

Spectrum Allocation for Multi-Operator Device-to-Device Communication

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Abstract—In order to harvest the business potential of device-to-device (D2D) communication, direct communication between devices subscribed to different mobile operators should be supported. This would also support meeting requirements resulting from D2D relevant scenarios, like vehicle-to-vehicle communication. In this paper, we propose to allocate the multi-operator D2D communication over dedicated cellular spectral resources contributed from both operators. Ideally, the operators should negotiate about the amount of spectrum to contribute, without revealing proprietary information to each other and/or to other parties. One possible way to do that is to use the sequence of operators' best responses, i.e., the operators make offers about the amount of spectrum to contribute using a sequential updating procedure until reaching consensus. Besides spectrum allocation, we need a mode selection scheme for the multi-operator D2D users. We use a stochastic geometry framework to capture the impact of mode selection on the distribution of D2D users and assess the performance of the best response iteration algorithm. With the performance metrics considered in the paper, we show that the best response iteration has a unique Nash equilibrium that can be reached from any initial strategy. In general, asymmetric operators would contribute unequal amounts of spectrum for multi-operator D2D communication. Provided that the multi-operator D2D density is not negligible, we show that both operators may experience significant performance gains as compared to the scheme without spectrum sharing.

Index Terms—Co-primary spectrum sharing, Multi-operator D2D communication, Sub-modular games.

I. INTRODUCTION

Device-to-device (D2D) communication refers to the situation where two user equipments bypass the cellular base station (BS) and core network and establish a direct local communication link. D2D communication has been introduced as one of the highlights of 3GPP Release 12 [1]–[3]. It offers several advantages, e.g., higher transmission rates due to proximity, reduced battery consumption at the mobile handsets, reuse and hop gain, etc. [4], [5]. Along with benefits, D2D support brings also new challenges because the cellular network needs to cope with new interference situations, e.g., cross-tier interference between cellular and D2D users [5].

In order to alleviate the interference problems introduced by D2D, a great deal of mode selection and resource allocation algorithms have been proposed, see for instance [6]–[8]. Unfortunately, almost all existing algorithms are applicable to single-operator networks. However, without multi-operator support, i.e., when the two ends in the D2D pair have subscriptions with different mobile network operators (MNOs), the

business potential of commercial D2D communication would be very limited.

The only available studies for multi-operator D2D can be found in [9], [10]. Both patents designed protocols to setup a D2D communication session considering different MNOs. However, they are not seen to address how the spectrum is allocated, which spectrum is used and how the communication mode is selected. In this paper, we assume that multi-operator D2D discovery has been handled, using for instance the protocol in [9], and we propose algorithms for multi-operator D2D spectrum allocation and mode selection.

D2D communication can be enabled either over licensed or unlicensed spectrum. D2D communication in unlicensed bands would suffer from unpredictable interference. Also it poses a requirement for two wireless communication interfaces and efficient power management at the D2D users [11]. Due to these reasons, at this moment, licensed spectrum seems to be the way forward to enable D2D communication, especially considering safety related scenarios such as vehicle-to-vehicle communication. In the cellular band, either dedicated spectrum can be allocated to the D2D users (a.k.a. D2D overlay) or D2D and cellular users can be allocated over the same resources (a.k.a. D2D underlay). In a multi-operator D2D underlay, cellular users may suffer from inter-operator interference, and in order to resolve it, information exchange between the MNOs might be needed. Due to the fact that MNOs may not be willing to reveal proprietary information to the competitors, we believe that, at a first stage, the overlay multi-operator D2D scheme would be easier to implement.

In this paper, we consider a scenario with two MNOs and we identify how much spectrum each MNO should commit for overlay multi-operator D2D communication. The MNOs need not contribute equal amount of spectrum as they may have different network utilities and loads. We formulate a game where the MNOs make offers about the amount of spectrum they want to contribute and use a sequential best response to each other. The MNOs neither need to exchange proprietary information nor to communicate with an external party. Also, they are not forced to take any action. If one party does not experience performance gain as compared to no sharing, the agreement can break and the communication among the devices is routed to the cellular infrastructure.

