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We study the value of commitment in contests and tournaments when there are costs for the follower to observe the leader's behavior. In a contest, the follower can pay to observe the leader's effort but cannot observe the effectiveness of that effort. In a tournament, the follower can pay to observe the effectiveness of the leader's effort but not the effort itself. We show that this distinction matters significantly: When observation is costly, the value of commitment vanishes entirely in sequential and endogenous move contests, regardless of the size of the observation cost. By contrast, in tournaments, the value of commitment is preserved completely, provided that the observation costs are sufficiently small.

The Value of Commitment in Contests and Tournaments when Observation is Costly[*]

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Abstract

We study the value of commitment in contests and tournaments when there are costs for the follower to observe the leader's behavior. In a contest, the follower can pay to observe the leader's effort but cannot observe the effectiveness of that effort. In a tournament, the follower can pay to observe the effectiveness of the leader's effort but not the effort itself. We show that this distinction matters significantly: When observation is costly, the value of commitment vanishes entirely in sequential and endogenous move contests, regardless of the size of the observation cost. By contrast, in tournaments, the value of commitment is preserved completely, provided that the observation costs are sufficiently small.

JEL Classification Nos.: H10

Keywords: Contests, Tournaments, Rent-Seeking, Commitment, Costly Leader Games

1 Introduction

The modeling framework of contests and tournaments, settings where players under-take effort or expend resources in pursuit of some "prize," has been usefully applied in numerous economic settings. These include patent races, allocating resources in elections, the private funding of public goods, designing incentive contracts in firms, and golf tournaments. (See, for instance, Taylor, 1995; Snyder, 1989; Morgan and Sefton, 2000; Lazear and Rosen, 1981; and Ehrenberg and Bognanno, 1990.)

Early modeling of contests focused on equilibrium when moves are made simultaneously (see, for example, Tullock 1980, 1985). The usual justification for this modeling choice was that there is positive value to commitment and, therefore, if a player faces the choice of moving at the same time as a rival or moving earlier, he prefers to move earlier. Since all competing parties share the same incentive, they will all race to move at the earliest possible moment. Unless there is some institutional feature preventing some parties from moving at the same time as others, then, arguably, the right model is that of simultaneous moves.

Baik and Shogren (1992) and Leininger (1993) question this argument. In both of these papers, the order of moves is endogenous and a sequential move contest emerges. The reason is that, even though both players indeed prefer moving first over moving simultaneously, the favorite prefers moving second over moving first, while the underdog prefers moving first over moving second.

While the literature on contests is vast, the literature on *sequential* contests—and the value of commitment—is much smaller.[1] The earliest analysis of sequential contests is offered by Dixit (1987), who derives conditions under which there is a

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[1]See Nitzan (1994) for an excellent survey, particularly with respect to modeling rent-seeking.

positive value of commitment.[2] That is, he shows when a player benefits from moving first rather than moving simultaneously.

The central theme of this line of the contest literature is that the timing of moves matters—actions in earlier stages of the contest have strategic effects on those in later stages of the contest, and this affects equilibrium outcomes.

A key assumption shared by all of these models is that moves made in earlier periods of the game are costlessly observed by players moving later in the game. In this paper, we relax this assumption. Instead, we suppose that a player must pay a small cost to observe the actions of other players in preceding stages of the game.

Our main finding is that, in contests, there is no value to commitment whatsoever when observation is costly—even if the observation costs are arbitrarily small. More precisely, efforts and payoffs in all subgame perfect equilibria of sequential and endogenous move contests are identical to those in the Nash equilibrium of the standard simultaneous contest.

The intuition for this result is most easily gained from the sequential contest: When the sequential contest is "well-behaved" in the sense of being strictly concave, the second player has a unique best response to any effort choice by the first player. Further, given that player 2 is choosing a best response, the first player also has a unique payoff maximizing effort level. This implies that, in any subgame perfect equilibrium of the sequential contest, the first player is playing a pure strategy. But if player 1 is playing a pure strategy, then, in equilibrium, obtaining information about player 1's choice is of no value to player 2 in . This is because player 2 can perfectly "solve" what player 1 did, such that there is no point in paying the observation cost. Therefore, in equilibrium, the second player will *never* pay to observe the first player's

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[2]See also Baik and Shogren (1992), Baye and Shin (1999), and Dixit (1999) for additional comments on Dixit (1987).

choice—even if the cost of doing so is arbitrarily small. This, in turn, destroys any strategic effect player 1's move could have on player 2. Hence, there is no value to commitment.

The key problem for the first player is that, by playing a pure strategy, she destroys the incentive for the second player ever to pay to observe. Indeed, one may well wonder why the first mover does not create her own "noise" by playing a mixed strategy, as this could restore the value of commitment. The reason is that, since her optimization problem is "well-behaved" (in the sense of being strictly concave), she cannot resist the temptation to "purify" any mixed strategy and instead play her unique, most profitable action. Thus, somewhat ironically, it ultimately is the "well-behavedness" of the first player's problem that destroys her value of commitment.

A modeling framework closely related to that of contests was introduced in the important paper of Lazear and Rosen (1981) on rank-order tournaments. In a tournament, players also compete by exerting effort. This effort, plus a random noise component, then determines a player's "output." Prizes are awarded on the basis of the rank-ordering of outputs. That is, first prize is awarded to the player with the highest output, second prize to the player with the second-highest output, and so on.

