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Digital watermarking in the fractional Fourier transformation domain

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An application of the fractional Fourier transform for the multimedia copyright protection is proposed in the paper. The watermark robustness as well as statistical performance are considered.

1. Introduction

Embedding of a watermark signal is an interesting research and application field in the copyright protection of multimedia signals (images, sounds, movies) [1–4]. The goal is to embed the watermark that is imperceptible in the image, while the copyright holder can detect its existence, by using proper private information—key. Commonly used methods are based on embedding watermark signals in the space or spatial-frequency domain [1]. Recently, watermarking in the combined space/spatial-frequency domain has been defined [5]. In this paper, we will consider image watermarking in the fractional Fourier transformation (FRFT) domain. This approach uses combination of the space and spatial/frequency domains, without introducing the multidimensional Radon-Wigner distribution [5].

The paper is organized as follows. The FRFT is described in Section 2. Watermark embedding in the FRFT domain is considered in Section 3. Numerical examples are given in Section 4.

2. The fractional Fourier transform

For the analysis of images one can use a two-dimensional FRFT, defined by:

$$S_{\alpha_x,\alpha_y}(u_x, u_y) = FRFT_{\alpha_y}^{t_y \to u_y}\{FRFT_{\alpha_x}^{t_x \to u_x}\{s(t_x, t_y)\}\} \tag{1}$$

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where $s(t_x, t_y)$ is a 2D signal and (α_x, α_y) are transformation angles. The symbol $FRFT_{\alpha}^{t\to u}$ denotes one-dimensional FRFT. It is defined as:

$$X_\alpha(u)=\int_{-\infty}^{\infty} x(t)K_\alpha(u,t)dt,\ X_\alpha(u)=FRFT_\alpha^{t\to u}\{x(t)\} \tag{2}$$

where:

$$K_\alpha(u,t)=\begin{cases} \sqrt{\dfrac{1-j\cot\alpha}{2\pi}}e^{j(t^2+u^2)\cot\alpha/2-jut\ \csc\alpha} & \text{if } \alpha \text{ is not multiple of } \pi \\ \delta(t-u) & \text{if } \alpha \text{ is multiple of } 2\pi \\ \delta(t+u) & \text{if } \alpha+\pi \text{ is multiple of } 2\pi. \end{cases} \tag{3}$$

The FRFT can be understood as a rotation of the signal for an arbitrary angle α in the time-frequency plain [6,7]. The inverse FRFT can be treated as a rotation for angle $-\alpha$: $x(t)=FRFT_{-\alpha}^{u\to t}\{X_\alpha(u)\}$. The FRFT is additive with respect to the angle, i.e., the FRFT for angle β of $X_\alpha(u)$ is equal to $X_{\alpha+\beta}(u)$. Note that the Fourier transform (FT) $X(\omega)$ of a signal $x(t)$ is a special case of the FRFT for $\alpha=\pi/2$.

Obviously, the FRFT transformation domain is a combination of the time and frequency domains. For angles α close to $\alpha=\pi/2$, for example for $3\pi/4\geq|\alpha|\geq\pi/4$, we can consider the FRFT as the transformation being dominantly in the frequency domain. On the other hand, for small α, $|\alpha|<\pi/4$, the FRFT is dominantly in the time domain. For the watermarking proposed in this paper, we will use the frequency domain dominant case. It must preserve the realness of a signal after the watermark is embedded in the FRFT domain. For this reason, when we embed a signal $Y(u)$ into the FRFT with angle α, at the same time we are to embed the signal $Y^*(u)$ into FRFT with angle $-\alpha$.

Recently, numerical realization of the discrete FRFT has been a very intensively studied research topic. An efficient numerical approach, presented in [8,9], is used in this paper for watermark embedding.