Besides spectrum allocation, the MNOs also need to agree about the mode selection scheme for the multi-operator D2D users. In principle, existing schemes for single-operator net-

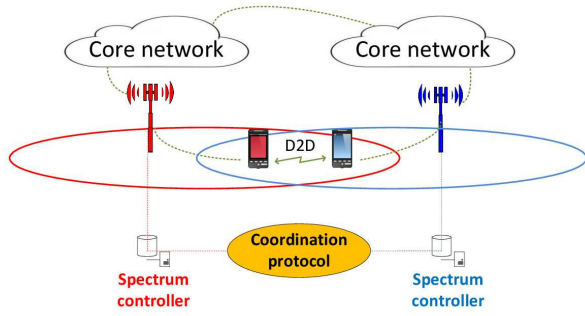


Fig. 1. User equipments registered to different MNOs communicating in a D2D manner.

works enabling D2D communication can be used. We consider the algorithm proposed in [8] where D2D communication mode is used only if the measured interference in the part of the cellular spectrum dedicated for D2D communication is low. We extend this algorithm to a multi-operator setting. Given the mode selection scheme, we allocate spectrum for multi-operator D2D users for maximizing the D2D user rate while also meeting MNO-specific user rate constraints. Under these conditions, it turns out that the formulated game is concave, satisfies the dominance solvability condition, and is also sub-modular. As a result, one can prove that the best response optimization converges monotonously to the unique Nash equilibrium (NE) from any initial point.

Using the proposed scheme we illustrate that both MNOs may experience significant performance gains as compared to no sharing. To the best of our knowledge, this is the first work which uses a non-cooperative game theoretical model for spectrum allocation for multi-operator D2D communication.

II. SYSTEM MODEL

We consider two MNOs, $i = \{1, 2\}$, enabling D2D communication. For the i -th operator, the BSs are distributed according to a Poisson point process (PPP) with density λ_i^b and, the cellular users are distributed according to a PPP with density λ_i^c . The distributions of BSs and cellular users are independent. A cellular user is associated with the nearest home-operator BS. An MNO has two types of D2D users: intra-operator D2D users, i.e., when the two ends in the D2D pair have subscriptions with the same MNO, and multi-operator D2D users. The locations of intra-operator D2D transmitters follow a PPP with density λ_i^d and the locations of multi-operator D2D transmitters follow a PPP with density λ . For the multi-operator D2D pairs, we assume that the densities of transmitters from different MNOs are equal, i.e., $\lambda/2$. As a result, for the i -th MNO, the ratio of multi-operator D2D to intra-operator D2D transmitters is $w_i = \lambda/(\lambda + 2\lambda_i^d)$.

Figure 1 illustrates the D2D concept towards a multi-operator scenario. Devices registered to different MNOs can communicate in a D2D manner or in infrastructure mode via the nearby serving BSs. The spectrum controller is an entity to coordinate the agreement about spectrum allocation and mode selection between the peer networks.

Figure 2 shows the spectrum allocation for the MNOs in

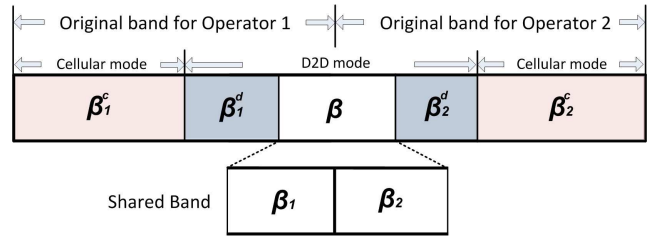


Fig. 2. Spectrum divisions for the MNOs.

case they employ the overlay principle also for intra-operator D2D communication. A fraction β_i^c of the MNO's spectrum is dedicated for cellular communication and a fraction β_i^d is dedicated for intra-operator D2D communication. Finally, an MNO contributes a fraction β_i of spectrum to the shared band, $\sum_i \beta_i$, where multi-operator D2D communication takes place. Obviously, $\beta_i^d + \beta_i^c + \beta_i = 1, \forall i$. D2D users selecting cellular transmission mode would be allocated to the β_i^c part of the spectrum. While in our analysis we assume FDD MNOs that contribute frequency resources for D2D communication, the same analysis is applicable to TDD MNOs that contribute time-frequency resource blocks. In the TDD case, multi-operator D2D support poses a requirement for time synchronization between the MNOs which is more challenging.