What distinguishes tournaments from contests? According to our terminology, in a sequential tournament, the *effectiveness* of the first-mover's effort is revealed to the second mover, rather than the effort itself. That is, the second long jumper gets to observe the distance jumped by the first, but not the underlying effort that produced the jump. By contrast, in a sequential contest it is effort that is observable, while its ultimate effectiveness remains unobservable until the very end of the contest. For instance, it may be easy to find out that a pressure group has hired a K Street lobbying firm for a certain amount of money. However, until the regulator has ruled

or Congress has voted, the effectiveness of that lobbying effort is not revealed. As we will show, when observation is costly, the unobservability of the effectiveness of effort plays a key role in the difference between the value of commitment in sequential contests as compared to sequential tournaments.

As Dixit (1987) points out, the modeling difference between contests and tournaments matters little when observation is costless. When observation is costly, however, this equivalence breaks down. Indeed, the value of commitment in tournaments is dramatically different compared to contests. Unlike in a contest, we show that the value of commitment is *completely preserved* in a sequential tournament, provided the cost of observation is sufficiently small.

What accounts for this difference? In a tournament, the second player can pay to observe the effectiveness of the first player's effort, i.e., his output. Owing to the outside noise, there is value to observing the effectiveness of the first player's output, even if the first player is playing a pure strategy. Thus, for a small enough cost, the second player will choose to observe, and since the incentive to observe remains intact, the value of commitment is preserved. Hence, our results highlight that it is the observability of the effectiveness of effort—rather than of the effort itself—that creates the value of commitment.

Returning to the question of the timing of moves in the contest literature, we conclude the following. Our results suggest that in circumstances where effectiveness is unobservable, as is arguably the case in many regulatory and legislative settings, timing does not matter at all; all subgame perfect equilibria of sequential and endogenous move contests correspond to Nash equilibria of the simultaneous contest. This, we believe, is an important new justification for focusing on that particular extensive form. However, we also show that this justification must be used with caution: In

tournaments, where it is effectiveness that is observable, the timing of moves clearly does matter.

The remainder of the paper proceeds as follows: The rest of this section contains a review of the related literature on the fragility of commitment. Section 2 presents the model of contests. Section 3 shows that in sequential and endogenous move contests with observation costs, the value of commitment vanishes completely—even when these costs are arbitrarily small. Section 4 shows that in tournaments with observation costs, the value of commitment is preserved completely when these costs are sufficiently small. Finally, Section 5 concludes. A generalization of our main result is contained in the Appendix.

Relationship to the Literature on the Fragility of Commitment

Unlike the contest literature, the prior literature on the fragility of the value of commitment has focused on finite games. Bagwell (1995) studies leader-follower games where, with small but positive probability, the follower receives the wrong signal as to the leader's action. The pure strategy Nash equilibrium outcomes of such a 'noisy leader game' turn out to be equal to the pure-strategy Nash equilibrium outcomes of the simultaneous game. In other words, the leader's value of commitment may not be robust to noise in the communication technology.[3]

Van Damme and Hurkens (1997) partly salvage the value of commitment in noisy leader games by showing that these games always have a mixed strategy equilibrium in which the value of commitment is preserved asymptotically when the noise vanishes. They refer to such an equilibrium as a 'noisy Stackelberg equilibrium' and develop a selection theory that selects them.[4] Finally, Maggi (1999) shows that the value of

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[3]See also Huck and Müller (2000) for experiments relating to Bagwell's game and Güth, Kirchsteiger, and Ritzberger (1998) for an n player extension.

[4]In contrast, Oechssler and Schlag (2000) show that an evolutionary selection procedure selects

commitment may be restored when there is private information on the part of the leader.

An alternative approach, and the modeling framework adopted here, is where observation is costly, as first suggested by Várdy (2004).[5] That paper studies bimatrix games in which the follower faces a small cost, ε, to observe the leader's action. He shows that the Bagwell and van Damme and Hurkens results carry over to this setting. Thus, commitment is destroyed completely if one restricts attention to pure strategy equilibria, but is restored if one allows for mixed strategies and observation costs are sufficiently low.

2   The Model

Two risk-neutral players, labeled 1 and 2, are competing to win some object. The players may be thought of as pressure groups and the object as a favorable piece of legislation, a monopoly concession, and so on. Let V_i denote the (positive and finite) value of the object to player i. The valuation that each player places on the object is commonly known. If i does not receive the object, his payoff is normalized to zero (exclusive of contest expenditures—more on this below).

Players compete for the object by making irreversible effort outlays. The effort of player i is denoted $x_i \in \mathbb{R}_0^+$. There is a continuously differentiable contest success function $P(x_1, x_2)$, which gives the probability that the object will be awarded to player 1 when efforts x_1 and x_2 are expended. The cost of effort is $C_i(x_i)$. Hence,

the pure strategy equilibrium, where the value of commitment is lost entirely.