3. Watermark embedding

For an image $s(n_x, n_y)$ we find FRFT for angles (α_x, α_y), followed by transformation coefficients reordering in nonincreasing sequence $S=\{S_i|S_i\geq S_{i-1}\}$. The first L coefficients are omitted and the watermark is embedded in the next M transformation coefficients. If the watermark were embedded in the highest coefficients, it could produce significant image deformation, while if it were embedded in the lowest coefficients it could be cleaned by lossy image compression or lowpass filtering, without significant image visual degradation. Therefore, watermark is embedded as [3]:

$$S_i^w=S_i+k_i'|Re\{S_i\}|+jk_i''|Im\{S_i\}|,\quad i=L+1, L+2, \ldots, L+M, \tag{4}$$

where (k_i', k_i''), $i=L+1, \ldots, L+M$ is a real-valued watermark key. Watermark detection must be reliable if the watermark key and positions of transformation coefficients are known. Let the watermark be a Gaussian white noise with variance σ^2, i.e. variances of k_i' and k_i'' are $\sigma^2/2$. A watermark detection check is performed comparing the detection value:

$$d=\sum_{i=L+1}^{L+M}[k_i'-jk_i'']S_i^{(a)} \tag{5}$$

with a chosen threshold. Here, $S_i^{(a)}$ denotes the FRFT of the target image with a possible attack.

In order to determine the statistical performance of the proposed algorithm, we will first assume that the watermarked image is not changed by attacks (common image processing algorithms) or communication channel noise. Then, the value of d is equal to:

$$d=\sum_{i=L+1}^{L+M}[k_i'-jk_i''][S_i+k_i'|Re\{S_i\}|+jk_i''|Im\{S_i\}|]. \tag{6}$$

Since the number of coefficients where the watermark is embedded (M) can be high (for a 256×256 image it can be a few thousand) then, for a watermark key uncorrelated with image, the mean value of d is given as:

$$E\{d\}=\frac{\sigma^2}{2}\sum_{i=L+1}^{L+M}[|Re\{S_i\}|+|Im\{S_i\}|]. \tag{7}$$

If there is no watermark (k_i', k_i'') in the image, $E\{d\}=0$. Variance of d is the same in both cases:

$$var\{d\}=\sigma^2\sum_{i=L+1}^{L+M}|S_i|^2. \tag{8}$$

Thus, the detection threshold should be chosen as $E\{d\}/2$, while the watermark key variance is chosen by a trade-off between watermark imperceptiveness and probability of false detection (false alarm). Because pirates can modify the image, by using some common image processing transforms (attacks), a value higher than $var\{d\}$ can be used for calculation of the false alarm probability.

4. Examples

The algorithm is tested on various standard test images and attacks. In the examples, standard test image Lena (256×256) is used [Fig. 1(a)]. Watermarked image Lena ($L=8000$, $M=8000$, $\sigma^2=0.04$, $\alpha_1=\alpha_2=0.375\pi$) is shown in Fig. 1(b). Detector responses in watermarked and non-watermarked images for

Figure 1. Test image 'Lena': a) Original image, b) watermarked image.

1000 different watermark keys are shown in Fig. 2(a). Further, we have supposed that a pirate knows watermark key and watermark key position, but that he doesn't know the transformation angles. Detector response over different transformation angles is shown in Fig. 2(b). From this figure, it is clear that for watermark detection it is necessary to know watermark angles as well. The watermark key consists of k_i' and k_i'', positions of embedded coefficients, and the angles (α_1, α_2). In this way, we can create more watermarks than in the FT or DCT domain, since we can use different angles for watermark embedding. Calculation complexity of the procedure for watermark embedding and detection is not significantly increased, since there are standard fast algorithms for the FRFT calculation [8,9].

This watermarking approach is robust on some common attacks (geometrical transform, filtering, histogram stretching, etc.). Watermarked image Lena embedded with white Gaussian noise with variance $\sigma_G^2 = 6000$ is shown in Fig. 3(a), while the cropped watermarked image is shown in Fig. 3(b). Detection responses,

