}$ (average return divided by its standard deviation) for $\tau = 1, .., 250$ days following the investment day. Peripheral portfolios from $X + Y$ hybrid measure are indicated with ($\square$). Central portfolios from $X + Y$ hybrid measure are indicated with ($\triangledown$). Peripheral portfolios from Betweenness Centrality index are indicated with ($\diamond$). Central portfolios from Betweenness Centrality index are indicated with ($\triangle$). Portfolio sizes are respectively $m = 5, 10, 20, 30$ stocks.
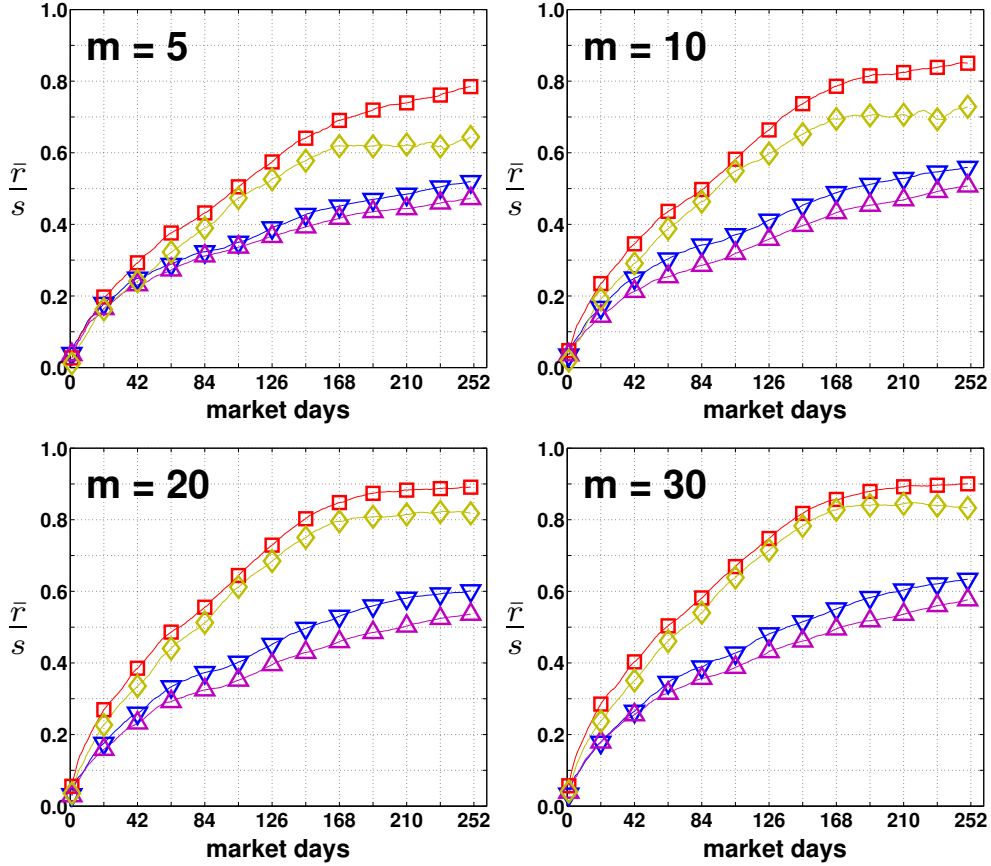
Figure S.8: Demonstration that uniformly weighted portfolios, constructed by using Betweenness Centrality index (C_{BC}^w) on PMFG to select peripheral or central vertices are less effective than portfolios constructed by using the hybrid measure $X + Y$ introduced in the paper. The plots report the 'signal-to-noise ratio' $\frac{\bar{r}(\tau)}{s(\tau)}$ (average return divided by its standard deviation) for $\tau = 1, .., 250$ days following the investment day. Peripheral portfolios from $X + Y$ hybrid measure are indicated with ($\square$). Central portfolios from $X + Y$ hybrid measure are indicated with ($\triangledown$). Peripheral portfolios from Betweenness Centrality index are indicated with ($\diamond$). Central portfolios from Betweenness Centrality index are indicated with ($\triangle$). Portfolio sizes are respectively $m = 5, 10, 20, 30$ stocks.

