Appendix to "Modified Sonine approximation for granular binary mixtures."

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Journal of Fluid Mechanics, vol. 623 (2009), pp. 387–411

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Chapman-Enskog theory for a granular binary mixture of hard spheres

We write now the set of coupled linear integral equations from which we solve the unknowns $\boldsymbol{\mathcal{A}}_i(\boldsymbol{V})$, $\mathcal{B}_i(\boldsymbol{V})$, $\boldsymbol{\mathcal{C}}_i(\boldsymbol{V})$, and $\boldsymbol{\mathcal{D}}_i(\boldsymbol{V})$ Garzó & Dufty (2002):

$$\left[-\zeta^{(0)}\left(T\partial_T+p\partial_p\right)+\mathcal{L}_1\right]\boldsymbol{\mathcal{A}}_1+\mathcal{M}_1\boldsymbol{\mathcal{A}}_2=\boldsymbol{A}_1+\left(\frac{\partial\zeta^{(0)}}{\partial x_1}\right)_{p,T}\left(p\boldsymbol{\mathcal{B}}_1+T\boldsymbol{\mathcal{C}}_1\right), \tag{1a}$$

$$\left[-\zeta^{(0)}\left(T\partial_T+p\partial_p\right)+\mathcal{L}_2\right]\boldsymbol{\mathcal{A}}_2+\mathcal{M}_2\boldsymbol{\mathcal{A}}_1=\boldsymbol{A}_2+\left(\frac{\partial\zeta^{(0)}}{\partial x_1}\right)_{p,T}\left(p\boldsymbol{\mathcal{B}}_2+T\boldsymbol{\mathcal{C}}_2\right), \tag{1b}$$

$$\left[-\zeta^{(0)}\left(T\partial_T+p\partial_p\right)+\mathcal{L}_1-2\zeta^{(0)}\right]\boldsymbol{\mathcal{B}}_1+\mathcal{M}_1\boldsymbol{\mathcal{B}}_2=\boldsymbol{B}_1+\frac{T\zeta^{(0)}}{p}\boldsymbol{\mathcal{C}}_1, \tag{2a}$$

$$\left[-\zeta^{(0)}\left(T\partial_T+p\partial_p\right)+\mathcal{L}_2-2\zeta^{(0)}\right]\boldsymbol{\mathcal{B}}_2+\mathcal{M}_2\boldsymbol{\mathcal{B}}_1=\boldsymbol{B}_2+\frac{T\zeta^{(0)}}{p}\boldsymbol{\mathcal{C}}_2, \tag{2b}$$

$$\left[-\zeta^{(0)}\left(T\partial_T+p\partial_p\right)+\mathcal{L}_1-\frac{1}{2}\zeta^{(0)}\right]\boldsymbol{\mathcal{C}}_1+\mathcal{M}_1\boldsymbol{\mathcal{C}}_2=\boldsymbol{C}_1-\frac{p\zeta^{(0)}}{2T}\boldsymbol{\mathcal{B}}_1, \tag{3a}$$

$$\left[-\zeta^{(0)}\left(T\partial_T+p\partial_p\right)+\mathcal{L}_2-\frac{1}{2}\zeta^{(0)}\right]\boldsymbol{\mathcal{C}}_2+\mathcal{M}_2\boldsymbol{\mathcal{C}}_1=\boldsymbol{C}_2-\frac{p\zeta^{(0)}}{2T}\boldsymbol{\mathcal{B}}_2, \tag{3b}$$

$$\left[-\zeta^{(0)}\left(T\partial_T+p\partial_p\right)+\mathcal{L}_1\right]\mathcal{D}_1+\mathcal{M}_1\mathcal{D}_2=\mathsf{D}_1, \tag{4a}$$

$$\left[-\zeta^{(0)}\left(T\partial_T+p\partial_p\right)+\mathcal{L}_2\right]\mathcal{D}_2+\mathcal{M}_2\mathcal{D}_1=\mathsf{D}_2. \tag{4b}$$