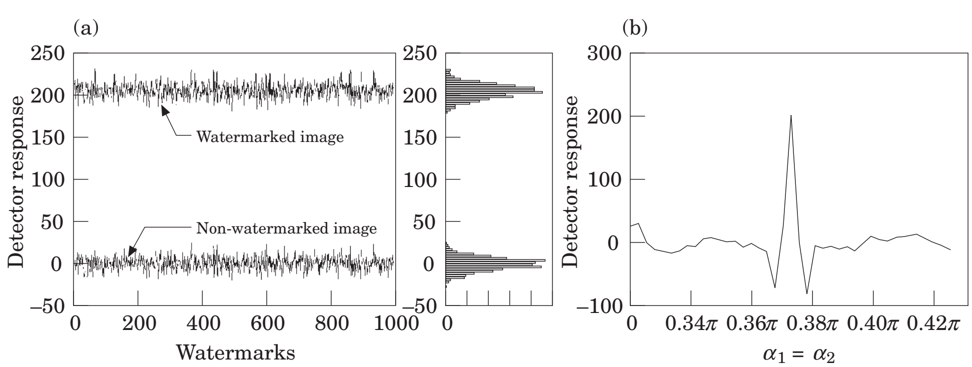

Figure 2. a) Statistical analysis of detecting watermark signal in watermarked and nonwatermarked image, b) detection of watermark signal using different transformation angles.

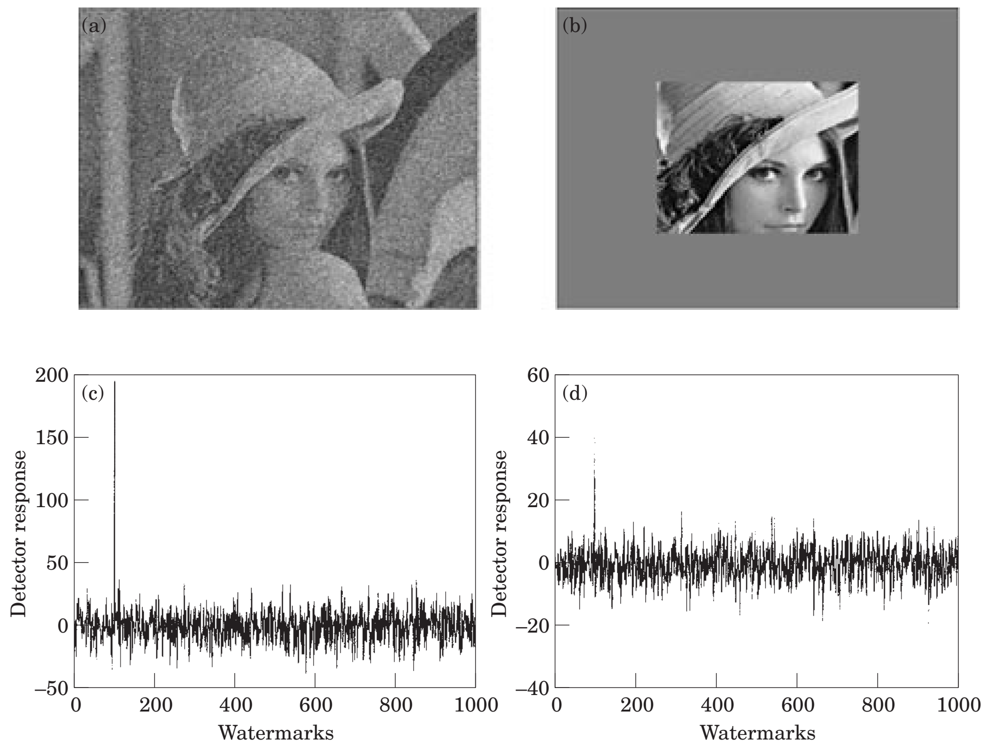

Figure 3. a) Noisy image 'Lena', b) cropped image 'Lena', c) detection of watermark after embedding of noise, d) detection of watermark after cropping.

in both cases, over 1000 different watermarks are shown in Fig. 3(c) and (d). Only the true watermark no. 100. is detected.

5. Conclusion

The FRFT transformation domain watermarking concept is proposed. It offers two more degrees of freedom, resulting in the possibility to generate more watermarks than in the FT and DCT domains. This watermarking is robust on some important attacks that could be performed by a pirate.

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