Regularized MMSE Multiuser Detection Using Covariance Matrix Tapering

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***Abstract*—The linear minimum mean-squared error (MMSE) detector for direct-sequence code-division multiple-access (DS-CDMA) systems relies on the inverse of the covariance matrix of the received signal. In multiuser environments, when few samples are available for the covariance estimation, the matrix ill-conditioning may produce large performance degradation. In order to cope with this effect, we propose a modified MMSE detector based on the covariance matrix tapering (CMT). This regularization technique modifies the estimated covariance matrix, thereby reducing its eigenvalue spread. Two different tapering matrices are introduced. The performance comparison with other regularization techniques is carried out by simulations, which show the effectiveness of the proposed technique.**

Keywords-CDMA; multiuser detection; MMSE; regularization; covariance matrix tapering

I. Introduction

Multiuser detection techniques [1] have the capability of reducing the multiple-access interference in direct-sequence code-division multiple-access (DS-CDMA) systems. Among the multiuser receivers proposed in the literature (see [1] and the references therein), the linear minimum mean-squared error (MMSE) detector offers a good trade-off between performance and complexity [2]. Indeed, the MMSE detector, implemented as a Wiener filter at the chip level, is moderately complex because it does not require the knowledge of the interfering users' parameters (spreading codes, channels, and timing) [2]. Specifically, the Wiener MMSE filter is obtained by multiplying the inverse of the covariance matrix of the received signal with the signature waveform vector of the user of interest [1]-[3], and therefore the interfering users' parameters are implicitly contained into the covariance matrix of the received signal.

In realistic scenarios, the signature waveform vector and the covariance matrix are not known and they must be estimated. The estimation errors, which depend on the employed estimation techniques, clearly become larger when a small sample set is used for the estimation. Examples where the detector is obtained by using small data sets include:

- transmissions of short data blocks;
- transmissions over slowly time varying channels, where it is convenient to use data blocks within the channel coherence time.

In multiuser CDMA environments, the covariance matrix is often ill-conditioned, being characterized by a high eigenvalue spread for high values of the signal-to-noise ratio (SNR). As a consequence of the high eigenvalue spread, the matrix inversion contained in the MMSE detector tends to enhance the covariance matrix estimation errors, causing a large performance degradation [3].

Different approaches are possible in order to counteract such degradation. A first option is to obtain a new covariance matrix estimate by exploiting the estimated channels of all the users [4]. Indeed, by using good channel estimates, like the ones obtained by subspace methods [5], the covariance estimation errors decrease [4]. However, this method requires the knowledge of channels, timing, and spreading codes of all the users, and therefore it is feasible in the uplink only. A second approach relies on regularization techniques [6], which improve the conditioning by modifying the covariance matrix. Another alternative is based on reduced rank techniques [2], which project the received signal onto a lower-dimensional subspace, thereby resulting in a covariance matrix (of the projected received signal) with smaller eigenvalue spread. Anyway, certain reduced rank techniques are sometimes classified as regularization methods [6].

In this paper, we focus on the regularization techniques for MMSE detectors. We show that the covariance matrix tapering (CMT), previously proposed in [7]-[9] for beamforming applications, is an effective technique also in the multiuser CDMA environment. In this context, we propose new tapering matrices, which outperform the tapering matrix commonly used for beamforming. Moreover, the performance of the CMT is compared by simulations to other regularization techniques.

II. MMSE Detection of CDMA Signals

A. DS-CDMA System Model

A DS-CDMA system with K active users is considered. Using a notation similar to [5], the transmitted signal of the kth user is expressed by

$$x_k(t) = A_k \sum_{i=0}^{P-1} b_k[i] s_k(t - iT_s - \tau_k), \qquad (1)$$

where P is the number of transmitted symbols within the channel coherence time, T_s is the symbol duration, A_k, $b_k[i]$, $s_k(t)$, and τ_k are the amplitude, the ith symbol, the spreading waveform, and the asynchronism delay of the user k, respectively. The symbols $\{b_k[i]\}$ are independent and equiprobable random variables drawn from the set $\{\pm 1\}$. The spreading waveform is expressed by

