Privacy-preserving Efficient Subset of Features Selection for Regression Models

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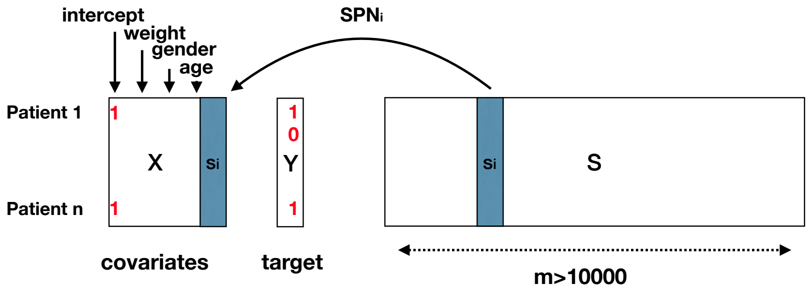
December 10, 2018

GWAS (find the best additional feature)



Question: Is the new future important?

Naive method: compute $stat_i$ for each i...

... that means compute more than 10000 logreg

Description of Idash 2018 Task 2

Goal:

Develop a secure parallel outsourcing solution to compute Genome Wide Association Studies (GWAS) based on linear/logistic regression using **homomorphically encrypted** data.

Challenge (informally):

- Currently: a logreg model, 250 patients, with 3 or 4 physical features.
- Which new feature, among 10000 possible genes (SNP), would improve the model?

Semi-parallel approach

- Don't do 10000 logregs...

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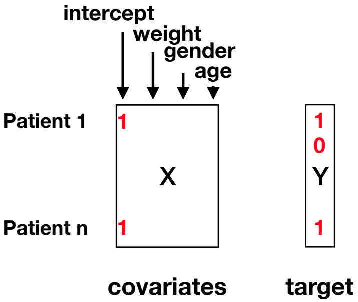
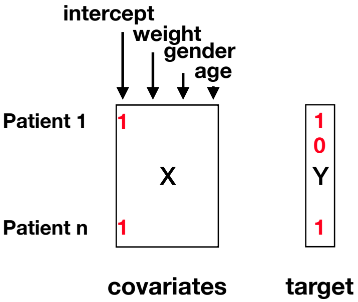
Logreg, IRLS, relevance of a feature



- Single Logistic regression:
  - Find θ s.t $Y = sign(X\theta)$
- IRLS:
  - Compute $grad = X^t(Y - p)$, with $p = \sigma(X\theta)$
  - Compute $Hessian = X^t diag(p(1-p))X$

Logreg, IRLS, relevance of a feature



Importance of the i^{th} feature:

- the i^{th} coeff is big: θ_i (numerator)
- the i^{th} error term is small: $(Hess^{-1})_{i,i}$ (denominator)

stat= ratio

- Single Logistic regression:
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- IRLS:
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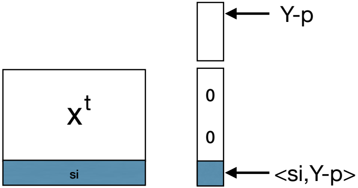
Semi-parallel GWAS (optimized)

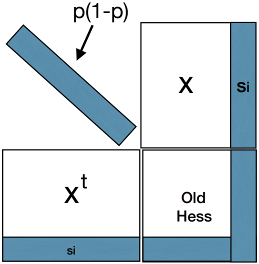
1. Do $logreg(X, y)$ without S
2. Once model is converged, add s_i

- Gradient:



They can be batch-computed: $(Y\text{-}p)^t S$

- Hessian:



MPC versus FHE

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FHE

- Long term storage
- Unique Cloud
- Slower and consumes more memory

MPC

- Faster than FHE
- More accuracy
- All data owner must participate

Fixed points versus Floating point

Floating point:

- $x = m.2^{\tau}$, with $m \in 2^{-\rho}.\mathbb{Z}$ and $\frac{1}{2} \leq |m| < 1$
- $\tau = \lceil log_2(x) \rceil$ data dependent and not public (not FHE-friendly)
- The exponent is always in sync with the data
  ex: $(1.23 \cdot 10^{-4}) * (7.24 \cdot 10^{-4}) = (8.90 \cdot 10^{-8})$

Fixed point:

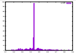
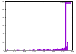
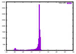
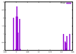
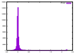
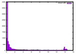
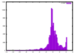
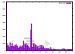
- $x = m.2^{\tau}$, with $m \in 2^{-\rho}.\mathbb{Z}$ and $0 \leq |m| < 1$,
- τ is public, thus FHE-friendly
- Risk of overflow (τ too small)
- Risk of underflow (τ too large)
  ex: $(0.000123 \cdot 10^{0}) * (0.000724 \cdot 10^{0}) = (0.000000 \cdot 10^{0})$

Plaintext parameters:

- $\rho \in \mathbb{N}$: bits of precision of the plaintext (≈ 15 bits)
- $\tau \in \mathbb{Z}$: slot exponent (order of magnitude of the complex values in each slot)

