

String stable integral control design for vehicle platoons with disturbances

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Abstract

This paper presents a control design with integral action for vehicle platoons with disturbance that ensures string stability of the closed loop and disturbance rejection. The addition of integral action and a coordinate change allows to develop sufficient smoothness conditions on the closed loop system to ensure that the closed loop system using the proposed controller is string stable in the presence of time-varying disturbances and able to reject constant disturbances. In addition, bounds for the tracking error of the platoon configuration are also given. Further, a case study is considered together with a suitable controller structure, which satisfies the required smoothness conditions. Simulation results illustrate the performance of the closed loop.

Key words: Multi-agent systems; String stability; Vehicle platoons

1 Introduction

Multi-agent systems are common in natural, social and engineering systems, including cooperative systems [1,15,6], intelligent transportation system [9,20], and aerial vehicles coordination [5,21]. Due to advantages of increasing traffic throughput and reducing fuel consumption [19], vehicle platooning has received recent attention, see [3] and references within. Generally, each platoon agent is coordinated to maintain a desired distance, or time headway, to its neighbours [24].

Research on vehicle platoons and intelligent transportation systems dates back to 1960s [17], which introduced the idea of controlling vehicle positions based on sensor and communication data. It was then realised that additional stability properties are desirable for vehicle platoons [22], in particular string stability. A platoon system is said to be string stable when disturbances and initial conditions are attenuated along the string, regardless of the string length [27,4].

The communication structure, spacing policy, and vehicle dynamics of platoons impact their behaviour and sta-

bility properties, see [11,26] and references within. The communication structure can be, among other topologies, unidirectional, where information travels only one way in the string [21,23], and bidirectional, in which information travels both ways along the string [24,1,16,14]. Furthermore, bidirectional strings can be divided into symmetric or asymmetric strings according to the symmetric or asymmetric coupling between preceding and following vehicles [14,18]. Spacing policies, in turn, determine the desired distance between vehicles, which can be a function of the vehicle speed [8]. The spacing policy affects the system stability and agents behaviour. It is shown in [24,2] that strings using only relative spacing information with constant spacing policy, and a double integrator model for the vehicle dynamics, are always L_2 string unstable for any linear controller. In general, vehicle dynamics are considered linear [7,10,21,23], which allows the use of transfer functions and H_∞ system norm string stability to study L_2 string stability. Nonlinear methods have been used in [16,12,18], which are results in control system with better performance.

In the weaker L_2 string stability setting, an alternative to L_∞ string stability of [27], it has been shown that symmetry in position coupling combined with asymmetry in velocity coupling improves the performance of heterogeneous, asymmetric, bidirectional platoons [14]. An extension of string stability that considered disturbances has been proposed in [4]. Recently, sufficient con-

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ditions that ensures disturbance string stability (DSS) were proposed in [18]. These conditions were used for control design, however, the presence of nonzero mean disturbances produces a nonzero steady-state error.

The key contribution of this paper is to propose a control design with integral action that provides disturbance string stability properties for a bidirectional platoon of vehicles with constant spacing policy subject to time varying disturbances. In addition, the proposed controller rejects constant disturbances. We use DSS sufficient conditions to obtain the controller gains, and then evaluate the performance of the proposed controller in simulation. Thus, we extend the results in [18] by considering a dynamic compensation of constant disturbances while ensuring DSS of the interconnected system.

The remaining of the paper is organised as follows. We introduce the notation and formulate the problem in Section 2. The control design and sufficient conditions for string stability are discussed in Section 3. We present a numerical example to illustrate the control system performance in Section 4. We wrap up the paper with the conclusions in Section 5.

2 Notation and Problem Formulation

2.1 Notation

Let x be a vector in \mathbb{R}^n and consider the signal $s(\cdot): \mathbb{R}_+ \rightarrow \mathbb{R}^n$. We define $|x|_2$ as the L_2 vector norm of x , and the L_∞ function norm $\|s(\cdot)\|_\infty = \sup_t |s(t)|_2$. We let A be a matrix in $\mathbb{R}^{n \times n}$ and define the L_2 matrix norm $\|A\|_2$ and the matrix measure $\mu_2(A)$, induced by the L_2 norm, as $\mu_2(A) = \max_i (\lambda_i[A]_s)$ and $[A]_s$ as the symmetric part of A , where $\lambda(A)$ are the eigenvalues of A . Additionally, the minimum and maximum singular value of the matrix A are $\sigma_{\min}(A)$ and $\sigma_{\max}(A)$. A function $\Gamma(\cdot): \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is of class- \mathcal{K} if it is strictly increasing and $\Gamma(0) = 0$, and of class- \mathcal{L} if it monotonically decreases to 0 as its parameter goes to infinity. Finally, a function $\gamma(\cdot, \cdot)$ is of class- \mathcal{KL} if $\gamma(\cdot, t)$ is a class- \mathcal{K} function $\forall t \geq 0$ and $\gamma(a, \cdot)$ is a class- \mathcal{L} function $\forall a \geq 0$.