$$s_k(t) = \sum_{j=0}^{N-1} c_k[j]\,\psi(t - jT_c)\,, \quad (2)$$

where N is the processing gain, $T_c = T_s / N$ is the chip duration, $\psi(t)$ is the chip pulse shaping waveform, assumed rectangular for the sake of simplicity, and $c_k[j] \in \{\pm 1/\sqrt{N}\}$ is the (j+1)th value of the short spreading code assigned to the kth user. The transmitted signal $x_k(t)$ passes through a slowly time varying multipath channel, assumed to be constant during the transmission of the P symbols, and characterized by an impulse response expressed by

$$g_k(\tau) = \sum_{q=1}^{Q_k} \beta_{q,k}\,\delta(\tau - \tau_{q,k})\,, \quad (3)$$

where Q_k is the number of paths, $\beta_{q,k}$ and $\tau_{q,k}$ are the complex gain and the propagation delay of the qth path of the kth user channel, respectively, and $\delta(\tau)$ is the Dirac delta function. Denoting with $n(t)$ the additive white Gaussian noise (AWGN) at the receiver side, the received signal $r(t)$, expressed by

$$r(t) = \sum_{k=1}^{K} \int_{-\infty}^{+\infty} g_k(\tau) x_k(t-\tau) d\tau + n(t)\,, \quad (4)$$

is firstly filtered by a chip-matched filter, and successively sampled at the chip rate $1/T_c$, thus obtaining

$$r_n[l] = \int_{lT_s+nT_c}^{lT_s+(n+1)T_c} r(t)\psi(t - lT_s - nT_c)dt \quad (5)$$

$$= \sum_{k=1}^{K} \sum_{i=0}^{P-1} b_k[i] h_{n,k}[l-i] + n_n[l]\,, \quad (6)$$

$$h_{n,k}[i] = \sum_{q=1}^{Q_k} \sum_{j=0}^{N-1} \beta_{q,k} A_k c_k[j] R_\psi(iT_s + (n-j)T_c - \tau_{q,k} - \tau_k)\,, \quad (7)$$

$$n_n[l] = \int_{lT_s+nT_c}^{lT_s+(n+1)T_c} n(t)\psi(t - lT_s - nT_c)dt\,, \quad (8)$$

where $R_\psi(\tau)$ is the autocorrelation function of the chip pulse shaping waveform $\psi(t)$. By setting $\underline{r}[l] = [r_0[l],...,r_{N-1}[l]]^T$, $\underline{n}[l] = [n_0[l],...,n_{N-1}[l]]^T$, $\underline{b}[l] = [b_1[l],...,b_K[l]]^T$, and by defining $\underline{H}[l]$ as the $N \times K$ matrix with elements $[\underline{H}[l]]_{n,k} = h_{n-1,k}[l]$, (6) can be rearranged as

$$\underline{r}[l] = \sum_{i=0}^{P-1} \underline{H}[l-i]\underline{b}[i] + \underline{n}[l]\,. \quad (9)$$

Taking into account that the K channels have a finite impulse response, with memory $L = \max_{q,k}\{\lceil(\tau_{q,k} + \tau_k)/T_s\rceil\}$ symbol intervals, it follows that $\underline{H}[l] = \mathbf{0}_{N\times K}$ when $l \geq L+1$. Assuming that the receiver window spans M symbol intervals, we obtain

$$\mathbf{r}[l] = \mathbf{H}\mathbf{b}[l] + \mathbf{n}[l]\,, \quad (10)$$

where the column vectors $\mathbf{r}[l] = [\underline{r}[l]^T,...,\underline{r}[l+M-1]^T]^T$ and $\mathbf{n}[l] = [\underline{n}[l]^T,...,\underline{n}[l+M-1]^T]^T$ have dimension MN, $E\{\mathbf{n}[l]\mathbf{n}[l]^H\} = \sigma^2 \mathbf{I}_{MN}$, $\mathbf{b}[l] = [\underline{b}[l-L]^T,...,\underline{b}[l+M-1]^T]^T$ is a column vector of size $(L+M)K$, and $\mathbf{H}$ is the $MN \times (L+M)K$ block Toeplitz channel matrix expressed by