In order to describe the quality-of-service offered to the D2D users, we assume that an MNO maintains a network utility function that is equal to the average D2D rate

$$U_i = (1 - w_i) Q_i^d + w_i Q_i^s, \quad i = \{1, 2\} \quad (1)$$

where Q_i^d is the average normalized rate for intra-operator D2D users and Q_i^s describes the same quantity for multi-operator D2D users. Also, an MNO must offer to its cellular users an average rate equal to a target rate, i.e., $Q_i^c = \tau_i$.

The average rate of cellular users, Q_i^c , can be obtained by scaling their average spectral efficiency, R_i^c , with the normalized bandwidth available for cellular transmissions, β_i^c . On the other hand, the D2D users may operate either in cellular or in D2D mode. Let us denote by q_i^d the fraction of intra-operator D2D users selecting D2D mode and by q the same quantity for multi-operator D2D users. The average rate for intra-operator D2D users, Q_i^d , should be computed as an average of their average spectral efficiencies in cellular mode, R_i^c , and D2D mode, R_i^d , scaled with the appropriate fractions of user density and cellular bandwidth. The average rate for multi-operator D2D users, Q_i^s , should be obtained in a similar manner. To sum up,

$$\begin{aligned} Q_i^c &= \beta_i^c R_i^c \\ Q_i^d &= \beta_i^c R_i^c (1 - q_i^d) + \beta_i^d R_i^d q_i^d \\ Q_i^s &= \beta_i^c R_i^c (1 - q) + \beta R q \end{aligned} \quad (2)$$

where R is the average spectral efficiency for multi-operator D2D users selecting D2D mode. The average spectral efficiencies for cellular and D2D users can be calculated as in [6]

$$\begin{aligned} R_i^c &= \nu_i \int_0^\infty \frac{P_i^c}{1+\gamma} d\gamma \\ R_i^d &= \int_0^\infty \frac{P_i^d}{1+\gamma} d\gamma \\ R &= \int_0^\infty \frac{P}{1+\gamma} d\gamma \end{aligned} \quad (3)$$

where γ is the SINR, \mathcal{P} is the coverage probability for multi-operator D2D users in D2D mode, i.e., the probability that the SINR at a typical multi-operator D2D user is larger than the SINR γ , \mathcal{P}_i^d describes the same quantity for intra-operator D2D users, \mathcal{P}_i^c is the coverage probability for cellular users and ν_i is the portion of time a user in cellular mode is active.

The coverage probability depends on the interference level at the user. The density of interferers can be computed only after the mode selection scheme is specified. There are many schemes available in the literature, e.g., based on the D2D pair distance [6] and/or the distance between the D2D transmitter and cellular BS [7]. In these cases, D2D pairs can be arbitrarily close to each other. In order to avoid that, in [8], it is proposed to select the mode based on the measured interference at the D2D transmitter. When the measured interference is below a threshold, there is indication there are few ongoing D2D communication pairs. Provided that the D2D pair distance d is small, low interference at the D2D transmitter necessitates low interference at the D2D receiver.

It is straightforward to extend the mode selection algorithm described in [8] in a multi-operator setting. The D2D transmitter measures the interference over the shared band, $\sum_i \beta_i$, and communicates a quantized version of it to its home BS. The BS decides about the mode using the same threshold-based test and communicates its decision back to the transmitter. In this paper, the measurement is assumed to be done at the D2D transmitter in order to simplify the mathematical analysis of outage probability but practical implementation could be based on measurements conducted by the D2D receiver. In that case, the mode would be selected at the home operator of the receiver. As long as the D2D pair distance d is short, we do not expect significant differences in the results.

In [8], it is shown that the locations of D2D transmitters selecting D2D communication mode follow a Matérn-hardcore point process (MPP) type II with hardcore distance δ . Using the properties of MPP type II, the hardcore distance can be mapped to the mode selection threshold, ϵ , using simplified methods as in [12]. Finally, the coverage probability for multi-operator D2D users in the presence of Rayleigh fading can be computed using a similar approach as in [8]

$$\mathcal{P} = e^{-\frac{\gamma \sigma^2 \beta}{P_t l(d)} - q \lambda \int_0^{2\pi} \int_{\delta}^{\infty} \frac{f(r)r}{1+f(r)} dr d\phi - cq \lambda \int_0^{2\pi} \int_{\delta}^{2\pi} \frac{f(r)r}{1+f(r)} dr d\phi} \quad (4)$$

where $\beta = \beta_1 + \beta_2$, P_t is the D2D transmit power level, σ^2 is the noise level calculated over the full cellular band, $c = 2\pi(4\pi/3 + \sqrt{3}/2)^{-1}$, $l(\cdot)$ is the distance-based pathloss and $f(r) = \gamma l(\sqrt{r^2 + d^2 - 2rd \cos \phi})/l(d)$.

