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# Towards Intelligent PID Control

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October 1989

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# Towards Intelligent PID Control

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*Abstract.* Auto-tuners for PID controllers have now been commercially available for a few years. These controllers are automating the task normally performed by an instrument engineer. The auto-tuners include some technique for extracting process dynamics from experiments and some control design method. They may even be able to select to use PI or PID control. For a higher degree of automation it is desirable to also automate tasks normally performed by process engineers. To do so it is necessary to provide the controllers with reasoning capability. This seems technically feasible with the increased computing power that is now available in single loop controllers. This paper describes a PID controller with such reasoning capabilities.

## 1. Introduction

In the design of an intelligent or knowledge based feedback controller (Åström et al 1986, Årzén 1987, 1988), it would be desirable to incorporate the expert knowledge of design engineers so that it can make decisions on the choice of control algorithm and provide diagnostics on the effectiveness of the control system. A system with such facilities would make the task of the operator and the instrument engineer more interesting. It would also make it possible for the instrument engineer to improve the reasoning of the system. For real-time implementation it would also be desirable to have as much deep knowledge as possible, in place of large number of possibly conflicting rules. It would also be desirable that the controller to a limited degree could explain its own reasoning, e.g. why derivative action was used. It should also be able to tell if PID is appropriate in the particular case and possibly also suggest alternative control schemes.

In this paper we attempt to develop formal tools to assess what can be achieved by PID control of a class of systems with the Ziegler-Nichols tuning formula and to characterize a class of systems where PID control is appropriate. Based on empirical results and approximate analytical study, we introduce two numbers, namely the normalized dead time  $\theta$  and the normalized process gain  $\mathcal{K}$ , to characterize the open loop process dynamics and two numbers, the peak load error  $\lambda$  and the normalized rise time  $\tau$ , to characterize the closed loop response. Simple methods of measuring these parameters are proposed. It is shown that  $\theta$  and  $\mathcal{K}$  are related and either of them can be used to predict the achievable performance

of PID controllers tuned by the Ziegler-Nichols formula. Using these relations the intelligent controller can thus interact with the operator and advise on choice of control algorithm.

We have established useful relations, which can be used to assess whether the PID controller is properly tuned. The simplicity of the relations allows the development of a first generation of intelligent controllers using current technology. Significant insight into the properties and heuristic aspects of PID control is gained. Such knowledge can be formally discussed and further refined. It is believed that the approach can be extended to other classes of systems and this is a topic of current research.

The paper is organized as follows. The restricted class of processes that we are concerned with is introduced in Section 2. Some useful dimensionless numbers are introduced in Section 3. In Sections 4 and 5 some relations between the features are derived by approximate analysis and empirical refinement based on simulation. The results are used in Section 6 to discuss the performance that may be achieved with PID control based on Ziegler-Nichols tuning. Some characteristics are given in Section 7.

## 2. Process Characteristics

It is assumed that the process dynamics is linear and stable. The characteristics will be further restricted both in the time and the frequency domain. Feedback with simple controllers only is considered.

### Time Domain Characterization

It will be assumed that the step response is monotone or essentially monotone, i.e. monotone except for a small initial part. Such systems can be characterized by the parameters  $K_p$ ,  $L$ , and  $T$ , where  $K_p$  is the static process gain,  $L$  is the apparent dead time and the  $T$  is the apparent time constant. The parameters  $T$  and  $L$  are obtained by graphical construction, where the tangent is drawn in the inflection point of the step response. The transfer function

$$G(s) = K_p \frac{e^{-sL}}{1 + sT} \quad (1)$$

is a crude analytic approximation of the the transfer function of the class of processes that we are considering. Notice however that the transfer functions considered are not restricted to this class.

The class of systems considered is the same as that used in the classical works on Ziegler-Nichols tuning. There are important classes of systems that are excluded, e.g., systems having integrators and systems with resonant poles. Systems having integrators may have monotone step responses but they are not stable. Systems with resonant poles do not have essentially monotone step response.

### Frequency Domain Characterization

A different frequency domain characterization of process dynamics will also be introduced. It is assumed that the Nyquist curve is monotone or essentially monotone, i.e. both the phase and the amplitude are monotone functions of the frequency. The first intersection with the negative real axis defines the ultimate frequency,  $\omega_u$ , and the ultimate gain,  $K_u$ . Lack of monotonicity can be accepted at frequencies, where the phase shift is larger than  $180^\circ$ .

