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LOW PEAK DERIVATIVE SUM OF SINES

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ABSTRACT

This paper proposes a method to generate multisines signals with reduced peak derivative. This is done by adjusting the phases in the Fourier domain with Genetic Algorithm such that in the time domain the maximum absolute signal's derivative is minimized. The performance evaluation is made by comparing the method to signals with the same spectrum but with phase adjusted randomly. It will be shown for an audio test signal case study of a multisines signal comprising 2500 sines that a reduction of 42% with respect to the mean maximum peak derivative and 31% with respect to the minimum maximum peak derivative over 1 million random trials. The proposed method is contrasted with a low crest factor signal method. It is found that there is a positive correlation between peak amplitude and peak derivative when signals are generated with random phases. Moreover, the signals obtained by minimizing either the peak amplitude of the peak derivative had also good performances with respect to the non-optimized criterion.

Index Terms— Excitation Signal, Crests Factor, LTI, Nonlinear, Genetic Algorithms, Room Impulse Response

1. INTRODUCTION

The multisine signals [1-3] are used for the estimation of frequency impulse response in numerous applications such as in telecommunications [4], radar technologies [5], biomedical measurements [6, 7], and for audio system parameter identification [8, 9].

Multisines signals have been the subject of numerous researches [3, 10-14], especially with respect of the minimization of their crest factor (CF), a measure of the extreme value of a signal for a given power spectrum [10] and is still an open problem [7, 14, 15]. The main approach is to adjust the phases of the multisine components in order to reduce the extreme values. Both analytical [10] and numerical methods [12] have been used for the CF minimization problem. The analytical methods generally have good performances and are fast but are most effective when the sines have equal amplitude. Numerical methods are slow and prone to converge to a local minimum, but are very flexible, allowing the use of any spectral shape [7, 15]. Excellent performances are attainable when initializing the numerical methods with results obtained by analytical ones [12]. However, this again gives better performances to cases with sines of equal amplitude. The reduction of the CF of a test signal is beneficial when nonlinearity involving the amplitude of the signal is present [9], to reduce the effect of the nonlinearity when estimating the linear frequency response [1]. Perhaps the simplest and best example of such nonlinearities is when a limit is imposed to the signal [11].

However, the CF minimization, while reducing the impact of the signal amplitude related nonlinearities, does not address nonlinearities caused by the derivative of the signal. Such derivative related nonlinearities are found in a wide variety of systems. Among them, systems involving operational amplifiers, where slew rate is important, have a limit to the derivative before distortion occurs. The operational amplifier is probably the simplest and best example of nonlinearities that are induced by the signal's derivative. Also, with antennas used for wideband communication, non-static nonlinearities can be important [16]. Such nonlinearities which include memory are related to the signal's derivative. As well, loudspeakers are well-known to be nonlinear. Some aspects of their nonlinearity are related to the amplitude, but others are associated with the cone velocity. For example, the electro-dynamic coupling factor has nonlinear relations with both the cone displacement and velocity [17].

In this paper, the minimization of the peak derivative of a multisine test signal is proposed to mitigate system nonlinearity linked to the derivative of its input. The method should have applications in frequency response estimation for systems that involve nonlinearities caused by high signal's derivative such as operational amplifiers with slew rate, wideband communication with antennas which includes nonlinearities with memory and loudspeakers with velocity related nonlinearities.

The methods developed for CF minimization are natural candidates for peak derivative minimization. However, the derivative's emphasis on the high frequencies restricts the use of the methods developed for constant spectrums. For example, if a multisine signal has equal amplitude components, the multisine resulting of its derivative will not. Hence, the analytical methods that need equal amplitude are not well suited for the peak derivative minimization. A possible exception is the case of pink noise (1/f noise, flicker noise) where the derivative yields a constant spectrum.