In the above equations, $\zeta^{(0)}$ is the cooling rate of the HCS and the inhomogeneous terms $\boldsymbol{A}_i$, $\boldsymbol{B}_i$, $\boldsymbol{C}_i$, and D_i are given by

$$\boldsymbol{A}_i(\boldsymbol{V})=-\left(\frac{\partial}{\partial x_1}f_i^{(0)}\right)_{p,T}\boldsymbol{V}, \tag{5}$$

$$\boldsymbol{B}_i(\boldsymbol{V})=-\frac{1}{p}\left[f_i^{(0)}\boldsymbol{V}+\frac{nT}{\rho}\left(\frac{\partial}{\partial\boldsymbol{V}}f_i^{(0)}\right)\right], \tag{6}$$

$$\boldsymbol{C}_i(\boldsymbol{V})=\frac{1}{T}\left[f_i^{(0)}+\frac{1}{2}\frac{\partial}{\partial\boldsymbol{V}}\cdot\left(\boldsymbol{V}f_i^{(0)}\right)\right]\boldsymbol{V}, \tag{7}$$

$$\mathsf{D}_i(\boldsymbol{V})=\boldsymbol{V}\frac{\partial}{\partial\boldsymbol{V}}f_i^{(0)}-\frac{1}{d}\mathsf{I}\boldsymbol{V}\cdot\frac{\partial}{\partial\boldsymbol{V}}f_i^{(0)}, \tag{8}$$

where I denotes the unit tensor in d dimensions. In addition, we have introduced the linearized Boltzmann collision operators

$$\mathcal{L}_1 X = -\left(J_{11}[f_1^{(0)}, X] + J_{11}[X, f_1^{(0)}] + J_{12}[X, f_2^{(0)}] \right) \ , \tag{9}$$

$$\mathcal{M}_1 X = -J_{12}[f_1^{(0)}, X]. \tag{10}$$

The corresponding expressions for the operators $\mathcal{L}_2$ and $\mathcal{M}_2$ can be easily obtained from (9) and (10) by just making the changes $1 \leftrightarrow 2$.

Also, the procedure to get the leading order contributions to the NS transport coefficients in the *modified* first Sonine approximation follows similar mathematical steps as the ones previously used in the *standard* first Sonine approximation. Only some technical details will be provided here.

Our *modified* Sonine approximation consists in taking $f_i^{(0)}$ as the weight function in the Sonine expansion used in the functions $\boldsymbol{\mathcal{A}}_i$, $\boldsymbol{\mathcal{B}}_i$, $\boldsymbol{\mathcal{C}}_i$, $\mathcal{D}_{i,k\ell}(\boldsymbol{V})$, instead of the simpler Maxwellian form $f_{i,M}$. Thus, in the case of the mass flux, the quantities $\boldsymbol{\mathcal{A}}_i$, $\boldsymbol{\mathcal{B}}_i$, $\boldsymbol{\mathcal{C}}_i$ are approximated by the lowest degree polynomials

$$\boldsymbol{\mathcal{A}}_1(\boldsymbol{V}) \rightarrow -f_1^{(0)} \boldsymbol{V} \frac{m_1 m_2 n}{\rho n_1 T_1} D, \quad \boldsymbol{\mathcal{A}}_2(\boldsymbol{V}) \rightarrow f_2^{(0)} \boldsymbol{V} \frac{m_1 m_2 n}{\rho n_2 T_2} D \tag{11}$$

$$\boldsymbol{\mathcal{B}}_1(\boldsymbol{V}) \rightarrow -f_1^{(0)} \boldsymbol{V} \frac{\rho}{p n_1 T_1} D_p, \quad \boldsymbol{\mathcal{B}}_2(\boldsymbol{V}) \rightarrow f_2^{(0)} \boldsymbol{V} \frac{\rho}{p n_2 T_2} D_p \tag{12}$$

$$\boldsymbol{\mathcal{C}}_1(\boldsymbol{V}) \rightarrow -f_1^{(0)} \boldsymbol{V} \frac{\rho}{T n_1 T_1} D', \quad \boldsymbol{\mathcal{C}}_2(\boldsymbol{V}) \rightarrow f_2^{(0)} \boldsymbol{V} \frac{\rho}{T n_2 T_2} D'. \tag{13}$$