## 3. Features

Dimension-free parameters, like Reynold's numbers, have found much use in many branches of engineering. They have however not been much used in automatic control. In this section it is attempted to introduce some numbers that are useful in assessing control system performance.

### Normalized Dead-time

The normalized dead-time is defined as the ratio of the apparent dead-time and the apparent time constant, or formally

$$\theta = \frac{L}{T} \quad (2)$$

This number is thus easily obtained from a record of the step response. It has been known from practical experience that the normalized dead-time may be used as a measure of the difficulty of controlling a process. Processes with a small  $\theta$  are easy to control and processes with a large  $\theta$  are difficult to control.

The parameter  $\theta$  was actually called the *controllability ratio* by Deshpande and Ash (1981). Fertik (1975) introduced the name *process controllability* for the quantity  $\theta/(1 + \theta)$ . To avoid possible confusion with the standard terminology of modern control theory we will use the word *normalized dead time*.

### Normalized Process Gain

The process gain  $K_p$  is not dimension-free. It can however be made dimension free by multiplication with a suitable controller gain. The ultimate gain  $K_u$ , i.e., the controller gain that makes the process unstable under proportional feedback control, is a suitable normalization factor. The normalized process gain,  $\mathcal{K}$ , can be defined as

$$\mathcal{K} = K_p K_u \quad (3)$$

This number is easily obtained from the Nyquist curve. It also has a physical interpretation as the largest process loop gain that can be achieved under proportional control. The number is useful to assess the control performance. Roughly speaking, a large value indicates that the process is easy to control while a small value indicates that the process is difficult to control.

The normalized process gain is directly obtained from a Nyquist curve of the process. It can also be obtained from an experiment with relay feedback, see Åström and Hägglund (1984).

Since the processes we consider are stable they have a static error under proportional feedback. The static error obtained for a unit step command

$$e_s = \frac{1}{1 + K_p K_c} > \frac{1}{1 + \mathcal{K}} \quad (4)$$

where  $K_c$  is the proportional gain used. The inequality follows because  $K_p K_c < \mathcal{K}$ . The number  $\mathcal{K}$  can thus be used to estimate the static error achievable under proportional control and also to determine if integral action is required to satisfy the specifications on static error.

### Peak Load Disturbance Error

The response to step load disturbances is an important factor when evaluating control systems. The effect of a load disturbance depends on where the disturbance acts on the system. In this section it will be assumed that the disturbance acts on the process input. With a controller without integral action a unit step disturbance in the load gives the static error

$$e_l = \frac{K_p}{1 + K_u K_c} > \frac{K_p}{1 + \mathcal{K}} \quad (5)$$

The quantity  $e_l/K_p$  is dimension-free.

When a controller with integral action is used the static error due to a step load disturbance is zero. A meaningful measure is then the maximum error due to a load disturbance. To obtain a dimension-free quantity it is also divided by the process gain. The following variable is thus obtained

$$\lambda = \frac{1}{l_0 K_p} \max e(t) \quad (6)$$

where  $l_0$  is the amplitude of the step disturbance.

### Normalized Closed Loop Rise Time

The closed loop rise time is a measure of the response speed of the closed loop system. Again, to obtain a dimension-free parameter it will be normalized by the apparent dead time  $L$  of the open loop system. The parameter is thus

$$\tau = \frac{t_r}{L} \quad (7)$$

## 4. Empirics

The Ziegler-Nichols closed-loop tuning procedure was applied to a large number of different processes. It was attempted to correlate the observed properties of the open and closed loop systems to the features introduced in Section 3. In this section we will present the empirical results. Processes with the transfer functions

$$G_1(s) = \frac{e^{-sD}}{(1+s)^2} \quad (8)$$

$$G_2(s) = \frac{1}{(1+s)^n}, \quad 3 < n < 20 \quad (9)$$

$$G_3(s) = \frac{1-\alpha s}{(1+s)^3}, \quad 0 < \alpha < 2.5 \quad (10)$$

will be investigated. These models cover a wide range of dynamic characteristics such as pure dead-time and nonminimum phase response. The main features of the models are summarized in Åström et al (1988).