2.2 Problem Formulation

We consider an interconnected system composed of $N \geq 1$ agents whose dynamics can be described as follows

$$\begin{aligned} \dot{x}_{1i} &= f_{1i}(x_{1i}, x_{2i}) \\ \dot{x}_{2i} &= f_{2i}(x_{1i}, x_{2i}) + u_i + d_i, \end{aligned} \quad (1)$$

for all $i = \{1, \dots, N\}$, with $x_{1i}, x_{2i} \in \mathbb{R}^n$, $u_i \in \mathbb{R}^n$ and $d_i \in \mathbb{R}^n$, where $n \geq 1$. The state vector, control input and the disturbance of the i th vehicle are $x_i = [x_{1i}^T \ x_{2i}^T]^T$, u_i and d_i respectively. The disturbances has

two components $d_i = w_i(t) + \bar{w}_i$, where $w_i(t)$ and \bar{w}_i are the time varying and constant components of the disturbance. The smooth functions $f_{1,2i}: \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}^n$ describe the system dynamics $f_i(x_i) = [f_{1i}^T \ f_{2i}^T]^T$. We define a virtual vehicle as the reference state x_0 .

A desired property for interconnected vehicle systems is the string stability property, which will ensure for example that disturbances are not amplified when they propagate along the string while maintaining a desired configuration. We express the control objective requiring that the state x_i should converge to the desired configuration x_i^* , where x_i^* verifies $\dot{x}_i^* = f_i(x_i^*)$, as well as it is a solution of the system in the absence of disturbances.

A definition of string stability expressed in ε - δ form was proposed by Swaroop and Hedrick [27] and implies that the state deviations from the origin are not amplified along the platoon. In [4], Besselink and Johansson proposed the concept of disturbance string stability to capture the effects of external disturbances. This definition is an extension of classical string stability but expressed in term of class- \mathcal{K} and class- \mathcal{KL} functions. In this paper, we use the disturbance string stability definition in [4].

Definition 1 (Disturbance String Stability)

Consider the system (1) and assume that x_i^* is a solution to its unperturbed dynamics. Then, the equilibrium x_i^* is said to be disturbance string stable if there exists a \mathcal{KL} function γ and a \mathcal{K} function Γ such that, for any disturbance d_i and initial conditions the estimate

$$\begin{aligned} \sup_i |x_i(t) - x_i^*(t)|_2 &\leq \gamma \left(\sup_i |x_i(0) - x_i^*(0)|_2, t \right) \\ &+ \Gamma \left(\sup_i \|d_i(t)\|_\infty \right) \end{aligned} \quad (2)$$

is verified for all $t > 0$ and $N \in \mathbb{N}$. The functions $\gamma(\cdot, \cdot)$ and $\Gamma(\cdot)$ are the same for any platoon length N , and thus the estimate of the state error norm is independent of the number of agents.

Notice that (2) ensures asymptotic stability for the undisturbed case (see [25, Section 2.5]), which implies $x_i \rightarrow x_i^*$ as $t \rightarrow \infty$.

We propose the following form for the controller u_i

$$u_i = h_{i,i-1}(t, x_i, x_{i-1}) + \varepsilon_i h_{i,i+1}(t, x_i, x_{i+1}) + h_i^0(t, x_i, x_0) + k\zeta_i \quad (3)$$

$$\begin{aligned} \dot{\zeta}_i &= g_{i,i-1}(t, x_i, x_{i-1}) + \varepsilon_i g_{i,i+1}(t, x_i, x_{i+1}) \\ &+ g_i^0(t, x_i, x_0) \end{aligned} \quad (4)$$

where the smooth functions $h_{i,j}: \mathbb{R}_+ \times \mathbb{R}^{2n} \times \mathbb{R}^{2n} \rightarrow \mathbb{R}^n$ represent the coupling functions between neighbour vehicles i and j , while $h_i^0: \mathbb{R}_+ \times \mathbb{R}^{2n} \times \mathbb{R}^{2n} \rightarrow \mathbb{R}^n$ is the

coupling of vehicle i with the reference state x_0 , and $\varepsilon_i \in [0, 1]$ is the symmetry constant for vehicle i , which weights its coupling with the following vehicle. The controller state $\zeta \in \mathbb{R}^n$ allows for integral action to compensate disturbances. The constant k is the integral action gain and the smooth functions $g_{i,j}: \mathbb{R}_+ \times \mathbb{R}^{2n} \times \mathbb{R}^{2n} \rightarrow \mathbb{R}^n$ shape the integral action dynamics (4).

The problem to be solved is the design of the dynamic controller in the form (3)-(4) for the i th agent of a bidirectionally interconnected system (1) that ensures disturbance string stability and thus (2) is satisfied.