$$\mathbf{H} = \begin{bmatrix} \underline{H}[L] & \cdots & \cdots & \underline{H}[0] & & \mathbf{0}_{N\times K} \\ & \ddots & & & \ddots & \\ \mathbf{0}_{N\times K} & & \underline{H}[L] & \cdots & \cdots & \underline{H}[0] \end{bmatrix}. \quad (11)$$

B. MMSE Detection with Estimated Covariance Matrix

For the adopted binary phase-shift keying (BPSK) modulation, the decision rule of a linear receiver is expressed by

$$\hat{b}_k[l] = \mathrm{sign}(\,\mathrm{Re}(\mathbf{w}_k^H \mathbf{r}[l]))\,, \quad (12)$$

where $\mathbf{w}_k$ is a column vector that represents the detector of the user k. The detector that minimizes the mean-squared error $E\{|\,b_k[l] - \mathbf{w}_k^H \mathbf{r}[l]\,|^2\}$ can be obtained as a Wiener filter [1]-[3], as expressed by

$$\mathbf{w}_{\mathrm{MMSE},k} = \mathbf{R}^{-1}\mathbf{h}_k\,, \quad (13)$$

where $\mathbf{R} = E\{\mathbf{r}[l]\mathbf{r}[l]^H\} = \mathbf{H}\mathbf{H}^H + \sigma^2\mathbf{I}_{MN}$ is the $MN \times MN$ covariance matrix of the received signal, and $\mathbf{h}_k$, obtained as the (KL+k)th column of $\mathbf{H}$, is the signature waveform (i.e., the channel-affected spreading code) of the user k. The estimated version of the MMSE receiver is expressed by

$$\hat{\mathbf{w}}_{\mathrm{SMI},k} = \hat{\mathbf{R}}^{-1}\hat{\mathbf{h}}_k\,, \quad (14)$$

where

$$\hat{\mathbf{R}} = \frac{1}{P}\sum_{l=0}^{P-1} \mathbf{r}[l]\mathbf{r}[l]^H \quad (15)$$

is obtained without explicit knowledge of $\mathbf{H}$ and of σ^2, and $\hat{\mathbf{h}}_k$ is obtained by using a training sequence [10] or a blind channel estimation technique [5]. The receiver (14), obtained by inverting (15), is known as sample matrix inversion (SMI) or direct matrix inversion (DMI) detector.

In many practical situations, the value of P has to be kept small, leading to non negligible covariance matrix estimation errors. As an example, when $P < MN$, $\hat{\mathbf{R}}$ is not full rank and therefore not invertible. As a rule of thumb, we can assume that the covariance matrix estimation errors are small when $P > 6MN$ [11]. However, when the system is not fully loaded, the matrix $\mathbf{H}$ can be tall, with $D = \mathrm{rank}(\mathbf{H})$ strictly lower than MN. In such a situation, the covariance matrix $\mathbf{R}$ is often ill-conditioned, especially when the SNR is high. Therefore, the matrix inversion contained in (14) amplifies the small covariance matrix estimation errors, according to the eigenvalue spread of $\mathbf{R}$ [3], causing significant performance degradation.

It is noteworthy that a high eigenvalue spread of $\mathbf{R}$ produces also an amplification of the possible signature waveform mismatch $\mathbf{\Delta h}_k$ [12][3]. Moreover, it should be pointed out that the problem of a small P is exacerbated when multiple antennas are used at the receiver side, since the dimension of $\mathbf{R}$ increases by a factor equal to the number of the receiving antennas [13]. Hence, the regularization techniques introduced in the next section can also be applied in CDMA systems with multiple receiving antennas.