The normalized apparent dead-time was measured from the step responses. The ultimate gain was determined by simulation. Parameters of PID regulators were determined by a straight forward application of the Ziegler-Nichols closed-loop method without fine tuning, i.e. with values of proportional gain  $K_c$ , integral time  $T_i$  and derivative time  $T_d$  set as  $0.6K_u$ ,  $0.5T_u$ , and  $0.125T_u$  respectively. The closed loop performance is judged based on the closed loop step and load responses.

The results obtained are summarized in Tables 1-3. The tables give a parameter that characterizes the process, the ultimate period  $T_u$ , the overshoot  $os$ , the undershoot  $us$  of the closed loop step response, the apparent normalized dead-time  $\theta = L/T$ , the normalized loop gain  $\mathcal{K}$ , the normalized closed loop rise time  $\tau = t_r/L$ , and the normalized peak load error  $\mathcal{K}\lambda$ .

The results for the first process are summarized in Table 1. The closed loop behavior was judged to be satisfactory for  $0.15 < \theta < 0.6$ . The overshoot for  $\theta$  in the low range is too high. This is however easily reduced by using the setpoint weighting factor modification, see Åström and Hägglund (1988). For large values of  $\theta$  there is a pronounced undershoot in the step response. The results for the second process are summarized in Table 2. The closed loop behavior was judged to be satisfactory for  $0.22 < \theta < 0.64$ . The overshoot for  $\theta$  in the low range is too high. This is however easily reduced by using the setpoint

| $D$ | $T_u$ | $os$ | $us$ | $\theta$ | $\mathcal{K}$ | $\tau$ | $\mathcal{K}\lambda$ |
|-----|-------|------|------|----------|---------------|--------|----------------------|
| 0.1 | 1.4   | 75   | 26   | 0.15     | 21            | 0.80   | 1.26                 |
| 0.2 | 2.0   | 60   | 14   | 0.19     | 10.5          | 0.95   | 1.57                 |
| 0.4 | 2.8   | 50   | 5    | 0.26     | 5.7           | 1.0    | 1.53                 |
| 0.6 | 3.6   | 35   | 2    | 0.34     | 4.0           | 0.94   | 1.48                 |
| 1.0 | 4.8   | 26   | 3    | 0.49     | 2.7           | 1.02   | 1.40                 |
| 1.5 | 6.0   | 19   | 9    | 0.69     | 2.0           | 0.93   | 1.35                 |
| 2.0 | 7.2   | 14   | 14   | 0.89     | 1.7           | 0.85   | 1.26                 |
| 2.5 | 8.3   | 12   | 17   | 1.09     | 1.5           | 0.82   | 1.28                 |
| 3.0 | 9.4   | 20   | 20   | 1.26     | 1.4           | 0.79   | 1.25                 |

Table 1. Experimental results for a system with the  $G(s) = e^{-sD}/(s+1)^2$ .

weighting factor modification. For large values of  $\theta$  there is a pronounced undershoot in the step response. Similar results are obtained for the third process as summarized in Table 3.

| $n$ | $T_u$ | $os$ | $us$ | $\theta$ | $\mathcal{K}$ | $\tau$ | $\mathcal{K}\lambda$ |
|-----|-------|------|------|----------|---------------|--------|----------------------|
| 3   | 3.7   | 50   | 13   | 0.22     | 8.0           | 1.07   | 1.52                 |
| 4   | 6.0   | 40   | 10   | 0.32     | 4.0           | 1.16   | 1.40                 |
| 6   | 10.6  | 26   | 11   | 0.49     | 2.4           | 1.14   | 1.30                 |
| 8   | 14.6  | 17   | 14   | 0.64     | 1.88          | 1.08   | 1.18                 |
| 10  | 18.8  | 12   | 17   | 0.76     | 1.60          | 0.96   | 1.18                 |
| 15  | 29.0  | 0    | 24   | 1.05     | 1.36          | 0.9    | 1.16                 |
| 20  | 39.0  | 5    | 30   | 1.28     | 1.25          | 0.8    | 1.14                 |

Table 2. Experimental results for a system with the  $G(s) = 1/(s+1)^n$ .