3 Controller Design

The control objective is to drive the states of the system (1) to the desired configuration x_i^* , while satisfying the estimate (2), that is disturbance string stability is ensured.

3.1 Sufficient Conditions for String Stability

Consider the system (1) in closed loop with the controller (3) and $k = 0$ (no integral action), then a set of sufficient conditions for string stability are [18]:

- C1** $h_{i,i-1}(t, x_i^*, x_{i-1}^*) = 0$, $h_{i,i+1}(t, x_i^*, x_{i+1}^*) = 0$, and $h_i^0(t, x_i^*, x_0) = 0$;
C2 for some $c \neq 0$ and $b > 0$

$$\begin{aligned} \mu_2 \left(\frac{\partial f_i(x_i)}{\partial x_i} + \frac{\partial u_i}{\partial x_i} \right) &\leq -c^2, \\ \max \left\{ \left\| \frac{\partial h_{i,i-1}}{\partial x_{i-1}} \right\|_2, \left\| \frac{\partial h_{i,i+1}}{\partial x_{i+1}} \right\|_2 \right\} &\leq b, \\ \text{for all } x_i, x_{i-1}, x_{i+1} &\in \mathbb{R}^{2n}; \end{aligned} \quad (5)$$

- C3** $\varepsilon_i < \frac{c^2}{b} - 1$.

The following proposition formalise the result in [18].

Proposition 2 (Sufficient Conditions for DSS)

Assume that the coupling functions $h_{i,j}$ in (3) are designed such that conditions **C1**, **C2**, and **C3** are satisfied. Then, the trajectories of the system (1) in closed loop with the controller (3), with $k = 0$, satisfy

$$\begin{aligned} \sup_i |x_i(t) - x_i^*(t)|_2 &\leq e^{-c^2 t} \sup_i |x_i(0) - x_i^*(0)|_2 \\ &+ \frac{1 - e^{-c^2 t}}{c^2} \sup_i \|d_i(t)\|_\infty \end{aligned} \quad (6)$$

where $\bar{c}^2 = c^2 - b(1 + \max_i \varepsilon_i)$, which ensures DSS.

PROOF. It follows directly from [18, Theorem 1]. \square

The sufficient conditions **C1**, **C2**, and **C3** in Proposition 2 provide a tool to select controllers that ensure string stability of an interconnected system. The proposition also ensures that the states are ultimately bounded for bounded disturbances. However, under the classical scenario of constant disturbances, when a platoon of vehicles hits a sloping road for instance, the states will not converge to the desired values and the state error will not vanish. Moreover, the error will be bounded by the maximum value of the disturbance weighted by an increasing function of time. This behaviour is undesirable and the controller should be able to compensate, at least, for constant disturbances. We propose to design a controller with the integral action capable of rejecting constant disturbances and preserving the DSS property.

3.2 String Stable Control with Constant Disturbance Rejection

We augment the system (1) with the state $\xi_i \in \mathbb{R}^n$ and consider the controller (3)-(4). Then, the dynamics of the closed loop can be written as follows

$$\dot{z}_i = \phi_i(z_i) + v_i + p_i, \quad (7)$$

where $z_i = [x_i^T \ \xi_i^T]^T$ is the state vector and $\xi_i = \zeta_i + k^{-1} \bar{w}_i$. Also, we define $v_i = [0_{1 \times n} \ v_{x,i}^T \ v_{\zeta,i}^T]^T$, where $0_{a \times b}$ is the a -by- b matrix of zeros and $a, b \in \mathbb{N}$, with $v_{x,i} = h_{i,i-1} + \varepsilon_i h_{i,i+1} + h_i^0 + k \xi_i$ and $v_{\zeta,i} = g_{i,i-1} + \varepsilon_i g_{i,i+1} + g_i^0$, and the vector $p_i = [0_{1 \times n} \ w_i^T \ 0_{1 \times n}]^T$ contains the time varying disturbance. The system dynamics is defined by $\phi_i: \mathbb{R}^{3n} \rightarrow \mathbb{R}^{3n}$. Then, the reference for the new state vector becomes $z_i^* = [x_i^{*T} \ 0_{1 \times n}^T]^T$. Note that we dropped the function dependencies to simplify the notation.

We aim at designing v_i such that the closed loop system (7) is disturbance string stable. Moreover, if conditions **C1**, **C2**, and **C3** (applied to (7)) are satisfied, the inclusion of $\xi_i \in \mathbb{R}^n$ and the change of coordinates lead the controller to reject constant disturbances whilst guaranteeing disturbance string stability. However, directly satisfying those conditions is in general a difficult task.