Choice of slot exponent

The slot exponent τ that defines the plaintext interval must be carefully estimated.

variable	avg	stdev	min	max	dist
p	0.440816	0.0975715	0.176397	0.853487	
w	0.236977	0.0201871	0.125047	0.25	
z_i^*	-3.33092	7.36068	-30.9426	31.2008	
G	0.0577846	0.0953495	-0.011997	0.236977	
A	0.0621965	0.301255	-0.317312	2.236	
$(s_i^*)^2$	2.44243	4.11085	0.111961	14.5044	
$\log(stat_i)$	0.200039	1.84459	-13.7207	4.36158	
$p-value$	0.310218	0.24083	0	0.999163	

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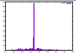
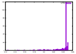
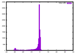
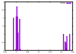
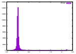
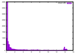
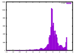
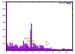
p	0.440816	0.0975715	0.176397	0.853487	
w	0.236977	0.0201871	0.125047	0.25	
z_i^*	-3.33092	7.36068	-30.9426	31.2008	
G	0.0577846	0.0953495	-0.011997	0.236977	
A	0.0621965	0.301255	-0.317312	2.236	
$(s_i^*)^2$	2.44243	4.11085	0.111961	14.5044	
$\log(stat_i)$	0.200039	1.84459	-13.7207	4.36158	
$p-value$	0.310218	0.24083	0	0.999163	

Numerical stability

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Not stable

Increase the precision of the algorithm, but that implies bigger parameters.

Stable

Use stable computation with negative feedback
(e.g. gradient descent)
Smaller parameters

FHE Solution

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FHE parameters:

- $L \in \mathbb{N}$: level exponent of the ciphertext ($\alpha = 2^{-(L+\rho)}$: noise rate)
- $N = f(\lambda, \alpha)$: key size, with λ the security parameter

The `lwe-estimator` script was used to assert the security.
(conform to HE security standardization white paper)

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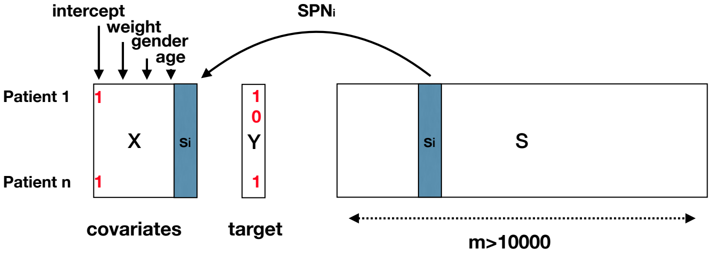


Input:

- $X \in \mathcal{M}_{n,k+1}(\mathbb{R})$ input matrix
- $y \in \mathbb{B}^n$ binary vector
- $S \in \mathcal{M}_{n,m}(\mathbb{R})$ assumed binary

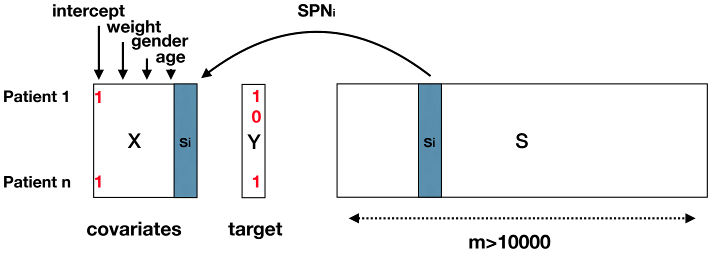
Output:

- $stat \in \mathbb{R}^m$
  with $stat_i = \frac{\mathbf{z}^*{}_i}{\sqrt{\mathbf{s}^*{}_i^2}}$

Key points of our solution:

- Make plaintext algorithm FHE friendly
- Use hybrid homomorphic encryption

Plaintext algorithm in FHE solution



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Optimization of plaintext algorithm

Make the plaintext algorithm FHE friendly

- Find simple geometric equivalents of the formula
- Find approximation with lower multiplicative depth
- Replace feature scaling of X with orthogonalization

Algorithm in plaintext

Algorithm 2 Plaintext algorithm

$\beta^{(0)} = (0, \dots, 0)$
for $t = 1$ to iters **do**
  $\beta^{(t)} \leftarrow \beta^{(t-1)} + \text{step} \cdot X^T. \left(\mathbf{y} - \sigma(X\beta^{(t-1)})\right)$
end for ▷ logreg
$\mathbf{p} \leftarrow \sigma(X\beta^{(\text{iters})})$
$\mathbf{z}^* \leftarrow (\mathbf{y} - \mathbf{p})^T.S$ ▷ numerator
$W \leftarrow \text{diag}(p * (1-p))$
$G \leftarrow X^T.W.X \approx \frac{1}{4} * Id$ (assumed that X orthogonal)
$A \leftarrow X^T.W.S$
$\mathbf{s}^{*\mathbf{2}} = \text{colsums}(W \cdot (S \odot S)) - \text{colsums}(A \odot G^{-1}A)$
$(\approx A[0] * \sqrt{n} - 4 * \text{colsums}(A \odot A))$ ▷ denominator
$\mathbf{r}_i = [2 \cdot \log(|\mathbf{z}^*{}_i|) - \log(|\mathbf{s}^{*\mathbf{2}}{}_i|)]$ for each $i \in [1, m]$ ▷ log of stat

$$stat_i = \frac{\mathbf{z}^*{}_i}{\sqrt{\mathbf{s}^*{}_i^2}} = \frac{1}{2}\exp(\mathbf{r}_i)$$

$$\text{p-value}_i = \text{p-Norm}(stat_i)$$

Algorithm in plaintext

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Algorith

for loops
(better with fast bootstrapping)

