# CLOCK CHARACTERIZATION TUTORIAL

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## ABSTRACT

Managers are often required to make key program decisions based on the performance of some elements of a large system. This paper is intended to assist the manager in this important task in so far as it relates to the proper use of precise and accurate clocks. An intuitive approach will be used to show how a clock s stability is measured, why it is measured the way it is, and why it is described the way it is. An intuitive explanation of the meaning of time domain and frequency domain measures as well as why they are used will also be given.

Explanations of when an "Allan variance plot should be used and when it should not be used will also be given. The relationship of the rms time error of a clock to a $\sigma_y(\tau)$ diagram will also be given. The environmental sensitivities of a clock are often the most important effects determining its performance. Typical environmental parameters of concern and nominal sensitivity values for commonly used clocks will be reviewed.

## SYSTEMATIC AND RANDOM DEVIATIONS IN CLOCKS

This paper is tutorial in nature with a minimum of mathematics -- the goal being to characterize clock behavior. First, time deviations or frequency deviations in clocks may be categorized into two types: systematic deviations and random deviations. The systematic deviations come in a variety of forms. Typical examples are frequency sidebands, diurnal or annual variations in a clock s behavior, time offset, frequency offset and frequency drift. Figure 1 illustrates some of these.

If a clock has a frequency offset, the time deviations will appear as a ramp function. On the other hand, if a clock has a frequency drift, then the resulting time deviations will appear as a quadratic time function --the time deviations will be proportional to the square of the running time. There are many other systematic effects that are very important to consider in understanding a clock s characteristics and Figure 1 is a very simplistic picture or nominal model of most precision oscillators. The random fluctuations or deviations in precision oscillators can often be characterized by power law spectra. In other words, if the time residuals are examined, after removing the systematic effects, one or more of the power law spectra shown in Figure 2 are typically observed. The meaning of power law spectra is that if a Fourier analysis or spectral density analysis is proportional to $f^\beta$; $\beta$ designates the particular power law process ($\beta$ = 0, -1,-2,-3,-4 and $\omega = 2\pi f$). The first process shown in Figure 2 is called white noise phase modulation (PM)., The noise is typically observed in the short term fluctuations, for example, of an active hydrogen maser for sample times of from one second to about 100 seconds. This noise is also observed In quartz crystal oscillators for sample times in the vicinity of a millisecond and shorter. The flicker noise PM, $f^{-1}$, is the second line in Figure 2. This kind of noise is often found for sample times of one millisecond to one second in quartz crystal oscillators. The $f^{-2}$ or random walk PM indicated by the third line is what is observed for the time deviations of rubidium, cesium, or hydrogen frequency standards. If the first difference is taken of a series of discrete time readings from the third line, then the result is proportional to the frequency deviations, which will be an $f^0$ process or a white noise frequency modulation (FM) process. In other words the time and the frequency are related through a derivative or an integral depending upon which way one goes. The derivative of the time deviations yields the frequency deviations, and the integral of the frequency deviations yields the time deviations. So, random walk time deviations result from white noise FM. In general, the spectral density of the frequency fluctuations is $\omega^2$ times the spectral density of the time fluctuations. The fourth line in Figure 2 is an $f^{-3}$ process. If this were representative of the time fluctuations then the frequency would be an $f^{-1}$ or a flicker noise FM process. This process is typical of the output of a quartz crystal oscillator for sample times longer than one second or the output of rubidium or cesium standards in the long term (on the order of a few hours, few days, or few weeks depending upon which standard). We find that in very long term, most atomic clocks have an $f^{-4}$ type behavior for the time fluctuations -- making the frequency fluctuations an $f^{-2}$ process or random walk FM. These five power law processes are very typical and one or more of them are appropriate models for essentially every precision oscillator. Characterizing the kind of power-law process thus becomes an important part of characterizing the performance of a. clock (1). Once a clock has been characterized in terms of its systematic and its random characteristics then a time deviation model can be developed. A very simple and useful model that is commonly used is given by the following equation (2):

$$x(t) = x_o + y_o \cdot t + 1/2 D \cdot t^2 + \varepsilon(t) \qquad (1)$$

where x(t) is the time deviation of the clock at time t, xo is the synchronization error at t = o, and $y_o$ is the syntonization error at t = o, which produces a linear ramp in the time deviations. D is the frequency drift term, which is almost always an applicable model element in commercial standards. This $1/2Dt^2$ term in the time deviation due to the frequency drift yields a quadratic time deviation. Lastly, the $\varepsilon(t)$ term contains all of the random fluctuations. It is this term which is typically characterized by one or more of the various power law processes. Once a clock has been fully characterized, then it is possible to do optimum time prediction. Shown in Figure 3 are some examples. The even power law spectra have simple algorithms for prediction. In the case of white noise PM, the optimum predictor is the simple mean. In the case of random walk PM, it is the last value. In the case of random walk FM, an $f^{-4}$ process on the time, the optimum predictor is the last slope. In the case of flicker noise, the prediction algorithms are significantly more complicated but not intractable, and ARIMA techniques can be employed in order to develop optimum prediction algorithms (3).

THE CONCEPT OF AN ALLAN VARIANCE

Figure 4 illustrates a simulated random walk PM process. Suppose this process is the time difference between two clocks or the time of a clock with repsect to a perfect clock. Again this process is typical of the time deviations for rubidium, cesium, and passive hydrogen clocks. Choose a sample time $\tau$, as indicated, and note the three time deviation readings ($x_1$, $x_2$, and $x_3$) indiaated by the circles and spaced by the interval $\tau$. The frequency deviation $y_1$ is proportional to the slope between $x_1$ and $x_2$ ($y_1 = (x_2 - X_1)/T$). Similarly $y_2$ is proportional to the slope between $x_2$ and x3 ($y_2 = (x_3 - x_2)/\tau$). The difference in slope, Ay, is a measure of the frequency change from the first $\tau$ sample interval to the next adjacent $\tau$ sample interval. With a fixed value of $\tau$, imagine averaging through the entire data set for all possible readings of $x_1$, $x_2$, and x3 displaced by $\tau$ each yielding a $\Delta y$. The average squared value of Ay divided by 2 is called the "Allan variance . In theory, it is the average over all time. In practice, finite data sets yield rapidly converging estimates. The square root of the Allan variance is *denoted* $\sigma_y(\tau)$; $\sigma_y(\tau)$ is an efficient estimator of the power law spectra model for the data. How $\sigma_y(\tau)$ changes with $\tau$ indicates the exponent for the power law process. In fact, in the case of random walk FM, $\sigma_y(\tau)$ is statistically the most efficient estimator of this power law process. If power law processes are good models for a clocks random fluctuations, which they typically are, then the Allan variance analysis is faster and more accurate in estimating the process than the Fast Fourier Transform. Some virtues of the Allan variance are: It is theoretically and straightforwardly relatable to the power law spectral type ($\beta = -\mu - 3$, $-2 < \mu < 2$, where $\mu$ is the exponent of $\tau$). Once a data set is stored in a computer it is simple to compute $\sigma(\tau)$ as a function of $\tau$. The difference in frequency, $\Delta y$, is often closely related to the actual physical process of interest, e.g. frequency change after a radar return delayed by, $\tau$, effects of oscillator instabilities in a servo with loop time constant $\tau$, the change in frequency after a calibration over an interval $\tau$, etc. Some drawbacks of the Allan variance are: it is transparent to periodic deviations where the period is equal to the sample time $\tau$. It is ambiguous at $\mu = -2$, i.e. $\sigma_y(\tau) \sim \tau^{-1}$ may be either white noise PM or Flicker noise PM. Remembering from above the relationship between the spectral density of the frequency deviations and the spectral density of the time deviations, if $S_y(f) \sim f^{\beta}$ and $S_x(f) \sim f^{\alpha}$, then $\alpha = \beta + 2$ and $\alpha = -\mu - 1$ ($-2 < \mu < 2$), where $S_x(f)$ is the spectral density of the time deviations and $S_y(f)$ is the spectral density of the fractional frequency deviations. Figure 5 shows the noise type and the relationship between $\mu$ and $\alpha$. There are some ways around the ambiguity problem at $\mu = -2$. For noise processes where $\alpha > 1$ there is a bandwidth dependence (4). A software trick can be employed to effectively vary the bandwidth rather than doing it with the hardware. Rather than calculating $\sigma_y(\tau)$ from individual phase points the phase can be averaged over an interval $\tau$. Hence the $x_1$, $x_2$, and $x_3$ from Figure 4 become phase or time difference averages. As $\tau$ increases the effective measurement system bandwidth decreases. This technique removes the ambiguity problem. We have called this the modified Allan variance or modified $\sigma_y(\tau)$ analysis technique. Figure 6 shows the $\mu$, $\alpha$ mapping for the modified Allan variance. For white noise PM, $\mu$ is equal to -3, and for flicker noise PM, $\mu$ is equal to -2.