Similar to [18], we consider a heterogeneous car platoon system where $\phi_i(z_i) = F z_i$ and we design a controller v_i that meets the control objectives. The system dynamics can be written as

$$\dot{z}_i = F z_i + \frac{1}{m_i} v_i + \frac{1}{m_i} p_i, \quad (8)$$

with $F = [0_{3n \times n} \ [I_n \ 0_{n \times 2n}]^T \ 0_{3n \times n}]$, where I_n is the n -by- n identity matrix, and m_i is the mass of vehicle i . It is convenient to write $v_i = m_i [H_{i,i-1} + \varepsilon_i H_{i,i+1} + H_i^0]$, with and $H_{i,i-1} = [0_{1 \times n} \ h_{i,i-1}^T \ g_{i,i-1}^T]^T$, $H_{i,i+1} = [0_{1 \times n} \ h_{i,i+1}^T \ g_{i,i+1}^T]^T$, and $H_i^0 = [0_{1 \times n} \ [h_i^0 + k \xi_i]^T \ g_i^0]^T$.

Now we modify the sufficient conditions **C1**, **C2**, and **C3** through their application to a linear transformation of the closed loop system (8). This facilitates finding the controller gains that satisfy the conditions by posing them as constraints of an optimisation problem.

Proposition 3 (Transformed Sufficient Conditions)

Consider the system (8) in closed loop with the controller (3)-(4). Let T_i be the transformation matrix, with the coupling constant matrices $\alpha_i \in \mathbb{R}^{n \times n}$ and $\beta_i \in \mathbb{R}^{n \times n}$

$$T_i \triangleq \begin{bmatrix} I_n & \alpha_i & 0_{n \times n} \\ 0_{n \times n} & I_n & \beta_i \\ 0_{n \times n} & 0_{n \times n} & I_n \end{bmatrix}. \quad (9)$$

Assume the following sufficient conditions are satisfied,

C1* $H_{i,i-1}(t, z_{i-1}^*, z_i^*) = 0$, $H_{i,i+1}(t, z_{i+1}^*, z_i^*) = 0$, and $H_i^0(t, z_i^*, z_0) = 0$;

C2* for some $c \neq 0$ and $b > 0$

$$\begin{aligned} \mu_2(J_{i,i}) &\leq -c^2, \\ \max \{ \|J_{i,i-1}\|_2, \|J_{i,i+1}\|_2 \} &\leq b, \end{aligned} \quad (10)$$

for all $z_i, z_{i-1}, z_{i+1} \in \mathbb{R}^{3n}$;

C3* $\varepsilon_i < \frac{c^2}{b} - 1$.

where the Jacobian J of the closed loop system (8), is given by the matrices $J_{i,i}(\alpha_i, \beta_i, z_i, z_{i-1}, z_{i+1}, \varepsilon_i)$, $J_{i,i-1}(\alpha_i, \beta_i, z_i, z_{i-1})$ and $J_{i,i+1}(\alpha_i, \beta_i, z_i, z_{i+1})$ below

$$\begin{aligned} J_{i,i} &= T_i F T_i^{-1} \\ &+ \begin{bmatrix} \frac{\partial \tilde{v}_i}{\partial z_i} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} & \frac{\partial \tilde{v}_i}{\partial z_i} \begin{bmatrix} -\alpha_i \\ 1 \\ 0 \end{bmatrix} & \frac{\partial \tilde{v}_i}{\partial z_i} \begin{bmatrix} \alpha_i \beta_i \\ -\beta_i \\ 1 \end{bmatrix} \end{bmatrix} \\ J_{i,i\pm 1} &= \begin{bmatrix} \frac{\partial \tilde{H}}{\partial z} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} & \frac{\partial \tilde{H}}{\partial z} \begin{bmatrix} -\alpha_i \\ 1 \\ 0 \end{bmatrix} & \frac{\partial \tilde{H}}{\partial z} \begin{bmatrix} \alpha_i \beta_i \\ -\beta_i \\ 1 \end{bmatrix} \end{bmatrix}, \end{aligned} \quad (11)$$

where $\frac{\partial \tilde{H}}{\partial z} \triangleq \frac{\partial T_i H_{i\pm 1}}{\partial z_{i\pm 1}}$ and $\tilde{v}_i \triangleq T_i v_i$.

Define $\tilde{z}_i \triangleq T_i z_i$ and $\tilde{p}_i \triangleq T_i p_i$. Then, the following properties hold true.

(i) The dynamics of the transformed system are

$$\dot{\tilde{z}}_i = T_i F T_i^{-1} \tilde{z}_i + \tilde{v}_i + \tilde{p}_i \quad (12)$$

with the new state vector \tilde{z}_i defined as

$$\tilde{z}_i = \begin{bmatrix} x_{1i} + \alpha_i x_{2i} \\ x_{2i} + \beta_i \xi_i \\ \xi_i \end{bmatrix} = \begin{bmatrix} \tilde{z}_i^1 \\ \tilde{z}_i^2 \\ \tilde{z}_i^3 \end{bmatrix}. \quad (13)$$

The transformed unperturbed closed loop dynamics are

$$\begin{aligned} \dot{\tilde{z}}_i &= T_i F T_i^{-1} \tilde{z}_i + T_i [H_{i,i-1}(t, T_i^{-1} \tilde{z}_i^*, T_{i-1}^{-1} \tilde{z}_{i-1}^*) \\ &+ \varepsilon_i H_{i,i+1}(t, T_i^{-1} \tilde{z}_i^*, T_{i+1}^{-1} \tilde{z}_{i+1}^*) + H_i^0(t, T_i^{-1} \tilde{z}_i^*, z_0)] \end{aligned} \quad (14)$$

and the transformed desired configuration is $\tilde{z}_i^* = T_i z_i^*$.

