

# Use of a modified SIR-V model to quantify the effect of vaccination strategies on hospital demand during the Covid-19 pandemic

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**Abstract**—A novel compartmental model that includes vaccination strategy, permanence in hospital wards and tracing of infected individuals has been implemented to forecast hospital overload caused by COVID-19 pandemics in Italy. The model parameters were calibrated according to available data on cases, hospital admissions, and number of deaths in Italy during the second wave, and were validated in the timeframe corresponding to the first successive wave where vaccination campaign was fully operational. This model allowed quantifying the decrease of hospital demand in Italy associated with the vaccination campaign.

**Clinical relevance** This study provides evidence for the ability of deterministic SIR-based models to accurately forecast hospital demand dynamics, and support informed decisions regarding dimensioning of hospital personnel and technologies to respond to large-scale epidemics, even when vaccination campaigns are available.

## I. INTRODUCTION

Large-scale epidemics are generally characterised by slow onsets and exponential growths, with long-lasting effects [1]. The generally observed gradual increase in the number of cases allows governments to gradually implement mitigation strategies to counter the oncoming wave, but the continued spread of the disease and the possibility of a delayed recovery of patients can easily lead hospital wards (HW) and Intensive Care Units (ICU) to overload. The COVID-19 pandemic, in particular, has put such facilities to the test, both in terms of involved personnel and technology [2].

In the virtual absence of effective treatments, non-pharmaceutical interventions (NPIs, which may include limiting gatherings at a maximum specified number, mandating personal protective equipment use, defining minimum interpersonal physical distances, and enforcing lockdowns) aimed to reduce the transmission between hosts, while research into the development and production of effective vaccines was sped up, based on the assumption that the availability of an effective vaccine can control the spread of the virus in the population. With regard to the recent pandemics, vaccination campaigns started in a few countries about a year after the epidemics outbreak [3], and nearly all countries have now

implemented mass vaccination campaigns, with an overall coverage of around 60% of the entire world population [4].

Both NPIs and vaccination campaigns contribute to reduce hospital demand, easing support for patients with the infection, and lowering the likelihood of a less effective treatment for patients accessing hospitals for other conditions.

From a scientific standpoint, epidemiology comes in help, by providing mathematical models that can be used to study the epidemic and predict its evolution over time [5]. Compartmental models provide the community with simulations of various strategies that policymakers can use to optimise interventions and allocate resources [6]. Calibrating these models based on actual data makes it possible to determine optimal values for some parameters of interest regarding the evolution of an epidemics, such as the infection rate, infectious period, case fatality rate. From this optimisation, the evolution over time of the reproduction number can be estimated to monitor and forecast the trend of the transmission of such diseases [7].

Since the outbreak of COVID-19 epidemics, a vast number of compartmental models have been appearing to study the ongoing infectious disease [8], and they have been proven capable of suggesting which interventions are most likely to effectively reduce the number of cases [9]: most of them stemmed from the traditional SIR model, where population is divided into three stocks or compartments (susceptible - S, infected - I, and recovered - R), and transitions from one compartment to another are regulated based on parameters that take into account specific temporal rates.

Elaborations of such models were introduced to account for incubation periods (thus introducing the additional exposed - E compartment), as well as the possible lethality associated with the disease (thus introducing the additional deceased - D *sink* compartment). In an effort to simulate the pathway of infected individuals across different stages of the disease, additional compartments were introduced to mimic the admission to different hospital departments, usually distinguishing between regular HWs, and ICUs [10]. In some cases, models diverged from the traditional deterministic transition process introducing statistical variations that would account for uncertainties associated with the disease dynamics [11]. The presence of interventions within the evolution of the pandemics led to the introduction of dynamic variations over time of the main parameters of transmission [12], based on the assumption that a static model is not able to accurately follow variations of the dynamics associated with the enforcement of such interventions.

In the present contribution, we will present a modified SIR

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deterministic model that includes a few factors reflecting the in-field observations of the management of pandemics at the national level: 1) the presence of compartments associated with different stages of hospitalisations, 2) the introduction of vaccination during the evolution of pandemics, and 3) the presence of a non-ideal ability to identify new cases.

We will focus on the modelling of COVID-19 evolution and prediction of the associated health system outcomes (admissions to hospital and deaths) during the ongoing vaccination campaign in Italy. The rest of the study is organised as follows. The following section will contain the description of the proposed SIR-V model, and the implemented procedure for the parameter calibration. Procedure for the validation of the model will be presented in the ensuing section. Results from the validation procedure will be given in section IV, while discussion on the results and the limitations of the study will be drawn in section V.

