

## Decomposition of the cubic function using knots

The two knots are respectively denoted  $K_1$  and  $K_2$ , with  $K_1 = 5.333 h$  and  $K_2 = 17.333 h$ . Therefore the function  $f_i(t)$  is composed of three cubic functions  $f_{i1}(t)$ ,  $f_{i2}(t)$  and  $f_{i3}(t)$  for a gene  $G_i$  in the whole time interval:

$$f_{i1}(t) = \beta_{i10} + \beta_{i11} t + \beta_{i12} t^2 + \beta_{i13} t^3, \quad (0 \leq t < K_1)$$

$$f_{i2}(t) = \beta_{i20} + \beta_{i21} t + \beta_{i22} t^2 + \beta_{i23} t^3, \quad (K_1 \leq t < K_2)$$

$$f_{i3}(t) = \beta_{i30} + \beta_{i31} t + \beta_{i32} t^2 + \beta_{i33} t^3, \quad (K_2 \leq t \leq 32).$$

The three functions  $f_{i1}(t)$ ,  $f_{i2}(t)$  and  $f_{i3}(t)$  should satisfy the conditions such that themselves, their first and second derivatives are continuous at each knot. Also their second derivatives are null at times 0 h and 32 h. These constrains can be written as:

$$\begin{aligned} f_{i1}(K_1) &= f_{i2}(K_1), & \frac{df_{i1}(K_1)}{dt} &= \frac{df_{i2}(K_1)}{dt}, & \frac{d^2 f_{i1}(K_1)}{dt^2} &= \frac{d^2 f_{i2}(K_1)}{dt^2}, & \frac{d^2 f_{i1}(0)}{dt^2} &= 0, \\ f_{i2}(K_2) &= f_{i3}(K_2), & \frac{df_{i2}(K_2)}{dt} &= \frac{df_{i3}(K_2)}{dt}, & \frac{d^2 f_{i2}(K_2)}{dt^2} &= \frac{d^2 f_{i3}(K_2)}{dt^2}, & \frac{d^2 f_{i3}(32)}{dt^2} &= 0. \end{aligned}$$