$\beta^{(0}$

for $t = 1$ to iters **do**

$\quad \beta^{(t)} \leftarrow \beta^{(t-1)} + \text{step} \cdot X^T . \left(\mathbf{y} - \sigma(X\beta^{(t-1)}) \right)$

enc ▷ logreg

continuous non-polynomial functions
(Approx numbers, or Lookup tables)

$\mathbf{p}$

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$\mathbf{s}^{*\mathbf{2}} = \text{colsums}(W \cdot (S \odot S)) - \text{colsums}(A \odot G^{-1} A)$

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individual non-linear operations in small dimension
(lookup tables)

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$W \leftarrow$ diag(

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multiplication with fresh ciphertexts
(better with TFHE's external product)

$A \leftarrow X^T.W.S$

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4: **en**

individual non-linear operations in small dimension
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5: $\mathbf{p}$
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12: $\mathbf{r}_i = [2 \cdot \log(|\mathbf{z^*}_i|) - \log(|\mathbf{s^{*2}}_i|)]$ for each $i \in [1, m]$ ▷ log of stat

very large dimension
(fully packed SIMD)

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continuous function batched on a large vector

Algorithm in plaintext

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continuous function batched on a large vector

Which fully homomoprhic scheme should we choose?

Each library has its own strengths

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Strengths of HE libraries

- BGV/Helib: SIMD finite field arithmetic
- B/FV, Seal: SIMD vector $\mod t$
- HEAAN: SIMD fixed point arithmetic
- TFHE: single evaluation, boolean logic, comparison, threshold, complex circuits
- etc...

How to get all the benefits without the limitations?

Solution: Chimera

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Idea:

- Unified plaintext space over the Torus
- Switch between ciphertext representations
- Implement bridges between TFHE, B/FV and HEAAN

For this use-case

We use the switch between TFHE and HEAAN!

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Chimera solution

1. Initial Logreg on matrix X and vector y
    - adapt lib TFHE + logreg
2. Mass Linear algebra computations
    - implement Chimera (version 2 of TFHE)
3. Batch Logarithm computation
    - adapt lib HEAAN

Benchmarks (Idash Bootstrapped)

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Steps	Timing (4 cores)	Timing (96 cores)	RAM
KeyGen	5.5 mins	2.0 mins	4.4 GB
Encryption	7.2 mins	1.3 mins	8.6 GB
Cloud Computation	3h06	10.2 mins	7.8 GB

- Input ciphertext: 5GB (enc X, y, S)
- Final ciphertext: 640KB (enc numerator + denominator)

Benchmarks (with new optimizations)

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$k = 3,\ n = 250,\ m = 10000$

Steps	**Timing (4 cores)**	**Timing (96 cores)**	**RAM**
KeyGen	5.5 mins	2.0 mins	4.4 GB
Encryption	7.2 mins	1.3 mins	8.6 GB
Cloud Computation	35 mins	3 mins	7.8 GB

$k = 7,\ n = 250,\ m = 10000$

Steps	**Timing (4 cores)**	**Timing (96 cores)**	**RAM**
KeyGen	5.5 mins	2.0 mins	4.4 GB
Encryption	7.2 mins	1.3 mins	8.6 GB
Cloud Computation	41 mins	3.1 mins	7.8 GB

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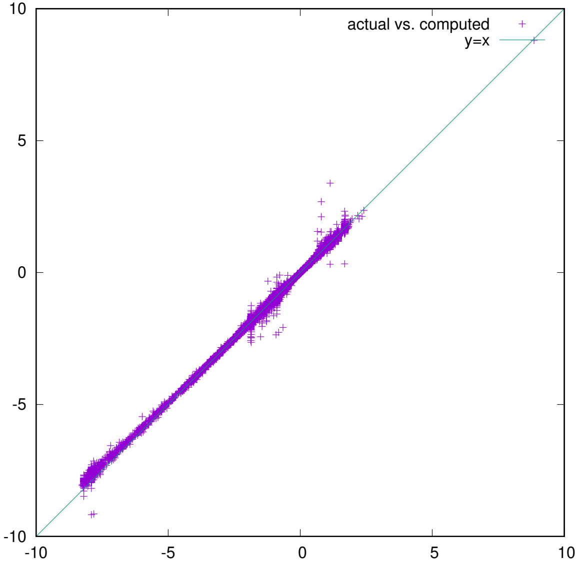
Numerical Accuracy (FHE has noise)



Questions?