## II. MODEL DESCRIPTION

### A. Description of the compartmental architecture

The model was introduced to provide a description of the epidemic, with a specific focus on the hospital demand in terms of both ordinary HWs and ICUs. The model's backbone is a traditional SIR, with an additional number of compartments fitted for the aims of the work. On top of this, the model also accounted for those asymptomatic infections that escaped testing, but played a significant role in feeding the epidemic with new infections. Indeed, these asymptomatic individuals, being undetected, might freely proceed with their every-day lives, and are able to spread the infection as long as they recover; in the meantime, they will experience neither isolation nor hospitalisation. For this reason, this contingency was included in the model.

All of the above considerations led to the introduction of two distinct infection terms: one corresponding to the detected infections and the other corresponding to the escaped ones. Basically, infections are assumed to be either detected or undetected. The former will cause individuals to be either asymptomatic or symptomatic and, depending on the severity of symptoms, they may be subject to home isolation or hospitalisation. The latter remains active during the entire period of infectivity. A critical point worth mentioning is that isolated/hospitalised infected individuals are cut apart from the infection chain, meaning that they no longer contribute to spread the disease (assuming a perfect isolation both at home and in the hospital), whereas the undetected asymptomatic individuals still continue to spread the virus until they naturally recover. For the sake of simplicity, we hypothesised that susceptible individuals being infected are able to spread the virus with no delay (i.e. no added Exposed compartment).

The overall structure of the different compartments with the corresponding connections is shown in Fig. 1, where:

- S is the compartment for susceptible individuals;
- V is the compartment for vaccinated individuals;
- I is the compartment of infected individuals;
- U is a compartment collecting the undetected infected individuals;

- J is the compartment of detected infected individuals in isolation;
- ICU is the compartment of infected individuals admitted to intensive care units;
- HW is the compartment of infected individuals admitted to regular hospital wards;
- $HW_R$  is the compartment of individuals recovered after ICU admission, and admitted to hospital wards;
- D is the compartment of the deceased individuals;
- R is the compartment of the recovered individuals.

Regarding connections between compartments, these are regulated from the following parameters:

- Transmission rate,  $\beta$
- Share of undetected infections,  $und$
- Recovery rate of undetected asymptomatic individuals,  $\gamma_4$  ( $days^{-1}$ )
- Isolation rate,  $\sigma_1$  ( $days^{-1}$ )
- Recovery rate from isolation,  $\gamma_1$  ( $days^{-1}$ )
- Share of severe symptomatic individuals,  $p$
- Hospitalisation rate,  $\sigma_2$  ( $days^{-1}$ )
- Share of hospitalised individuals in ordinary ward,  $q$
- Recovery rate from ordinary ward,  $\gamma_2$  ( $days^{-1}$ )
- Death rate from ordinary ward,  $\gamma_{2d}$  ( $days^{-1}$ )
- Share of people recovering from ordinary ward,  $k$
- Share of deceased from ICU,  $b$
- Death rate,  $i$  ( $days^{-1}$ )
- Recovery rate from ICU,  $\sigma_3$  ( $days^{-1}$ )
- Recovery rate from the ward after ICU,  $\gamma_3$  ( $days^{-1}$ )
- Vaccination rate,  $P$
- Vaccine effectiveness,  $eff$
- Days before immunity,  $T$

With reference to the components of the model architecture, the compartment U may be considered as a virtual one, since, while still populating the share of infected individuals, no member of the compartment is moved towards other compartments (such as hospitalisations and isolations) until recovery. Regarding the I compartment, we opted for making this compartment neglecting the incubation latency, as we were not necessarily interested in the specific dynamics of such transition, which would instead further complicate the proposed model. Regarding the different isolation compartments, we made the following assumptions: individuals are detected all outside the hospital, and all of them remain in isolation. A share of them is then admitted to the hospital after some days. This time constant is common to both hospitalisation admission (i.e. HW and ICU), and we assumed that individuals in HW can either die or recover, but no direct connection from HW to ICU was considered. People in ICU, if recovering, will re-join the hospital ward for a few days before leaving the hospital as recovered individuals. In the model, deaths are assumed to come either from regular wards or from ICUs, but not directly from the isolation compartment, based on some evidence that, in the second wave in Italy, the share of deaths at home or in nursing homes not passing through the hospitals was a small minority.