TIME PREDICTION ERROR OF A CLOCK

Another concept which has become useful is the computation of the time error of prediction. In the case of white noise FM and random walk noise FM, $\tau \sigma_y(\tau)$ is the optimum time error of prediction. For white *noise PM* the optimum value achievable is $\tau \sigma_y(\tau)/\sqrt{3}$, and for flicker noise FM it is $\tau \sigma_y(\tau)/\sqrt{\ln 2}$.

Applying some of the above concepts to state of the art frequency standards yields the frequency stability plot shown in Figure 7 (5), and the corresponding RMS time prediction error plot shown in Figure 8. The RMS time prediction error plot is based on reference (2) and is a function of the levels of noise in the clock and also the uncertainties associated with determining the systematic deviations due to a finite data length.

## ENVIRONMENTAL EFFORTS ON CLOCKS

From a management point of view, the characteristics of the various clocks should be related to the needs of a particular program. It is important to keep in mind that, in practice, systematic and environmental effects often are the predominant influence on the time and frequency deviations of a clock. The reliability of a clock is often a basic issue, and the manager should assure himself that adequate reliability has been documented. The manager also needs to ask the following questions in each application that he may have. What are the environmental conditions, the rapidity of the temperature changes, the magnetic field conditions, the shock and vibration conditions, and the humidity conditions? How do these conditions affect the clock s performance? All clocks are affected at some level by changes in the above environmental parameters plus some others as well. Some clocks are affected by barometric pressure. Vibration can be extremely important. In some clocks the servos will unlock, for example, if a 1 kHz vibration is present. (6) Some clocks are accoustically sensitive. What is the gravitational or g sensitivity? What are the cost, size, weight and power requirements? Line voltage power fluctuations can affect clocks. Changes in the dc power can affect some clocks. We have found that a good clock environment can improve clock performance considerably, and we have provided a highly controlled environment for the NBS clock ensemble to improve the performance over that obtained in typical laboratory environments. Another very important question to ask is what is a clock s lifetime? Redundancy and/or multiple clocks are sometimes necessary to overcome lifetime and reliability problems. It is important to take a system s approach in establishing the best clock(s) and clock(s) configuration. In some cases the program needs are for synchroniztion to UTC, in other cases the needs are for syntonization, i.e. the frequencies within a system need to agree. Often the need is for time or frequency self consistency within a program, e.g. GPS requires time consistency. It seems many people are buying cesium standards as a panacea, when in fact they may not be solving the problem at hand. Buying a cesium standard does not guarantee synchronization. However, it does guarantee syntonization within some accuracy. All clocks will diverge and eventually depart from synchronization tolerance. It's just a matter of time! Knowing a clock s characteristics, the system requirements, and the environmental conditions will allow the manager to know the best clock or clocks to buy and the best way to implement them. For example, a rubidium clock coupled to a GPS receiver (used in the common-view mode with UTC(USNO MC) or with UTC(NBS) would have better short term and better long term stability than any commercial cesium clock available. The stability would be somewhat worse in the vicinity of $\tau$ equal to one day. In practice, there are some problems with this idea, but it illustrates the point.

Lastly, Figures 9 through 13 show nominal values for some important clock coefficients that managers and design engineers need to properly assess when evaluating which clock or clocks will best serve their needs. These are only nominal values and there will be exceptions. A band of values is listed for these coefficients ranging from nominal best performance available from laboratory-type standards through the range of typical values observed and specified for commercially available standards.

## ACKNOWLEDGEMENTS

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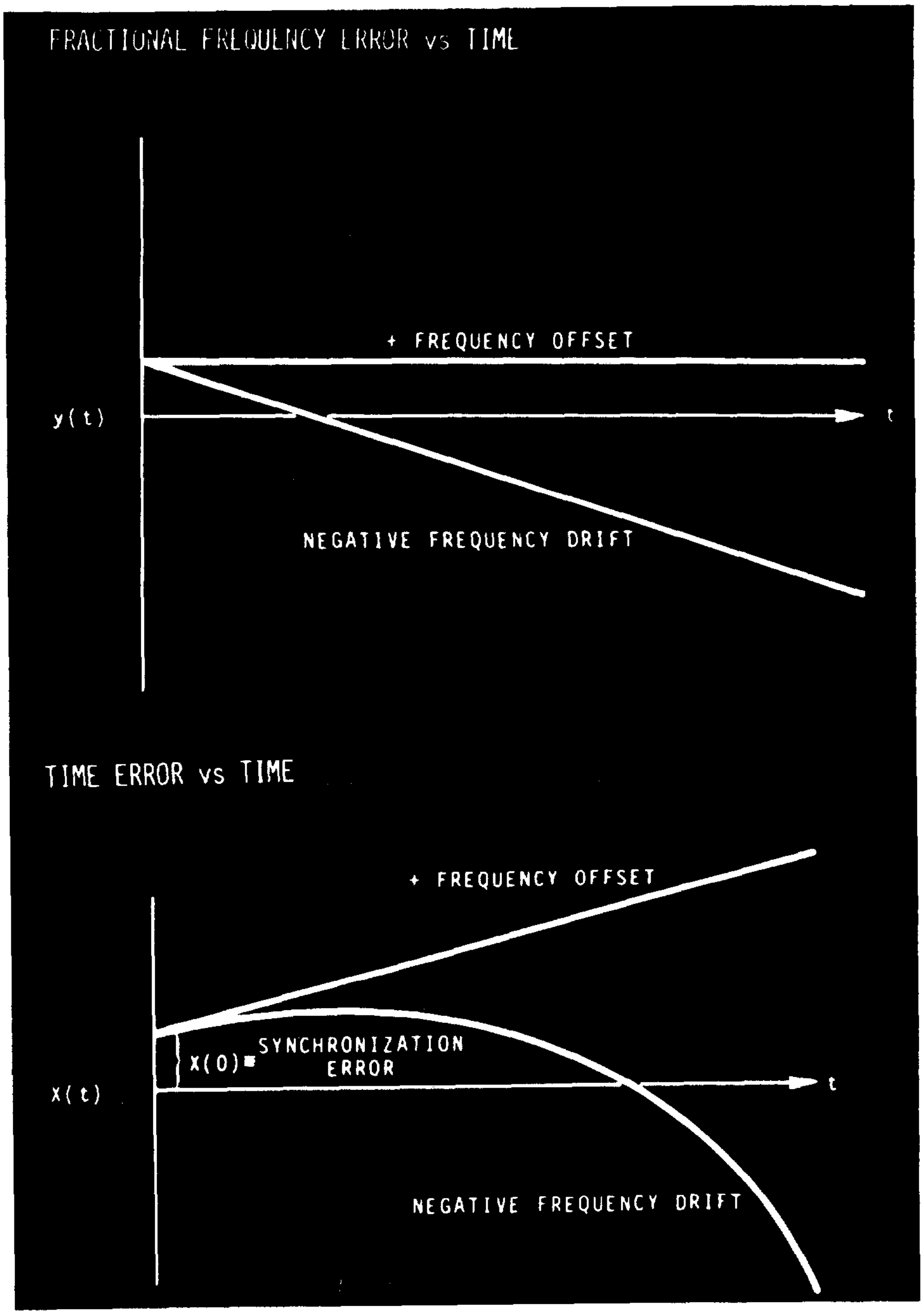


