Dis&approval voting: a characterization*

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Abstract

The voting rule considered in this paper belongs to a large class of voting systems, called "range voting" or "utilitarian voting", where each voter rates each candidate with the help of a given evaluation scale and the winner is the candidate with the highest total score. In approval voting the evaluation scale only consists of two levels: 1 (approval) and 0 (non approval). However non approval may mean disapproval or just indifference or even absence of sufficient knowledge for evaluating the candidate. In this paper we propose a characterization of a rule (that we refer to as dis&approval voting) that allows for a third level in the evaluation scale. The three levels have the following interpretation: 1 means approval, 0 means indifference, abstention or 'do not know', and -1 means disapproval.

1 Introduction

Consider a situation where voters are asked to answer the question: "would each of the following candidates be a good president/head?" The question addressed to voters is thus an absolute one.[1] Voters can answer in a positive, negative or null manner on each candidate. Voters are warned that when a candidate receives a blank or a spoiled vote, the latter option is marked by default.

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[1] That is, voters are not asked to compare candidate but grade them instead. For a relative question where voters are asked to give their best and worst candidates, see García-Lapresta et al. (2010).

Various procedures to aggregate ballots with different levels have been studied. Yilmaz (1999) proposes the 'approval-Condorcet-elimination' procedure. Ju (2005) deals with social choice rules mapping each profile into a single alternative. Aleskerov et al. (2007) axiomatize the 'threshold rule'. Balinski and Laraki (2007) propose 'majority judgment'. Gaertner and Xu (2012) axiomatize a ranking rule with a fixed electorate where voters place the candidates into a fixed number of categories. We investigate the 'dis&approval rule', that selects the candidates who obtain the largest difference between the number of positive votes and the number of negative votes.

Earlier references have studied the dis&approval rule. Felsenthal (1989) compares it with approval voting from a voter's point of view. A controversial conclusion is that no rational voter will choose to 'abstain' because 'abstaining' is a dominated strategy. Nevertheless there is experimental evidence that people do use abstention if they are offered that option, e.g., the framed field experiment during the French presidential elections by Baujard et al. (2012). A possible reason for this behavior is that these people prefer to abstain rather than expressing a random or a false opinion. Hillinger (2004, 2005) advocates for using the dis&approval rule for general elections under the term 'evaluative voting'. Lepelley and Smaoui (2012) study some of its properties. However the literature has not provided a characterization of this rule. We focus on providing one such characterization.

Young (1974) characterizes the Borda count with the help of four axioms. Fishburn (1978a, 1978b) provides the analogue axioms for ballot aggregation that characterize approval voting. Alós-Ferrer (2006) shows that one of Fishburn's axioms (neutrality) is unnecessary and provides a characterization[2] based on faithfulness, cancellation and consistency. Faithfulness requires that if the society consists of one individual, his or her approved candidate(s) is (are) selected. Cancellation requires that whenever all candidates receive the same number of approvals, the full set of candidates is selected. Consistency requires that whenever there are common selected candidates for two disjoint societies, those candidates that were selected for both of the original societies are exactly the candidates that are selected for the joint society. In this paper we provide a characterization of the dis&approval rule in similar terms. However we make note that the adapted version of cancellation to our context with three-option ballots is not sufficient for our purpose, and a stronger (under adapted faithfulness and consistency) requirement is introduced in its place, namely a 'compensation' property. This requires that if all candidates receive as many approvals as disapprovals, then the full set of candidates is selected.

This paper is organized as follows. Section 2 introduces the setting and notation. Section 3 provides a characterization of the dis&approval rule and the discussion about the necessity of replacing 'cancellation' by 'compensation'.

[2]For a review of the axiomations of approval voting see Xu (2010). More generally Ju (2010) provides a survey on "simple preferences", i.e., the case when individuals have either dichotomous or trichotomous preferences.

In Section 4 we check the independence of the axioms in our characterization. Section 5 concludes with some remarks.