Note that equations (11)–(13) are consistent with the orthogonality conditions (3.5)–(3.7). The expressions (3.11)–(3.13) for D, D_p, and D' can be easily obtained when one multiplies the integral equations (1)–(3) by $m_1 \boldsymbol{V}$ and integrates over $\boldsymbol{V}$. In order to obtain γ_1 and the partial derivatives appearing in these integral equations we use the first order Sonine approximations of the partial cooling rates (A2)–(A4) in the condition $\zeta_1^{(0)} = \zeta_2^{(0)}$ (A12). The expression of the collisional frequency ν_D appearing in (3.11)–(3.13) is given by

$$\nu_D = \frac{1}{d n_1 T_1 \nu_0} \int d\boldsymbol{V}_1 m_1 \boldsymbol{V}_1 \cdot \left[\mathcal{L}_1 (f_1^{(0)} \boldsymbol{V}_1) - \delta \gamma \mathcal{M}_1 (f_2^{(0)} \boldsymbol{V}_2) \right] , \tag{14}$$

where $\delta = n_1/n_2$ and $\gamma = T_1/T_2$. The evaluation of the collision integral (14) is made in the next section and the result is given by (B1). Using all these results together in (3.11)–(3.13), we can obtain the explicit dependence of D, D_p, and D' on the parameters of the mixture.

In the case of the shear viscosity, the simplest approximation for the function $\mathcal{D}_{i,k\ell}$ is

$$\mathcal{D}_{i,k\ell}(\boldsymbol{V}) \rightarrow -f_i^{(0)} \frac{\eta_i}{T} R_{i,k\ell}(\boldsymbol{V}), \quad (i = 1, 2) \tag{15}$$

where

$$R_{i,k\ell}(\boldsymbol{V}) = m_i \left(V_k V_\ell - \frac{1}{d} V^2 \delta_{k\ell} \right) , \tag{16}$$

and

$$\eta_i = -\frac{1}{(d-1)(d+2)} \frac{T}{n_i T_i^2} \frac{1}{1 + \frac{c_i}{2}} \int d\boldsymbol{v} R_{i,k\ell}(\boldsymbol{V}) \mathcal{D}_{i,k\ell}(\boldsymbol{V}). \tag{17}$$

The choice (16) preserves the solubility conditions (3.5)–(3.7). The shear viscosity coefficient is given by

$$\eta=\sum_{i=1}^{2}\frac{n_iT_i^2}{T}\left(1+\frac{c_i}{2}\right)\eta_i. \tag{18}$$

Analogously to the case of the transport coefficients associated with the mass flux, the coefficients η_i are determined from the integral equations (4) when one takes into account the modified first Sonine approximation (15) for $\mathcal{D}_{i,k\ell}$. After some calculations, one gets the expressions (3.15a) and (3.15b) for $\eta_1^*=(1+\frac{c_1}{2})\eta_1$ and $\eta_2^*=(1+\frac{c_2}{2})\eta_2$, respectively, where

$$\tau_{11}=\frac{1}{(d-1)(d+2)}\frac{1}{1+\frac{c_1}{2}}\frac{1}{n_1T_1^2\nu_0}\int d\boldsymbol{v}_1R_{1,k\ell}\mathcal{L}_1\left(f_1^{(0)}R_{1,k\ell}\right), \tag{19}$$

$$\tau_{12}=\frac{1}{(d-1)(d+2)}\frac{1}{1+\frac{c_2}{2}}\frac{1}{n_1T_1^2\nu_0}\int d\boldsymbol{v}_1R_{1,k\ell}\mathcal{M}_1\left(f_2^{(0)}R_{2,k\ell}\right), \tag{20}$$

The integrals (19), (20) are calculated analogously to the integral (14), that is explained in the next section.