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Figure S.9: Demonstration that Markowitz weighted portfolios with no-short-selling, constructed by using Eigenvector Centrality index (C_E^w) on PMFG to select peripheral or central vertices are less effective than portfolios constructed by using the hybrid measure $X + Y$ introduced in the paper. The plots report the 'signal-to-noise ratio' $\frac{\bar{r}(\tau)}{s(\tau)}$ (average return divided by its standard deviation) for $\tau = 1, .., 250$ days following the investment day. Peripheral portfolios from $X + Y$ hybrid measure are indicated with ($\square$). Central portfolios from $X + Y$ hybrid measure are indicated with ($\triangledown$). Peripheral portfolios from Eigenvector Centrality index are indicated with ($\diamond$). Central portfolios from Eigenvector Centrality index are indicated with ($\triangle$). Portfolio sizes are respectively $m = 5, 10, 20, 30$ stocks.
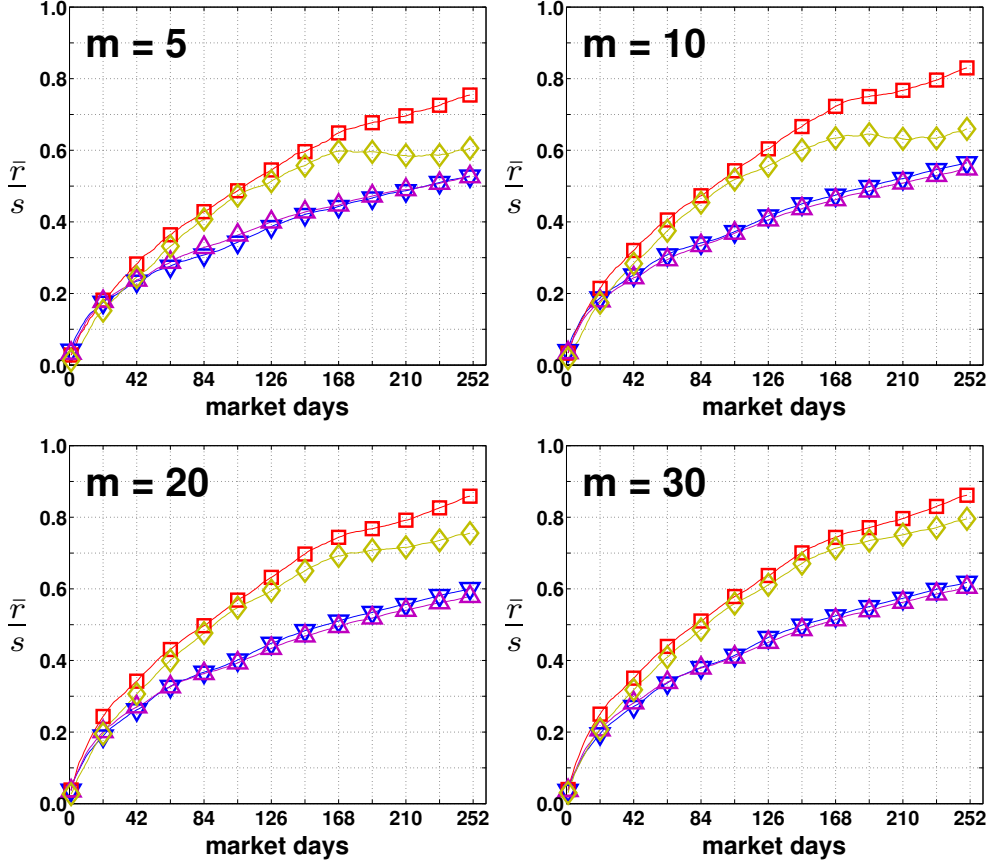
Figure S.10: Demonstration that the uniformly weighted portfolios, constructed by using Eigenvector Centrality index (C_E^w) on PMFG to select peripheral or central vertices are less effective than portfolios constructed by using the hybrid measure $X+Y$ introduced in the paper. The plots report the 'signal-to-noise ratio' $\frac{\bar{r}(\tau)}{s(\tau)}$ (average return divided by its standard deviation) for $\tau = 1, .., 250$ days following the investment day. Peripheral portfolios from $X + Y$ hybrid measure are indicated with ($\square$). Central portfolios from $X + Y$ hybrid measure are indicated with (∇). Peripheral portfolios from Eigenvector Centrality index are indicated with ($\diamond$). Central portfolios from Eigenvector Centrality index are indicated with ($\triangle$). Portfolio sizes are respectively $m = 5, 10, 20, 30$ stocks.

