Frontiers of Information Technology & Electronic Engineering
www.jzus.zju.edu.cn; engineering.cae.cn; www.springerlink.com
ISSN 2095-9184 (print); ISSN 2095-9230 (online)
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Supplementary materials for

Bin HE, Hongtao SU, 2022. Supermodular interference suppression game for multistatic MIMO radar networks and multiple jammers with multiple targets. *Front Inform Technol Electron Eng*, 23(4):617-629.
https://doi.org/10.1631/FITEE.2000652

Proof S1 Proof of Eq. (23)

The first partial derivative of the utility function with respect to $p_{\mathrm{R}(kl)}$ is obtained, and now make it zero:

$$\frac{\partial U_{\mathrm{R}(klj)}(p_{\mathrm{R}(kl)},p_{\mathrm{J}(jk)})}{\partial p_{\mathrm{R}(kl)}}=\frac{\frac{\partial\gamma_{\mathrm{R}(kl)}}{\partial p_{\mathrm{R}(kl)}}}{\gamma_{\mathrm{R}(kl)}-\gamma_{\mathrm{R}(kl)}^{\min}}-\varepsilon_{kl}=\frac{\gamma_{\mathrm{R}(kl)}}{p_{\mathrm{R}(kl)}\left(\gamma_{\mathrm{R}(kl)}-\gamma_{\mathrm{R}(kl)}^{\min}\right)}-\varepsilon_{kl}=0, \tag{S1}$$

$$\Rightarrow p_{\mathrm{R}(kl)}(\gamma_{\mathrm{R}(kl)}-\gamma_{\mathrm{R}(kl)}^{\min})\varepsilon_{kl}=\gamma_{\mathrm{R}(kl)},$$
$$\Rightarrow \gamma_{\mathrm{R}(kl)}(\varepsilon_{kl}p_{\mathrm{R}(kl)}-1)=\varepsilon_{kl}\gamma_{\mathrm{R}(kl)}^{\min}p_{\mathrm{R}(kl)}.$$

Substituting Eqs. (15) and (16) into Eq. (S1), Eq. (23) is obtained.

Proof S2 Proof of Eq. (25)

The first partial derivative of the utility function with respect to $p_{\mathrm{J}(jk)}$ is obtained, and now make it zero:

$$\frac{\partial U_{\mathrm{R}(klj)}(p_{\mathrm{R}(kl)},p_{\mathrm{J}(jk)})}{\partial p_{\mathrm{J}(jk)}}=\frac{\frac{\partial\gamma_{\mathrm{R}(kl)}}{\partial p_{\mathrm{J}(jk)}}}{\gamma_{\mathrm{R}(kl)}-\gamma_{\mathrm{R}(kl)}^{\min}}+\omega_{jk}=\frac{-\frac{p_{\mathrm{R}(kl)}\left|\boldsymbol{w}_{r(kl)}^{\mathrm{H}}\hat{\boldsymbol{h}}_{t(kkl)}\right|^2\left|\boldsymbol{w}_{r(kl)}^{\mathrm{H}}\hat{\boldsymbol{c}}_{t(jk)}\right|^2}{I_{\mathrm{R}(kl)}^2}}{\gamma_{\mathrm{R}(kl)}-\gamma_{\mathrm{R}(kl)}^{\min}}+\omega_{jk}=0, \tag{S2}$$

$$\Rightarrow I_{\mathrm{R}(kl)}^2(\gamma_{\mathrm{R}(kl)}-\gamma_{\mathrm{R}(kl)}^{\min})=\frac{p_{\mathrm{R}(kl)}\left|\boldsymbol{w}_{r(kl)}^{\mathrm{H}}\hat{\boldsymbol{h}}_{t(kkl)}\right|^2\left|\boldsymbol{w}_{r(kl)}^{\mathrm{H}}\hat{\boldsymbol{c}}_{t(jk)}\right|^2}{\omega_{jk}}.$$

Substituting Eqs. (15) and (16) into Eq. (S2), Eq. (25) is obtained.