The case of the heat flux is more involved since it requires going up to the second Sonine polynomial approximation. In this case, the quantities $\boldsymbol{\mathcal{A}}_i$, $\boldsymbol{\mathcal{B}}_i$, $\boldsymbol{\mathcal{C}}_i$ are taken to be

$$\boldsymbol{\mathcal{A}}_1(\boldsymbol{V})\to f_1^{(0)}\left[-\frac{m_1m_2n}{\rho n_1T_1}D\boldsymbol{V}+d_1''\overline{\boldsymbol{S}}_1(\boldsymbol{V})\right],\quad \boldsymbol{\mathcal{A}}_2(\boldsymbol{V})\to f_2^{(0)}\left[\frac{m_1m_2n}{\rho n_2T_2}D\boldsymbol{V}+d_2''\overline{\boldsymbol{S}}_2(\boldsymbol{V})\right] \tag{21}$$

$$\boldsymbol{\mathcal{B}}_1(\boldsymbol{V})\to f_1^{(0)}\left[-\frac{\rho}{pn_1T_1}D_p\boldsymbol{V}+\ell_1\overline{\boldsymbol{S}}_1(\boldsymbol{V})\right],\quad \boldsymbol{\mathcal{B}}_2(\boldsymbol{V})\to f_2^{(0)}\left[\frac{\rho}{pn_2T_2}D_p\boldsymbol{V}+\ell_2\overline{\boldsymbol{S}}_2(\boldsymbol{V})\right] \tag{22}$$

$$\boldsymbol{\mathcal{C}}_1(\boldsymbol{V})\to f_1^{(0)}\left[-\frac{\rho}{Tn_1T_1}D'\boldsymbol{V}+\lambda_1\overline{\boldsymbol{S}}_1(\boldsymbol{V})\right],\quad \boldsymbol{\mathcal{C}}_2(\boldsymbol{V})\to f_2^{(0)}\left[\frac{\rho}{Tn_2T_2}D'\boldsymbol{V}+\lambda_2\overline{\boldsymbol{S}}_2(\boldsymbol{V})\right]. \tag{23}$$

In these equations, it is understood that D, D_p and D' are given by (3.11), (3.12), and (3.13), respectively. The (modified) Sonine polynomial $\overline{\boldsymbol{S}}_i(\boldsymbol{V})$ has the same polynomial structure as the standard one $\boldsymbol{S}_i(\boldsymbol{V})$, but is chosen to verify the conditions (3.5)–(3.7). A simple calculation yields

$$\overline{\boldsymbol{S}}_i(\boldsymbol{V})=\boldsymbol{S}_i(\boldsymbol{V})-\frac{d+2}{4}c_iT_i\boldsymbol{V}, \tag{24}$$

where

$$\boldsymbol{S}_i(\boldsymbol{V})=\left(\frac{1}{2}m_iV^2-\frac{d+2}{2}T_i\right)\boldsymbol{V}. \tag{25}$$

The coefficients d_i'', ℓ_i and λ_i are defined as

$$\begin{pmatrix} d_i'' \\ \ell_i \\ \lambda_i \end{pmatrix}=\frac{2}{d(d+2)}\frac{m_i}{n_iT_i^3}\frac{1}{1+\frac{d+8}{4}c_i}\int d\boldsymbol{v}\,\overline{\boldsymbol{S}}_i(\boldsymbol{V})\cdot\begin{pmatrix} \boldsymbol{\mathcal{A}}_i \\ \boldsymbol{\mathcal{B}}_i \\ \boldsymbol{\mathcal{C}}_i \end{pmatrix}, \tag{26}$$

where nonlinear terms in c_i and the sixth cumulants of $f_i^{(0)}$ have been neglected in these relations. Let us introduce the dimensionless coefficients d_i^*, ℓ_i^*, and λ_i^*:

$$d_i^*\equiv\left(1+\frac{d+8}{4}c_i\right)T\nu_0d_i'',\quad \ell_i^*\equiv\left(1+\frac{d+8}{4}c_i\right)pT\nu_0\ell_i,\quad \lambda_i^*\equiv\left(1+\frac{d+8}{4}c_i\right)T^2\nu_0\lambda_i. \tag{27}$$