To show the differences and similarities between the CF minimization and peak derivative minimization problems, the same algorithm is used in both cases. Since the use of the analytical method should apply advantageously to the CF minimization than to the peak derivative minimization, a fully numerical method is preferred. Therefore, a GA approach is used, similar to the one proposed in [12], but without the initialization phase that used analytical approach. The results will be compared with signals with randomly generated phases and with a GA CF minimization result. It is found that a correlation exists between the amplitude CF and the derivative CF. The possibility of using multi-objective criterion [18-20] for control over the trade-off between the amplitude CF and the derivative CF will be discussed.

Although the method presented in this paper has numerous applications, the example given in was developed as a test signal for room impulse response (RIR) estimation [21]. There are many methods to generate test signals for RIR with different properties [8, 22]. The multisine test signal is not the most common. Popular alternatives to the multisine signals are the Chirp signals [23] and the maximal length sequences (MLS) [24, 25]. Chirp signals have a great (low value) CF. They also allow eases of separation of the linear RIR from the non-linear distortions introduced by the loudspeakers. However, they are well-known for their high annoyance level in audio testing context [26]. MLS provides white noise type signals, much less unpleasant. However, with the MLS, the CF is not controlled and the RIR estimation much more sensitive to loudspeaker nonlinearities. The multisine approach does not have the annoyance factor of the Chirp signals and can have reduced CF, mitigating the loudspeaker nonlinear distortions with respect to displacement amplitude. The multisine test signals are therefore relevant to RIR estimation.

The paper is organized as follows: Section 2 recalls the CF optimization by explaining the multisine signal, the CF and the GA. Section 3 describes the proposed derivative minimization method, the performance metrics and the application example. Section 4 presents and discusses the results. Finally, the conclusions are drawn in Section 5.

2. CREST FACTOR OPTIMIZATION

The fundamentals of multisine signals optimization by genetic algorithms are described in this section.

2.1. Multisine Signals

A multisine signal is simply a signal composed of a certain quantity of sinusoids. However, in the context of CF optimization, there is a harmonic relationship between the frequencies involved. To ensure that the CF factor stays constant with time, the signal must be periodic. The multisine is sometimes written as a sum of sines or cosines. The signal s of length N at sample n with fundamental frequency f_1, K components of amplitude a_k and initial phase φ_k is written:

$$s(n) = \sum_{k=1}^{K} a_k \sin(2\pi k f_1 n/N + \varphi_k). \qquad (1)$$

It is generally preferable to keep K<<N. This makes possible the evaluation of the CF without oversampling. If the number *K* is high, the computation should be carried by mean of the Inverse Fast Fourier Transform (IFFT). This is particularly useful when multiple signals must be generated, as in the GA optimization method described later. Let's recall that in order to obtain a real signal, the phase spectrum must be conjugate symmetric.

2.2. Crest Factor, Peak Value

The CF is a measure of the relation between the extreme value of the signal with respect to its RMS, or standard deviation when the mean is null. In [10], the relation involves the difference between the maximum and minimum values while [11]. The CF is sometimes normalized with respect to the CF of a pure sine wave, which is the square-root of 2. In this paper, the simple peak value is preferred, with no normalization with respect to the standard deviation of the CF of the sine wave. This will be simpler to compare with the peak derivative. However, it should be noted that there is no impact regarding the improvement with respect to randomly generated phase signals.

2.3. Randomly Generated Signals

The CF of the test signal can be reduced by adjusting the phases of the sines. A simple approach is to generate multiple signals with the same amplitude but random phases. Some signals will have higher CF, some will have lower CF. The signal with the lowest CF can then be selected.

2.4. Genetic Algorithms Optimization

The nonlinear nature of the CF minimization problem leads naturally to consider GA [27] for minimization of the CF. This has been done successfully in [12]. The general idea of the GA method applied to the CF minimization problem is to generate an initial population of multisine signals with randomly generated phases. The signals with the best CF are used to generate a second generation of signals. The crossover between the selected signals and mutations are used to create new diversity in the new population. The selection, crossover and mutation process is performed multiple times (multiple generations). The main challenge in using the GA is to find a set of meta-parameters, such as the number of generations, the crossover ratio, the mutation ratio, the mutation method and elitism (number of best signals that are passed to the next generation untouched.)