| $\alpha$ | $T_u$ | $os$ | $us$ | $\theta$ | $\mathcal{K}$ | $\tau$ | $\mathcal{K}\lambda$ |
|----------|-------|------|------|----------|---------------|--------|----------------------|
| 0        | 3.7   | 50   | 13   | 0.22     | 8             | 1.15   | 1.52                 |
| 0.1      | 3.8   | 50   | 15   | 0.23     | 6.2           | 1.09   | 1.49                 |
| 0.25     | 4.3   | 48   | 11   | 0.28     | 4.5           | 1.09   | 1.44                 |
| 0.5      | 5.0   | 38   | 3.8  | 0.38     | 3.2           | 1.16   | 1.41                 |
| 1.0      | 6.0   | 21   | 3.8  | 0.58     | 2.0           | 0.98   | 1.34                 |
| 1.5      | 6.5   | 9.6  | 7.7  | 0.76     | 1.45          | 0.89   | 1.30                 |
| 2.0      | 7.0   | -1.9 | 16   | 0.98     | 1.15          | 0.84   | 1.24                 |

Table 3. Experimental results for a system with the  $G(s) = (1-\alpha s)/(s+1)^3$ .

## 5. Relations

We have thus introduced two normalized numbers, namely the normalized dead-time  $\theta$  and the normalized process gain  $\mathcal{K}$ , to characterize the open loop dynamics and two numbers, the peak load error  $\lambda$  and the normalized closed loop rise time  $\tau$  to characterize the closed loop response. Some relations between these numbers will now be established. In doing so we will also develop an intuitive feel for the meaning of the numbers.

### Normalized Dead-time and Process Gain

As can be seen from Tables 1-3 there appears to be a relation between normalized process gain  $\mathcal{K}$  and normalized dead time  $\theta$ . For specific systems it is possible to find the relations exactly, see Åström et

al (1988). For first order systems with dead time we have:

$$\theta = \frac{\omega_u L}{\omega_u T} = \frac{\pi - \arctan \sqrt{\mathcal{K}^2 - 1}}{\sqrt{\mathcal{K}^2 - 1}} \quad (11)$$

See Åström et al (1988). This relation is shown graphically in Figure 1.

It is possible to find exact expressions for the relations between  $\mathcal{K}$  and  $\theta$  for the processes given by equations (8), (9) and (10). They can also be obtained experimentally as discussed in Section 4. The relations are shown in Figure 1. The graphs indicate that for processes with higher order dynamics the product  $\mathcal{K}\theta$  is approximately constant. This is important because it means that the normalized process gain  $\mathcal{K}$  can be used instead of the normalized dead time  $\theta$  to assess achievable performance.

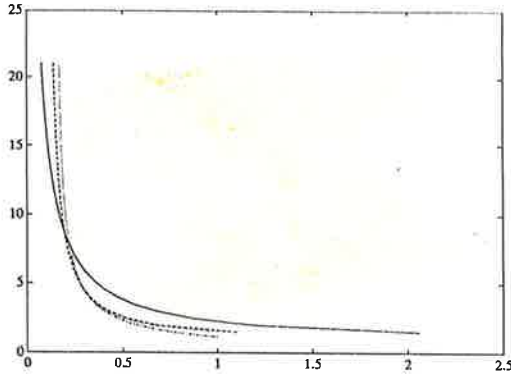


Figure 1. The normalized process gain  $\mathcal{K}$  as a function of apparent normalized dead time  $\theta$  for systems (1), (8), (9), and (10).

### Peak Load Error and Normalized Dead-time

Consider the closed loop system obtained with the process and the controller. Assume that the disturbance enters at the plant input with PID control and Ziegler-Nichols tuning. The transfer function from the load disturbance to the output is

$$G_d(s) = \frac{1}{G_c(s)} \frac{G_p(s)G_c(s)}{1 + G_p(s)G_c(s)} \quad (12)$$

A PID controller with Ziegler/Nichols tuning has the transfer function

$$G_c(s) = \frac{K_c(s + \alpha)^2}{2\alpha s}$$

where

$$\alpha = \frac{1}{2T_d} = \frac{4}{T_u}$$

This choice gives good rejection of load disturbances as discussed by Hang (1989). With Ziegler-Nichols tuning the closed loop system has a bandwidth  $\omega \approx 7.4/T_u$ . The transfer function (12) can then be approximated by

$$G_d(s) \approx \frac{1}{G_r(s)} = \frac{2\alpha s}{K_r(s + \alpha)^2}$$

The corresponding unit step response is

$$H(t) = \frac{2\alpha t}{K_r} e^{-\alpha t}$$

It has a maximum

$$\frac{2}{eK_r} = \frac{0.74}{K_r} = \frac{1.23}{K_u} \quad (13)$$

at

$$t = 2T_d$$

We thus find that the parameter  $\mathcal{K}\lambda$  can be expected to be constant. This is also supported by the experimental results given in Tables 1-3 which gives

$$\mathcal{K}\lambda \approx 1.3 \quad (14)$$

The knowledge of  $\lambda$  can be used by an intelligent controller to check if a PID controller with Ziegler-Nichols tuning can be used to satisfy the given specifications to peak load error. From the analysis we also find that the peak error occurs  $T_u/4$  time units after the step disturbance is applied.