III. Regularized MMSE Detectors

Regularization techniques [6] deal with ill-conditioned problems by substituting the matrix $\mathbf{R}$ with a matrix characterized by a smaller eigenvalue spread. Consequently, the variance of the estimation errors decreases, at the cost of introducing some bias in the detector estimate. The goal is to find a good trade-off between bias and variance, constructing the regularized matrix by judiciously modifying $\hat{\mathbf{R}}$.

A. Regularization by Covariance Matrix Loading

Although not widely recognized, some regularization techniques have already been used in multiuser detection. Indeed, the constrained minimum output energy (CMOE) receiver in [12][14] exploits a particular form of *Tikhonov regularization* [6] replacing the matrix $\mathbf{R}$ with $\mathbf{R}+\nu\mathbf{I}_{MN}$, as expressed by

$$\hat{\mathbf{w}}_{\text{CMOE},k} = (\hat{\mathbf{R}}+\nu\mathbf{I}_{MN})^{-1}\hat{\mathbf{h}}_k, \quad (16)$$

where $\nu = \alpha\,\text{tr}(\hat{\mathbf{R}})$ is a positive parameter. By using the eigenvalue decomposition (EVD) $\mathbf{R} = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^H$, it is easy to verify that the CMOE detector attenuates the eigenvectors associated with the small eigenvalues, lowering the eigenvalue spread value from $\chi(\mathbf{R}) = \lambda_{\max}(\mathbf{R})/\sigma^2$ to $\chi(\mathbf{R}+\nu\mathbf{I}_{MN}) = (\lambda_{\max}(\mathbf{R})+\nu)/(\sigma^2+\nu)$, where $\lambda_{\max}(\mathbf{R})$ is the largest eigenvalue of $\mathbf{R}$. In the array processing literature, this detector is known as diagonal loaded SMI (LSMI) [11].

B. Regularization by Eigenvalue Decomposition Truncation

A different regularized detector can be obtained by applying the EVD to $\mathbf{R}$ and neglecting the eigenvectors associated with the $MN-r$ smallest eigenvalues. Using this kind of regularization, usually referred as *truncated singular value decomposition* (TSVD) [6], the reduced rank detector of the user *k* can be expressed as

$$\hat{\mathbf{w}}_{\text{TSVD},k} = \hat{\mathbf{U}}_r\hat{\mathbf{\Lambda}}_r^{-1}\hat{\mathbf{U}}_r^H\hat{\mathbf{h}}_k, \quad (17)$$

where $\mathbf{U}_r$ contains only the selected r eigenvectors. If r is equal to $D = \text{rank}(\mathbf{H})$, the detector is constrained to lie in the signal subspace [5]. However, a choice $r < D$ could allow better performance in the presence of covariance estimation errors [6]. In this way, the eigenvalue spread is reduced from $\chi(\mathbf{R}) = \lambda_{\max}(\mathbf{R})/\sigma^2$ to $\chi(\mathbf{U}_r\mathbf{\Lambda}_r\mathbf{U}_r^H) = \lambda_{\max}(\mathbf{R})/\lambda_r(\mathbf{R})$, where $\lambda_r(\mathbf{R}) > \sigma^2$ is the *r*th largest eigenvalue of $\mathbf{R}$.

C. Regularization by Covariance Matrix Tapering

In this subsection, we propose a new multiuser detector based on the CMT approach, originally proposed in [7]-[9] for robust beamforming in order to widen the nulls of the antenna array pattern. The basic idea of CMT is to multiply the elements of $\mathbf{R}$ with different weights, attenuating those elements far apart from the main diagonal. In mathematical terms, the CMT detector can be expressed as

$$\hat{\mathbf{w}}_{\text{CMT},k} = (\hat{\mathbf{R}}\circ\mathbf{T})^{-1}\hat{\mathbf{h}}_k, \quad (18)$$

where the symbol $\circ$ represents the Hadamard (element-wise) product [15] between matrices, and $\mathbf{T}$ is the tapering matrix (real, symmetric, and Toeplitz). In the following, we show that the CMT approach gives rise to an eigenvalue spread $\chi(\mathbf{R}\circ\mathbf{T})$ smaller than $\chi(\mathbf{R})$, thus enabling the bias-variance trade-off.