Figure 1. Frequency, y(t), and time, x(t) deviation due to frequency offset and to frequency drift in a clock.

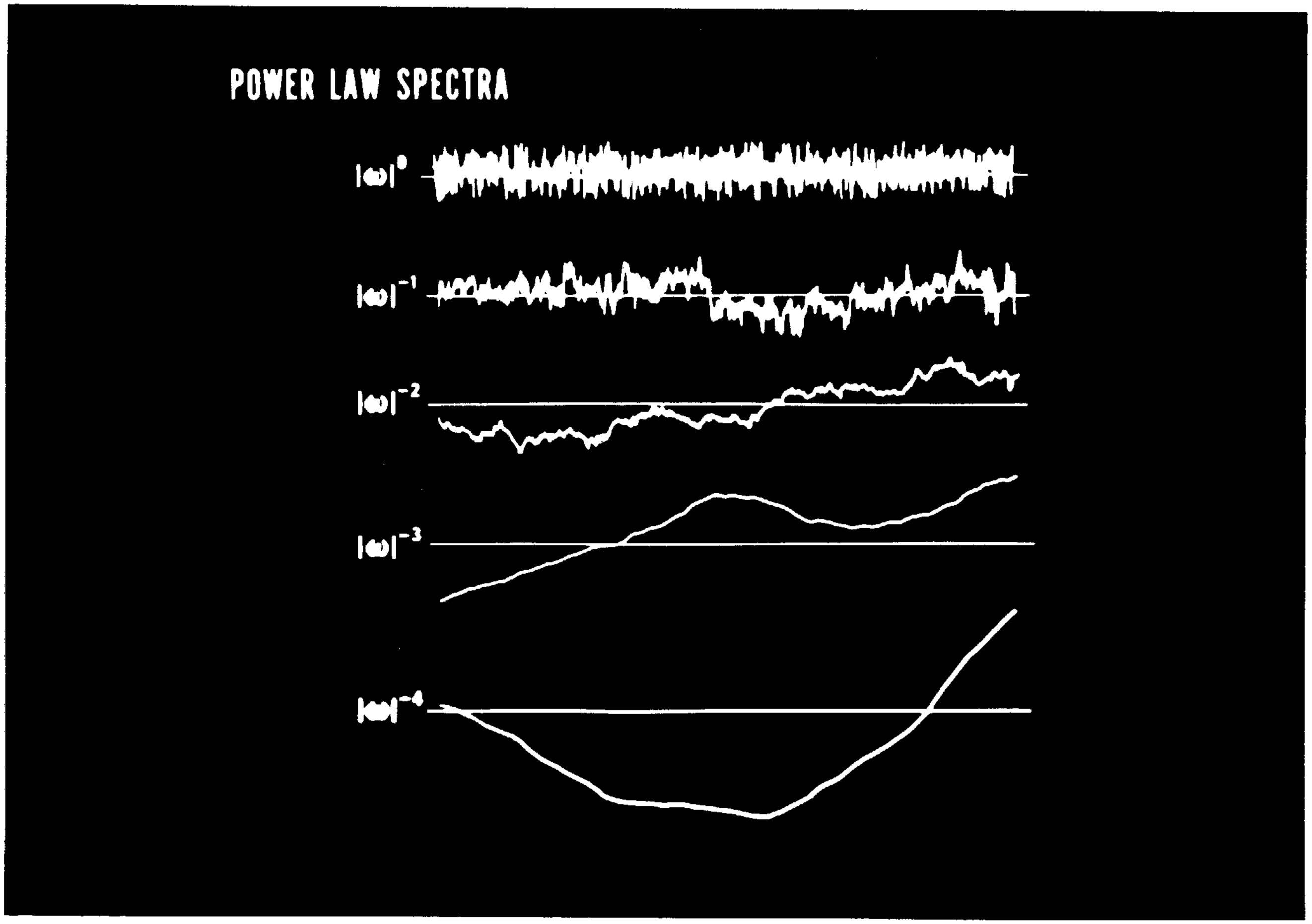


Figure 2. Simulated random processes commonly occurring in the output signal of atomic clocks. Power law spectra $S(\omega)$, are proportional to $\omega$ to some exponent, where $\omega$ is the Fourier frequency.