### Closed Loop Rise Time

The experimental results given in Tables 1-3 show that the normalized rise time is approximately constant. Hence

$$\tau \approx 1 \quad (15)$$

In physical terms this implies that  $t_r \approx L$ , compare with equation (7). This means that the Ziegler-Nichols method gives a closed loop system with a rise time approximately equal to the apparent dead-time of the open loop system.

## 6. Ziegler-Nichols Tuning

The results obtained will now be used to evaluate PID controllers with Ziegler-Nichols tuning. We can first observe that the Ziegler-Nichols tuning procedure is very simple. It is based on a simple characterization of the process dynamics, either parameters  $a$  and  $L$  from the step response or the critical point on the Nyquist curve parameterized in  $K_u$  and  $\omega_u$ . We have also obtained two relations  $\tau \approx 1$  and  $\mathcal{K}\lambda \approx 1.3$  which characterizes the closed loop performance. The condition  $\tau \approx 1$  implies that Ziegler-Nichols tuning tries to make the closed loop rise time equal to the apparent dead-time.

### When can Ziegler-Nichols Tuning be used?

The results obtained show that Ziegler-Nichols tuning will give good results under certain conditions and that these conditions can be characterized by one parameter,  $\theta$ , or  $\mathcal{K} = K_u K_p$ .

The results are summarized in Table 4. Four cases are introduced in the table. They are classified as follows:

**Case 1,  $\theta < 0.15$  or  $\mathcal{K} > 20$ :** Ziegler-Nichols tuning may not give the best results in this case. The reason is that it is possible to use comparatively high loop gains. There are many possible choices of controllers. A P or PD controller may be adequate if the requirements on static errors are not too stringent. A proportional controller could be chosen if a static error around 10% is tolerable. (This estimate is based on the assumption that the controller gain is half of the ultimate gain). If smaller static errors are required it is necessary to use integral action. In some cases performance can be increased significantly by using derivative action or even more complicated control laws. Temperature control where the dynamics is dominated by one large time constant is a typical case. We have observed that the derivative time  $T_d = T_i/4$  obtained by the Ziegler-Nichols rule is too large in this case. It gives a long tail in the step response; a better value is  $T_d = T_i/8$ .  $\square$

**Case 2,  $0.15 < \theta < 0.6$  or  $2 < \mathcal{K} < 20$ :** This is the prime application area for PID controllers with Ziegler-Nichols tuning. It works well in this case. Derivative action is often very helpful.  $\square$

**Case 3,  $0.6 < \theta < 1$  or  $1.5 < \mathcal{K} < 2$ :** When  $\theta$  approaches 1 Ziegler-Nichols tuning becomes less useful. This is easy to understand if we recall that the tuning procedure tries to make closed loop rise time equal to the apparent dead time. It is difficult to achieve tight control with Ziegler-Nichols tuned PID controllers. Other tuning methods and other controller structures like Smith predictors, pole placement, or feedforward could be considered.  $\square$

**Case 4,  $\theta > 1$  or  $\mathcal{K} < 1.5$ :** PID control based on Ziegler-Nichols tuning is not recommended when  $\theta$  is larger than 1. The reason why PID controllers work so poorly for  $\theta > 0.6$  is partly due to inherent limitations of PID controllers and partly due to the Ziegler-Nichols tuning procedure. Modifications of the Ziegler-Nichols rule were proposed by Cohen-Coon (1953). By choosing other tuning methods it is however possible to tune PID controllers to work satisfactorily even for  $\theta = 10$ , see Åström (1988).  $\square$

A parallel effort by Hang and Åström (1988) has gone further than merely using  $\theta$  to predict the effectiveness of the Ziegler-Nichols tuning formula. The following modification to eliminate manual fine tuning has been recommended. When  $\theta < 0.6$  the main drawback of the Ziegler-Nichols formula

is excessive overshoot. This can be overcome by setpoint weighting where the weighting factor is a simple function of  $\theta$ . When  $\theta > 0.6$  the integral time computed by the Ziegler-Nichols formula needs to be modified by a factor which again can be expressed as a simple function of  $\theta$ . These modifications are essential to obtain high quality PID control without manual fine tuning.