(ii) The following estimate is satisfied

$$\begin{aligned} \sup_i |\tilde{z}_i(t) - \tilde{z}_i^*(t)|_2 &\leq e^{-c^2 t} \sup_i |\tilde{z}_i(0) - \tilde{z}_i^*(0)|_2 \\ &+ \frac{1 - e^{-c^2 t}}{c^2} \sup_i \|\tilde{m}_i(t)\|_\infty. \end{aligned} \quad (15)$$

PROOF. First note that (13) follows from simple algebra and (12) follows by substituting $z_i = T^{-1} \tilde{z}_i$ into (8), which proves property (i).

To prove property (ii), we show that the conditions **C1***, **C2*** and **C3*** are verified if and only if the conditions **C1**, **C2** and **C3** are verified for the dynamics (14), which include the integral state ζ_i and dynamic controller (3)-(4). Thus, by Proposition 2, the closed loop system in coordinates \tilde{z}_i is DSS. Notice that **C1*** results from applying **C1** to the transformed closed loop dynamics (14). Hence, condition **C1** is satisfied for the transformed closed loop dynamics if and only if **C1*** is satisfied. Now, to compute **C2**, we differentiate the transformed closed loop dynamics (12), and obtain the Jacobian $\tilde{J} \in \mathbb{R}^{3nN \times 3nN}$, defined by the matrices $\tilde{J}_{i,i} \in \mathbb{R}^{3n \times 3n}$, $\tilde{J}_{i,i-1} \in \mathbb{R}^{3n \times 3n}$ and $\tilde{J}_{i,i+1} \in \mathbb{R}^{3n \times 3n}$ defined below

$$\tilde{J}_{i,i} = \frac{\partial(T_i F \tilde{z}_i T_i^{-1})}{\partial \tilde{z}_i} + \frac{\partial \tilde{v}_i}{\partial \tilde{z}_i}, \quad \tilde{J}_{i,i\pm 1} = \frac{\partial T_i H_{i\pm 1}}{\partial \tilde{z}_{i\pm 1}}, \quad (16)$$

which can be expressed in terms of the state vector z_i of (8), as $\partial(T_i F \tilde{z}_i T_i^{-1})/\partial \tilde{z}_i = T_i F T_i^{-1}$ and

$$\frac{\partial \tilde{v}_i}{\partial \tilde{z}_i} = \begin{bmatrix} \frac{\partial \tilde{v}_i}{\partial \tilde{z}_i^1} & \frac{\partial \tilde{v}_i}{\partial \tilde{z}_i^2} & \frac{\partial \tilde{v}_i}{\partial \tilde{z}_i^3} \end{bmatrix} = \begin{bmatrix} \frac{\partial \tilde{v}_i}{\partial z_i} \frac{\partial z_i}{\partial \tilde{z}_i^1} & \frac{\partial \tilde{v}_i}{\partial z_i} \frac{\partial z_i}{\partial \tilde{z}_i^2} & \frac{\partial \tilde{v}_i}{\partial z_i} \frac{\partial z_i}{\partial \tilde{z}_i^3} \end{bmatrix}$$

$$\begin{aligned} \frac{\partial T_i H_{i\pm 1}}{\partial \tilde{z}_{i\pm 1}} &= \begin{bmatrix} \frac{\partial T_i H_i}{\partial \tilde{z}_{i\pm 1}^1} & \frac{\partial T_i H_i}{\partial \tilde{z}_{i\pm 1}^2} & \frac{\partial T_i H_i}{\partial \tilde{z}_{i\pm 1}^3} \end{bmatrix} \\ &= \begin{bmatrix} \frac{\partial T_i H_i}{\partial z_{i\pm 1}} \frac{\partial z_{i\pm 1}}{\partial \tilde{z}_{i\pm 1}^1} & \frac{\partial T_i H_i}{\partial z_{i\pm 1}} \frac{\partial z_{i\pm 1}}{\partial \tilde{z}_{i\pm 1}^2} & \frac{\partial T_i H_i}{\partial z_{i\pm 1}} \frac{\partial z_{i\pm 1}}{\partial \tilde{z}_{i\pm 1}^3} \end{bmatrix}. \end{aligned}$$

By solving the partial derivatives above, we obtain the Jacobian matrix J in (11), which is written in terms of the state vector z_i , obtaining also condition **C2***. This implies that **C2** is satisfied for the transformed closed loop dynamics if and only if **C2*** is satisfied. Condition **C3*** follows directly from **C3**. Finally, the inequality (15) results by application of Proposition 2. \square

It is important to note that to use the sufficient conditions **C1***, **C2***, and **C3***, it is not necessary to compute the transformed system, but only the Jacobian J and the transformation matrix T_i . Also, the change of coordinates and the transformation allows us to find controllers independent of the constant disturbance \bar{w}_i . Now we show that if the transformed system (12) is disturbance string stable, so is the system (8).