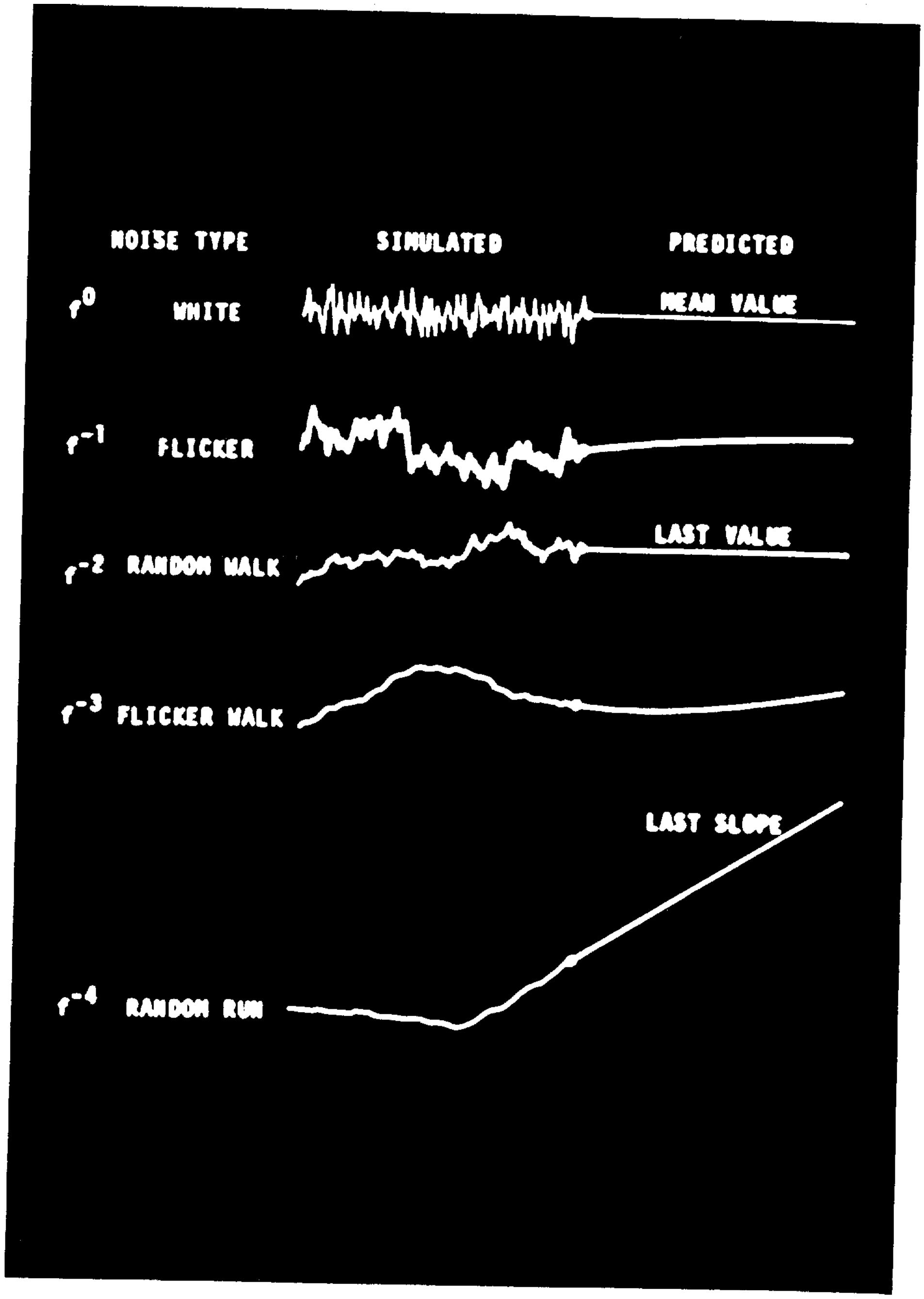


Figure 3. Simulated power law noise processes with their optimum predicted estimates.

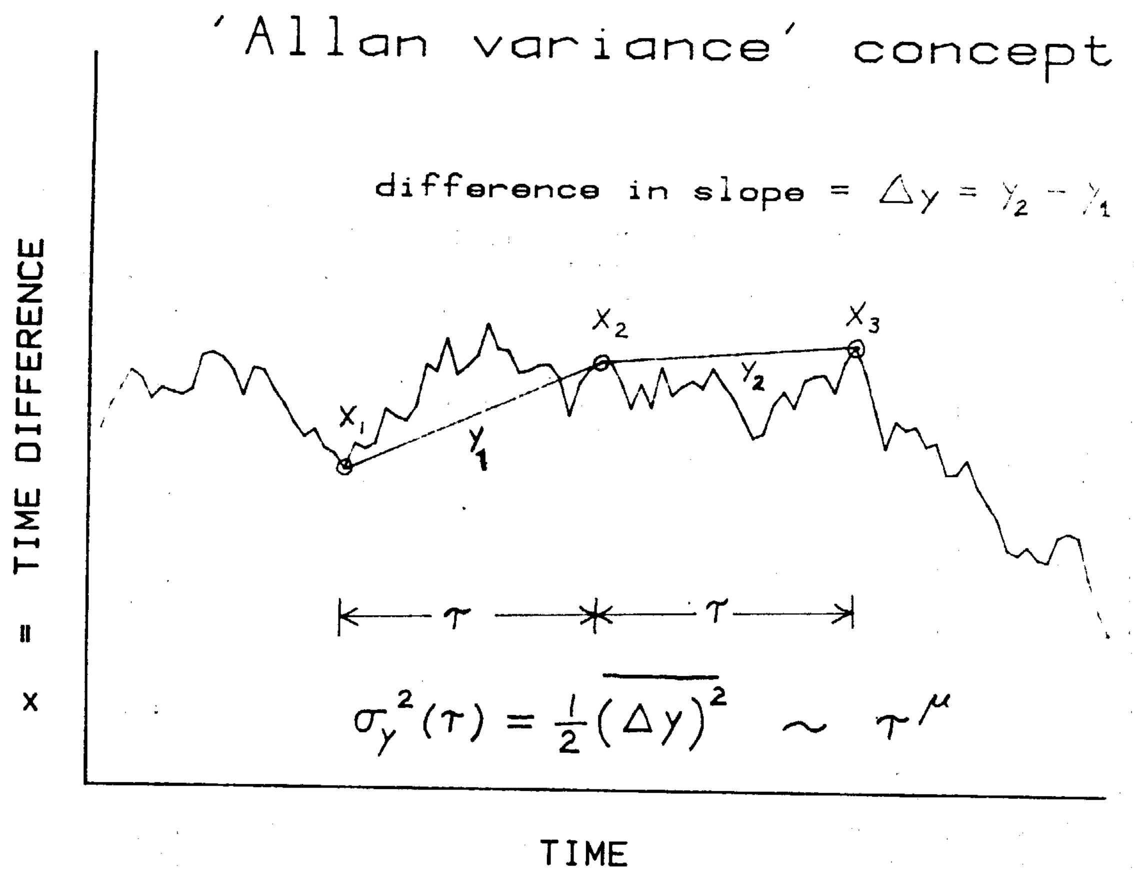


Figure 4. Pictorial of computation of "Allan variance".

