

```

1  /* Main procedure */
2   $bc[i] \leftarrow 0$ , for  $i = 0..n - 1$ ;
3  Set the adjacency arrays  $a_1[i]$ ,  $a_2[i]$ , for  $i = 0..2m - 1$ ;
4   $d[i], \sigma[i], \delta[i] \leftarrow 0$ , for  $i = 0..n - 1$ ;
5   $p[i][j] \leftarrow 0$ , for  $i = 0..n - 1, j = 0..n - 1$ ;
6  Set up APSP kernel execution configuration:  $grid_1, threads_1$ ;
7  Set up back propagation kernel execution configuration:  $grid_2, threads_2$ ;
8  for  $i \in 0..n - 1$  do
9      /* APSP */
10      $continue \leftarrow true$ ;
11      $dist \leftarrow 0$ ;
12     while  $continue$  do
13         |  $apsp\_kernel\langle\langle\langle grid_1, threads_1 \rangle\rangle\rangle(a_1, a_2, d, \sigma, p, done, dist)$ ;
14         |  $dist++$ ;
15     end
16     /* Back propagation */
17      $done \leftarrow false$ ;
18     while  $dist > 1$  do
19         |  $back\_prop\_kernel\langle\langle\langle grid_1, threads_1 \rangle\rangle\rangle(a_1, a_2, d, \sigma, \delta, p, dist)$ ;
20         | Sync thread blocks;
21         |  $back\_sum\_kernel\langle\langle\langle grid_2, threads_2 \rangle\rangle\rangle(i, dist, d, \delta, bc)$ ;
22         |  $dist--$ ;
23     end
24 end
25 return  $bc$ 

26 /* APSP kernel */
27 procedure  $apsp\_kernel(a_1, a_2, d, \sigma, p, done, dist)$ 
28 foreach  $thread\ i$  do
29     |  $u \leftarrow a_1[i], w \leftarrow a_2[i]$ ; /* set the node ids for edge  $i$  */
30     | if  $d[u] == dist$  then
31         | | if  $d[w] == -1$  then
32             | | |  $continue \leftarrow true; d[w] \leftarrow dist + 1$ ;
33         | | end
34         | | if  $d[w] == dist + 1$  then
35             | | |  $p[w][u] \leftarrow 1$ ;
36             | | |  $atomicAdd(\sigma[w], \sigma[u])$ ;
37         | | end
38     | end
39 end

40 /* Back propagation kernel */
41 procedure  $back\_prop\_kernel(a_1, a_2, d, \sigma, \delta, p, dist)$ 
42 foreach  $thread\ i$  do
43     |  $u \leftarrow a_1[i]; w \leftarrow a_2[i]$ ; /* set the node ids for edge  $i$  */
44     | if  $d[u] == dist - 1$  then
45         | | if  $p[u][w] == 1$  then
46             | | |  $atomicAdd(\delta[w], \sigma[w] / \sigma[u] * (1 + \delta[u]))$ ;
47         | | end
48     | end
49 end

50 procedure  $back\_sum\_kernel(s, dist, d, \delta, bc, n)$ 
51 foreach  $thread\ i$  do
52     | if  $i \neq s \ \&\& \ d[i] == dist - 1$  then
53         | |  $bc[i] \leftarrow bc[i] + \delta[i]$ ;
54     | end
55 end

```