The main challenge in using the GA is to find a set of meta-parameters, such as the number of generations, the cross-over ratio, the mutation ratio, the mutation method and elitism (number of best signals that are passed to the next generation untouched.) It should be mentioned that the initial population in [12] was generated by another method [11] which gave better initial population performances. This approach is not used in this paper.

3. PROPOSED METHOD

This section presents the proposed maximum absolute derivative minimization problem and describes how its performances are evaluated. Also, an example of the method application is given.

3.1. Maximum Derivative Minimization Problem

The approach to minimizing the maximum derivative of a test signal with a fixed power spectrum proposed is illustrated the GA optimization method to adjust the phases of the multisine signal. An initial population of N_I individuals is generated. Each individual consists of a vector $\boldsymbol{\varphi}$ containing the K phases φ_k.For each individual, the fitness function is evaluated. This fitness function is the maximum value of the absolute value of the derivative of the signal obtained with the prescribed power spectrum and an individual's phases. The best individuals are used to generate the following generation of individuals. The process is repeated as much as needed.

The signal's derivative is not defined in discreate time. We used a sampled derivative:

$$s^{'}(n) = \left.\frac{ds(t)}{dt}\right|_{t=n} . \quad (2)$$

Applying this with (1), the following derivative signal is obtained:

$$s^{'(n)} = \frac{2\pi f_1}{N^2}\sum_{k=1}^{K} k a_k \cos\left(2\pi k f_1 n/N + \varphi_k\right). \quad (3)$$

The fitness function g that must be minimized is:

$$g(\boldsymbol{\varphi}) = \max_{\mathrm{n}}\left(\left|s^{'}(n)\right|\right). \quad (4)$$

It worth noting that if the number of multisine components K is high, the computation of s' can be faster using the IFFT.

3.2. Performance Metrics

Since no method have been proposed yet to reduce the peak derivative, the performances are principally compared to signals with randomly generated phases. This approach is found notably in [12] where 100 random trials were performed. In this paper, with more powerful computer than in [12], it is very reasonable to do the trials over 1 million trials. For each trial, the absolute peak and the peak derivative are computed. The performances of the GA optimized signals are compared to 1) the mean absolute peak and peak-derivative and 2) the minimum CF and peak-derivative obtained from the random trials. The relationship between the optimized and random signals are further presented with histograms and a scatter plot.

3.1. Room Impulse Response Test Signal

The design of a test signal for audio RIR is used as a typical application example. The parameters of the test signal will hence be selected in order to be useful for audio systems. Therefore, $K = 2500$ sinusoidal components, from 1Hz to 2500 Hz by steps of 1 Hz, a constant amplitude a_k of the components, a sampling of 1 second at the sampling frequency of 44.1 kHz ($N = 44100$ samples) is desired.

4. RESULTS AND DISCUSSION

The results are presented and discussed in this section.

Fig. 1 shows the fitness function during the GA optimization algorithm for the minimization of the peak derivative. The GA was performed by Matlab® with Global Optimization Toolbox. The number of generations was 5000 but only 1000 were shown. No visible improvement was obtained after. The population size was 3000. Uniform initialization and mutation with a rate of 0.05 were selected along with an elitism of a single individual. The same GA options were used for the minimization of the peak amplitude (CF). In the graphic of Fig. 1, the initial population (generation 0) is representative of the randomly generated phases, with the mean on the above curve and the minimum over 3000 trials in the lower curve. The fitness function of the derivative has a high value, in the order of 1E4, because of the multiplication by the frequency involved with the derivative.

Fig. 2 shows the power spectrum of the signals (a) and their derivative (b). It should be remarked that the power spectrums are identical for all the signals, optimized or not, since only the phases are different between them. The constant value of the signal's power spectrum is selected in order that the signal has a root mean square (RMS) value of 1. The effect of the signal derivative in the Fourier domain is a linear multiplication. For this reason, in the band of interest, the derivative's power spectrum is a parabola.