Proposition 4 (DSS of the Augmented System) *Consider the system (8) in closed loop with the controller (3)-(4), and assume that the functions H_{ij} are such that the conditions **C1***, **C2***, and **C3*** are satisfied. Then,*

$$\begin{aligned} \sup_i |z(t) - z^*(t)|_2 &\leq K e^{-\bar{c}^2 t} \sup_i |z(0) - z^*(0)|_2 \\ &+ K \frac{1 - e^{-\bar{c}^2 t}}{\bar{c}^2} \sup_i \|w_i(t)\|_\infty \end{aligned} \quad (17)$$

where $\bar{c}^2 = c^2 - b(1 + \max_i \varepsilon_i)$ and $K = \frac{\max_i \{\sigma_{\max}(T_i)\}}{\min_i \{\sigma_{\min}(T_i)\}}$.

PROOF. Under the assumption, the conditions in Proposition 3 are satisfied, and thus (15) holds true. Then, we define $A_i \triangleq T_i^T T_i$ and $\sigma(T_i) = \sqrt{\lambda(A_i)}$ to obtain the following bound for the quadratic form $z_i^T A_i z_i$

$$\begin{aligned} \underline{\sigma} \sup_i |z_i(t) - z_i^*(t)|_2 &\leq \sup_i |\tilde{z}_i(t) - \tilde{z}_i^*(t)|_2 \\ \sup_i |\tilde{z}_i(0) - \tilde{z}_i^*(0)|_2 &\leq \bar{\sigma} \sup_i |z_i(0) - z_i^*(0)|_2 \\ \sup_i \|\tilde{p}_i(t)\|_\infty &\leq \bar{\sigma} \|p_i(t)\|_\infty \end{aligned} \quad (18)$$

where $\bar{\sigma} = \max_i \{\sigma_{\max}(T_i)\}$ and $\underline{\sigma} = \min_i \{\sigma_{\min}(T_i)\}$.

Hence, by using (18) in (15) and noting that $\sup_i \|p_i(t)\|_\infty = \sup_i \|w_i(t)\|_\infty$, we obtain

$$\begin{aligned} \sup_i |z_i(t) - z_i^*(t)|_2 &\leq \frac{\bar{\sigma}}{\underline{\sigma}} e^{-\bar{c}^2 t} \sup_i |z_i(0) - z_i^*(0)|_2 \\ &+ \frac{\bar{\sigma}}{\underline{\sigma}} \frac{1 - e^{-\bar{c}^2 t}}{\bar{c}^2} \|w_i(t)\|_\infty \end{aligned}$$

from where we obtain (17) by setting $K = \frac{\bar{\sigma}}{\underline{\sigma}}$. \square

Proposition 3 shows DSS of the closed loop dynamics (8), which includes the integral action. Now, we will show that DSS holds for the original states of the system (1).

Corollary 5 (DSS of the Original System) *Consider the system (1) in closed loop with the controller (3)-(4). If the functions H_{ij} are such that the conditions **C1***, **C2***, and **C3*** are satisfied, then the state errors can be bounded as follows*

$$\begin{aligned} \sup_i |x_i(t) - x_i^*(t)|_2 &\leq K e^{-\bar{c}^2 t} \sup_i |x_i(0) - x_i^*(0)|_2 \\ &+ K e^{-\bar{c}^2 t} \sup_i |\zeta_i(0) + k^{-1} \bar{w}_i|_2 \\ &+ K \frac{1 - e^{-\bar{c}^2 t}}{\bar{c}^2} \sup_i \|w_i(t)\|_\infty \end{aligned} \quad (19)$$

where $\bar{c}^2 = c^2 - b(1 + \max_i \varepsilon_i)$ and $K = \frac{\max_i \{\sigma_{\max}(T_i)\}}{\min_i \{\sigma_{\min}(T_i)\}}$.