S.5 Markowitz Portfolio Selection Problem

Markowitz seminal work [7] and subsequent Capital Asset Pricing Model (CAPM) contributions [8, 9, 10, 11, 12, 13, 14] propose to reduce risk by minimizing the variance of a portfolio subject to some constraits. A portfolio variance is a function of stocks' variances, covariances and correlations. Markowitz portfolio optimization problem can be written in a general form as:

$$\min_{\mathbf{q}} \quad \mathbf{q}^T \mathbf{V} \mathbf{q}$$
$$\text{s.t.:} \tag{S.1}$$
$$\mathbf{g}(\mathbf{q}) \geq \mathbf{0}$$
$$\mathbf{h}(\mathbf{q}) = \mathbf{0}$$

where $\mathbf{V}$ is the covariance matrix, $\mathbf{q}$ is a vector of weights representing, for each security, the percentage of the total wealth invested; $\mathbf{q}^T \mathbf{V} \mathbf{q}$, the objective function, is the total portfolio variance. Inequality and equality constraints, are $\mathbf{g}(\mathbf{q}) \geq \mathbf{0}$ and $\mathbf{h}(\mathbf{q}) = \mathbf{0}$. The problem consists in minimizing the portfolio's risk, assumed to be adequately estimated by the portfolio expected variance, subject to some budget constraint (e.g. $\mathbf{q}^T \mathbf{u} = 1$), the attainment of a certain expected return performance (e.g. $\mathbf{q}^T \bar{\mathbf{r}} \geq r^*$, where $\bar{\mathbf{r}}$ is a vector of securities's expected returns, $\mathbf{q}^T \bar{\mathbf{r}}$ is the portfolio return and r^* is a desired return performance), or other constraints. Other main assumptions of the model are returns being jointly normally distributed (when used in a constraint), correlations and variances being stable over time; no transaction costs; investors are rational, price takers, profit maximizing, risk-averse, endowed with complete unbiased information, able to lend and borrow unlimited amounts of funds at the risk free rate of interest; securities are infinitely divisible.

The solution of the problem is the vector $\mathbf{q}$, which is a function of sample variances, covariances, correlations and any other parameter introduced in the constraint (such as average returns). Correlations influence the curvature of the Efficient Frontier of Investments, i.e. the locus of a portfolio's minimum expected variances for any level of targeted expected return.

We define the Markowitz problem without short sales ($\mathcal{P}_{ns}$) and with short sales ($\mathcal{P}_s$) as:

$\mathcal{P}_{ns}$	$\mathcal{P}_s$
$\min\limits_{\mathbf{q_{ns}}} \quad \dfrac{1}{2}\, \mathbf{q_{ns}}^T\, \bar{\mathbf{V}}^{\mathbf{w}}\, \mathbf{q_{ns}}$	$\min\limits_{\mathbf{q_s}} \quad \dfrac{1}{2}\, \mathbf{q_s}^T\, \bar{\mathbf{V}}^{\mathbf{w}}\, \mathbf{q_s}$
s.t.:	s.t.:
$\mathbf{q_{ns}} \geq \mathbf{0}$	$\mathbf{q_s}^T \bar{\mathbf{r}} \geq \mathbf{q_{ns}^*}^T \bar{\mathbf{r}}$
$\mathbf{q_{ns}}^T \mathbf{u} = 1$	$\mathbf{q_s}^T \mathbf{u} = 1$

where $\mathbf{q_{ns}}$ is constrained to be non-negative (being short-selling not allowed); $\bar{\mathbf{r}}$ is the vector of expected returns; $\mathbf{q_{ns}^*}$ is the vector of weights solving $\mathcal{P}_{ns}$. The solutions are unique: $\mathbf{q_{ns}^*}$ corresponds to the minimum-variance point over the Efficient Frontier and is also known as the global minimum variance portfolio.

In the present paper the two problems have been solved numerically using Matlab function "quadprog", setting the number of maximum iterations ($MaxIter$) at 2000 and termination tolerance on the constraint violation ($TolCon$) at $2.2204e-014$. The starting point for $\mathcal{P}_{ns}$ was $\mathbf{q_{ns}^o} = \frac{1}{N}\mathbf{u}$; and for $\mathcal{P}_s$ was $\mathbf{q_s^o} = \mathbf{q_{ns}^*}$. In all instances the optimization was successful and a solution was found within few iterations.