Fig. 3 presents the signals in the time domain. In (a), an example of a randomly generated signal is given. In (b), the signal has been optimized for peak amplitude (CF) minimization while in (c), for peak derivative minimization. The signals in (d-f) are the derivative of their left counterparts. It is clear that the signal in (b) and (f) have a

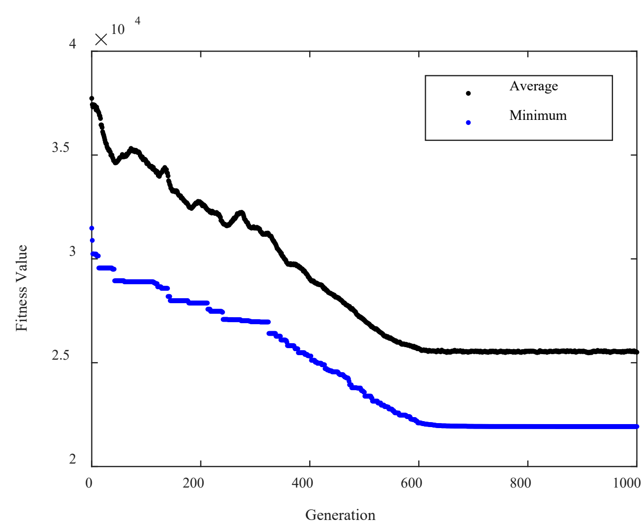

Fig. 1. Average and best (Minimum) performances for each generation for peak derivative reduction with GA.

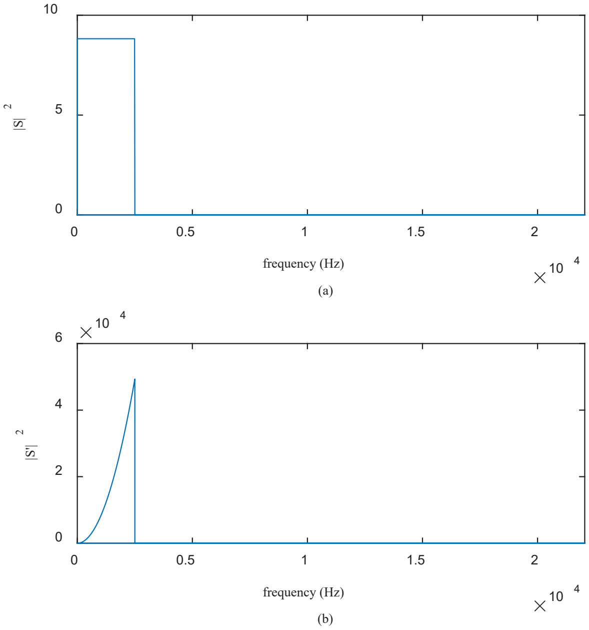

Fig. 2. Power spectrum of a signal *s* (a) and of the derivative *s'*. The power spectrums are identical for every signal since only the phases are different.

limited range while other signals do not have the same behavior.

Fig. 4 shows the distributions of the of the random signal peak value (CF) in (a) over 10^6 trials and the distribution of their derivative in (b). The peak value of the signal optimized for the peak value is shown in (a) and the peak derivative for the signal optimized for the peak derivative is shown in (b). These are shown as a version of a one bin histogram, scaled for better visualization. It is clear that the optimized versions perform better than the best randomly generated signal in each case.

Fig. 5 display the random signals as a scatter point of the peak derivative with respect to the peak value (CF). Correlation between the amplitude CF and the derivative CF was found for the randomly generated signals of 0.28. On the same plot are shown the two optimized signals' features. It was also found that the signal optimized with respect to the amplitude CF had an excellent derivative CF, as well as the signal optimized with respect to the derivative CF had also a good amplitude CF.