Theorem 1 (Schur product theorem) [9]: If $\mathbf{R}$ and $\mathbf{T}$ are $MN\times MN$ positive semidefinite (p.s.d.) matrices, then $\mathbf{R}\circ\mathbf{T}$ is p.s.d.. Moreover, if $\mathbf{R}$ is positive definite (p.d.) and $\mathbf{T}$ is p.s.d. with no zero entries on the main diagonal, then $\mathbf{R}\circ\mathbf{T}$ is p.d..

Proof: See [15].

Definition 1: An $MN\times MN$ matrix $\mathbf{T}$ is a *correlation matrix* if $\mathbf{T}$ is p.s.d. with $\mathbf{I}_{MN}\circ\mathbf{T} = \mathbf{I}_{MN}$, i.e., with all ones on the main diagonal.

Theorem 2 (Eigenvalue spread theorem): If $\mathbf{R}$ is an $MN\times MN$ p.d. matrix and $\mathbf{T}$ is an $MN\times MN$ correlation matrix, then

$$\chi(\mathbf{R}\circ\mathbf{T}) \le \chi(\mathbf{R}), \quad (19)$$

where $\chi(\mathbf{R})$ is the eigenvalue spread of $\mathbf{R}$.

Proof: See the Appendix.

Theorem 2 proves that the CMT certainly is a regularization technique, but it does not guide us in the design of a suitable tapering matrix. In [7][8], the tapering matrix is chosen as

$$[\mathbf{T}_{\text{sinc},\alpha}]_{m,n} = \text{sinc}(\alpha\,|m-n|) = \frac{\sin(\alpha\pi|m-n|)}{\alpha\pi|m-n|}, \quad (20)$$

where $[\mathbf{T}]_{m,n}$ is the (*m*,*n*)th element of $\mathbf{T}$, and $\alpha > 0$ is the regularization parameter. Anyway, by Theorem 2, any correlation matrix can be selected. For example, the Tikhonov regularization can be interpreted as a particular way to enhance the main diagonal of $\mathbf{R}$ with respect to the other diagonals. Hence, a reasonable choice of the matrix $\mathbf{T}$ should produce an attenuation that increases when moving away from the main diagonal. As a consequence, the elements of the tapering matrix should be chosen as

$$[\mathbf{T}]_{m,n} = f(|m-n|), \quad (21)$$

where $f(x)$ is a non increasing weighting function of x.

It should be noted that, if α is not too high, the second derivative of the sinc function in (20) (regarded as a function of a continuous variable) is negative, at least for small values of the argument. This implies that the decreasing rate of the weighting function increases with $|m-n|$, or, equivalently, that the diagonals close to the main diagonal are weighted with weights close to 1, which is the weight of the main diagonal. Therefore, in order to test different tapering matrix families, we propose a matrix $\mathbf{T}$ whose function (21) has a second derivative equal to zero (i.e., the diagonals are weighted linearly), as expressed by

$$[\mathbf{T}_{\text{tri},\alpha}]_{m,n} = \text{clip}(1-\alpha\,|m-n|), \quad (22)$$

where the clipping function $\text{clip}(x)$ is defined as

$$\text{clip}(x) = \begin{cases} 0 & x<0 \\ x & 0\le x\le 1 \\ 1 & x>1 \end{cases} \quad (23)$$

in order to guarantee that all the weights fall in the $[0,1]$ set. Furthermore, we also propose a matrix $\mathbf{T}$ whose function (21) has a positive second derivative, as expressed by