PROOF. First note that $z_i = [x_i^T \ \xi_i^T]^T$ implies that

$$\sup_i |x_i(t) - x_i^*(t)|_2 \leq \sup_i |z_i(t) - z_i^*(t)|_2. \quad (20)$$

As all the assumptions of Proposition 4 are satisfied, inequality (17) holds. Thus, using (17) in (20), we obtain

$$\begin{aligned} \sup_i |x_i(t) - x_i^*(t)|_2 &\leq \sup_i |z_i(t) - z_i^*(t)|_2 \\ &\leq K e^{-\bar{c}^2 t} \sup_i |z_i(0) - z_i^*(0)|_2 \\ &+ K \frac{1 - e^{-\bar{c}^2 t}}{\bar{c}^2} \sup_i \|w_i(t)\|_\infty. \end{aligned} \quad (21)$$

Also, using the triangle inequality, we can write $\sup_i |z_i(0) - z_i^*(0)|_2 \leq \sup_i |x_i(0) - x_i^*(0)|_2 + \sup_i |\xi_i(0)|_2$. Then from (21) and the fact that $\xi_i(0) = \zeta_i(0) + k^{-1} \bar{w}_i$, we obtain (19), which completes the proof. \square

Corollary 5 proves that the state errors of the original system (1) are bounded by the initial errors, the initial integral action deviations from their respective equilibria, and by the infinite norm of the time-variant disturbances. Also, the integral action ensures that when the agents are subject to constant disturbances, the states converge to their desired values. The bound (19) is comparable to the bound in Corollary 1 of [18], which is

$$\begin{aligned} \sup_i |x_i(t) - x_i^*(t)|_2 &\leq K e^{-\bar{c}^2 t} \sup_i |x_i(0) - x_i^*(0)|_2 \\ &+ K \frac{1 - e^{-\bar{c}^2 t}}{\bar{c}^2} \sup_i \|\bar{w}_i + w_i(t)\|_\infty. \end{aligned} \quad (22)$$

4 Simulations

In this section, we consider $x_i = [q_i \ \dot{q}_i]^T$, where $q_i, \dot{q}_i \in \mathbb{R}$ are the position and speed of vehicle i in a vehicle platoon, whose closed loop dynamics is described by system (1), with $n = 1$, $f_{1i} = x_{2i}$ and $f_{2i} = 0$, and controller (3)-(4), with the coupling functions below

$$\begin{aligned} h_{i,i-1} &= h_i^p(q_{i-1} - q_i - \delta_{i,i-1}) + K_i^v(\dot{q}_{i-1} - \dot{q}_i) \\ h_{i,i+1} &= h_i^p(q_{i+1} - q_i + \delta_{i+1,i}) + K_i^v(\dot{q}_{i+1} - \dot{q}_i) \\ h_i^0 &= K_i^{p0}(q_0 - q_i - \delta_{i,0}) + K_i^{v0}(\dot{q}_0 - \dot{q}_i) \end{aligned} \quad (23)$$

where $\delta_{i,j}$ is the desired spacing, while $\delta_{i,0} = \sum_{j=1}^i \delta_{j,j-1}$ is the distance to the reference x_0 . The functions that shape the integral action dynamics are

$$\begin{aligned} g_{i,i-1} &= g_i^p(q_{i-1} - q_i - \delta_{i,i-1}) + G_i^v(\dot{q}_{i-1} - \dot{q}_i) \\ g_{i,i+1} &= g_i^p(q_{i+1} - q_i + \delta_{i+1,i}) + G_i^v(\dot{q}_{i+1} - \dot{q}_i) \\ g_i^0 &= G_i^{p0}(q_0 - q_i - \delta_{i,0}) + G_i^{v0}(\dot{q}_0 - \dot{q}_i) \end{aligned} \quad (24)$$

where we selected $h_i^p(x) = K_{1i}^p \tanh(K_{2i}^p x)$ and $g_i^p(x) = G_{1i}^p \tanh(G_{2i}^p x)$. The controller gains are $K_{1i}^p, K_{2i}^p, K_i^v, K_i^{p0}, K_i^{v0}, k, G_{1i}^p, G_{2i}^p, G_i^v, G_i^{p0},$ and G_i^{v0} . We compute the partial derivatives in (11) and using CVX, a package for specifying and solving convex programs [13], find the controller gains that satisfy the conditions **C1***, **C2*** and **C3*** so that (17) holds. The controller gains are $\alpha_i = 0.3, \beta_i = -0.4, \varepsilon_i = 1, K_{1i}^p = K_{2i}^p = 0.1188, K_i^v = 0.0121, K_i^{p0} = 0.6, K_i^{v0} = 0.6, k = 0.2508, G_i^p = 0.01, G_i^v = 0.01, G_i^{p0} = 0.2881,$ and $G_i^{v0} = 0.3420$.