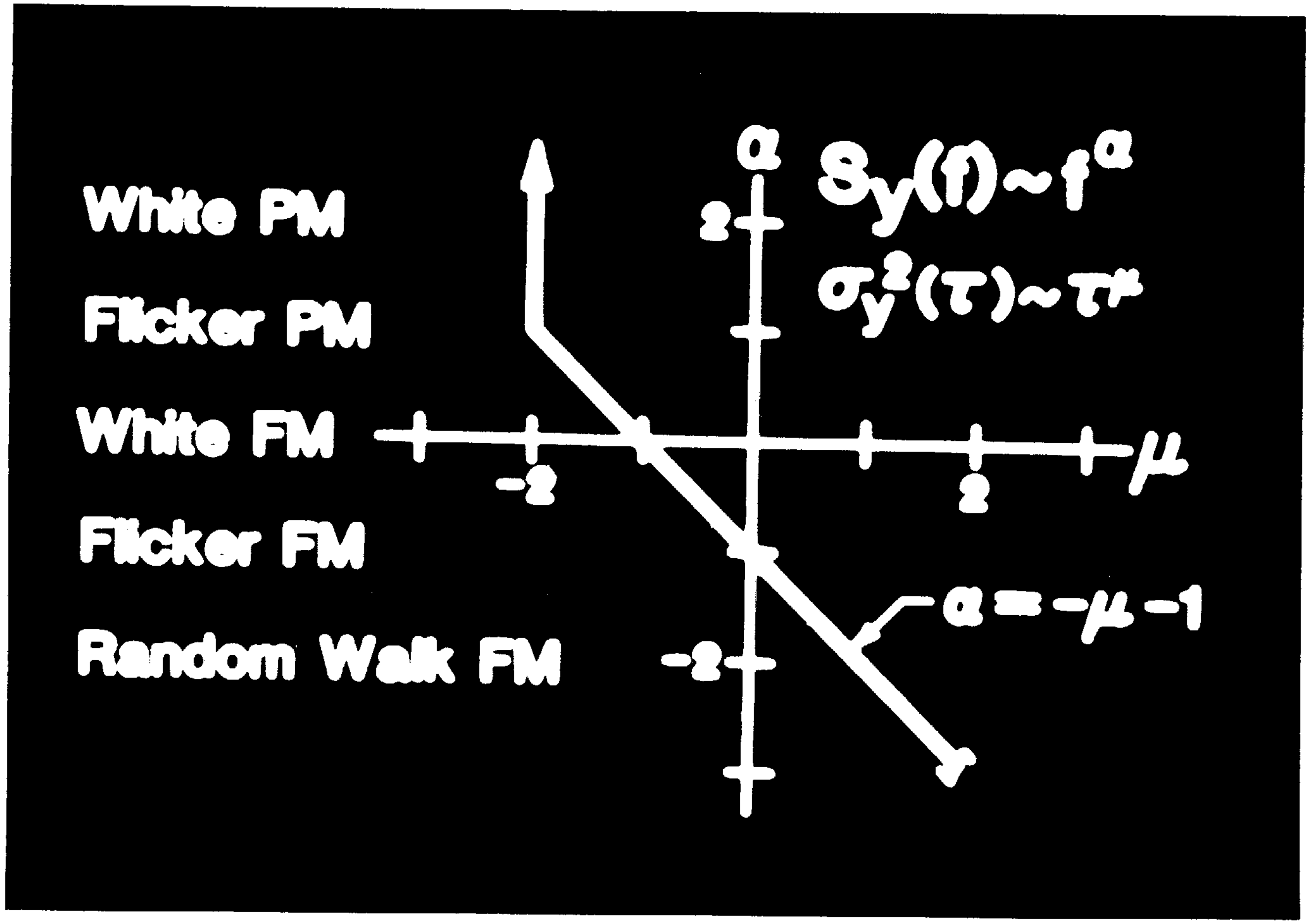


Figure 5. Various power law noise types commonly occurring in the precision clocks and the mapping between the exponent α of the frequency domain measure and the exponent μ of the time domain measure.

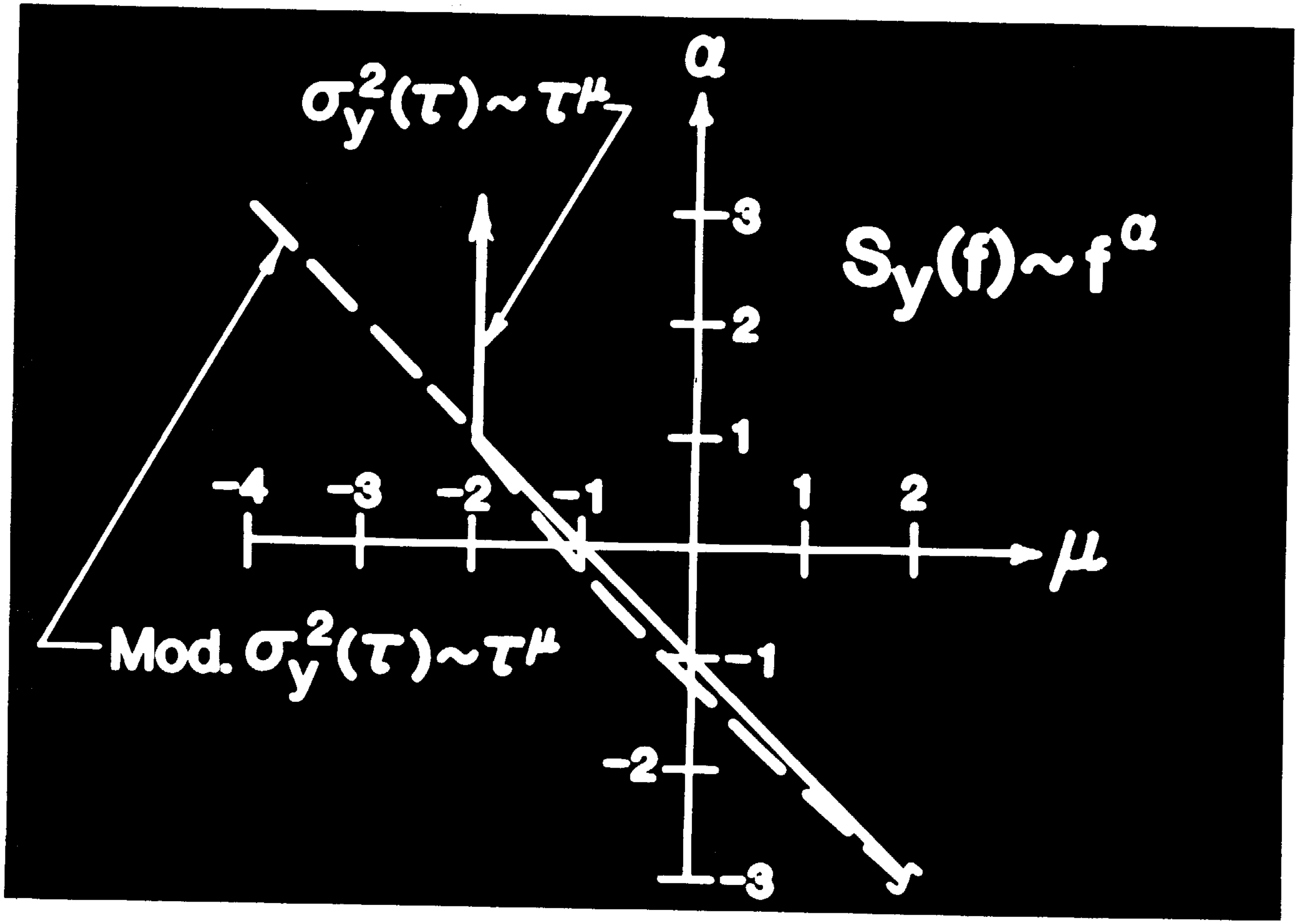


Figure 6. The mapping of the $\tau$ exponent $\mu$ of the Allan variance and of the modified Allan variance to the power law spectrum exponent $\alpha$.