The coverage probability for intra-operator D2D users can be expressed in a form similar to equation (4) after replacing q by q_i^d , λ by λ_i^d and β by β_i^d . Finally, the coverage probability for the cellular uplink has been derived in [8]. If a power law model for the distance-based pathloss is used, with pathloss exponent a , the coverage probability can be written as

$$\mathcal{P}_i^c = \left(1 + \alpha_i \frac{2\gamma}{a-2} {}_2F_1\left(1, \frac{a-2}{a}, 2 - \frac{2}{a}, -\gamma\right)\right)^{-1} \quad (5)$$

where α_i is the probability a BS is active and ${}_2F_1$ is the Gaussian hypergeometric function. Note that the activity probability α_i should take into account not only the densities of cellular

users but also the densities of intra-operator D2D and multi-operator D2D users selecting cellular communication mode, i.e., $(1 - q_i^d)\lambda_i^d$ and $(1 - q)\lambda/2$ respectively.

III. MULTI-OPERATOR D2D SPECTRUM SHARING GAME

We consider a strategic non-cooperative spectrum sharing game between two MNOs, $\mathcal{G} = (\mathcal{I}, \mathcal{S}, \mathcal{U})$, where \mathcal{I} is the set of MNOs, $\mathcal{S} = \mathcal{S}_1 \times \mathcal{S}_2$ is the set of the joint strategies and $\mathcal{U} = [U_1, U_2]$ is the vector of utility functions. The strategy space for an MNO represents the spectrum fraction contributed for the shared band, i.e., $\mathcal{S}_i = \{\beta_i : 0 \leq \beta_i \leq u_i\}$, $i = \{1, 2\}$. The upper limit of the strategy space, u_i , depends on the MNO-specific constraints which are presented below.

In this paper, we assume that the mode selection threshold, ϵ^d , for the spectrum band dedicated to intra-operator D2D has been decided by each MNO, and the mode selection threshold, ϵ , for the shared band dedicated to multi-operator D2D has been agreed between the MNOs or imposed by the regulator. Note that given the decision thresholds, ϵ^d and ϵ , an MNO is able to compute the densities of intra-operator D2D and multi-operator D2D users selecting cellular communication mode.

In a non-cooperative game, each player sets its strategy profile to maximize its own utility function. For the game in question, an MNO maximizes the average D2D user rate, see equation (1), under operator-specific constraints for cellular users and intra-operator D2D users. For instance, the average cellular user rate should be equal to a target rate, $Q_i^c = \tau_i$, and the intra-operator D2D rate should be higher than a constraint, $\beta_i^d R_i^d \geq \mu_i^d$. We assume that without spectrum sharing, i.e., $\beta_i = 0$, these constraints are satisfied. To sum up, an MNO will identify the amount of spectrum to contribute for multi-operator D2D communication, β_i , as the solution of the following optimization problem.

$$\text{Maximize : } U_i. \quad (6a)$$

$$\text{Subject to : } Q_i^c = \tau_i \quad (6b)$$

$$\beta_i^d R_i^d \geq \mu_i^d. \quad (6c)$$

Since the inter-operator spectrum sharing game should be distributed and non-cooperative, one possible way to reach to a consensus is the best response iteration. According to it, given the opponent's proposal β_j , the i -th MNO identifies its contribution β_i for maximizing U_i . In order to identify the optimal spectrum fraction β_i we use the following steps: Given the thresholds ϵ_i^d and ϵ , the i -th MNO computes the fractions of intra-operator D2D and multi-operator D2D users in cellular communication mode, i.e., $(1 - q_i^d)\lambda_i^d$ and $(1 - q)\lambda/2$ respectively, thus able to identify the required amount of spectrum for cellular users, β_i^c , based on the equality constraint (6b). Then, the spectrum fractions, β_i and β_i^d , can be related as $\beta_i = 1 - \beta_i^c - \beta_i^d$. In the Appendix, we show that the utility and the left-hand side of the constraint in (6c) are both concave in β_i . If there is always a β_i such that the constraint in (6c) is strictly satisfied, the first-order conditions are both necessary and sufficient. As a result, the optimal β_i can be identified using standard convex optimization tools at a low complexity.