The coupled set of six equations verifying the (reduced) coefficients $\{d_1^*, d_2^*, \ell_1^*, \ell_2^*, \lambda_1^*, \lambda_2^*\}$ can be obtained by taking the modified Sonine approximation (21)–(23) in the integral equations (1)–(3), multiplying these equations by $\overline{\boldsymbol{S}}_i$ and integrating over velocity. By using matrix notation, the coupled set of six equations for the above six quantities can be written as

$$\Lambda_{\sigma\sigma'} X_{\sigma'} = Y_\sigma, \tag{28}$$

where $X_{\sigma'}$ is the column matrix defined by the set $\{d_1^*, d_2^*, \ell_1^*, \ell_2^*, \lambda_1^*, \lambda_2^*\}$ and $\Lambda_{\sigma\sigma'}$ is the square matrix

$$\Lambda = \begin{pmatrix} \nu_{11} - \frac{3}{2}\zeta^* & \nu_{12} & -\left(\frac{\partial \zeta^*}{\partial x_1}\right)_{p,T} & 0 & -\left(\frac{\partial \zeta^*}{\partial x_1}\right)_{p,T} & 0 \\ \nu_{21} & \nu_{22} - \frac{3}{2}\zeta^* & 0 & -\left(\frac{\partial \zeta^*}{\partial x_1}\right)_{p,T} & 0 & -\left(\frac{\partial \zeta^*}{\partial x_1}\right)_{p,T} \\ 0 & 0 & \nu_{11} - \frac{5}{2}\zeta^* & \nu_{12} & -\zeta^* & 0 \\ 0 & 0 & \nu_{21} & \nu_{22} - \frac{5}{2}\zeta^* & 0 & -\zeta^* \\ 0 & 0 & \zeta^*/2 & 0 & \nu_{11} - \zeta^* & \nu_{12} \\ 0 & 0 & 0 & \zeta^*/2 & \nu_{21} & \nu_{22} - \zeta^* \end{pmatrix}, \tag{29}$$

and the column matrix $\boldsymbol{Y}$ is given by (B7)–(B12). The value of ω_{12} is given by

$$\omega_{12} = \frac{2}{d(d+2)} \frac{m_1}{n_1 T_1^2 \nu_0} \left[\int d\boldsymbol{v}_1 \overline{\boldsymbol{S}}_1 \cdot \mathcal{L}_1(f_1^{(0)} \boldsymbol{V}_1) - \delta\gamma \int d\boldsymbol{v}_1 \overline{\boldsymbol{S}}_1 \cdot \mathcal{M}_1(f_2^{(0)} \boldsymbol{V}_2) \right]. \tag{30}$$

The corresponding expression for ω_{21} can be deduced from (30) by interchanging $1 \leftrightarrow 2$. The solution to (28) is

$$X_\sigma = \left(\Lambda^{-1}\right)_{\sigma\sigma'} Y_{\sigma'}. \tag{31}$$

From this relation one gets the expressions (3.19), (3.20), and (3.21) for the coefficients d_i^*, ℓ_i^* and λ_i^*, respectively. In these expressions, the (reduced) collision frequencies ν_{ij} are given by the integrals

$$\nu_{11} = \frac{2}{d(d+2)} \frac{1}{1 + \frac{d+8}{4} c_1} \frac{m_1}{n_1 T_1^3 \nu_0} \int d\boldsymbol{v}_1 \overline{\boldsymbol{S}}_1 \cdot \mathcal{L}_1 \left(f_1^{(0)} \overline{\boldsymbol{S}}_1 \right), \tag{32}$$

$$\nu_{12} = \frac{2}{d(d+2)} \frac{1}{1 + \frac{d+8}{4} c_2} \frac{m_1}{n_1 T_1^3 \nu_0} \int d\boldsymbol{v}_1 \overline{\boldsymbol{S}}_1 \cdot \mathcal{M}_1 \left(f_2^{(0)} \overline{\boldsymbol{S}}_2 \right), \tag{33}$$

whose calculation is analogous to that of ν_D, carried out in the next section.