Proof S3 Proof of inequality (28)

The second-order mixed partial derivative of the utility function to the strategies of both sides of the radars and jammers is greater than or equal to zero:

$$\frac{\partial U_{\mathrm{R}(klj)}^2(p_{\mathrm{R}(kl)},p_{\mathrm{J}(jk)})}{\partial p_{\mathrm{J}(jk)}\partial p_{\mathrm{R}(kl)}}=\frac{\partial\left(\frac{\gamma_{\mathrm{R}(kl)}}{p_{\mathrm{R}(kl)}\left(\gamma_{\mathrm{R}(kl)}-\gamma_{\mathrm{R}(kl)}^{\min}\right)}-\varepsilon_{kl}\right)}{\partial p_{\mathrm{J}(jk)}}=-\frac{\frac{\partial\gamma_{\mathrm{R}(kl)}}{\partial p_{\mathrm{J}(jk)}}\gamma_{\mathrm{R}(kl)}^{\min}}{p_{\mathrm{R}(kl)}\left(\gamma_{\mathrm{R}(kl)}-\gamma_{\mathrm{R}(kl)}^{\min}\right)}$$
$$=\frac{\left|\boldsymbol{w}_{r(kl)}^{\mathrm{H}}\hat{\boldsymbol{h}}_{t(kkl)}\right|^2\left|\boldsymbol{w}_{r(kl)}^{\mathrm{H}}\hat{\boldsymbol{c}}_{t(jk)}\right|^2\gamma_{\mathrm{R}(kl)}^{\min}}{I_{\mathrm{R}(kl)}^2\left(\gamma_{\mathrm{R}(kl)}-\gamma_{\mathrm{R}(kl)}^{\min}\right)}\geq 0. \tag{S3}$$

Proof S4 Proof of Eq. (29)

BR function (29) of the radar has three properties of the standard function for all $\boldsymbol{p}_{\mathrm{R}} > 0$ and $\boldsymbol{p}_{\mathrm{J}} > 0$:
(1) Positivity. For any $p_{\mathrm{R}(kl)} > 0$ and $p_{\mathrm{J}(jk)} > 0$, we have $\mathrm{BR}_{\mathrm{R}}\left(p_{\mathrm{J}(jk)}\right) > 0$.
(2) Monotonicity. If $p_{\mathrm{J}(jk)} \leq p'_{\mathrm{J}(jk)}$, then

$$\mathrm{BR}_{\mathrm{R}}\left(p_{\mathrm{J}(jk)}\right) - \mathrm{BR}_{\mathrm{R}}\left(p'_{\mathrm{J}(jk)}\right) = \frac{\gamma_{\mathrm{R}(kl)}^{\min}\left|\boldsymbol{w}_{r(kl)}^{\mathrm{H}}\hat{\boldsymbol{c}}_{t(jk)}\right|^2}{\left|\boldsymbol{w}_{r(kl)}^{\mathrm{H}}\hat{\boldsymbol{h}}_{t(kkl)}\right|^2}\left(p_{\mathrm{J}(jk)} - p'_{\mathrm{J}(jk)}\right) \leq 0. \tag{S4}$$

(3) Scalability. For all $a > 1$, we have

$$a\mathrm{BR}_{\mathrm{R}}\left(p_{\mathrm{J}(jk)}\right) - \mathrm{BR}_{\mathrm{R}}\left(ap_{\mathrm{J}(jk)}\right) = (a-1)\frac{\gamma_{\mathrm{R}(kl)}^{\min}\hat{\eta}_{\mathrm{R}(kl)}}{\left|\boldsymbol{w}_{r(kl)}^{\mathrm{H}}\hat{\boldsymbol{h}}_{t(kkl)}\right|^2} > 0. \tag{S5}$$