Some of the assumptions made may be considered as not

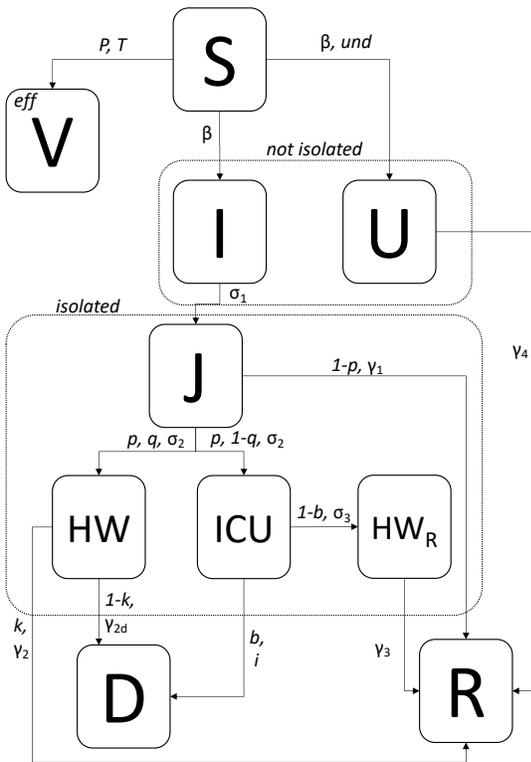


Fig. 1. Structure of the proposed model. Parameters for transitions between compartments are defined in the main text.

reflecting reality in conditions of overload (such as those observed during the first outbreak of COVID-19), but they more probably adhere to the actual conditions when the epidemic wave can be managed by the hospital network system. Following the same reasoning, we assumed that the transition to the deceased compartment can only originate from hospitalised individuals.

### B. Calibration of model parameters

A total number of 18 parameters were thus introduced in the model. We decided to perform the calibration of the model, by considering Italy government choices for a subset of the parameters, and taking into account the observed data in the timeframe August-December 2020 for a second subset of them. In particular, regarding observed data, we made reference to four monitored variables: detected cases, hospital wards presences, intensive care units presences, and deaths. These were made available online by the Italian Civil Protection Department [13]. As it can be seen in Fig. 2, the chosen timeframe allowed us to identify the onset of the second wave. The first timeframe ended at December 2020, in order to avoid two confounding factors: 1) the initiation of the vaccination campaign, and 2) the onset of a third sub-wave. Both events can be approximately located in the first two weeks of January 2021.

Observed data for the four variables were then analytically fitted with a gaussian model, to reduce uncertainties associated with data collection. Out of 18 parameters, 3 of them

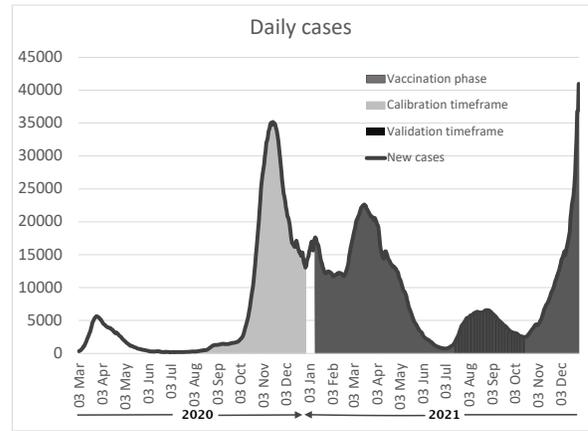


Fig. 2. COVID-19 daily detected cases in Italy. Model calibration and vaccination campaign timeframes are shaded with different grey levels.

( $P, T, eff$ ) were left apart because, referring to vaccination strategy and efficacy characteristics, they were not available at the time of the second wave. For the remaining 15 parameters, we fixed 10 of them, leaving the calibration procedure to identify optimal values for 5 relevant parameters of interest, which were: the transmission rate  $\beta$ , the share of detected individuals requiring hospitalisation  $p$ , the share of individuals admitted to hospital and not requiring intensive care  $q$ , the share of patients in regular wards recovering from the disease  $k$ , and the share of people in ICU which would not recover  $b$ .