[5]See also Morgan and Várdy (2004) for experiments relating to the fragility of commitment in this framework.

player 1's expected payoff is:

$$E\pi^1 = P(x_1, x_2) V_1 - C_1(x_1)$$

while player 2's expected payoff is

$$E\pi^2 = [1 - P(x_1, x_2)] V_2 - C_2(x_2)$$

Following Dixit and much of the contest literature, in the main text we assume that the contest success function takes the Logit form; that is

$$P(x_1, x_2) = \begin{cases} \frac{f_1(x_1)}{f_1(x_1)+f_2(x_2)} & \text{if} \quad (x_1, x_2) \neq (0,0) \\ \frac{1}{2} & \text{otherwise} \end{cases}$$

where $f_i(0) = 0$, $f_i'(\cdot) > 0$, and $f_i''(\cdot) \leq 0$. Here, we also assume that $C_i(x_i) = x_i$; that is, the cost of effort is equal to the effort itself.

Note that, although the contest success function is assumed rather than derived from some optimization problem, Clark and Riis (1998) and Skaperdas (1996) offer axiomatic foundations for the Logit form of contest success functions. Moreover, in the Appendix, we offer general conditions on payoffs such that the conclusions derived in the main text continue to hold.

Simultaneous Contests

Consider the case where x_1 and x_2 are selected simultaneously. The following facts are known in the contest literature (see, e.g., Yildrim, 2003):

Fact 1. Player i's problem is strictly concave in x_i.

Fact 1 implies that for each x_j, player i has a unique best response $x_i(x_j)$ satisfying $\partial E\pi^i / \partial x_i = 0$. Together with the earlier assumptions made on f_i and C_i, this implies

that $x_i(x_j)$ is continuously differentiable and bounded. Hence, a pure strategy Nash equilibrium is a pair (x_1^*, x_2^*) satisfying

$$\frac{f_2(x_2^*) f_1'(x_1^*)}{(f_1(x_1^*) + f_2(x_2^*))^2} V_1 - 1 = 0$$

$$\frac{f_1(x_1^*) f_2'(x_2^*)}{(f_1(x_1^*) + f_2(x_2^*))^2} V_2 - 1 = 0$$

Fact 2. The best response function $x_i(x_j)$ is strictly increasing when efforts x_i, x_j are such that $f_i(x_i) > f_j(x_j)$, reaches its maximum when $f_i(x_i) = f_j(x_j)$, and is strictly decreasing when $f_i(x_i) < f_j(x_j)$.

Fact 2 implies that the best-response functions $x_1(x_2)$, $x_2(x_1)$ cross the locust where $f_i(x_i) = f_j(x_j)$ exactly once. Therefore, the Nash equilibrium (x_1^*, x_2^*) exists and is unique.

Sequential Contests

Next, suppose that the contest is played sequentially. That is, player 1 chooses x_1, player 2 costlessly and perfectly observes its value, and then chooses x_2.

Note that player 2's best response function, $x_2(x_1)$, is identical to his best response function in the simultaneous contest. Player 1's optimization problem is to choose x_1 to maximize $E\pi^1$ recognizing that player 2 will be playing a best response to x_1.

Fact 3. Given that player 2 is playing a best response, player 1's problem is strictly concave in x_1.

From Facts 3 it now follows that there exists a unique subgame perfect equilibrium in the sequential contest, which we shall denote by $(x_1^{**}, x_2(x_1^{**}))$.

Finally, following Dixit (1987), we say that there is *value to commitment* if the profits of the first mover in the subgame perfect equilibrium of the sequential contest

are higher than in any pure strategy Nash equilibrium of the simultaneous contest. It is easily checked that this corresponds to $f_1 (x_1^*) \neq f_2 (x_2^*)$ for the Nash equilibrium pair x_1^*, x_2^*. We assume that this condition holds.

3 Contests with Observation Costs

How does the value of commitment change when there are positive observation costs? We first examine the case of sequential contests in section 3.1. Next, in section 3.2, we study an extension where the order of moves is determined endogenously. We show that the insights of the sequential contest analysis continue to apply. Finally, in section 3.3, we summarize the main findings and highlight the relationship to the extant literature on the value of commitment.

3.1 Sequential Contests with Observation Costs

Consider the sequential contest but suppose that, prior to deciding on x_2, player 2 must decide whether to pay a cost $\varepsilon > 0$ to observe player 1's choice. If player 2 pays this cost, then player 1's choice is revealed to him. If not, then player 2 obtains no information about 1's choice.

We now present the main result of the paper: The value of commitment vanishes completely when observation is costly. Formally,

Proposition 1 *Fix $\varepsilon > 0$. In any subgame perfect equilibrium of the sequential contest with observation costs, there is no value to commitment.*

To establish Proposition 1, we show that all subgame perfect equilibria of the sequential contest with observation costs correspond to the Nash equilibrium of the simultaneous contest.

First, we prove the following lemma.

Lemma 1 *Fix $\varepsilon > 0$. In any pure strategy subgame perfect equilibrium of the sequential contest with observation costs, player 2 never pays to observe 1's choice.*

Proof. By definition, in any pure strategy subgame perfect equilibrium, player 1 chooses some effort level, x_1, with probability 1. Let $\hat{x}_1$ denote player 2's conjecture about x_1. In equilibrium, $\hat{x}_1 = x_1$. That is, player 2's conjecture about x_1 is correct. This implies that player 2's expected payoff conditional on observing player 1's effort is the same as his expected payoff conditional on not observing player 1's effort. But the cost of observing, ε, is strictly greater than 0. Therefore, in any pure strategy subgame perfect equilibrium, player 2 chooses to never observe player 1's effort. ∎

Given that player 1 anticipates that player 2 never observes player 1's effort, player 1's choice of x_1 satisfies the first order condition

$$\frac{f_2 \left(\hat{x}_2\right) f_1' \left(x_1\right)}{\left(f_1 \left(x_1\right) + f_2 \left(\hat{x}_2\right)\right)^2} V_1 - 1 = 0$$

Here, $\hat{x}_2$ is player 1's conjecture about player 2's choice, where player 2's choice cannot depend on x_1 because he does not observe x_1.