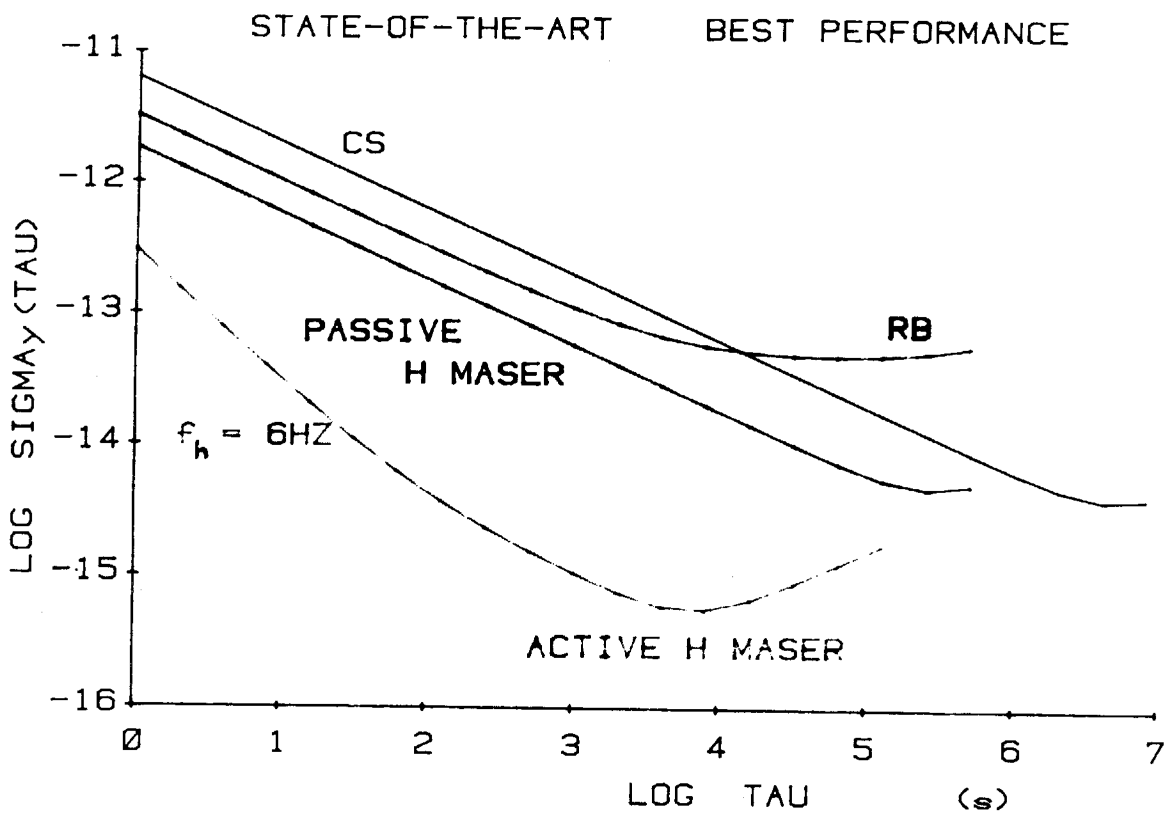


Figure 7. Frequency stability of some of the best performing frequency standards from each of four main types: CS = cesium beam, RB = rubidium gas cell, H = hydrogen (active and passive). Note: $\log 10^{I} = I$; e.g. $\log 10^{-14} = -14$.

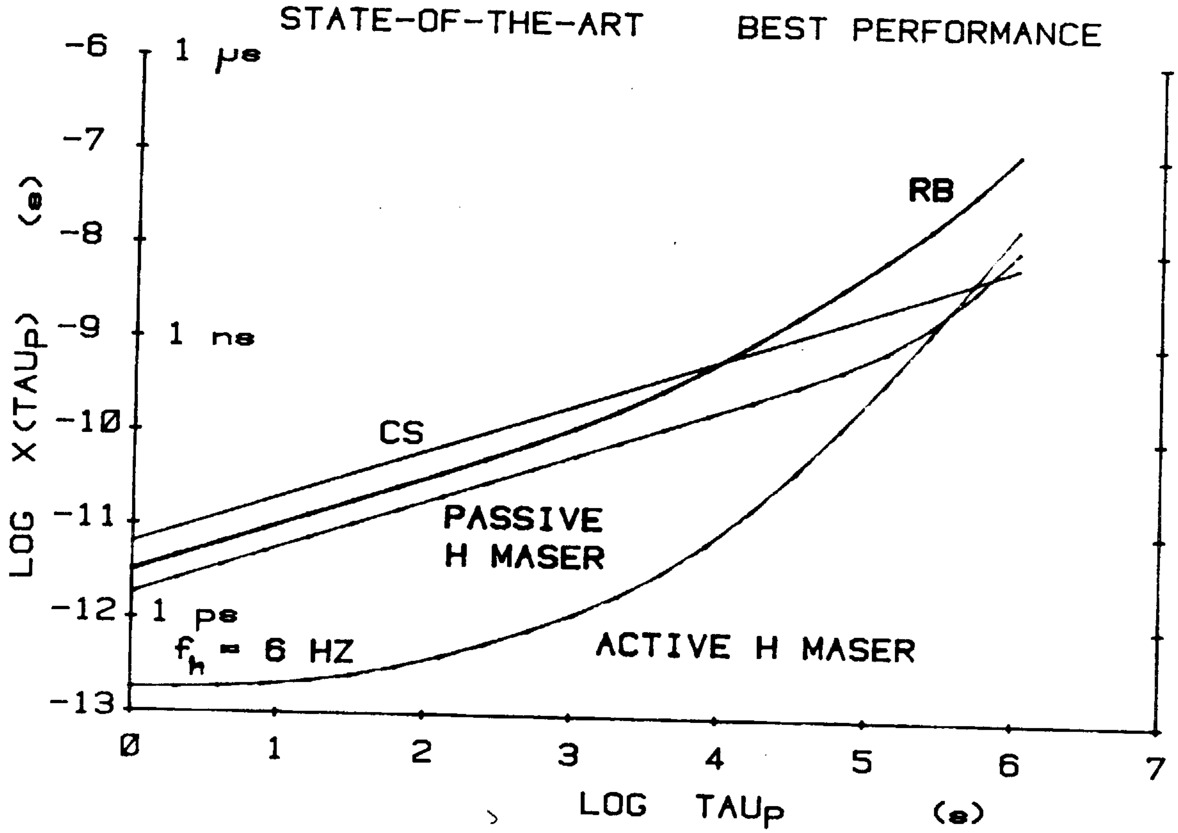


Figure 8. RMS time prediction error plot for the same frequency standards as shown in Figure 7. Note: $\log 10^{I} = I$; e.g. $\log 10^{-14} = -14$.