2 Notation and terminology

Consider a fixed set $C = \{x_1, ..., x_m\}$ of $m \geqslant 2$ candidates and an electorate of n voters. Voters are asked to cast a (ternary) ballot T where, for each candidate, they can vote either "in favor", or "against", or "indifferent, abstain or do not know". Voters are warned that when a candidate receives a blank vote, the latter option is marked by default. Thus a voter's ternary ballot T on the fixed set of candidates can be represented by a 3-partition of C, and we denote by T^+ the candidates whom the voter approves, T^- are the candidates whom the voter disapproves, and T^0 are the remaining candidates (i.e., those who either leave the voter indifferent, or on whom the voter does not emit an opinion, or on whom the voter admits ignorance). We let T_D be the ternary ballot with $T^- = C$, and T_I is the ternary ballot with $T^0 = C$. A voter that casts T_I is called unconcerned. If a voter does not show up then she is unconcerned, i.e., her ternary ballot is T_I. Also, with each ternary ballot T we associate its reverse ballot $T_\pm$ such that $T^+_\pm = T^-$ and $T^-_\pm = T^+$ (thus $T^0_\pm = T^0$).

Let $\mathcal{T}$ denote the set of all ternary ballots on the set of candidates C. In line with the inspiring Alós-Ferrer (2006), an *anonymous voter response profile of ternary ballots* (or voter response profile for simplicity) is a mapping $\pi : \mathcal{T} \longrightarrow \mathbb{N}$. We interpret $\pi(T)$ as the number of voters that cast ballot T, thus the concept of a voter response profile incorporates anonymity. Clearly, one cannot recover the original list of ternary ballots from its induced voter response profile (of ternary ballots). The class of all voter response profiles is denoted by Π, thus we are allowing for variable electorate size and the number of voters in the electorate is $\sum_{T\in\mathcal{T}} \pi(T)$. Every $T \in \mathcal{T}$ can be identified with π^T such that $\pi^T(T) = 1$, $\pi^T(T') = 0$ if $T' \neq T, T' \in \mathcal{T}$. Summing up two voter response profiles corresponds to merging the vote profiles of two disjoint societies. With each $\pi \in \Pi$ we associate $\pi_\pm \in \Pi$ such that $\pi_\pm(T) = \pi(T_\pm)$ for all $T \in \mathcal{T}$.

Definition 1 *A ballot aggregation function on voter response profiles is a correspondence W that assigns a non-empty set of candidates to every $\pi \in \Pi$.*

Each W given by Definition 1 is anonymous since voter response profiles of ternary ballots capture the number of ballots of each type without any reference to specific voters. Furthermore, anonymous ballot aggregation functions on vote profiles (i.e., on the actual list of ballots cast by the electorate) can be defined from Definition 1 because vote profiles naturally induce voter response profiles.

The following tallies are defined for each voter response profile $\pi \in \Pi$ and $x_i \in C$: candidate x_i's number of supporters: $n_i^+(\pi) = \sum_{T\in\mathcal{T},\, x_i\in T^+} \pi(T)$, candidate x_i's number of rejecters: $n_i^-(\pi) = \sum_{T\in\mathcal{T},\, x_i\in T^-} \pi(T)$, and the number of indifferent voters towards candidate x_i: $n_i^0(\pi) = \sum_{T\in\mathcal{T},\, x_i\in T^0} \pi(T)$. Therefore for each electorate with n voters, $n_i^+(\pi) + n_i^0(\pi) + n_i^-(\pi) = n$ for every $x_i \in C$.