The remaining parameters were fixed based on data available from the literature [14, 15, 16]. Calibration was then performed, by minimising an error function obtained by combining two shape parameters for each of the monitored variables (active infections, HW presences, ICU presences, and cumulative deaths): the relative error in correspondence to the maximum value, and the relative difference in temporal duration in correspondence to half the maximum value.

TABLE I  
VALUES OF THE MODEL PARAMETERS

Parameter	Value
Fixed parameter values	
$und$	0.500
$\sigma_1$	0.200
$\sigma_2$	0.250 days <sup>-1</sup>
$\sigma_3$	0.071 days <sup>-1</sup>
$\gamma_1$	0.333 days <sup>-1</sup>
$\gamma_2$	0.055 days <sup>-1</sup>
$\gamma_{2d}$	0.142 days <sup>-1</sup>
$\gamma_3$	0.333 days <sup>-1</sup>
$\gamma_4$	0.142 days <sup>-1</sup>
$i$	0.080
Calibrated parameter values	
$\beta$	0.197 days <sup>-1</sup>
$p$	0.007
$q$	0.885
$b$	0.347
$k$	0.900

### III. MODEL VALIDATION

The procedure for model validation was performed again on data for the timeframe July 15-October 13, 2021. Also in this case, gaussian fitting was performed for all variables of interest. Same values for the fixed parameters were used, but in this case, parameters on vaccination strategy and effectiveness were added to the model: a vaccination rate  $P$  corresponding to a daily number of vaccinated individuals fixed at 300.000, according to the average amount of vaccinations that were performed in Italy during the general population coverage phase; a vaccine effectiveness  $eff$  in terms of protection from infection set to 0.90, to reflect the claimed values given before the onset of waning effect, and an average latency from immunity  $T$  set to 14 days.

Two important assumptions were made for the vaccine: the first was that the vaccine provided immunity against the infection with effectiveness  $eff$  without specific differences regarding the protection from infection, symptoms, or hospitalisations, an assumption that, however, might not be necessarily granted over time [17]; the second was that the immunity coming from the vaccine was considered long-lasting within the studied timeframe (three months), without waning, even if this assumption may not hold true for longer timeframes.

### IV. VALIDATION PHASE RESULTS

Thus, the fitting model applied on the wave corresponding to the period July-October 2021 yielded the calculated values for the parameters reported in Table 2.

TABLE II  
VALUES OF THE MODEL PARAMETERS (VALIDATION PHASE)

Parameter	Value
$und$	0.600
$\beta$	$0.326 \text{ days}^{-1}$
$p$	0.004
$q$	0.882
$b$	0.700
$k$	0.940

In particular, most parameters showed values different from those in the timeframe of the second wave, with the exclusion of the relative shares of people in hospital who were in either ordinary wards or intensive care units. On the contrary, the transmission rate  $\beta$  almost doubled, the share of detected people who needed hospitalisation was relevantly lower, while the relative share of deaths coming from ICUs resulted relatively higher in the second timeframe as compared to the second wave. Also, it was necessary to increase the share of undetected individuals to 0.6. With the chosen parameters, the model fitted the observed data accurately, with a negligible relative error at maximum value, and a good accuracy in terms of time evolution for detected cases and ordinary hospitalisations, despite a slightly larger wave duration for both hospital wards and intensive care units presences (see Fig. 3). Then, to assess the ability of the vaccination strategy in reducing the burden on hospital

facilities and decreasing fatalities, two additional simulations were run changing only the number of daily vaccinated individuals (doubling and then halving the value of the actual vaccination rate  $P$ , respectively). As it can be seen from Fig. 4, even in presence of a small wave such as the one experienced in the summer-early autumn 2021 in Italy, the effect of a possible reduction of the vaccination rate at 150.000 vaccinations/day would have been not negligible, with an approximate increase of 15% of hospital presences for both HWs and ICUs, and a further predicted increase of around 500 deaths.