Agent 2's optimal choice of x_2 satisfies the first order condition

$$\frac{f_1 \left(\hat{x}_1\right) f_2' \left(x_2\right)}{\left(f_1 \left(\hat{x}_1\right) + f_2 \left(x_2\right)\right)^2} V_2 - 1 = 0$$

where $\hat{x}_1$ is player 2's conjecture about player 1's choice.

Now notice that the resulting first-order conditions, in combination with the equilibrium restrictions $\hat{x}_1 = x_1$ and $\hat{x}_2 = x_2$, are identical to those for the unique Nash equilibrium of the simultaneous move contest. Thus, we have:

Lemma 2 *The effort levels in any pure strategy subgame perfect equilibrium of the sequential contest with observation costs are identical to the effort levels in the unique pure strategy Nash equilibrium of the simultaneous move contest.*

Next, we turn to mixed strategy equilibria.

As usual, we begin with player 2. Conditional on having observed player 1's choice, player 2's best response is pure, because his problem is strictly concave. If player 2 chooses not to observe, then he forms some beliefs about player 1's choice. We represent these beliefs by the cumulative distribution function $H(\cdot)$. In that case, player 2's optimization problem is

$$
\begin{aligned}
\max_{x_2} E\pi^2 &= V_2 \int_{x_1} \frac{f_2(x_2)}{f_1(x_1) + f_2(x_2)} dH(x_1) - x_2 \\
&= \int_{x_1} \left[\frac{f_2(x_2)}{f_1(x_1) + f_2(x_2)} V_2 - x_2 \right] dH(x_1)
\end{aligned}
$$

Note that this optimization problem is also strictly concave in x_2, as it consists of a convex combination of expressions that we know to be strictly concave in x_2. This implies that player 2 best responds to beliefs H with some pure action, $\hat{x}_2$. We conclude player 2 always plays a pure continuation strategy, whether he decides to observe or not.

Now we turn to player 1. Suppose that player 1 believes that player 2 pays to observe player 1's effort with probability p. Then, player 1's optimization problem is

$$
\begin{aligned}
\max_{x_1} E\pi^1 &= V_1 \left[p \frac{f_1(x_1)}{f_1(x_1) + f_2(x_2(x_1))} + (1-p) \frac{f_1(x_1)}{f_1(x_1) + f_2(\hat{x}_2)} \right] - (x_1) \\
&= p \left[\frac{f_1(x_1)}{f_1(x_1) + f_2(x_2(x_1))} V_1 - x_1 \right] + (1-p) \left[\frac{f_1(x_1)}{f_1(x_1) + f_2(\hat{x}_2)} V_1 - x_1 \right]
\end{aligned}
$$

This optimization problem is strictly concave in x_1, as it is the convex combination of two strictly concave expressions. Therefore, for all $\hat{x}_2$, a unique x_1 maximizes this

expression. Hence, player 1 plays a pure strategy in any subgame perfect equilibrium, which implies that $p = 0$. Therefore, player 2 also plays a pure strategy. We conclude:

Lemma 3 *All subgame perfect equilibria of the sequential contest with observation costs are in pure strategies.*

Now, Lemmas 2 and 3 imply Proposition 1. This completes the proof of the main result.

3.2 An Extension: Contests with Endogenous Moves

Next, we consider an extension where the sequence of moves is endogenously determined. That is, players can expend effort in any one of the two periods. Depending on the timing of their moves, players end up moving simultaneously or sequentially, with either player leading. We assume that in period 2, players do know whether their rival has acted in period 1. But to learn how much effort he expended, players must incur a cost ε. As described in the introduction, many contests have this flavor. For instance, in lobbying settings, it may be quite obvious that a rival lobbying group has been active, but to determine the extent or intensity of its lobbying, one may well have to do some research.

Our main result in this section is to show that the conclusion of Proposition 1 continues to hold in this setting.

Proposition 2 *Fix $\varepsilon > 0$. In any subgame perfect equilibrium of the endogenous move contest with observation costs, there is no value to commitment.*

To establish this result, we show that all equilibrium effort levels undertaken in the first period are 'pure', i.e., non-random. Hence, in equilibrium, there is no incentive

for a player moving in the second period to pay ε to observe his rival's first period effort level, since that effort level is already 'known' with certainty.

First consider the situation of a player i moving in the second period. If his rival j is moving in the second period as well, then both players play the unique Nash equilibrium strategies of the simultaneous game.