The Markowitz problem has been solved by using the average correlations with shrinkage $\bar{\mathbf{R}}^{\mathbf{w}}$, defined in Eq. 1 in the main paper. Consistently with $\bar{\mathbf{R}}^{\mathbf{w}}$, we have defined the average weighted covariance matrix with shrinkage as

$$\bar{\mathbf{V}}^{\mathbf{w}} = \mathbf{P}^{\mathbf{w}\frac{1}{2}}\, \bar{\mathbf{R}}^{\mathbf{w}}\, \mathbf{P}^{\mathbf{w}\frac{1}{2}} \tag{S.2}$$

where $\mathbf{P}^{\mathbf{w}}$ is a diagonal matrix with average weighted variances over the main diagonal defined as $\left(\bar{\hat{s}}_k^w\right)^2 = \frac{1}{\tau+1}\sum\limits_{h=t-\tau}^{t}\left(\hat{s}_{kh}^w\right)^2$. Matrix $\bar{\mathbf{V}}^{\mathbf{w}}$ is generally full-rank and numerically stable. The condition number of $\bar{\mathbf{V}}^{\mathbf{w}}$ is similar to that of $\bar{\mathbf{R}}^{\mathbf{w}}$. Note that a sum of covariance matrices does not generally enjoy the same properties as $\bar{\mathbf{V}}^{\mathbf{w}}$.

S.6 Comparison of portfolio composition

While MSTs might be preferable to $PMFG$s for the greater simplicity of their graphic representation, the latter offers a richer description of the sys-

Table S.10: Indexes of coincidence between MST and $PMFG$ peripheries.

| | MST vs. $PMFG$ peripheries. No short sales. | | | | | MST vs. $PMFG$ peripheries. Short sales. | | | | |
	5 stocks	10 stocks	20 stocks	30 stocks	40 stocks	5 stocks	10 stocks	20 stocks	30 stocks	40 stocks
$mean$	33.27%	37.91%	43.34%	48.17%	52.75%	33.22%	37.71%	42.71%	46.94%	50.86%
$CI_{2.5}$	0.00%	1.97%	10.84%	17.27%	22.92%	0.00%	2.41%	12.23%	19.72%	26.23%
$CI_{97.5}$	86.76%	81.08%	79.35%	79.45%	82.00%	86.31%	78.98%	74.86%	73.65%	73.83%

tem of relations between stocks.

In order to quantify the differences in the selection of peripheral portfolios from MST and $PMFG$ let us introduce a measure of coincidence. Specifically, let $q^1, q^2 \in \Re^{N \times 1}$ be two vectors solving the Markowitz problem subject to two different sets of constraints, with $\sum_{i=1}^{N} q_i^j = 1$, for $j = \{1, 2\}$. We introduce a measure of coincidence between q^1 and q^2 in order to compare the composition of the two portfolios. Let $l_j = \sum_{i=1}^{N} |q_i^j|$ be the amount of total transactions implied by q^j. Then we define the following index of coincidence:

$$\mathsf{c}_{12} = \frac{\sum_{i=1}^{N} \frac{1}{2} \left[1 + sign\left(q_i^1\right) + sign\left(q_i^2\right) \right] \times \min\left(|q_i^1|, |q_i^2|\right)}{\sqrt{l_1 \times l_2}} \tag{S.3}$$

$\mathsf{c}_{12} = 1$ if and only if q^1 and q^2 are identical and $\mathsf{c}_{12} = 0$ if, $\forall i = \{1, 2, \ldots, N\}$, q_i^1 and q_i^2 are either both zero, have different signs or one is nonnull when the other is zero.

For all time periods, we have compared MST vs. $PMFG$ peripheral portfolios: the table reports the average and 95% confidence intervals of the corresponding coincidence indices, which are reported in Table S.10. The coincidence index shows that, on average, a large share of MST and $PMFG$ peripheral portfolios is not coincident. The average coincidence index increases with the number of stocks.

S.7 Indices of centrality and peripherality

Indices of centrality and peripherality have been calculated with the following $MATLAB$ code.