Table 4 indicates that a broad classification of Ziegler-Nichols tuned PID controllers can be made based on the normalized dead-time. This observation is useful if we try to build control systems with decision aids where the instrument engineer or the operator is advised also on controller selection.

### Implications for Smart Controllers

There are several simple auto-tuners that are based on the Ziegler-Nichols tuning procedure. A drawback with them is that they are unable to reason about the achievable performance. The result of this paper indicates that there is a simple modification. By determining one of the parameters  $\theta$  or  $\mathcal{K}$  it is thus a simple matter to provide facilities so that a simple auto-tuner can select the controller form P, PI, or PID and also give indications if a more sophisticated control law would be useful. For an auto-tuner based on the transient method this can be achieved by determining not only  $a$  and  $L$  but also  $K_p$  and including a logic based on Table 4. For relay based auto-tuners it is necessary to complement the determination of  $\omega_u$  and  $K_u$  with determination of  $K_p$ . This can easily be made from measurement of average values of inputs and outputs in steady state operation. It is also possible to modify the relay tuning so that the static gain is also determined. The accuracy of the tuning formula over a wide range of  $\theta$ -values can be markedly improved by the use of the correlation formula of Hang and Åström (1988) as discussed above.

### On-line Assessment of Control Performance

The results of this paper can also be used to evaluate performance of feedback loops under closed loop operation. Consider, e.g., the relation (15) for the normalized rise time. The rise time can be measured when the set point is changed. If the controller is properly tuned then the closed loop rise time should be equal to the apparent dead time. If

| $\theta$                 | Tight Control is Not Required | Tight Control is Required |                      |   |
|--------------------------|-------------------------------|---------------------------|----------------------|---|
|                          |                               | High Measurement Noise    | Low Saturation Limit | Low Measurement Noise and High Saturation Limit |
| Class I $< 0.15$         | P                             | PI                        | PI or PID            | P or PI   |
| Class II $0.15 \sim 0.6$ | PI                            | PI                        | PI or PID            | PID   |
| Class III $0.6 \sim 1$   | I or PI                       | I + A                     | PI + A               | PI or PID + A + C                               |
| Class IV $> 1$           | I                             | I + B + C                 | PI + B + C           | PI + B + D                                      |

Table 4. A: Feedforward compensation recommended, B: Feedforward compensation essential, C: Dead-time compensation recommended, D: Dead-time compensation essential.



the actual rise time is significantly different, say 50% larger, it indicates that the loop is poorly tuned. This type of assessment is particularly useful when the damping is adequate but it is not certain whether the control is too sluggish. Note that the Foxboro's EXACT adaptive controller, based on pattern recognition, Bristol (1977), cannot make this kind of judgement. Similarly the relation (13) can be used by introducing a perturbation at the controller output. If the maximum error deviates from that predicted by (13) we can suspect that the loop is poorly tuned.

## 7. Conclusions

In this paper it has been attempted to analyze simple feedback loops with PID controllers that are tuned using the Ziegler-Nichols closed loop method. It has been shown that there are some quantities that are useful to assess achievable performance and to select suitable controllers. These quantities are the *normalized process gain* ( $K$ ), the *normalized dead-time* ( $\theta$ ), the *normalized closed loop rise time* ( $\tau$ ), and the *peak load error* ( $\lambda$ ). Simple methods to determine these parameters have also been suggested.

It has been shown that  $K$  and  $\theta$  are related and that they can be used to assess the control problem. A small  $\theta$  indicates that tight control is possible with P or PI control but also that significant improvements is sometimes possible with more sophisticated control laws. Processes with  $\theta$  in the range from 0.15 to 0.6 can be controlled well by PID controllers with Ziegler-Nichols tuning. The results show clearly that Ziegler-Nichols tuning gives poor results when the normalized dead-time  $\theta$  is larger than 0.6. There are also relations like  $\tau \approx 1$  and  $K\lambda \approx 1.3$ , that may be used to assess the closed loop response time and the load rejection properties. The results indicate that it would be useful to determine at least one of the parameters  $K$  or  $\theta$  in connection with controller tuning because these parameters are so important for assessment of achievable performance.

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