Now suppose that player i has moved in the first period, but player j has not. Then, conditional on observing player i's effort level, player j's best response in period 2 is pure, because his problem is strictly concave. Conditional on not observing player i's effort level, player j forms some beliefs about player i's choice of effort, represented by the cdf $H(\cdot)$. In that case, player j's optimization problem is

$$
\begin{aligned}
\max_{x_j} E\pi^j &= V_j \int_{x_i} \frac{f_j(x_j)}{f_i(x_i) + f_j(x_j)} dH(x_i) - x_j \\
&= \int_{x_i} \left[\frac{f_j(x_j)}{f_i(x_i) + f_j(x_j)} V_j - x_j \right] dH(x_i)
\end{aligned}
$$

Since this problem is strictly concave, player's j's best response to any beliefs H about i's effort outlay in the first period must be pure. Thus, under all circumstances, player j plays some pure effort level in period 2.

Next, consider the situation facing player i when moving in period 1. Suppose that his beliefs are as follows: Player j also moves in the first period with probability q_j. Conditional on moving in the first period, player j chooses an effort level according to some cdf $L(x_j)$. Conditional on moving in the second period, player j chooses to observe player i's effort with probability p_j and best responds conditional on observing. With probability $1 - p_j$ player j does not observe and chooses some (pure) effort level $\hat{x}_j$. Thus, player i chooses x_i to maximize

$$
\max_{x_i} E\pi^i = V_i \left[q_j \int_{x_j} \frac{f_i(x_i)}{f_i(x_i) + f_j(x_j)} dL(x_j) + (1 - q_j) \left\{ \begin{array}{l} p \frac{f_i(x_i)}{f_i(x_i) + f_j(x_j(x_i))} + \\ (1 - p) \frac{f_i(x_i)}{f_i(x_i) + f_j(\hat{x}_j)} \end{array} \right\} \right] - x_i
$$

$$
\begin{aligned}
= \;\; & q_j \int_{x_j} \left(\frac{f_i(x_i)}{f_i(x_i) + f_j(x_j)} V_i - x_i \right) dL(x_j) \\
& + (1 - q_j) \, p_j \left(\frac{f_i(x_i)}{f_i(x_i) + f_j(x_j(x_i))} V_i - x_i \right) \\
& + (1 - q_j)(1 - p_j) \left(\frac{f_i(x_i)}{f_i(x_i) + f_j(\hat{x}_j)} V_i - x_i \right)
\end{aligned}
$$

Notice that $E\pi^i$ is a convex combination of three expressions that have previously been shown to be strictly concave in x_i. Therefore, player i's optimization problem is strictly concave as well and, as a consequence, he must choose a pure effort level x_i when going first in any subgame perfect equilibrium.

Of course, this then implies that, in equilibrium, it is never optimal for a player moving second to pay the observation cost to learn about the effort undertaken by a player moving first. Thus, the players' effort in all subgame perfect equilibria of contests with endogenous timing of moves coincides with their effort in the unique Nash equilibrium of the corresponding simultaneous game. Hence, there is no value to commitment.

Assumptions 1-3, given in the Appendix, which imply generalize Proposition 1, are also sufficient for Proposition 2 to hold more generally as well.

3.3   Discussion

In the case of finite games, Bagwell (1995) first observed that, if one restricts attention to pure strategy equilibria, the value of commitment in sequential move games of complete information is fragile to small perturbations of the game where the second player only imperfectly observes the first player's move. Várdy (2004) made a similar observation for finite games with observation costs. However, it is well known that the value of commitment in these settings is restored if one also allows for mixed strategies.

The cases of sequential and endogenous move contests—games with continuous strategy spaces—lead to a stronger result. Propositions 1 and 2 show that the value of commitment vanishes regardless of whether one includes mixed strategies.

Why is it that allowing for mixed strategies does not restore the value of commitments in contests? The intuition is grasped most easily from the sequential contest.

In games with finite strategy spaces, it is always possible to find two or more actions for player 1 over which she is indifferent in equilibrium. This allows player 1 to "commit" to playing a mixed strategy which, in turn, creates positive value to observing player 1's action. Indeed, if the cost is sufficiently small, this induces player 2 to observe players 1's choice, thus restoring the "transmission path" for commitment to have value.

In the continuous case, by contrast, precisely because the game is "well-behaved" —in the sense that the first player's payoff is strictly concave regardless of the strategy profile of the second player— the first player cannot credibly commit to anything other than her *unique* best response. In other words, mixing is not incentive compatible for the first player.

One might wonder whether the result obtained in Proposition 1 is specific to the Logit form of the contest success function and the linear cost of effort. As the intuition above suggests, it is not. In the Appendix, we offer general conditions on contest success and cost of effort functions such that Proposition 1 continues to hold.

Returning to the problem at hand, it seems clear that what the first player needs is a mechanism to credibly commit to unpredictable behavior and thereby induce player 2 to pay to observe 1's actions. In the next section we show that sequential rank-order tournaments —games which often have been viewed as isomorphic to contests (see, for instance Dixit, 1987)— offer an avenue to restore the value of commitment.

4 Tournaments with Observation Costs

Our model of tournaments closely follows that of Lazear and Rosen (1981). In Lazear and Rosen, effort x_i has an effectiveness $y_i = x_i + \delta_i$, which may be interpreted as output. That is, the output generated by a player's effort depends on the effort itself as well as on a random component δ_i. The object is then awarded to the player with the greater output. That is, player i wins when $y_i > y_j$ and loses otherwise.[6] We assume that for both players, δ_i is drawn from the same atomless distribution, F, with support Δ.