In a non-cooperative game, some of the most important questions are the existence and uniqueness of the NE. In case

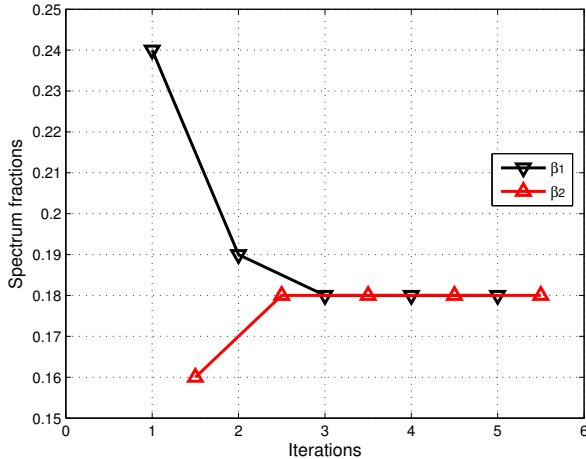


Fig. 3. Example convergence to the unique NE for symmetric operators, when the mode selection threshold in the shared band is $\epsilon = -72$ dBm and in the intra-operator D2D band is $\epsilon_i^d = -75$ dBm for both MNOs.

there are multiple Nash equilibria, the selected equilibrium would depend on the initial strategy profile [13], [14]. This property might be undesirable because the performance of an MNO would depend on the selection order between the MNOs and also on their initial proposals for spectrum contribution in the shared band.

Proposition 1. *The formulated game has a unique NE and the best response iteration converges to it from any initial point.*

Proof: According to [14, Theorem 1], a NE exists for a concave game. In [15, Theorem 4.1], the dominance solvability condition under which a concave game has a unique NE and the best response iteration converges to it from any initial point is proposed. In the Appendix, we prove that the game is concave and the utility function U_i subjecting to MNO-specific constraints satisfies the dominance solvability condition. ■

Proposition 2. *Using the best response iteration, the MNOs converge monotonously to the NE.*

Proof: According to [13, Algorithm I], the best response algorithm converges monotonically to a NE, from any initial strategy profile, if the utility U_i is sub-modular function and the strategy space satisfies the descending property. The monotonicity would be in the opposite direction for the MNOs. According to [16], the function U_i is sub-modular in the strategy set of the two players if the first-order cross derivative is negative. Following an approach similar to the one followed in equations (7) and (8), one can show that $\frac{\partial^2 U_i}{\partial \beta_i \partial \beta_j} < 0$. Also, due to the fact that the strategy space does not depend on the opponent behavior, but only on the operator-specific constraints, the descending property is satisfied. ■

After the best response converges, it is natural to assume that the agreement will break if the utility of an MNO is lower than the utility corresponding to no spectrum sharing, $U_i < U_{0,i}$.

In general, the MNOs may have different network utility functions and constraints. Because of that, the best response iteration cannot be used to infer information about the network

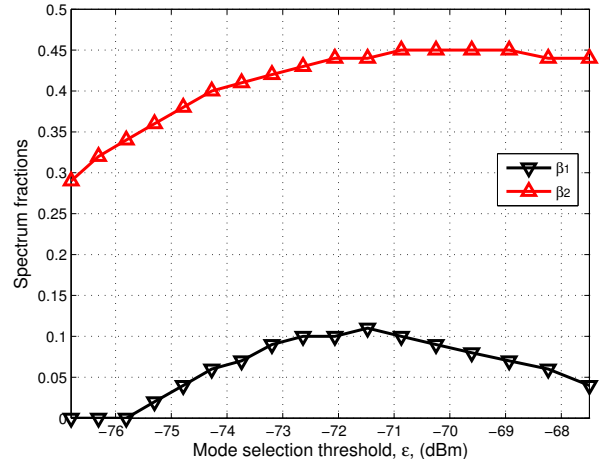


Fig. 4. Spectrum fractions, β_i , for multi-operator D2D w.r.t the mode selection threshold in the shared band. $\lambda_1^d = 1$, $\lambda_2^d = 0.8$. The mode selection threshold in the intra-operator D2D band is $\epsilon_i^d = -75$ dBm for both MNOs.

state of the opponent MNO.