Table 1 summarizes the important results. The minimum and the mean of the peak and peak derivative for the random signals are reported. The peak and peak derivative performances of both optimized signals are shown. However, the interpretation of the results is more easily performed with the relative reduction in peaks given in Table 2. It can be seen that the reduction of the peak derivative of the optimized signal with respect to the peak derivative is of 42% relative to the mean of the random signals' peak derivative and 31% relative to the minimum of the random signals' peak derivative. Also, for the same signal, the performances with respect to the peak amplitude are almost as good as the best randomly generated signal. The same kinds of results were observed for the peak amplitude optimized signal: There is a similar proportional reduction for the peak derivative optimization as found in for the amplitude peak.

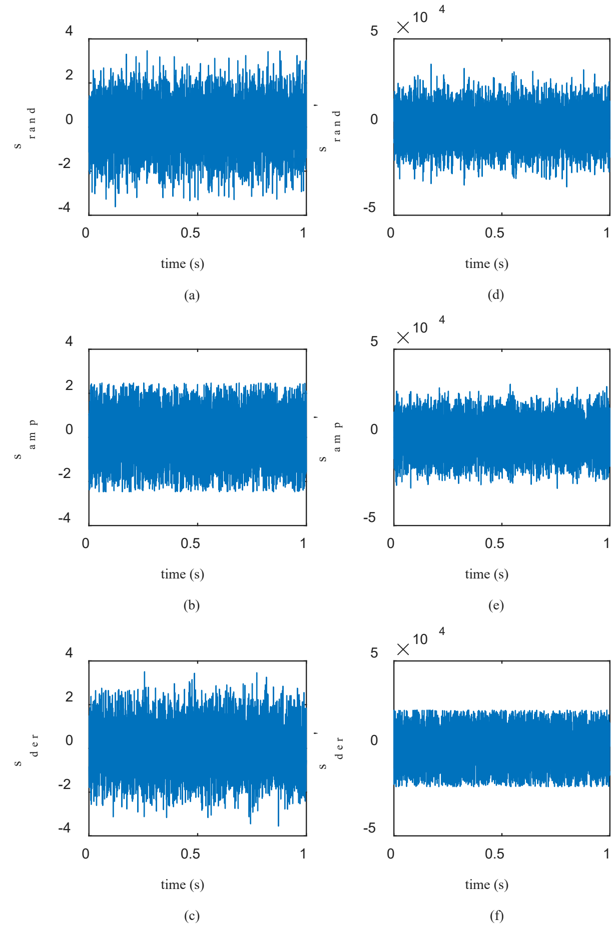

Fig. 3. Time domain signals and their derivatives. An example of random signal is given in (a) with its derivative in (d). The signal optimized for the CF reduction (peak amplitude) is found in (b) and its derivative in (e). The signal optimized for peak derivative is found in (c) and its derivative in (f).

The correlation between the peak value and peak derivative is probably linked to the fact that if there is a strong peak in the signal, it has similarities with a filtered Dirac function. The vicinity of the filtered Dirac impulse is subject to have strong derivative.

It is expected that trade-off between the amplitude CF and the derivative CF could be desired. This could be achieved by using multi-objective criterion in the GA's fitness function [18-20]. For this purpose, the flexibility of the GA's method should be contrasted to that of the analytical approach methods. As presented in [7, 15], numerical methods can address problems with sparse or non-constant power spectrums. To this, this paper adds the multi-objective criterion combining the crest of the signal and its derivative. Further generalization could be considered with multi-objective criterion combining multiple orders of the signal derivative.