$$[\mathbf{T}_{\exp,\alpha}]_{m,n} = e^{-\alpha|m-n|}. \quad (24)$$

In all the cases, the parameter $\alpha > 0$ controls the amount of regularization. As it happens for the CMOE receiver, the CMT receiver is equivalent to the SMI receiver when the regularization parameter is set to zero, while it tends to the RAKE receiver for high values of the regularization parameter. Indeed, if $\alpha = 0$, the tapering matrices defined by (20), (22), and (24) become equal to $\mathbf{T} = \mathbf{1}_{MN\times MN}$, i.e., to the all-one matrix. In this case, no regularization is applied. On the contrary, for increasing α, the tapering matrices defined by (22) and (24) tend to the identity matrix $\mathbf{I}_{MN}$. In this case, only the elements of the main diagonal of $\hat{\mathbf{R}}$ are selected, and, since these elements are approximately equal, the CMT receiver in (18) is practically a scaled version of $\hat{\mathbf{w}}_{\text{RAKE},k} = \hat{\mathbf{h}}_k$. When such amount of regularization is applied, the eigenvalue spread $\chi(\mathbf{R}\circ\mathbf{T})$ is roughly equal to one, but the receiver has lost all the interference suppression capabilities of the MMSE receiver.

Of course, neither $\alpha = 0$ nor $\alpha \to +\infty$ are optimal in the short data record case. The optimum value of α depends not only on the chosen tapering matrix, but also on the scenario. Obviously, when P increases, the optimum value of α should decrease. Different algorithms for the automatic choice of α can be derived by exploiting the methods employed for other regularization techniques [6]. These algorithms are still under investigation, and therefore they are not discussed in the present paper.

IV. Simulation Results

In this section, we present some simulation results in order to compare the performance of the different regularized detectors. We consider a downlink situation with a base station that transmits data to K active users. Gold sequences of length $N = 31$ have been chosen for the short spreading codes $\{c_k[j]\}$. The amplitudes of the $Q_k = 15$ chip-spaced channel paths are modeled as independent zero-mean complex Gaussian random variables with variance $E\{|\beta_{q,k}|^2\} = 1/Q_k$. Since the channel memory in symbol intervals is $L = 1$, the size of the receiving window has been fixed to $M = 2$ symbol intervals, and therefore the dimension of $\mathbf{R}$ is equal to $MN = 62$.

In the first scenario, we consider $K = 10$ users with equal powers ($A_k = A$) and data blocks of length $P = 6MN = 372$. The SNR is defined as $\text{SNR} = A^2/\sigma^2$. We also assume $\hat{\mathbf{h}}_k = \mathbf{h}_k$. Although the perfect channel knowledge is realistic only in AWGN channels (where $\mathbf{h}_k$ represents the user code), we want to focus on the effects of the covariance matrix estimation errors. Fig. 1 shows the bit-error rate (BER) of the CMT detector (18) with exponential law (24) as a function of the regularization parameter α. It is evident that there exists an optimum value of $\alpha \approx 0.05$ that minimizes the BER. This optimum value seems to be approximately constant over the SNR. Moreover, it is noteworthy that the CMT detector BER is smaller than the SMI detector BER (obtained by CMT with $\alpha = 0$) for all the values of α in the range $0 < \alpha \le 0.4$. This fact implies that also a non optimal choice of α allows improved performance with respect to the SMI. Values of α higher than 0.4 should not be considered, since the CMT detector becomes very close to the RAKE receiver.

Fig. 2 compares the BER of the regularized detectors expressed by (16), (17), and (18). In order to have a fair comparison, we have chosen the regularization parameter that gives the best performance for each detector. It is noteworthy that the two CMT detectors proposed in this paper outperform the CMT detector with the sinc profile (20). Among all the estimated detectors, the CMT with exponential profile (24) gives the best performance when $\text{SNR} > 15$ dB, producing the same BER of the CMOE detector (16) at lower SNR.

Fig. 3 shows the tapering profiles (20), (22), and (24) for the best α for each CMT detector. It should be noted that all the three optimum profiles highly attenuate the elements of $\hat{\mathbf{R}}$ that are very far from the main diagonal (i.e., when $|m-n| \ge 45$). Indeed, since the multipath channel does not span a whole symbol interval, the last rows of $\underline{H}[L]$ are equal to the all-zero matrix, and hence the exact $\mathbf{R}$ contains a zero matrix block in the north-east (and in the south-west) corner.