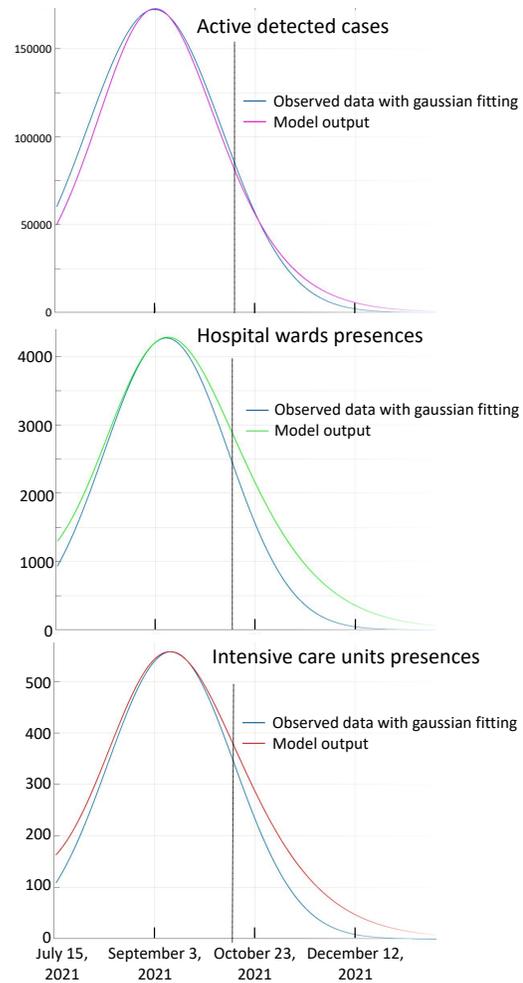


Fig. 3. Comparison of SIR-V model output with gaussian fit on observed data in the timeframe July 15-December 15. The thin black line identifies the time limit for gaussian fitting on observed data (October 15).

### V. DISCUSSION AND CONCLUSIONS

The proposed SIR-V model was able to capture a share of the modifications associated with the pandemics in the two timeframes corresponding to September-December 2020 and July-October 2021. In particular, the model captured the increased transmission rate observed in the latter wave, probably due to the increased infectivity of the delta variant [18], which was most prevalent at that time. The model was

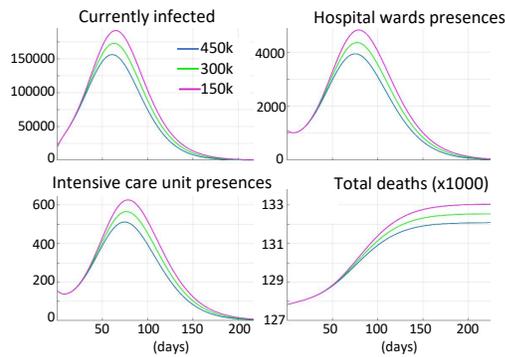


Fig. 4. Effect of varying vaccination rates on the predicted cases, presences in hospital departments, and cumulated deaths.

also able to display a higher resilience of the system to treat the hospitalised subjects, with a modification of the shares of patients needing intensive care, and a smaller share of patients in ordinary wards who would have not recovered. On top of this, the presence of the vaccination campaign was proved to be effective, as can be noticed by the modifications between the two timeframes (in terms of ratio between cases and deaths). Considering that no major modifications were made to most of the other dynamics parameters, this evidence suggested that the reduced severity of cases displayed by the model in the second timeframe was a further confirmation of the vaccine effectiveness. The change of the undetected share of asymptomatic individuals, increased at 0.6 from 0.5 passing from the first to the second timeframe, could also be associated with the introduction of the vaccination campaign and the immunity passport enforcement: if on one side, escapes from detection might have been less frequent, considering that unvaccinated individuals needed to be tested for attending a variety of activities in that timeframe, on the other side there were lower chances for a vaccinated individual to be infected, since, at the central time of the second timeframe, two thirds of the population were vaccinated.

#### A. Study limitations and conclusions

The deterministic model has a number of limitations that need to be taken into account. First, the model parameters were kept fixed in each evolution, while it is reasonable to accept that a number of them may vary as the result of possible containment strategies in response to an outbreak. For example, we can consider the transmission rate which decreases when lockdowns are enforced, or the undetected share, which may substantially vary if the strategy and quantity of testing is modified during the outbreak. Second, the introduction of the vaccination effect could be elaborated at will, by considering that its efficacy might be different if considered as immunity from infection, or in terms of protection from infectiousness, or as protection from symptoms. Third, vaccine could change (i.e. decrease) its efficacy over time, allowing individuals which were removed from the susceptible compartment to re-enter in the same compartment while in the same outbreak. Finally, modifications of the viral

agent were not considered in this model, and, although they were indirectly confirmed across different outbreaks, they could not be verified within each outbreak. Despite all these limitations the model was able to predict the actual hospital demand variables and outcomes for the second timeframe, and it indirectly confirmed the efficacy of vaccination campaigns to reduce the hospital burden and make the outbreaks more easily governed.

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