Supplemental material

Theorem 5. *For any* $p(y|\boldsymbol{x}) \in [\epsilon, 1]$ *and* $T \geq 1$*, the* L_{ICCE} *loss is bounded by:*

$$0 \leq L_{ICCE}(f(\boldsymbol{x}), y) \leq B \tag{12}$$

where $B = (1-\epsilon)\max(\frac{e^{T(\epsilon-0.5)}}{\epsilon}, e^{0.5T})$ *(The proof see Appendix).*

Proof. Observe that, since the $\frac{e^{T(p(y|\boldsymbol{x})-0.5)}}{p(y|\boldsymbol{x})} > 0$ for all $p(y|\boldsymbol{x}) \in [\epsilon, 1]$, $L_{ICCE} \geq 0$ for all $p(y|\boldsymbol{x}) \in [\epsilon, 1]$. Moreover, since

$$\max_{p(y|\boldsymbol{x})\in[\epsilon,1]} \frac{e^{T(p(y|\boldsymbol{x})-0.5)}}{p(y|\boldsymbol{x})} = \max(\frac{e^{T(\epsilon-0.5)}}{\epsilon}, e^{0.5T})$$

hence, we have

$$\begin{aligned} L_{ICCE}(f(\boldsymbol{x}), y) &= \int_{p(y|\boldsymbol{x})}^{1} \frac{e^{T(p(y|\boldsymbol{x})-0.5)}}{p(y|\boldsymbol{x})} dp(y|\boldsymbol{x}) \\ &\leq \int_{p(y|\boldsymbol{x})}^{1} \max(\frac{e^{T(\epsilon-0.5)}}{\epsilon}, e^{0.5T}) \\ &= (1-x)\max(\frac{e^{T(\epsilon-0.5)}}{\epsilon}, e^{0.5T}) \\ &\leq (1-\epsilon)\max(\frac{e^{T(\epsilon-0.5)}}{\epsilon}, e^{0.5T}) \end{aligned}$$

Based on the Theorem 5, we will show the L_{ICCE} loss with respect to all classes is bounded under condition $p(y|\boldsymbol{x}) \in [\epsilon, 1]$ and $T \geq 1$.