IV. NUMERICAL ILLUSTRATION

We consider two MNOs with BS density $\lambda_i^b = 1/(\pi 200^2)$, $i = \{1, 2\}$, cellular user density $\lambda_i^c = \lambda_i^b$ and multi-operator D2D density, $\lambda = 4\lambda_i^b$. We evaluate the performance of the spectrum allocation scheme for different intra-operator D2D densities. We fix $\lambda_1^d = \lambda_i^b$ and we vary λ_2^d . We take 3GPP propagation environment [18] into account with pathloss equation in dB: $37.6 \log_{10}(r) + 15.3$ for the cellular mode and, $40.0 \log_{10}(r) + 28$ for the D2D mode, where r is the distance in meters. The D2D link distance is fixed to $d = 30$ m. We use fixed transmit power levels equal to 23 dBm for the cellular mode and 20 dBm for the D2D mode. The normalized target rates for intra-operator D2D users and cellular users are $\tau_i = \mu_i^d = 0.3$, $i = \{1, 2\}$.

Figure 3 shows an example convergence of the best response iteration to the unique NE for $\lambda_1^d = \lambda_2^d$. If the MNOs are symmetric, they should contribute equal amounts of spectrum in the shared band. The initial strategy profiles are randomly selected. The MNO 1 first solves the optimization problem (6) assuming that the MNO 2 does not contribute any spectrum in the shared band. Then, the MNO 2 optimizes for β_2 given that the MNO 1 contributes spectrum fraction $\beta_1 = 0.24$ and so forth. According to Proposition 2, one operator converges monotonously increasing and the other monotonously decreasing to the NE.

Next, we make the operators non-symmetric assuming that the MNO 2 has less intra-operator D2D users, $\lambda_2^d = 0.8$. In Figure 4, we depict the spectrum fractions contributed by the MNOs with respect to the mode selection threshold in the shared band. In general, asymmetric MNOs would contribute unequal amounts of spectrum. In our example, the MNO 2 has less network load than MNO 1 and, because of that, it has the capacity to contribute more spectrum in the shared band. Also, one can point out that a low mode selection threshold results in more users in cellular communication mode and thus,

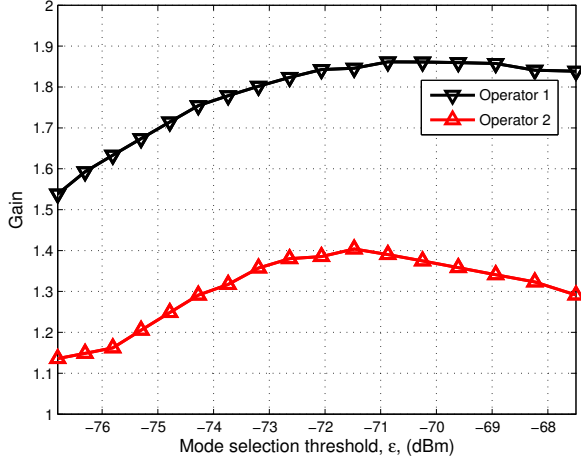


Fig. 5. Performance gain for the MNOs as compared to the case without spectrum sharing. $\lambda_1^d = 1$, $\lambda_2^d = 0.8$. The mode selection threshold in the intra-operator D2D band is $\epsilon_i^d = -75$ dBm for both MNOs.

more spectrum resources should be reserved by the MNOs to meet their cellular user rate constraints. As the mode selection threshold increases, the associated bandwidth for multi-operator D2D support increases too. However, increasing the mode selection threshold beyond certain point has adverse effects, since the increased self-interference among the multi-operator D2D pairs starts reducing their rate performance.

Figure 5 shows the sum rate gain for the MNOs as compared to the case without multi-operator D2D support, i.e., all multi-operator transmissions are routed through the BSs. Both MNOs experience performance gain. The MNO 2 that has less network load contributes the higher fraction of spectrum in the shared band, see Figure 4. Because of that, the MNO 1 enjoys more benefit from spectrum sharing than the MNO 2.