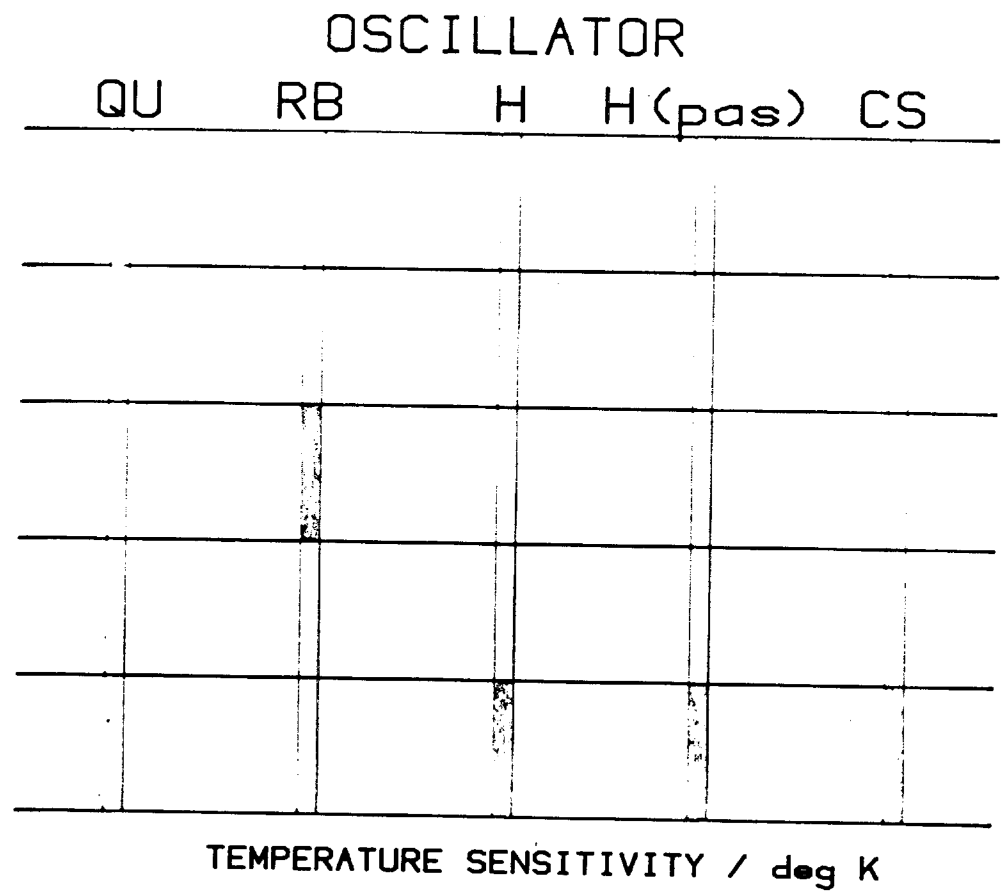


Figure 9. Nominal values for the temperature coefficient for the frequency standards: QU = quartz crystal, RB = rubidium gas cell, H = active hydrogen maser, H(pas) = passive hydrogen maser, and CS = cesium beam.

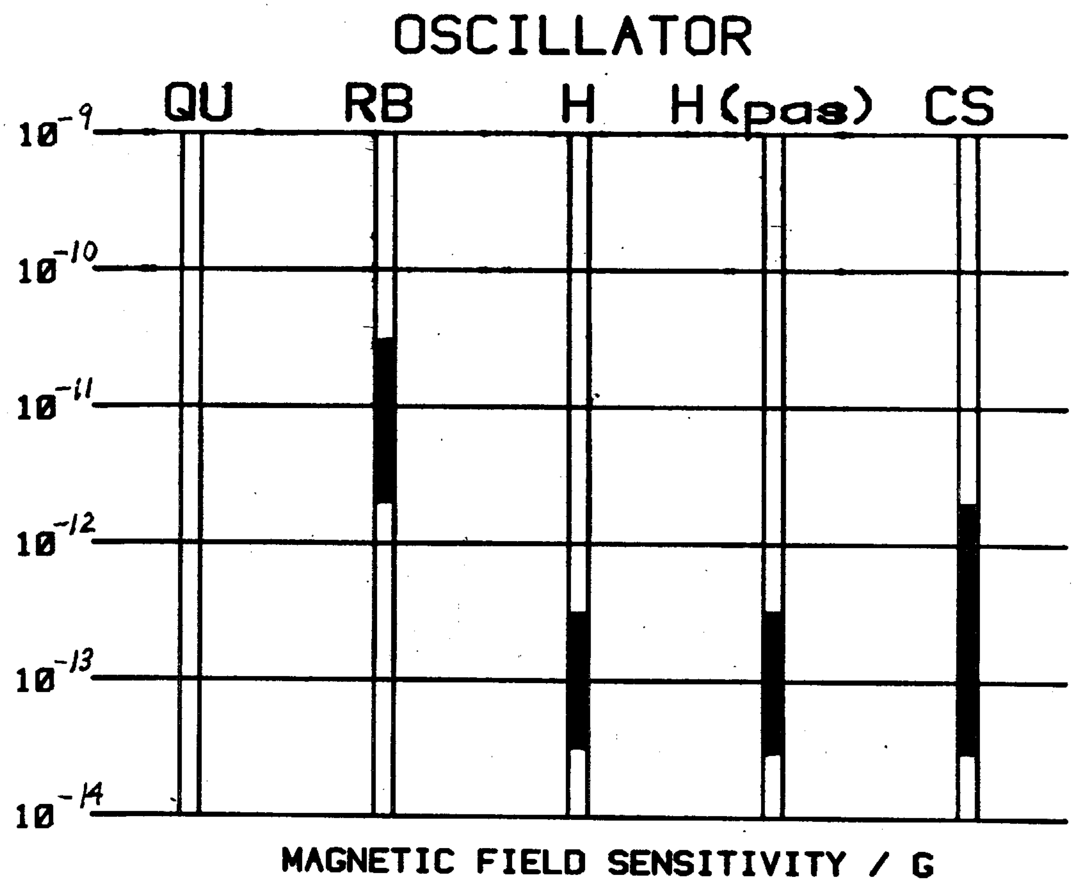


Figure 10. Nominal values for the magnetic field sensitivity for the frequency standards: QU = quartz crystal, RB = rubidium gas cell, H = active hydrogen maser, H(pas) = passive hydrogen maser, and CS = cesium beam.

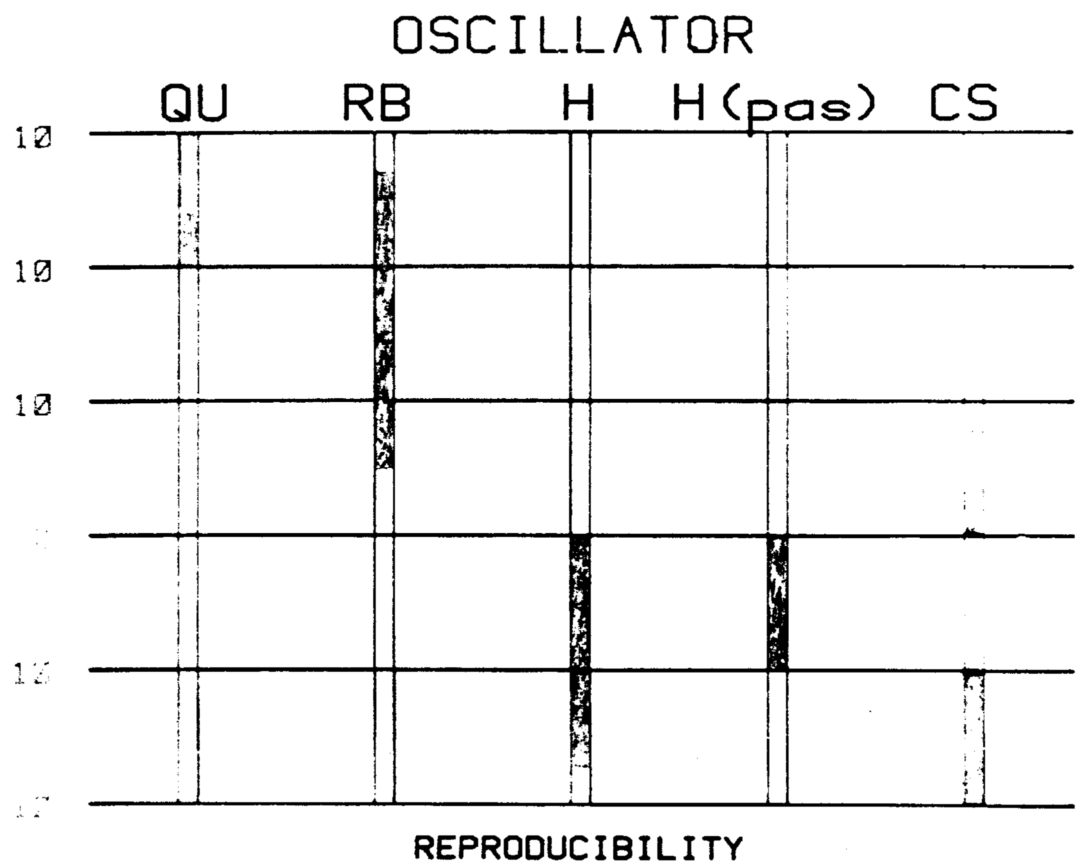


Figure 11. Nominal capability of a frequency standard to reproduce the same frequency after a period of time for the standards: QU = quartz crystal, RB = rubidium gas cell, H = active hydrogen maser, H(pas) = passive hydrogen maser, and CS = cesium beam.

