Multi-class Classification on Riemannian Manifolds for Surveillance Scenarios: Supplementary Material

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Abstract. We propose the full demonstration of some important facts related to the geometry of Sym_d^+ and then we present some additional qualitative examples of head pose detection and classification in different crowded low resolution scenarios.

1 The sectional curvature of $\mathbf{Sym_d^+}$ is non-positive

In this section we recall some concepts introduced in Section 3.1 of the paper and provide a detailed proof of the fact that the sectional curvature $\kappa_{I_d}(x,y) \leq 0$, where κ_{I_d} is defined in Equation (7) of the paper:

$$\begin{aligned}\kappa_{I_d}(x,y) = \frac{\langle R(x,y)x,y\rangle}{\| x \|^2 \| y \|^2 - \langle x,y\rangle^2} = \frac{Tr([[x,y],x]y)}{Tr(x^2)Tr(y^2)-(Tr(xy))^2} = \\ = 2\,\frac{Tr((xy)^2 - x^2y^2)}{Tr(x^2)Tr(y^2)-(Tr(xy))^2}. \qquad (1)\end{aligned}$$

We wish to show that

$$Tr((xy)^2) \leq Tr(x^2y^2) \qquad (2)$$

and equality holds if and only if $[x,y]=0$. This will follow from the following immediate consequence of the Schwarz inequality for (real) inner products:

$$\langle z,k\rangle \leq \| z \|^2, \text{ if } \| z \| = \| k \| \qquad (3)$$

(equality holding if and only if $k=z$). Indeed, upon setting $z=xy$, $k=yx$, $\langle z,k\rangle = Tr(z^Tk) = Tr(k^Tz)$, and using $x^T=x$, $y^T=y$, we easily attain the conclusion, since on the one hand we have:

$$\begin{aligned}Tr((xy)^2) = Tr(xyxy) = Tr(x^Ty^Txy) = \\ = Tr((yx)^Txy) = Tr(k^Tz) = \langle z,k\rangle \qquad (4)\end{aligned}$$

and, on the other hand:

$$\begin{aligned}Tr(x^2y^2) = Tr(xxyy) = Tr(yxxy) = Tr(y^Tx^Txy) = \\ = Tr((xy)^Txy) = Tr(z^2) = \| z \|^2 \qquad (5)\end{aligned}$$

and one has of course

$$\| k \|^2 = Tr((yx)^T(yx)) = Tr(x^T y^T yx) = Tr(xyyx) = Tr(x^2 y^2) = \| z \|^2 \qquad (6)$$

By Preissmann's theorem [1], any two points of a complete simply connected manifold with non-positive sectional curvature are connected by precisely one geodesic, which is minimizing. Given a geodesic triangle with sides of length a, b, c, and angle θ opposite to (the side with length) c, the following inequality holds

$$a^2 + b^2 - 2ab \cos\theta \leq c^2 \qquad (7)$$

which, when applied to the geodesic triangle with vertices I_d, X_1, X_2 (θ being the angle between $x_1 = \log X_1$ and $x_2 = \log X_2$, is tantamount to the assertion

$$d_{\mathcal{E}}(x_1, x_2) \leq d(x_1, x_2) \qquad (8)$$

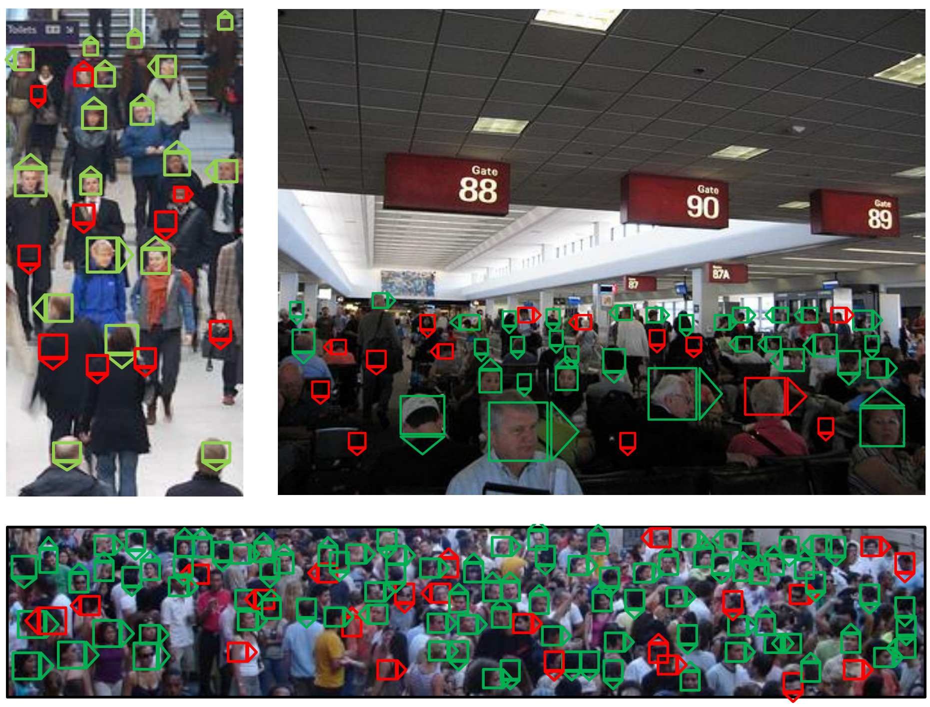
where

$$d_{\mathcal{E}}(x_1, x_2) = \sqrt{Tr[(x_1 - x_2)^2]} \qquad (9)$$

is the Euclidean distance between x_1 and x_2. If the sectional curvature is "small" – where with small we mean around 10^{-3} – one can replace the "true" distance by the Euclidean one. Also notice that if the two matrices are diagonal, the two distances coincide (the sectional curvature vanishes, so one actually works on a plane).

2 Qualitative Examples of Head Detection and Classification

In the following images, we use the classifier plus detector of the second experiment of Section 4.3 to detect heads in the image and determine the head orientation. We use detection windows that scale from [10×10] to [80×80] pixels.



References

1. Chavel, I.: Riemannian Geometry - A modern introduction. Cambridge University Press, Cambridge (2006)