Complex indices of centrality and peripherality for the vertices of a network

```
% Calculates centrality indices as used in the paper
% F. Pozzi, T. Di Matteo, T. Aste, "Spread of risk across
% financial markets: better to invest in the peripheries".
%
% INPUT
% G is a Planar Maximally Filtered Graph, stored in the form of a
% symmetric, square, N-by-N sparse matrix filtering a correlation
% matrix (data must be correlations).
%
% OUTPUT
% X, Y, XpY and XmY are, respectively, X, Y, (X + Y) and (X - Y)
% in the paper. A vertex characterized by high (low) ranking in terms
% of (X + Y) is likely to be a central (peripheral) vertex; a vertex
% characterized by high (low) ranking in terms of (X - Y) is likely
% to possess many unimportant (few important) connections. "High
% ranking" means "low score" (i.e. the most central vertex is assigned
% a small score). In detail:

%
% A small value of X indicates high connectedness whereas a large
% value indicates low connectedness
%
% A small value of Y indicates low eccentricity whereas a large
% value indicates high eccentricity
%
% A small value of XpY indicates high overall centrality whereas a
% large value indicates low overall centrality
%
% A small value of XmY indicates many low-quality connections
% whereas a large value indicates few high-quality connections
%
% Note: this code makes use of David Gleich's MATLAB BGL
% (FEX 10922, available at
% www.mathworks.com/matlabcentral/fileexchange/10922-matlabbgl)
%
% EYAMPLE using pmfg by Tomaso Aste (FEX 27360, available at
% www.mathworks.com/matlabcentral/fileexchange/27360-pmfg)
```

```matlab
% y = corrcoef(cumsum(randn(100, 30)));
% G = pmfg(y);
% [X Y XpY XmY] = centrinds(G);
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function [X Y XpY XmY] = centrinds(G)
 PMFG_Top = (G ~= 0) * 1;    % Topological planar
 PMFG_GeoW = G;     % Geodesic planar - weights
 PMFG_GeoW(PMFG_GeoW ~= 0) = 1 + ...
    PMFG_GeoW(PMFG_GeoW ~= 0);    % weights in [0, 2]
 PMFG_GeoD = G;     % Geodesic planar - distances
 PMFG_GeoD(PMFG_GeoD ~= 0) = sqrt(2 * (1 - ...
    PMFG_GeoD(PMFG_GeoD ~= 0)));    % distances in [0, 2]

 ShPTop = all_shortest_paths(PMFG_Top);    % Top. Sh. Paths
 ShPGeo = all_shortest_paths(PMFG_GeoD);    % Geom. Sh. Paths

% 1. Topological Degree
 DgrTopPMFG = full(sum(PMFG_Top));

% 2. Geometrical Degree: sum of weights (possibly negative)
 DgrGeoPMFG = full(sum(PMFG_GeoW));

% 3. Betweenness (based on Topological Shortest Paths)
 BtwTopPMFG = betweenness_centrality(PMFG_Top);

% 4. Betweenness (based on Geometrical Shortest Paths)
 BtwGeoPMFG = betweenness_centrality(PMFG_GeoD);

% 5. Eccentricity (based on Topological Shortest Paths)
 ExxTopPMFG = max(ShPTop);    % eccentricity of vertexes

% 6. Eccentricity (based on Geometrical Shortest Paths)
 ExxGeoPMFG = max(ShPGeo);    % eccentricity of vertexes

% 7. Closeness (based on Topological Shortest Paths)
 ClsTopPMFG = mean(ShPTop);

% 8. Closeness (based on Geometrical Shortest Paths)
 ClsGeoPMFG = mean(ShPGeo);
```

```
% 9. Topological Eigenvector Centrality
[eigvec, eigval] = eigs(PMFG_Top); eigval = sum(eigval);
index = find(eigval == max(max(eigval)));

if all(round(eigvec(:, index) * 1e8) <= 0);
  EigTopPMFG = -eigvec(:, index);
else
  EigTopPMFG = eigvec(:, index);
end;

% 10. Geometrical Eigenvector Centrality
[eigvec, eigval] = eigs(PMFG_GeoW); eigval = sum(eigval);
index = find(eigval == max(max(eigval)));

if all(round(eigvec(:, index) * 1e8) <= 0);
  EigGeoPMFG = -eigvec(:, index);
else
  EigGeoPMFG = eigvec(:, index);
end;

% Calculate rankings
DgrGeoPMFG_rnks = tiedrank(-DgrGeoPMFG);
DgrTopPMFG_rnks = tiedrank(-DgrTopPMFG);
BtwTopPMFG_rnks = tiedrank(-BtwTopPMFG)';
BtwGeoPMFG_rnks = tiedrank(-BtwGeoPMFG)';
ExxGeoPMFG_rnks = tiedrank(ExxGeoPMFG);
ExxTopPMFG_rnks = tiedrank(ExxTopPMFG);
ClsGeoPMFG_rnks = tiedrank(ClsGeoPMFG);
ClsTopPMFG_rnks = tiedrank(ClsTopPMFG);
EigGeoPMFG_rnks = tiedrank(-EigGeoPMFG)';
EigTopPMFG_rnks = tiedrank(-EigTopPMFG)';

% Calculate indices
n = size(G, 1);
X = (DgrGeoPMFG_rnks + DgrTopPMFG_rnks + ...
    BtwGeoPMFG_rnks + BtwTopPMFG_rnks - 4) / 4 / (n - 1);
Y = (ExxGeoPMFG_rnks + ExxTopPMFG_rnks + ...
    ClsGeoPMFG_rnks + ClsTopPMFG_rnks + ...
    EigGeoPMFG_rnks + EigTopPMFG_rnks - ) / 6 / (n - 1);
XpY = X + Y; XmY = X - Y;
```

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