Finally, we fix the mode selection threshold in the shared band, $\epsilon = -72$ dBm, and assess the performance gains for varying density of D2D users for the MNO 2. In Figure 6, one can see that symmetric MNOs achieve around 50 % gain. For densities $\lambda_2^d > 1.2$, the MNO 2 does not have the capacity to contribute any spectrum, however, the MNO 1 still benefits by contributing a fraction $\beta_1 = 0.24$. In that case, the MNO 1 experiences 20% gain due to the proximity between multi-operator D2D users, while the gain for the MNO 2 is attributed to both proximity and spectrum sharing. The performance gain for both MNOs is high, i.e., close to 100%, only if the network load for the MNO 2 becomes low. In that case, the MNO 2 is able to contribute a high bandwidth fraction $\beta_2 = 0.52$, while for the MNO 1, $\beta_1 = 0.08$.

V. CONCLUSIONS

In this paper, we proposed a method for spectrum allocation for D2D communication considering different mobile network operators. In a multi-operator D2D setting, the operators may not be willing to reveal proprietary information to the competitor and/or to other parties. Because of that, we modeled their interaction as a non-cooperative game. An operator makes an

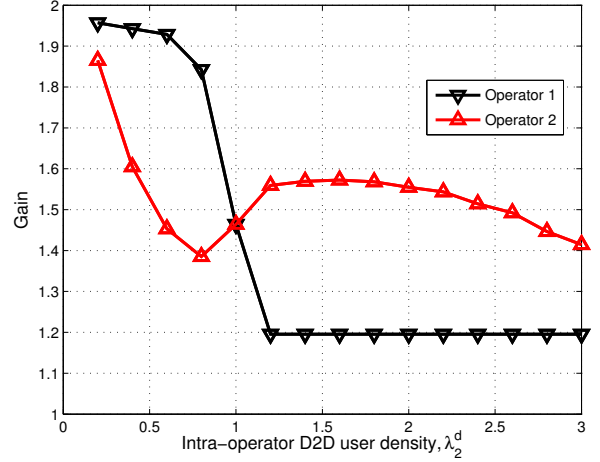


Fig. 6. Performance gain for the MNOs as compared to the case without spectrum sharing for varying density of D2D users for the MNO 2. $\lambda_1^d = 1$. The mode selection threshold in the shared band is $\epsilon = -72$ dBm and in the intra-operator D2D band is $\epsilon_i^d = -75$ dBm for both MNOs.

offer about the amount of spectrum to contribute for multi-operator D2D communication considering only its individual performance. While making the offer, it also takes into account the offer made by the opponent operator. With the performance metrics considered in the paper, we showed that the formulated game has a unique NE and the sequence of operators' best responses converges to it from any initial point. In general, asymmetric operators contribute unequal amount of spectrum. An operator may not contribute any spectrum at all but still, the opponent may have the incentive to be cooperative due to the D2D proximity gain. Provided that the multi-operator D2D density is not negligible, we showed that both operators may experience significant performance gains. The particular gain would depend on the operator-specific network load, utility and design constraints. As potential directions for future work, one may consider spectrum sharing for more than two MNOs enabling multi-operator D2D communication. One could study whether it is beneficial to construct a common pool of spectral resources or to realize multi-operator D2D by means of bilateral agreements between operators.

APPENDIX

First, we show that the utility U_i is concave in β_i by computing its second derivative

$$\begin{aligned} \frac{\partial^2 U_i}{\partial \beta_i^2} &= (1 - w_i) \frac{\partial^2 Q_i^d}{\partial \beta_i^2} + w_i \frac{\partial^2 Q_i^s}{\partial \beta_i^2} \\ &= \frac{(1 - w_i) q_i^d}{\eta} \int_0^\infty \frac{e^{-\frac{\gamma \beta_i^d}{\eta} - C(\gamma, q_i^d)}}{1 + \gamma} \left(\frac{\beta_i^d \gamma}{\eta} - 2 \right) \gamma d\gamma \\ &\quad + \frac{w_i q}{\eta} \int_0^\infty \frac{e^{-\frac{\gamma(\beta_i + \beta_j)}{\eta} - C(\gamma, q)}}{1 + \gamma} \left(\frac{(\beta_i + \beta_j) \gamma}{\eta} - 2 \right) \gamma d\gamma \end{aligned} \quad (7)$$

where $C(\gamma, q) = q\lambda \int_0^{2\pi} \int_0^\infty \frac{f(r)r}{1+f(r)} dr d\phi + cq\lambda \int_0^{2\pi} \int_0^\infty \frac{f(r)r}{1+f(r)} dr d\phi$, $C(\gamma, q_i^d)$ can be expressed in a similar manner and $\eta = \frac{P_{t1}(d)}{\sigma^2}$.

