Guided Image Filtering

Supplementary Technical Details

1 Derivation of the Filter Kernel

By definition the guided filter output q is computed through the following steps:

$$a_k = \frac{\frac{1}{|\omega|}\sum_{i\in\omega_k} I_i p_i - \mu_k \bar{p}_k}{\sigma_k^2 + \epsilon} \tag{1}$$

$$b_k = \bar{p}_k - a_k \mu_k \tag{2}$$

$$q_i = (\frac{1}{|\omega|}\sum_{k\in\omega_i} a_k) I_i + (\frac{1}{|\omega|}\sum_{k\in\omega_i} b_k). \tag{3}$$

As we indicate in the paper, the above equations can be written in the forms of filtering: $a_k = \sum_j A_{kj}(I)p_j$, $b_k = \sum_j B_{kj}(I)p_j$, and $q_i = \sum_i W_{ij}(I)p_j$. So the filter kernel can be computed by taking the partial derivation: $W_{ij} = \frac{\partial q_i}{\partial p_j}$.

Putting (2) into (3) and eliminating b, we obtain:

$$q_i = \frac{1}{|\omega|}\sum_{k\in\omega_i}(a_k(I_i - \mu_k) + \bar{p}_k). \tag{4}$$

Taking the derivative w.r.t. p_j, we have:

$$\frac{\partial q_i}{\partial p_j} = \frac{1}{|\omega|}\sum_{k\in\omega_i}(\frac{\partial a_k}{\partial p_j}(I_i - \mu_k) + \frac{\partial \bar{p}_k}{\partial p_j}) \tag{5}$$

In this equation, we have:

$$\frac{\partial \bar{p}_k}{\partial p_j} = \frac{1}{|\omega|}\delta_{j\in\omega_k} = \frac{1}{|\omega|}\delta_{k\in\omega_j}, \tag{6}$$

where $\delta_{j\in\omega_k}$ is one when j is in the window ω_k, and is zero otherwise. On the other hand, the partial derivative $\partial a_k/\partial p_j$ in (5) can be computed from (1):

$$\begin{aligned}\frac{\partial a_k}{\partial p_j} &= \frac{1}{\sigma_k^2+\epsilon}(\frac{1}{|\omega|}\sum_{i\in\omega_k}\frac{\partial p_i}{\partial p_j} I_i - \frac{\partial \bar{p}_k}{\partial p_j}\mu_k) \\ &= \frac{1}{\sigma_k^2+\epsilon}(\frac{1}{|\omega|} I_j - \frac{1}{|\omega|}\mu_k)\delta_{k\in\omega_j} \end{aligned} \tag{7}$$

Putting (6) and (7) into (5), we obtain:

$$\frac{\partial q_i}{\partial p_j} = \frac{1}{|\omega|^2}\sum_{k\in\omega_i, k\in\omega_j}(1 + \frac{(I_i-\mu_k)(I_j-\mu_k)}{\sigma_k^2+\epsilon}) \tag{8}$$

This is the expression of the filter kernel W_{ij}.

2 Approximate Solution to the Matting Laplacian Matrix

In the paper we have shown that: $\mathrm{L}_{ij} = |\omega|(\delta_{ij} - W_{ij})$. The matrix form of this equation is:

$$\mathrm{L} = |\omega|(\mathrm{U} - \mathrm{W}), \tag{9}$$

where U is a unit matrix of the same size as L. In [1], the alpha matte α is obtained by solving the following linear system:

$$(\mathrm{L} + \Lambda)\alpha = \Lambda\beta \tag{10}$$

This linear system can be solved by iterative methods. Now we approximate the solution by one step of the Jacobi method. A similar strategy is used in [2] to derive the bilateral filter from the viewpoint of optimization. We decompose W into a diagonal part W_d and an off-diagonal part W_o: $\mathrm{W} = \mathrm{W}_d + \mathrm{W}_o$. According to (9) and (10), we have:

$$(|\omega|\mathrm{U} - |\omega|\mathrm{W}_d - |\omega|\mathrm{W}_o + \Lambda)\alpha = \Lambda\beta. \tag{11}$$

Notice that only W_o is off-diagonal here. Using β as the initial guess of α, we compute one step of Jacobi iteration:

$$\alpha \approx (|\omega|\mathrm{U} - |\omega|\mathrm{W}_d + \Lambda)^{-1}(|\omega|\mathrm{W}_o + \Lambda)\beta \tag{12}$$

$$= (\mathrm{U} - \mathrm{W}_d + \frac{\Lambda}{|\omega|})^{-1}(\mathrm{W} - \mathrm{W}_d + \frac{\Lambda}{|\omega|})\beta \tag{13}$$

In this equation, we only need to perform the matrix multiplication $\mathrm{W}\beta$. The other matrices are all diagonal and point-wise operations. The matrix multiplication $\mathrm{W}\beta$ is indeed a guided filtering process. Because we only apply one step of Jacobi iteration to obtain the approximate solution, we require the initial guess β to be reasonably good. For example, it may be the binary segmentation mask of the object.

To further simplify this equation, we let the matrix Λ satisfy:$\Lambda = |\omega|\mathrm{W}_d$, or equivalently:

$$\Lambda_{ii} = \frac{1}{|\omega|}\sum_{k\in\omega_i}(1 + \frac{(I_i - \mu_k)^2}{\sigma_k^2 + \epsilon}) \tag{14}$$

Eqn.(13) is then reduced to:

$$\alpha \approx \mathrm{W}\beta \tag{15}$$

The diagonal matrices are eliminated and only the guided filter is remained. The expectation value of $\mathit{\Lambda}_{ii}$ in (14) is 2. This implies that the alpha matte is loosely constrained by β. In our matting application where β is a binary mask, the loose constraint is required because it allows the filter to change the values of the mask near the object boundaries. The loose constraint is also required in haze removal as in [3]. Therefore, the simplified equation (15) is applicable in both applications.

References

1. Levin, A., Lischinski, D., Weiss, Y.: A closed form solution to natural image matting. CVPR (2006)
2. Elad, M.: On the origin of the bilateral filter and ways to improve it. IEEE Transactions on Image Processing (2002)
3. He, K., Sun, J., Tang, X.: Single image haze removal using dark channel prior. CVPR (2009)