In the second scenario, we consider $K = 5$ equal power users and data blocks of length $P = 8MN = 496$. In this case, the signature waveform vector is simply estimated using a training sequence [10] of length $B = 2.5MN = 155$. Fig. 4 shows the BER performance of each optimum regularized detector. We omitted the CMT detector with triangular profile (22), whose BER for $\alpha = 6/(MN-1)$ is the same of the exponential CMT, and the CMT with sinc profile (20), which performs slightly worse. In this scenario, none of the detectors outperforms the others for all SNRs. At low SNR ($\text{SNR} < 15$ dB) the TSVD detector gives the best performance, while at medium SNR the CMOE detector outperforms the other ones. At high SNR ($\text{SNR} > 24$ dB) the CMT detector slightly outperforms the CMOE detector, but the BER improvement is smaller with respect to Fig. 2. Therefore, in order to achieve high performance, the CMT detector has to be used together with a better channel estimation technique, such the subspace-based one in [5]. Indeed, by applying the method in [5] using small values of P, at high SNR the covariance matrix estimation errors only are significant, while the signature waveform estimation errors are negligible [16].

V. Conclusions

In this paper, we have proposed a regularized MMSE detector for DS-CDMA systems with small data sets. We have proven that the CMT approach reduces the eigenvalue spread of the covariance matrix. Two new tapering matrices have been introduced. We have shown that the CMT detector outperforms other regularized detectors provided that the channel has been accurately estimated. The study of algorithms for the choice of the regularization parameter will be the subject of future works.

Appendix

In order to prove Theorem 2, we introduce the following theorem, whose proof can be found in [15].

Theorem 3 (Eigenvalue majorization theorem) [9]: If $\mathbf{R}$ is an $MN \times MN$ p.d. matrix and $\mathbf{T}$ is an $MN \times MN$ correlation matrix, then

$$\sum_{i=1}^{n} \lambda_i(\mathbf{R} \circ \mathbf{T}) \le \sum_{i=1}^{n} \lambda_i(\mathbf{R}), \quad \forall n = 1, \ldots, MN, \tag{25}$$

where $\lambda_i(\mathbf{R})$ is the ith eigenvalue of $\mathbf{R}$, with the eigenvalues ordered in non increasing order.

By using Theorem 3 with $n = 1$, because of the eigenvalues ordering, we have

$$\lambda_{\max}(\mathbf{R} \circ \mathbf{T}) \le \lambda_{\max}(\mathbf{R}). \tag{26}$$

Moreover, by Theorem 3 with $n = MN-1$, we obtain

$$\sum_{i=1}^{MN-1} \lambda_i(\mathbf{R} \circ \mathbf{T}) \le \sum_{i=1}^{MN-1} \lambda_i(\mathbf{R}). \tag{27}$$

Since $\mathbf{I}_{MN} \circ \mathbf{T} = \mathbf{I}_{MN}$, we have

$$\sum_{i=1}^{MN} \lambda_i(\mathbf{R} \circ \mathbf{T}) = \text{tr}(\mathbf{R} \circ \mathbf{T}) = \text{tr}(\mathbf{R}) = \sum_{i=1}^{MN} \lambda_i(\mathbf{R}), \tag{28}$$

and, combining (27) and (28), it follows

$$\lambda_{\min}(\mathbf{R} \circ \mathbf{T}) \ge \lambda_{\min}(\mathbf{R}). \tag{29}$$

Therefore, by (26) and (29), we have

$$\chi(\mathbf{R} \circ \mathbf{T}) = \frac{\lambda_{\max}(\mathbf{R} \circ \mathbf{T})}{\lambda_{\min}(\mathbf{R} \circ \mathbf{T})} \le \frac{\lambda_{\max}(\mathbf{R})}{\lambda_{\min}(\mathbf{R})} = \chi(\mathbf{R}), \tag{30}$$

which concludes the proof.