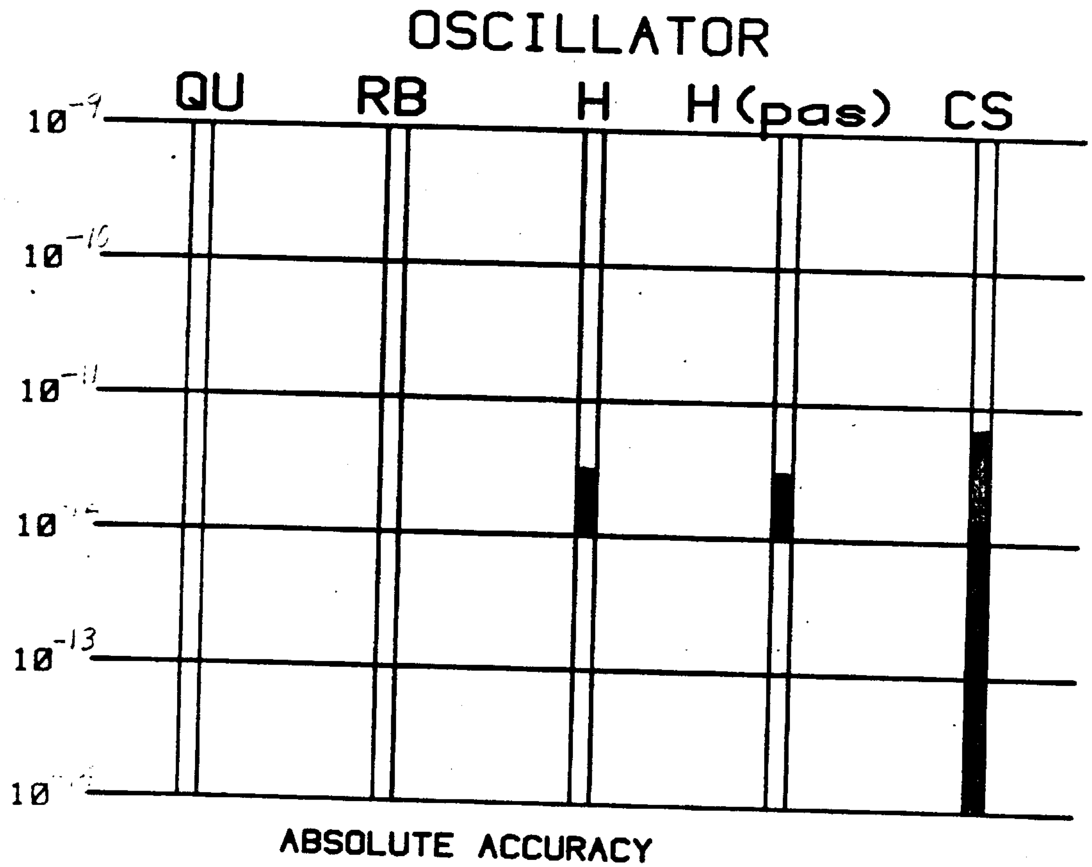


Figure 12. Nominal capability for a frequency standard to produce a frequency determined by the fundamental constants of nature for the standards: QU = quartz crystal, RB = rubidium gas cell, H = active hydrogen maser, H(pas) = passive hydrogen maser, and CS = cesium beam.

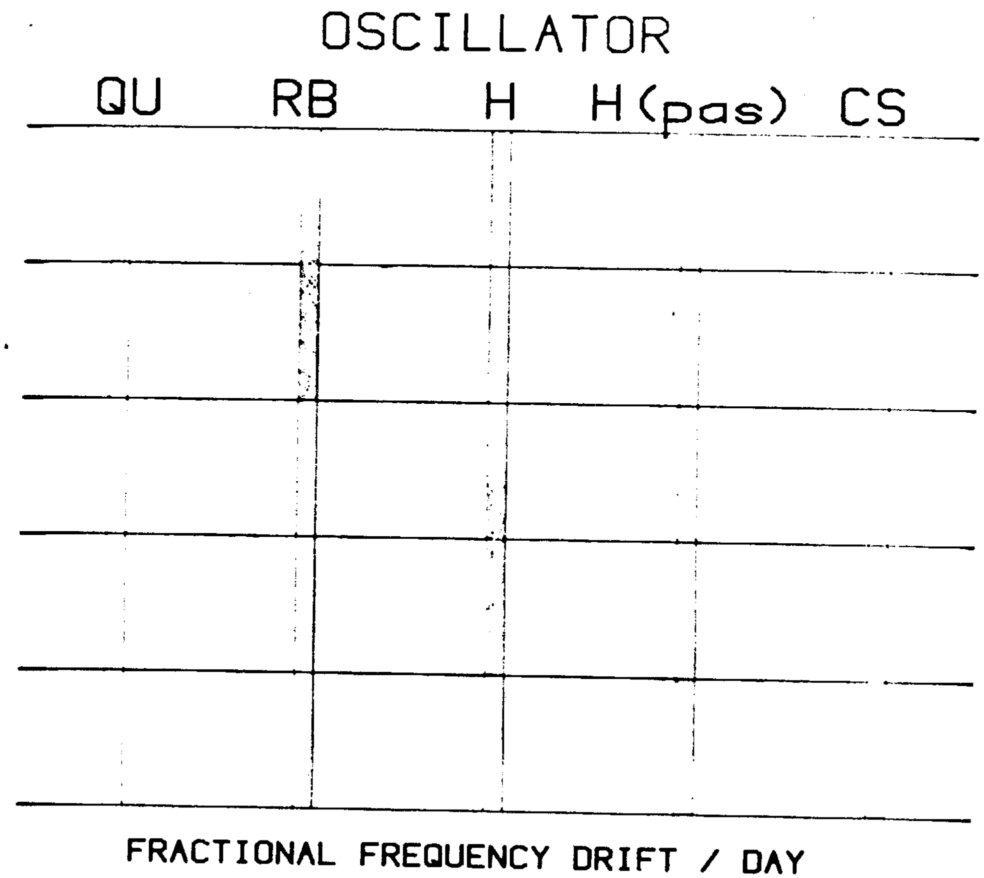


Figure 13. Nominal values (ignoring the sign) for the frequency drift for the frequency standards: QU = quartz crystal, RB = rubidium gas cell, H = active hydrogen maser, H(pas) = passive hydrogen maser, and CS = cesium beam.