The following related remark will be of use to produce voter response profiles in Example 1 and in the proof of Theorem 1:

Remark 1 *For each m-tuple of integer triplets $((a_1, b_1, c_1), \cdots, (a_m, b_m, c_m))$ verifying $a_i + b_i + c_i = n$ for all $i = 1, ..., m$, there exists (at least) one voter response profile $\pi \in \Pi$ with n voters such that: $n_i^+(\pi) = a_i$, $n_i^-(\pi) = b_i$, $n_i^0(\pi) = c_i$ for all $i = 1, ..., m$. To see it, one can consider the ordered profile in which every candidate x_i is approved by the first a_i voters, disapproved by the next b_i voters, and considered as indifferent by the last c_i voters.*

3 A characterization of the dis&approval voting rule

The dis&approval rule is the ballot aggregation function W_D where $W_D(\pi)$ is the subset of C formed by the candidates x_i that maximize $v_i(\pi) = n_i^+(\pi) - n_i^-(\pi)$: $W_D(\pi) = \arg\max_{x_i \in C} v_i(\pi)$. That is to say, the rule aggregates the ternary ballots by adding up numbers (where 1 is for the 'approve' option, -1 is for the 'disapprove' option, and 0 is attached to the remaining cases as expressed in the 'abstain' option), and then it selects the candidates with highest score. Since this rule is invariant under strictly positive affine transformations, the result of the computation does not change if we replace the $(1, 0, -1)$ scale by e.g., $(2, 1, 0)$.[3] The first case is supported by Hillinger (2004, Section 3, and 2005), who claimed "It is a common experience that in addition to feeling positive or negative about candidates or issues, we may also feel neutral." The second one is the subject of a study by Baujard and Igersheim (2011).

It is easy to check that the dis&approval rule satisfies the three axioms that characterize the approval rule (v. Alós-Ferrer, 2006) when they are extended to the ternary ballots framework as we proceed to specify. For ternary ballots we use the same terms, adding a * to avoid a confusing and unnecessary multiplicity of terms. Consistency can be adapted without changes. It requires that whenever there are common selected candidates for two disjoint societies, those candidates that were selected for both of the original societies are exactly those who are selected for the joint society. Formally:

Consistency*: for each $\pi, \pi' \in \Pi$, if $W(\pi) \cap W(\pi') \neq \varnothing$ then $W(\pi + \pi') = W(\pi) \cap W(\pi')$.

In the binary ballots framework cancellation requires that whenever all candidates receive the same number of approvals, the full set of candidates is selected. In order to adapt it to the ternary ballots framework, we require that whenever all candidates receive the same number of approvals and the same number of disapprovals, then the full set of candidates is selected. Formally:

[3] There is however evidence that the *votes* vary with the precise specification that is conveyed to the voters. The first scale tends to produce fewer 'penalising' or 'disapproval' votes than the second one: see Baujard et al. (2012).

Cancellation*: for each $\pi \in \Pi$, if for all $x_i, x_j \in C$ it is true that $n_i^+(\pi) = n_j^+(\pi)$ and $n_i^-(\pi) = n_j^-(\pi)$ then $W(\pi) = C$.

In the binary ballots framework faithfulness requires that if the society consists of one individual then his or her approved candidate(s) is (are) selected. In the ternary ballots framework we have to state what happens in case the individual approves no candidate. Under such circumstance, in our version all candidates are selected if all candidates are disapproved, otherwise the candidates for whom the individual is indifferent are selected.[4] Formally:

Faithfulness*: for each $T \in \mathcal{T}$,

$$W(\pi^T) = W(T) = \begin{cases} T^+ & \text{if } T^+ \neq \varnothing, \\ C & \text{if } T^- = C, \\ T^0 & \text{otherwise.} \end{cases}$$

Nonetheless these three properties, namely, consistency*, cancellation* and faithfulness*, do not characterize the dis&approval voting. Although they are implied by this rule, they do not uniquely determine it. We demonstrate this in Example 1 below:

Example 1 *Define* $W^*(\pi)$ *as the subset of* C *formed by the candidates* x_i *that maximize* $u_i(\pi) = 2\,n_i^+(\pi) - n_i^-(\pi)$. *This expression defines a ballot aggregation function that satisfies consistency*, cancellation* and faithfulness*. However* W^* *is different from the dis&approval rule. Consider* $m = 2$ *and by Remark 1, let* $\pi \in \Pi$ *be such that* $n_1^+(\pi) = 1$, $n_1^-(\pi) = 0$, $n_1^0(\pi) = 5$, *and* $n_2^+(\pi) = n_2^-(\pi) = 3$, $n_2^0(\pi) = 0$. *Then* $W^*(\pi) = \{x_2\}$ *because* $u_1(\pi) = 2 < 3 = u_2(\pi)$, *but* $W_D(\pi) = \{x_1\}$ *because* $v_1(\pi) = 1 > 0 = v_2(\pi)$.

In order to characterize the dis&approval rule we replace cancellation* with an alternative property, that we refer to as compensation*. It states that if all candidates receive the same number of approvals and of disapprovals then the set of selected candidates is the whole set of candidates. Formally:

Compensation*: for each $\pi \in \Pi$, $n_i^+(\pi) = n_i^-(\pi)$ for all $x_i \in C$ implies $W(\pi) = C$.

Observe that if W is a ballot aggregation function that satisfies compensation* then $W(T + T_\pm) = C$ for all $T \in \mathcal{T}$. Proposition 1 below demonstrates that compensation* is stronger than cancellation* in the presence of consistency* and faithfulness*. Then Lemma 1 proves that if we assume compensation* and consistency*, then the distribution of votes other than approved/disapproved does not affect the outcomes of W.

Proposition 1 *If a ballot aggregation function* W *satisfies consistency*, faithfulness*, and compensation*, then it satisfies cancellation*.*

[4]This is in line with the role of this property in the characterization by Alós-Ferrer (2006), as explained in his Footnote 3.

Proof. Suppose $\pi \in \Pi$ is such that for all $x_i, x_j \in C$ it is true that $n_i^+(\pi) = n_j^+(\pi)$ and $n_i^-(\pi) = n_j^-(\pi)$. We need to prove that $W(\pi) = C$. If $n_i^+(\pi) = n_i^-(\pi)$ for all $x_i \in C$ then $W(\pi) = C$ by compensation*. If $n_i^+(\pi) \neq n_i^-(\pi)$ for each $x_i \in C$ we can assume that $n_i^+(\pi) = n_i^-(\pi) + a$ for some fixed $a \geqslant 1$ and all $x_i \in C$, since the case $n_i^+(\pi) < n_i^-(\pi)$ is analogous. Define π^{a-} such that $\pi^{a-}(T_D) = a$, $\pi^{a-}(T) = 0$ when $T \neq T_D$. By faithfulness* we have $W(\pi^{a-}) = C$, and compensation* implies that $W(\pi + \pi^{a-}) = C$ as $n_i^+(\pi + \pi^{a-}) = n_j^-(\pi + \pi^{a-})$ for all $x_i, x_j \in C$. Then consistency* entails that $W(\pi) = C$ as $W(\pi + \pi^{a-}) = W(\pi) \cap W(\pi^{a-})$. This ends the proof. □

Lemma 1 *Suppose that a ballot aggregation function W satisfies compensation* and consistency*. Then, for each $\pi, \pi' \in \Pi$, if for all $x_i \in C$ it is true that $n_i^+(\pi) = n_i^+(\pi')$ and $n_i^-(\pi) = n_i^-(\pi')$, then $W(\pi) = W(\pi')$.*

Proof. Suppose $\pi, \pi' \in \Pi$ are such that for all $x_i \in C$ it is true that $n_i^+(\pi) = n_i^+(\pi')$ and $n_i^-(\pi) = n_i^-(\pi')$. Compensation* yields $W(\pi + \pi_\pm) = C = W(\pi' + \pi_\pm)$. Consistency* yields $W(\pi) = W(\pi + \pi' + \pi_\pm) = W(\pi')$. □

Lemma 1 ensures that unconcerned voters do not affect the outcomes of W, in the sense that for each $\pi \in \Pi$, $W(\pi) = W(\pi + T_I)$.