Collisional integrals

The different collision integrals defining the collision frequencies appearing along the main text are evaluated in this Appendix by using the modified first Sonine approximations for the functions $\{\boldsymbol{\mathcal{A}}_i, \boldsymbol{\mathcal{B}}_i, \boldsymbol{\mathcal{C}}_i, \mathcal{D}_i\}$. To simplify all the integrals, we use the property

$$\int d\boldsymbol{v}_1 h(\boldsymbol{V}_1) J_{ij} \left[\boldsymbol{v}_1 | f_i, f_j\right] = \sigma_{ij}^{d-1} \int d\boldsymbol{v}_1 \int d\boldsymbol{v}_2 f_i(\boldsymbol{V}_1) f_j(\boldsymbol{V}_2) \times \int d\widehat{\boldsymbol{\sigma}}\, \Theta(\widehat{\boldsymbol{\sigma}} \cdot \boldsymbol{g}_{12})(\widehat{\boldsymbol{\sigma}} \cdot \boldsymbol{g}_{12}) \left[h(\boldsymbol{V}_1'') - h(\boldsymbol{V}_1) \right], \tag{1}$$

with

$$\boldsymbol{V}_1'' = \boldsymbol{V}_1 - \mu_{ji}(1 + \alpha_{ij})(\widehat{\boldsymbol{\sigma}} \cdot \boldsymbol{g}_{12})\widehat{\boldsymbol{\sigma}}. \tag{2}$$

This result applies for both $i = j$ and $i \neq j$.

Let us start with the collision frequency ν_D defined in (14). Using the property (1) and performing the angular integration in (14) gives

$$\nu_D = \frac{m_1}{dn_1T_1}B_3\sigma_{12}^{d-1}\mu_{21}(1+\alpha_{12})\int d\boldsymbol{V}_1\int d\boldsymbol{V}_2\, g_{12}\Big[f_1^{(0)}(\boldsymbol{V}_1)f_2^{(0)}(\boldsymbol{V}_2)(\boldsymbol{V}_1\cdot\boldsymbol{g}_{12}) \\ -\delta\gamma f_1^{(0)}(\boldsymbol{V}_1)f_2^{(0)}(\boldsymbol{V}_2)(\boldsymbol{V}_2\cdot\boldsymbol{g}_{12})\Big], \tag{3}$$

where $\delta = n_1/n_2$ and (Ernst & Brito 2002)

$$B_k \equiv \int d\widehat{\boldsymbol{\sigma}}\,\Theta(\widehat{\boldsymbol{\sigma}}\cdot\boldsymbol{g}_{12})\,(\widehat{\boldsymbol{\sigma}}\cdot\widehat{\boldsymbol{g}}_{12})^k = \pi^{(d-1)/2}\frac{\Gamma\left(\frac{k+1}{2}\right)}{\Gamma\left(\frac{k+d}{2}\right)}. \tag{4}$$

Next, we introduce the reduced velocities $\boldsymbol{V}_i^* = \boldsymbol{V}_i/v_0$ and use the first Sonine approximation for $f_i^{(0)}$, equation (3.1). The latter form is conveniently rewritten as

$$f_i^{(0)}(\boldsymbol{V}_1) = n_i\left(\frac{m_i}{2\pi T_i}\right)^{d/2}\left(1+\frac{c_i}{4}\Delta_i\right)e^{-\theta_i V^{*2}}, \tag{5}$$

with

$$\Delta_i \equiv \theta_i^2\frac{\partial^2}{\partial\theta_i^2} + (d+2)\theta_i\frac{\partial}{\partial\theta_i} + \frac{d(d+2)}{4}. \tag{6}$$