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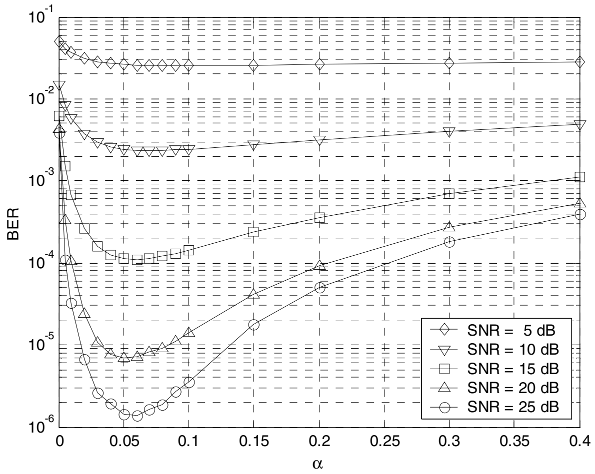

Figure 1. CMT detector BER as function of the regularization parameter.

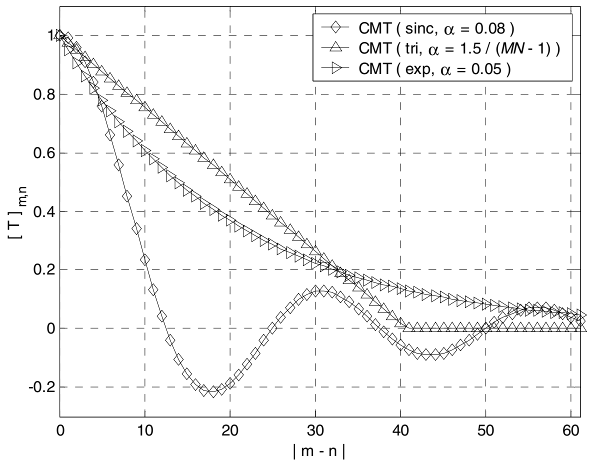

Figure 3. Tapering profiles of the CMT detectors in the first scenario.

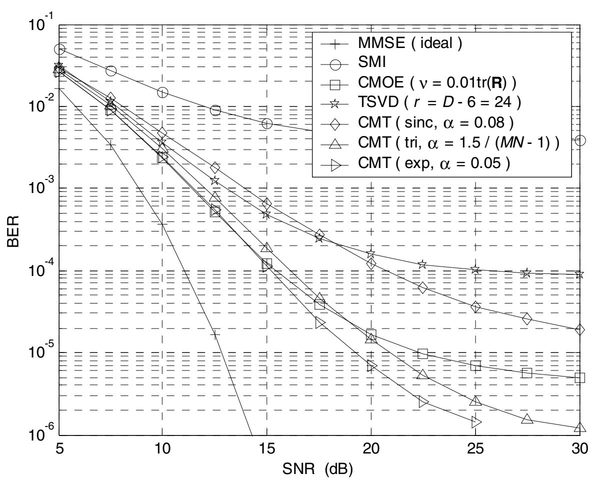

Figure 2. BER comparison in the first scenario.

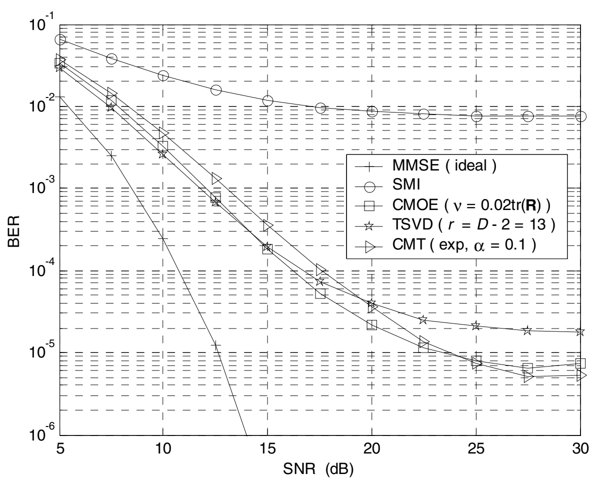

Figure 4. BER comparison in the second scenario.