Theorem 1 *The dis&approval rule is the only ballot aggregation function satisfying compensation*, faithfulness*, and consistency*.*[5]

Proof. We need to prove that if W is a ballot aggregation function that satisfies compensation*, faithfulness*, and consistency* then it coincides with the dis&approval rule W_D. Using Lemma 1 we deduce that for such purpose any $\pi \in \Pi$ is characterized by the m-tuple of triplets

$$(\,(n_1^+(\pi), n_1^-(\pi), n_1^0(\pi)),\ \dots,\ (n_m^+(\pi), n_m^-(\pi), n_m^0(\pi))\,)$$

We proceed to prove that for each $\pi \in \Pi$, $W(\pi)$ is the set of candidates x_i such that $v_i(\pi) = n_i^+(\pi) - n_i^-(\pi)$ is highest. Let us fix $\pi \in \Pi$.

From consistency*, we get $W(\pi) = W(\pi')$ with $\pi' = \pi + \pi = 2\pi$. Observe that $n_i^0(\pi')$ is an even integer for each $i = 1, ..., m$. Let $\alpha = \max\{n_i^0(\pi') : 1 \leqslant i \leqslant m\}$. We invoke Remark 1 to produce some auxiliary voter response profiles as follows.

Firstly, let $\tilde{\pi}$ be a voter response profile characterized by: for each $i = 1, ..., m$, $n_i^+(\tilde{\pi}) = \frac{n_i^0(\pi')}{2}$, $n_i^-(\tilde{\pi}) = \frac{n_i^0(\pi')}{2}$ and $n_i^0(\tilde{\pi}) = \alpha - n_i^0(\pi')$. From compensation* we obtain $W(\tilde{\pi}) = C$ and consistency* ensures

$$W(\pi) = W(\pi') = W(\pi' + \tilde{\pi}). \tag{1}$$

[5] We are indebted to an anonymous referee for providing the proof of Theorem 1 that we present here, which is more direct than our original argument. Furthermore, we stress that working with ballot aggregation functions on voter response profiles implicitly builds anonymity into the framework.

Secondly, when $\alpha > 0$ we first let $\bar{\pi}$ be a voter response profile characterized by: for each $i = 1, ..., m$, $n_i^+(\bar{\pi}) = 0$, $n_i^-(\bar{\pi}) = 0$ and $n_i^0(\bar{\pi}) = \alpha$. Then we let π_0 be a voter response profile characterized by: for each $i = 1, ..., m$, $n_i^+(\pi_0) = n_i^+(\pi') + \frac{n_i^0(\pi')}{2}$, $n_i^-(\pi_0) = n_i^-(\pi') + \frac{n_i^0(\pi')}{2}$ and $n_i^0(\pi_0) = 0$. We have $\pi' + \tilde{\pi} = \pi_0 + \bar{\pi}$ and from compensation* (or faithfulness*), $W(\bar{\pi}) = C$. Hence consistency* together with (1) yield $W(\pi) = W(\pi_0)$. When $\alpha = 0$ we just let $\pi_0 = \pi'$ thus $W(\pi) = W(\pi_0)$ holds true due to (1).

Because $n_i^+(\pi') - n_i^-(\pi') = 2\left(n_i^+(\pi) - n_i^-(\pi)\right) = n_i^+(\pi_0) - n_i^-(\pi_0)$, in order to prove that $W(\pi)$ is the set of candidates x_i such that $v_i(\pi)$ is maximal among C, it suffices to show that $W(\pi_0)$ is the set of candidates x_i for which $v_i(\pi_0) = n_i^+(\pi_0) - n_i^-(\pi_0)$ is maximal. To prove this we distinguish two cases.