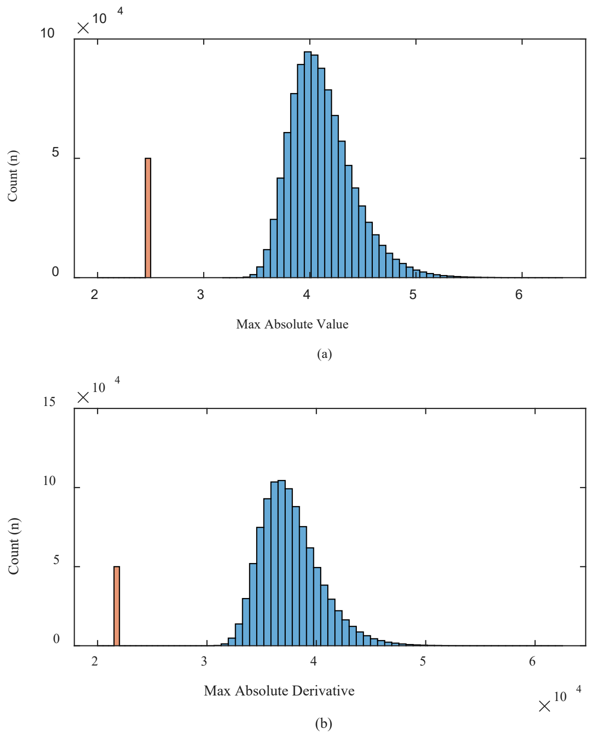

Fig. 4. Histograms of the extreme values obtained with random phase (distribution) and with GA optimization (single red value at the left). In (a), the optimization was the peak amplitude (CF) reduction. In (b), the optimization was the peak derivative reduction. The GA optimization results are shown in a histogram bin, but it has been scaled for visual purpose.

The method was developed with an example of a large number of components, which is generally a hard problem. Although the results were convincing, there is probably still place for better performances. Since the nature of the minimization problem, it is possible that only local minimums were attained. The GA's meta parameters can probably be improved to find a better solution. Initialization methods, such as in [12], could be applied. Also, research on analytical solutions, which takes into account the "blue-noise" nature of the derivative of a constant power spectrum signal is desirable. However, pink noise test signals for which the derivate signals have a constant power spectrum would readily benefit from all the CF minimization techniques already existing in the literature that requires a constant spectrum.

5. CONCLUSION

In this paper, a new optimization problem was proposed, leading to low derivative multisine signals. These signals can reduce the presence of nonlinearities linked to its derivative. Results showed for an audio test signal that a reduction of 42% could be achieved with respect to the average of randomly generated signals. It was also shown that the crest-factor and maximum derivative were correlated. Future research will focus on the trade-off between amplitude crest-factor and the peak derivative using multi-objective criterion.

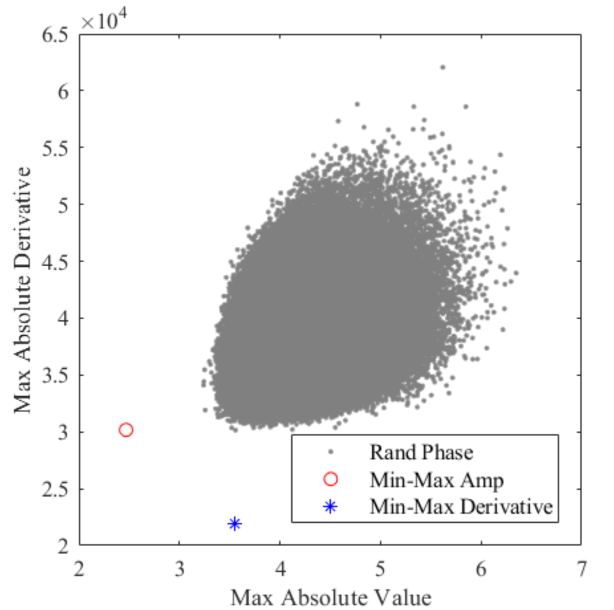

Fig. 5. Maximum absolute difference with respect to the maximum absolute value for random phase and the two GA optimization.

Table 1: Main Results

	Random phase		GA Optimized	
	min	mean	|s|	|s'|
max |s|	3.50	4.12	2.47	3.55
max |s'| ($\times 10^4$)	3.19	3.76	3.02	2.19

Table 2: Relative Reduction

GA Optimized	min |s|	mean |s|	min |s'|	mean |s'|
|s|	29.4%	40.0%	5.4%	19.7%
|s'|	-1.6%	13.6%	31.3%	41.7%

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