The first part of the right-hand side of the equation (7) can be treated as follow

$$\begin{aligned}
& \int_0^\infty \frac{e^{-\frac{\gamma\beta_i^d}{\eta} - C(\gamma, q_i^d)}}{1+\gamma} \left(\frac{\beta_i^d \gamma}{\eta} - 2 \right) \gamma d\gamma \\
& \stackrel{(p1)}{\leq} \int_0^\infty \frac{e^{-\frac{\gamma\beta_i^d}{\eta}}}{1+\gamma} \left(\frac{\beta_i^d \gamma}{\eta} - 2 \right) \gamma d\gamma \\
& \stackrel{(p2)}{=} -\frac{\rho+1}{\rho} + (\rho+2)e^{\rho} E_1(\rho) \\
& \stackrel{(p3)}{<} -\frac{\rho+1}{\rho} + (\rho+2) \left(\frac{1}{\rho+} \frac{1}{1+} \frac{1}{\rho} \right) = 0,
\end{aligned} \tag{8}$$

where $\rho = \frac{\beta_i^d}{\eta}$, inequality (p1) holds true due to $C(\gamma, q_i^d) > 0$ and equality (p2) uses that $\int_0^\infty \frac{e^{-\rho x}}{1+x} (\rho x - 2) x dx = -\frac{\rho+1}{\rho} + (\rho+2)e^{\rho} E_1(\rho)$ where $E_1(\rho) = \int_\rho^\infty \frac{e^{-\gamma}}{\gamma} d\gamma$ is the exponential integral. For $\rho > 0$, there is a continued fraction form expressed as $E_1(\rho) = e^{-\rho} \left(\frac{1}{\rho+} \frac{1}{1+} \frac{1}{\rho+} \frac{1}{1+} \dots \right)$ from [17, 5.1.22]. This continued fraction form is less than $e^{-\rho} \left(\frac{1}{\rho+} \frac{1}{1+} \frac{1}{\rho} \right)$. From this relation, inequality (p3) holds true.

In a similar manner one can show that the second term of the right-hand side of equation (7) is also negative. That completes the proof that the utility U_i is concave. Also, the utility U_i is continuous in β_j . The constraint (6b) just affects the upper limit of strategy space while the left-hand side of the constraint in (6c) is concave in β_i . That completes the proof that the game in question is concave.

The Lagrangian function of the optimization problem (6) is $L_i = U_i + \nu_i (\beta_i^c R_i^c - \tau_i^c) + \xi_i (\beta_i^d R_i^d - \mu_i^d)$, where $\nu_i \geq 0$ and $\xi_i \geq 0$ are the Lagrange multipliers. Then, the convergence of the best response to the unique NE can be proved by the diagonal dominance solvability condition of the Lagrangian.

$$\left| \frac{\partial^2 L_i}{\partial \beta_i^2} \right| > \left| \frac{\partial^2 L_i}{\partial \beta_i \partial \beta_j} \right|, \forall i \tag{9}$$

where

$$\frac{\partial^2 L_i}{\partial \beta_i^2} = (1-w_i) \frac{\partial^2 Q_i^d}{\partial \beta_i^2} + w_i \frac{\partial^2 Q_i^s}{\partial \beta_i^2} + \frac{\partial^2 \xi_i (\beta_i^d R_i^d - \mu_i^d)}{\partial \beta_i^2} \tag{10}$$

and

$$\frac{\partial^2 L_i}{\partial \beta_i \partial \beta_j} = \frac{w_i q}{\eta} \int_0^\infty \frac{e^{-\frac{\gamma(\beta_i + \beta_j)}{\eta} - C(\gamma, q)}}{1+\gamma} \left(\frac{(\beta_i + \beta_j) \gamma}{\eta} - 2 \right) \gamma d\gamma. \tag{11}$$

One can see that the second term in the right-hand side of equation (10) is negative and equal to right-hand side of equation (11). The first and third terms in the right-hand side of equation (10) are negative too thus, inequality (9) holds.

ACKNOWLEDGMENT

This work has been performed in the framework of the FP7 project ICT 317669 METIS, which is partly funded by

the European Union. Also, this work was supported by the Academy of Finland funded project SMACIW under Grant no. 265040.

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