We consider a string of $N = 5$ vehicles and compare the controllers C_1 and C_2 obtained using Corollary 1 in [18] and Proposition 4 respectively. The initial conditions are $x_i(0) = [q_0(0) - \delta_{i,0} + r, \dot{q}_0(0) + r]^T$, the inter-vehicle spacing is $\delta_{i,i-1} = \delta_{i+1,i} = 10$ m and the reference speed $\dot{q}_0 = 20$ m/s. We set the time-variant disturbance as $w_i(t) = r \sin(t) \exp(-0.1t)$ and the constant disturbance is $\bar{w}_i = (1 + r) \text{ m/s}^2$, where r is uniformly randomly generated in the interval $[-1, 1]$ and updated after each use. We considered the case where the controller uses the nominal vehicle mass $\hat{m}_i = 1000$ kg, while the mass in the model is $m_i = \hat{m}_i + 200r$.

Figure 1 compares the state error norm and the bounds (22) and (19). As expected, since the time-varying disturbances vanish, the state error norm converges to a constant when using the controller C_1 . However, the state error norm is suppressed when using the controller C_2 , which shows better performance and compensates for the constant disturbances. We notice that uncertainty in the masses is also compensated by the controller. Figure 2 shows the deviation of the inter-vehicle distances, that is $e_{i,i-1} = q_{i-1} - q_i - \delta_{i,i-1}$, from the desired value. We note that the controller C_1 cannot compensate for disturbances and the inter-vehicle distances

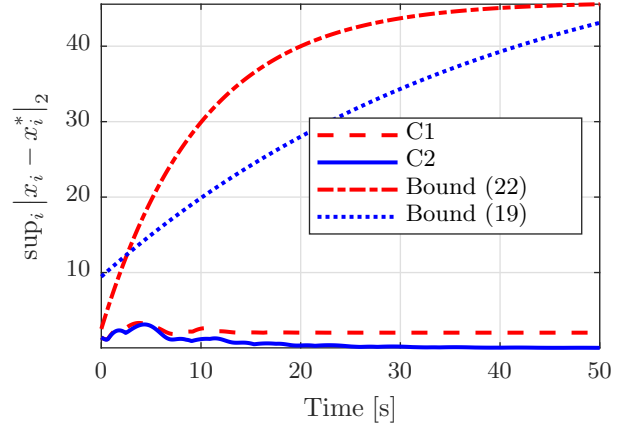


Fig. 1. State errors norm and bounds for C_1 and C_2 .

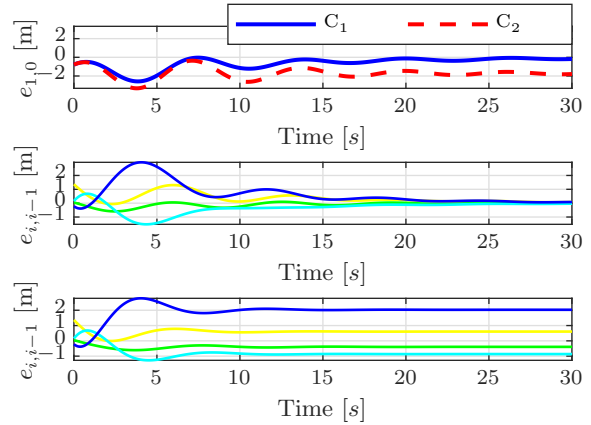


Fig. 2. Inter-vehicle distance error $e_{1,0}$ for C_1 and C_2 (top), and $e_{i,i-1}$ for C_2 (middle) and C_1 (bottom).

do not converge to the desired values, but controller C_2 does whilst ensuring a reasonable inter-vehicle distances during the transient. Figure 3 shows that the vehicles achieve the reference velocity. Also, notice that the integral action states converge to a value proportional to the constant disturbance, and the acceleration signals $a_i = u_i/m_i$ are smooth and within reasonable values.

5 Conclusion

In this paper, we presented sufficient conditions that guarantee disturbance string stable for a linear, asymmetric, bidirectional, interconnected system under nonlinear control with integral action. Under these conditions, the controller will ensure the state errors are bounded by functions of the initial conditions and the time-variant (zero mean) disturbance, whilst rejecting constant (non-zero mean) disturbance due to the addition of integral action. Future work will focus on developing sufficient conditions for controllers that use local information without global knowledge of reference signals.

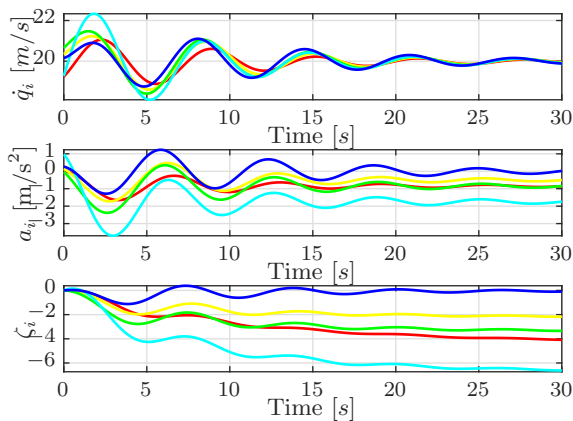


Fig. 3. Velocity (top), acceleration (middle) and integral state (bottom) time histories.

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