The cost to player i of exerting effort x_i is given by the function $C_i(x_i)$, where $C(0) = 0$ and $C(\cdot)$ is continuous, strictly increasing, and strictly concave. Thus, player i's expected payoff when she exerts effort x_i and player j exerts effort x_j is

$$E\pi^i = \Pr(y_i > y_j) V_i - C_i(x_i)$$

where $\Pr(y_i > y_j) = \Pr(x_i + \delta_i > x_j + \delta_j) = \Pr(\delta_j - \delta_i < x_i - x_j)$. We denote the cdf of $(\delta_j - \delta_i)$ by G, with associated density g.

Simultaneous Tournament

In a simultaneous tournament, both players choose effort sat the same time. Following this, the random variables δ_1 and δ_2 are realized and the player with the higher output is the winner. Lazear and Rosen (1981) show that, with sufficient structure, there exists a pure strategy equilibrium (x_1^*, x_2^*) characterized by the first-order conditions

$$g(x_1^* - x_2^*) V_1 - C_1'(x_1^*) = 0$$
$$g(x_1^* - x_2^*) V_2 - C_2'(x_2^*) = 0.$$

---

[6]We can safely ignore ties since F is atomless.

Sequential Tournament

In a sequential tournament, player 1 chooses x_1 and immediately thereafter, δ_1 is realized. The effectiveness, $y_1 = x_1 + \delta_1$, is then revealed to player 2. Upon observing y_1, player 2 chooses his effort x_2 and δ_2 is realized. Finally, players receive their payoffs.[7]

One can show that, with sufficient structure on F and C_i, there exists a subgame perfect equilibrium in pure strategies $(x_1^{**}, x_2^{**}(y_1))$ where commitment has value.[8] That is, $(x_1^{**}, x_2^{**}(y_1)) \neq (x_1^*, x_2^*)$ for any Nash equilibrium (x_1^*, x_2^*) of the simultaneous tournament.

Next, we consider the sequential tournament with observation costs. We show that the value of commitment is fully retained if ε is sufficiently small. The result follows easily from the following Lemma.

Lemma 4 *For a positive measure of δ_1 realizations, player 2's best response to $y_1 = x_1^{**} + \delta_1$ is not the same as his best response to $y_1 = x_1^{**}$.*

Proof. We prove the lemma by contradiction. Suppose that for almost all δ_1 realizations, player 2's best response to $y_1 = x_1^{**} + \delta_1$ is the same as his best response to $y_1 = x_1^{**}$. Or, in our notation: $x_2(x_1^{**} + \delta_1) = x_2(x_1^{**})$, for almost all δ_1. This implies

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[7]A different type of sequential tournament has been modeled by Dixit (1987). In his model, δ_1 and δ_2 are realized *after* both x_1 and x_2 have been chosen. In that case, a sequential tournament simply is a sequential contest with a particular functional form for the success function. However, for tournaments such as the olympic long jump, or the sales "contest" described in Glegarry Glenn Ross (Mamet, 1984), modeling δ_i as occurring contemporaneously with x_i seems to us more reasonable.

[8]Following Lazear and Rosen, consider the case where $\delta_i \backsim \mathcal{N}\left(0, \sigma^2\right)$, $V_1 \neq V_2$, and $C_i(x_i) = \frac{\gamma}{2}x_i^2$, where γ is a parameter describing the degree of convexity of the cost function. It may be verified that for sufficiently large values of γ and σ^2 there indeed exists a pure strategy subgame perfect equlibrium where commitment has value.

that, for almost all δ_1,

$$\frac{\partial x_2 \left(x_1^{**} + \delta_1\right)}{\partial x_1} = \frac{\partial x_2 \left(x_1^{**}\right)}{\partial \delta_1} = 0$$

Player 2's best response function, $x_2 \left(y_1\right)$, is characterized by

$$f \left(y_1 - x_2\right) V_2 - C_2' \left(x_2\right) = 0$$

where $y_1 = x_1 + \delta_1$. Therefore, the best response $x_2 \left(x_1^{**} + \delta_1\right)$ is characterized by

$$f \left(x_1^{**} + \delta_1 - x_2\right) V_2 - C_2' \left(x_2\right) = 0$$

Because $x_2 \left(x_1^{**} + \delta_1\right) = x_2 \left(x_1^{**}\right)$, it must be that $f \left(x_1^{**} + \delta_1 - x_2\right) = f \left(x_1^{**} - x_2\right)$ for almost all δ_1. This contradicts $f' \left(\cdot\right) > 0$. ∎

Lemma 4 implies that for sufficiently small ε, player 2 will always observe player 1's effectiveness if he believes that player 1 plays x_1^{**}. As a consequence, for sufficiently small observation costs, the sequential tournament with observation costs effectively reduces to a "vanilla" sequential tournament. Hence, we have:

Proposition 3 *For sufficiently small cost of observation, the value of commitment is preserved completely in the sequential tournament with observation costs.*

Formally: There exists a value, $k > 0$ such that when observation costs are less than k, effort levels $\left(x_1^{**}, x_2 \left(y_1\right)\right)$ with player 2 observing with probability one constitute a subgame perfect equilibrium of the sequential tournament with observation costs.