Case 1: $n_i^+(\pi_0) = 0$ for every x_i. Every candidate x_i maximizes $v_i(\pi_0)$ because $n_i^+(\pi_0)$ is constant across agents. Furthermore, all agents are disapproved by every voter, hence by faithfulness* and consistency* (or more directly, by compensation*), we obtain $W(\pi_0) = C$.

Case 2: $n_i^+(\pi_0) > 0$ for some $x_i \in C$. Let $\beta = \min\{n_i^-(\pi_0) : 1 \leqslant i \leqslant m\}$. When $\beta = 0$ we let $\pi_0' = \pi_0$. Otherwise let π_0' be a voter response profile characterized by $n_i^+(\pi_0') = n_i^+(\pi_0)$, $n_i^-(\pi_0') = n_i^-(\pi_0) - \beta$ and $n_i^0(\pi_0') = 0$ for each $i = 1, ..., m$, and let $\widehat{\pi}$ be a voter response profile characterized by $n_i^+(\widehat{\pi}) = n_i^0(\widehat{\pi}) = 0$ and $n_i^-(\widehat{\pi}) = \beta$ for each $i = 1, ..., m$,. When $\beta > 0$, from faithfulness* and consistency* we have $W(\widehat{\pi}) = C$, and because $\pi_0 = \pi_0' + \widehat{\pi}$, consistency* yields $W(\pi_0) = W(\pi_0')$. Irrespective of the value of β, observe that

$$W(\pi_0') = \bigcap_{\pi_0'(T) > 0} W(T) \tag{2}$$

because there is x_i with $x_i \in T^+$ whenever $\pi_0'(T) > 0$, hence consistency* applies.

By construction, a candidate x_i maximizes $v_i(\pi_0)$ along C if and only if $n_i^-(\pi_0') = 0$. Therefore x_i maximizes $v_i(\pi_0)$ along C if and only if $x_i \in T^+$ whenever $\pi_0'(T) > 0$. By appealing to faithfulness*, x_i maximizes $v_i(\pi_0)$ along C if and only if $x_i \in W(T)$ whenever $\pi_0'(T) > 0$. Now (2) proves that x_i maximizes $v_i(\pi_0)$ along C if and only if $x_i \in W(\pi_0')$. □

4 Independence of the axioms

In order to prove that the axioms in Theorem 1 are independent, we first observe that Example 1 verifies faithfulness*, consistency*, but not compensation*. Furthermore, the trivial rule $W_t(\pi) = C$ for each $\pi \in \Pi$ verifies consistency*, compensation*, but not faithfulness*. We complete the argument with the following example that proves that consistency* is independent of the conjunction of faithfulness* and compensation*:

Example 2 *Define a ballot aggregation function W_t^* as follows: the full set of candidates is always elected unless there is a single voter, in which case the rule is built to satisfy faithfulness*. In formal terms, for each $\pi \in \Pi$*

$$W_t^*(\pi) = \begin{cases} T_1^+ & \text{if } \sum_{T \in \mathcal{T}} \pi(T) = 1, \pi(T_1) = 1, \text{ and } T_1^+ \neq \varnothing, \\ T_1^0 & \text{if } \sum_{T \in \mathcal{T}} \pi(T) = 1, \pi(T_1) = 1, T_1^+ = \varnothing, \text{ and } T_1^0 \neq \varnothing, \\ C & \text{otherwise.} \end{cases}$$

Notice that W_t^* verifies compensation: provided $n_i^+(\pi) = n_i^-(\pi)$ for all $x_i \in C$, $W_t^*(\pi) = C$ arises both when the number of voters is greater than 1 (by construction) or exactly 1 (because the only possible case is that of a single unconcerned voter).*