Using (5) one gets

$$\nu_D = \frac{m_1}{dn_1T_1}B_3\sigma_{12}^{d-1}\mu_{21}(1+\alpha_{12})n_1n_2(\theta_1\theta_2)^{d/2}v_0^3\Big[\left(1+\frac{c_1}{4}\Delta_1+\frac{c_2}{4}\Delta_2\right)I_D^{(1)}(\theta_1,\theta_2) \\ -\delta\gamma\left(1+\frac{c_1}{4}\Delta_1+\frac{c_2}{4}\Delta_2\right)I_D^{(2)}(\theta_1,\theta_2)\Big], \tag{7}$$

where the integrals $I_D^{(1)}(\theta_1,\theta_2)$ and $I_D^{(2)}(\theta_1,\theta_2)$ are given by

$$I_D^{(1)}(\theta_1,\theta_2) = \pi^{-d/2}\int d\boldsymbol{V}_1^*\int d\boldsymbol{V}_2^* e^{-(\theta_1V_1^{*2}+\theta_2V_2^{*2})}g_{12}^*(\boldsymbol{V}_1^*\cdot\boldsymbol{g}_{12}^*), \tag{8}$$

$$I_D^{(2)}(\theta_1,\theta_2) = \pi^{-d/2}\int d\boldsymbol{V}_1^*\int d\boldsymbol{V}_2^* e^{-(\theta_1V_1^{*2}+\theta_2V_2^{*2})}g_{12}^*(\boldsymbol{V}_2^*\cdot\boldsymbol{g}_{12}^*), \tag{9}$$

with $\boldsymbol{g}_{12}^* \equiv \boldsymbol{g}_{12}/v_0$. Note that in (7) we have neglected nonlinear terms in c_i, i.e., $(1+\frac{c_1}{4}\Delta_1)(1+\frac{c_2}{4}\Delta_2) \to 1+\frac{c_1}{4}\Delta_1+\frac{c_2}{4}\Delta_2$. As in our previous works on granular mixtures (Garzó & Dufty 2002; Garzó & Montanero 2007), the integral $I_D(\theta_1,\theta_2)$ can be performed by the change of variables $\{\boldsymbol{V}_1^*,\boldsymbol{V}_2^*\} \to \{\boldsymbol{g}_{12}^*,\boldsymbol{z}\}$, where $\boldsymbol{z} \equiv \theta_1\boldsymbol{V}_1^*+\theta_2\boldsymbol{V}_2^*$ and the Jacobian is $(\theta_1+\theta_2)^{-d}$. With this change, the integrals $I_D^{(1)}$ and $I_D^{(2)}$ can be easily computed and the result is

$$I_D^{(1)}(\theta_1,\theta_2) = \frac{\Gamma\left(\frac{d+3}{2}\right)}{\Gamma\left(\frac{d}{2}\right)}(\theta_1+\theta_2)^{1/2}(\theta_1\theta_2)^{-(d+3)/2}\theta_2, \quad I_D^{(2)}(\theta_1,\theta_2) = I_D^{(1)}(\theta_2,\theta_1). \tag{10}$$

Use of this result in (7) gives

$$\nu_D = \frac{2\pi^{(d-1)/2}}{d\Gamma\left(\frac{d}{2}\right)}(1+\alpha_{12})\left(\frac{\theta_1+\theta_2}{\theta_1\theta_2}\right)^{1/2}\Big\{x_2\mu_{21}\left[1+\frac{1}{16}\frac{\theta_2(3\theta_2+4\theta_1)c_1-\theta_1^2c_2}{(\theta_1+\theta_2)^2}\right] \\ + x_1\mu_{12}\left[1+\frac{1}{16}\frac{\theta_1(3\theta_1+4\theta_2)c_2-\theta_2^2c_1}{(\theta_1+\theta_2)^2}\right]\Big\}. \tag{11}$$

The remaining collision frequencies can be obtained by following similar steps as those made in the case of ν_D.

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