5   Conclusions

We have shown that the value of commitment is very fragile in contests, but not in tournaments. In our terminology, a sequential contest is a competition where effort

is observable but its effectiveness is not. For instance, it may be easy to find out that a rival pressure group has hired an expensive lobbying firm. But, until the regulator has ruled or Congress has voted, the effectiveness of the lobbying effort is not fully revealed. In a sequential tournament, by contrast, it is the effectiveness of effort that is observable, while the underlying effort is not. For instance, a long jumper may readily observe the distance jumped by a rival jumper, even though the effort required to produce the jump remains unknown.

Previously, Bagwell and others have shown that the value of commitment is fragile in *finite* Cournot games. However, this result only holds if one limits attention to pure strategies. The value of commitment can always be restored in these games by allowing for mixed strategies. In this paper, by contrast, we have shown that for a general class of contests, mixed strategies offer no way out.

Ironically, the same conditions ensuring that a sequential contest is well-behaved also ensure the destruction of the value of commitment when the follower is obliged to pay even arbitrarily small costs to observe the action of the leader. For well-behaved tournaments, on the other hand, the value of commitment is perfectly preserved, provided that observation costs are sufficiently small.

The crucial difference between sequential contests and tournaments is in the *equilibrium* value of the information available to the second mover. In a sequential contest, the second mover gets to observe the first mover's effort, which has no value in equilibrium if the latter plays a pure strategy. In a tournament, effort is combined with exogenous noise to determine effectiveness. Hence, even if the first-mover plays a pure strategy, information about the – randomly determined – effectiveness of the first mover's effort is still valuable to the second mover. Indeed, in a sequential tournament with observation costs, the second mover cannot credibly commit to *not*

observing player 1's effectiveness—even though this would be desirable in reducing 1's advantage of moving first.

Why, then, does the first mover in a sequential contest not inject noise into her strategy to restore the value of observation for the second mover? While this would clearly be desirable, the first mover cannot overcome the temptation to "purify" her behavior and always play the unique pure strategy that maximizes her profits given her beliefs. Of course, the second mover anticipates this and the scenario unravels.

In an interesting paper, Yildrim (2005) studies contests with effort accumulation. In his two-period model, players have the option of exerting effort in both periods, such that a player's total effort is the sum of his efforts in the periods one and two. Because Yildrim restricts attention to pure strategy subgame perfect equilibria, it follows immediately from our Lemma 2 that these equilibria are not robust to the introduction of small observation costs. Whether there are mixed strategy equilibria that are robust to the introduction of observation costs remains an open question.

In this paper we have restricted attention to contests. However, it is quite obvious that the same techniques can be applied to other games where the first player's problem is strictly concave. An important example occurs in the standard Cournot quantity competition model with linear demand. In this setting, it is easy to see that the same conclusion is reached as in Proposition 1 is reached: In all subgame perfect equilibria of the sequential and endogenous move games with observation costs, quantities produced are identical to those in the unique Cournot-Nash equilibrium of the simultaneous move game—regardless of the size of observation costs.

Our results can be interpreted in two ways. First, they offer a justification for the attention that the extant literature has given to simultaneous contests. Indeed, in settings where effectiveness of lobbying is unobservable even though the lobbying

itself can be observed at some arbitrarily small cost, the sequential and endogenous contests are outcome equivalent to the simultaneous contest. Another way to read our results is that the fragility of the value of commitment depends crucially on modeling assumptions that are seemingly innocuous or even mathematically equivalent in the simultaneous case.

A   Appendix: Contests with General Payoff Structures

In this section, we derive general sufficient conditions for contest success and cost of effort functions such that Proposition 1 continues to hold.

First, suppose that the contest success function, P, is continuously differentiable and that $P_1 = \frac{\partial P(x_1, x_2)}{\partial x_1}$ and $P_2 = \frac{\partial P(x_1, x_2)}{\partial x_2}$ are such that $P_1 > 0$ and $P_2 < 0$ for all x_1, x_2. That is, agent 1's chances of winning are increasing in her own effort and decreasing in her rival's effort. Let $C_i(x_i)$ denote the cost of effort x_i to agent i and assume that $C_i(\cdot)$ is continuously differentiable, strictly increasing, and (weakly) convex. We also assume $\lim_{x_i \to \infty} C_i(x_i) = \infty$. Since V_i is bounded, this ensures that agents undertake finite effort levels.

Simultaneous Contest

Suppose that the two agents compete simultaneously. To ensure that this problem is well-behaved, we make the following regularity assumption.

Assumption 1. Agent i's problem is strictly concave. That is, for all x_1, x_2 :

$$P_{11}(x_1, x_2) V_1 - C_1''(x_1) < 0$$

and

$$-P_{22}(x_1, x_2) V_2 - C_2''(x_2) < 0$$

Assumption 1 is satisfied if C_i is sufficiently convex or $P_{11} < 0$ and $P_{22} > 0$. It guarantees that the first-order conditions are both necessary and sufficient for characterizing the best-response functions of the agents. Hence, a pure strategy Nash

equilibrium is a pair (x_1^*, x_2^*) satisfying

$$P_1(x_1^*, x_2^*) V_1 - C_1'(x_1^*) = 0$$

$$-P_2(x_1^*, x_2^*) V_2 - C_2'(x_1^*) = 0$$

Note that at least one such equilibrium exists, since the best response functions are bounded and continuous.

Sequential Contest

Next, suppose that the contest is played sequentially. That is, agent 1 chooses x_1, agent 2 costlessly and perfectly observes its value, and then chooses x_2.