However W_t^* does not verify consistency. Consider the ballot T_2 such that $T_2^+ = \{x_1\}$ and $T_2^0 = C \setminus \{x_1\}$, and let π be given by $\pi(T_2) = 2$, $\pi(T) = 0$ when $T \neq T_2$. Then we have $W_t^*(\pi^{T_2}) \cap W_t^*(\pi^{T_2}) \neq W_t^*(\pi^{T_2} + \pi^{T_2})$, because $W_t^*(\pi^{T_2}) \cap W_t^*(\pi^{T_2}) = \{x_1\}$ and $W_t^*(\pi^{T_2} + \pi^{T_2}) = W_t^*(\pi) = C$.*

5 Conclusion

The properties in our characterization of the dis&approval rule suffice to discuss on its virtues and faults, but of course they do not exhaust the list of its relevant attributes. They have further implications like cancellation and others, and knowing them helps to understand the normative behavior of the rule. To conclude let us review some of other classical properties. Dis&approval voting satisfies neutrality: the names of the candidates do not matter. It also satisfies unanimity: if all voters vote in favor of some candidate(s), then this (these) candidate(s) should be selected. Independence of irrelevant alternatives also holds true: if a non-selected candidate is removed from the list of candidates and this does not affect the voters' ballots for the remaining candidates, then the selected candidates do not change.

Furthermore, dis&approval voting satisfies some practical properties advanced by Brams and Fishburn (2005, p. 461) as arguments in favor of approval voting. Approval voting gives more flexible options than plurality voting, increases voter turnout, gives minority candidates their proper due, and is as eminently practicable. Dis&approval voting still enriches the options offered by approval voting by allowing voters to explicitly express disagreement with some (or all) candidates. If voters are better able to express their preferences they are more likely to vote. As under approval voting, minority candidates will not suffer. If supporters are allowed to vote for several candidates, they will not be tempted to desert a candidate who is weak in the polls (the so-called wasted vote). Finally, dis&approval voting is also simple for voters to understand and use. The thumb up or thumb down vote in Community Question Answering sites such as Yahoo! Answers illustrates that it is easy to ask users to cast a positive or negative vote (along with the possibility of abstaining).

Another interesting property stems from a practice explicitly used in countries such as UK, Australia, or Canada: pairing. The glossary of the UK Parliament defines pairing[6] as "an arrangement where one MP from a party agrees with an MP of an opposing party not to vote in a particular division. This gives both MPs the opportunity not to attend. [...] pairings can last for months or years." Formally, a voting rule W is not affected by pairing if for each $\pi \in \Pi$ and $T \in \mathcal{T}$, $W(\pi + T + T_{\pm}) = W(\pi)$. It is not difficult to check that dis&approval voting is not affected by pairing (because it both satisfies consistency and compensation).

In our view the most important practical property of dis&approval voting is that it explicitly allows voters to express dissatisfaction. If dissatisfaction with politicians is often observed in polls, no legitimate and explicit negative option is generally provided to electors. There are some exceptions. In the State of Nevada voters can express their disapproval of all official candidates with the "none of the above candidate" option (Arcelus, Mauser and Spindler, 1978). In 1987 the deputies elections were reorganized in the former Soviet Union. Under the new rule,[7] voters crossed off the names of those against whom they wished to vote (Hahn, 1988). The same rule is used in some Chinese village elections (Zhong and Chen, 2002). For some other historical examples of rules that include negative options, see Kang (2010). With an exception of the above examples the usual ways to voice in elections are absenteeism, spoiled or blank vote, or voting for an unviable candidate. These latter practices may be reduced under dis&approval voting, an hypothesis that would be worth testing.

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[7] If all voters have a clear-cut (i.e., positive or negative) opinion on each candidate, the rule that asks voters to cross out the names of candidates they do not want and then selects the candidates with the least number of crosses is equivalent to approval voting: candidates who are not crossed off the list are the approved candidates. It is not rare that voters are indifferent or even lack the sufficient knowledge to express a positive or a negative opinion on some candidates. If this is the case, then the two rules differ because the non disapproved candidates consist of the approved candidates and those who leave the voter indifferent.

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