Agent 2's best response function, which we denote by $x_2(x_1)$, is identical to his best response function in the simultaneous contest. It is characterized by the first order condition

$$P_2(x_1, x_2) V_2 - C_2'(x_1) = 0$$

Agent 1's optimization problem is to choose x_1 to maximize $E\pi^1$, recognizing the dependence of x_2 on x_1. To ensure that agent 1's optimization problem is well-behaved, we make the following assumption which is again satisfied provided that C_1 is sufficiently convex:

Assumption 2. Agent 1's problem is strictly concave. That is, for all x_1,

$$V_1 \left[\begin{array}{c} P_{11}(x_1, x_2(x_1)) + 2P_{12}(x_1, x_2(x_1)) \frac{\partial x_2}{\partial x_1} + \\ P_{22}(x_1, x_2(x_1)) \left(\frac{\partial x_2}{\partial x_1}\right)^2 + P_2(x_1, x_2(x_1)) \frac{\partial^2 x_2}{(\partial x_1)^2} \end{array} \right] - C_1''(x_1) < 0$$

where $x_2(x_1)$ is agent 2's best response to x_1.

If Assumptions 1 and 2 hold, then there exists a *unique* subgame perfect equilibrium, $(x_1^{**}, x_2(x_1^{**}))$, in the sequential contest. This equilibrium is characterized by the following first order conditions:

$$V_1 \left[P_1 \left(x_1^{**}, x_2 \left(x_1^{**} \right) \right) + P_2 \left(x_1^{**}, x_2 \left(x_1^{**} \right) \right) \frac{\partial x_2 \left(x_1^{**} \right)}{\partial x_1} \right] - C_1' \left(x_1^{**} \right) = 0$$

$$-P_2 \left(x_1, x_2 \right) V_2 - C_2' \left(x_2 \right) = 0$$

where

$$\frac{\partial x_2 \left(x_1^{**} \right)}{\partial x_1} = \frac{-P_{12} \left(x_1^{**}, x_2 \left(x_1^{**} \right) \right) V_2}{P_{22} \left(x_1^{**}, x_2 \left(x_1^{**} \right) \right) V_2 - C_2' \left(x_2 \left(x_1^{**} \right) \right)}$$

Value of Commitment

Fix the effort levels of the two agents at some pure strategy Nash equilibrium, $\left(x_1^*, x_2^* \right)$, of the simultaneous contest. The following assumption guarantees that there is positive value of commitment.

Assumption 3. For all $\left(x_1^*, x_2^* \right)$,

$$V_1 \left[P_2 \left(x_1^*, x_2 \left(x_1^* \right) \right) \frac{\partial x_2 \left(x_1^* \right)}{\partial x_1} \right] \neq 0$$

Here, $x_2 \left(x_1^* \right) = x_2^*$ by definition, and

$$\frac{\partial x_2 \left(x_1^* \right)}{\partial x_1} = \frac{-P_{12} \left(x_1^*, x_2 \left(x_1^* \right) \right) V_2}{P_{22} \left(x_1^*, x_2 \left(x_1^* \right) \right) V_2 - C_2' \left(x_2 \left(x_1^* \right) \right)}$$

Together, Assumptions 1-3 ensure that the class of contests we study excludes "pathological cases," where the first-mover's problem is ill-behaved, equilibrium only exists in mixed strategies, or the follower is non-reactive.

Sequential Contests with Costly Observation

Recall that our main result was:

In any subgame perfect equilibrium of the costly leader contest, there is no value to commitment.

To establish the result for the current, more general set up, we again show that all subgame perfect equilibria of the costly leader contest correspond to Nash equilibria of the simultaneous contest.

First, note that Lemmas 1 and 2 carry over immediately to the more general setting without requiring any change in their proofs. Thus, we need only worry about mixed strategy equilibria.

As usual, we begin with player 2. If player 2 chooses to observe, then, by Assumption 1, his problem is strictly concave. If player 2 chooses not to observe and believes that player 1's strategy is given by the cdf H, then player 2's problem is

$$
\begin{aligned}
\max_{x_2} E\pi^2 &= V_2 \int_{x_1} P(x_1, x_2) \, dH(x_1) - C_2(x_2) \\
&= \int_{x_1} [P(x_1, x_2) V_2 - C_2(x_2)] \, dH(x_1)
\end{aligned}
$$

and, again by assumption 1, this problem is strictly concave. Therefore, player 2 always plays a pure continuation strategy.

Turning to player 1, if she believes that player 2 observes with probability p, then her optimization becomes

$$
\begin{aligned}
\max_{x_1} E\pi^1 &= V_1 \left[pP(x_1, x_2(x_1)) + (1-p) P(x_1, \hat{x}_2) \right] - C_1(x_1) \\
&= p \left[P(x_1, x_2(x_1)) V_1 - C_1(x_1) \right] + (1-p) \left[P(x_1, \hat{x}_2) V_1 - C_1(x_1) \right]
\end{aligned}
$$

And, by Assumptions 1 and 2, this problem is strictly concave. Hence, player 1 plays a pure strategy in any subgame perfect equilibrium. Finally, this implies that $p = 0$. Therefore, all subgame perfect equilibria in the sequential contest with observation costs are in pure strategies and Proposition 1 follows.

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