Restoration Modeling and Optimization of Hybrid Overhead-Underground Power Distribution Systems

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***Abstract*—Disaster awareness increased in recent years among power system stakeholders to face many natural, technical, and malicious adversities. The smart distribution grid (SDG) is thereby at the core of proposed system enhancements, due to its high fragility as well as being the interface to most newly introduced grid applications (distributed energy resources, electrical vehicles, industrial Internet-of-Things, etc.). The SDG can be characterized by the type of lines composing the feeders (overhead and/or underground) and deployed intelligent electronic devices (IED) that allow efficient monitoring, protection, and control of the system. This paper proposes an optimization formulation to enhance the resilience of overhead and underground networks, while considering the coupling between power grid operation and the communicating remote-controlled switches (RCS). Novel *radiality* constraints are introduced to guarantee the tree structure during operation. Results from testing the model in a real network show the validity of proposed radiality constraints and quantify the gap in terms of achieved resilience between full overhead and hybrid overhead-underground networks.**

***Index Terms*—Smart Grid, Resilience, Overhead and Underground Networks, Radiality, Optimization, Communication**

NOMENCLATURE

Sets

N	Set of all power nodes (HV/MV SS, MV buses)
S	Set of HV/MV Substations (SS)
L	Set of all power lines
L^o, L^u	Set of overhead (o), underground (u) lines
L^m, L^r	Set of manual, remote switchable lines, resp.
L^{ar}, L^{cb}	Set of auto-reclosing, circuit-breaking lines, resp.
F	Set of failures in power lines

Parameters

M	Large number
l_l^m, l_l^r	1 if line l is manual, remote (resp.), 0 otherwise
l_l^{ar}, l_l^{cb}	1 if l is a recloser, a circuit breaker, 0 otherwise
f_l	1 if failure in line l, 0 otherwise
s_i	Binary parameter. 1 if i is a SS, 0 otherwise

Variables

$sw_{l,t}$	1 if switch at $l = (i, j)$ is closed at t, 0 otherwise
$sw_{ij,t}$	1 if directed switch (i, j) is closed at t, 0 otherwise
$p_{i,t}^{ns}$	Loss of active load at node i at time t
$d_{ij,t}$	1 if power flows from i to j at t, 0 otherwise
$a_{i,t}$	1 if bus i is available at t, 0 otherwise
$y_{i,t}$	1 if bus i is energized at t, 0 otherwise
$y_{i,t}^{dg}$	1 if a DG is connected at bus i at t, 0 otherwise

I. INTRODUCTION

Power systems use different types of conductors to carry electricity from generating units to customers. Underground lines are mostly used in metropolitan cities and urban areas based on their reduced losses and adequacy to a restrained public and private space, shared with other critical infrastructure assets [1]. Underground networks exhibit better robustness to many natural events like windstorms, hurricanes, and heavy snowfall; but resist less to heatwaves, flooding, earthquakes, etc., and may reduce the speed of recovery compared to overhead lines [2]. This motivates thorough evaluation of hazard threats in any given area before making the choice of the suitable conductor type to adopt [3]. However, despite the considerable proportion of underground networks in power systems, overhead lines dominate planning, operation, and restoration studies [1]. This work aims to fill this gap by proposing a general purpose model for hybrid overhead-underground configurations, applied to the case of resilience assessment. Performance disparity between fully overhead and hybrid configurations is investigated, and introduced radiality formulation is validated in a real network case study.

II. RELATED WORKS

The radial operation remains the dominating configuration in distribution grids despite proposals, more than a decade ago, to use meshed configurations given the growing penetration of distributed energy resources [4, 5]. As such, many works consider spanning tree (ST) constraints to guarantee radiality, which facilitates coordination of protection mechanisms and reduces short-circuit currents. Ref. [6] produced two necessary conditions for tree-like networks; i) The solution must have $N-1$ branches; ii) The solution must be connected. Authors in [7] specify detailed constraints for network radiality by introducing two binary variables corresponding to each line to indicate if the node at either end of the line is the parent of the other. However, typical distribution grids contain many feeders from one or multiple high-voltage/medium-voltage substations (HV/MV SS). Thus, ST constraints are not sufficient when there is more than one source, and the aforementioned radiality conditions update to; i) The solution must have $N - sg$ branches, where sg is the number of sub-graphs (or islands); ii) Each sub-graph in the solution is a connected tree.

The single commodity flow (SCF) model is widely adopted to extend the radiality conditions [8]. In such situation, two cases can be distinguished: Case 1 - The nominal configuration is a spanning tree, and new distributed generators (DGs) are deployed later, enforcing a spanning forest configuration; Case 2 - A real network case with a spanning forest layout, which should be kept following any network reconfiguration. SCF defines a fictitious network, identical to the considered SDG, where each sub-graph has one power source, and the remaining nodes are taken as load buses with a unit demand. Balance equations of commodity (i.e. Power) flow are used to express how each load demand is satisfied in the network, implying the existence of a path from a demanding load to the source node in every single sub-graph [9].

The same approach is used in [10] to generalize the radiality conditions to multi-source situations, which works well for the phases of *normal operation* and *service degradation* as the connected portion of the grid is either stable or shrinking, and there is prior knowledge on the number/composition of sub-graphs (sub-networks, islands). Later, the DSO deals with a variable network as the restoration is conducted through the opening and closing of switches, meaning that the number/composition of the network is unknown and to be optimized. This leads to modify the updated condition i) to:

$$\sum_{\forall (i,j)\in L} sw_{ij,t} = N - \sum_{i}^{N} root_{i,t}, \forall t \in T \tag{1}$$

where $sw_{ij,t}$ is the connection status of line $(i,j) \in L$ at time t, and $root_{i,t}$ indicates whether a power source (substation or DG unit) or a bus at node i is a root of an island at time t. Authors in [11] propose an adapted formulation to cope with the changing configuration by restricting the feasible solution to a subset from the ST of a fictitious network (the same as the SDG but without damages). The use of this approach is motivated for multi-feeder/multi-substation networks as power sources can be merged into a single node for radiality constraints, but still treated separately in operational constraints of the system [12].

Networked systems, like smart grids, are inherently prone to failure propagation due to numerous connections between involved elements [13, 14, 15]. As a result, three zones can be distinguished during a contingency event: i) Damaged zone: containing the initial failure and subsequent damages due to failure propagation; ii) Out-of-Service safe zone: part of the network, at first included in the damaged zone, but could be isolated from the damage using switches. Components in this zone can be reconnected through reconfiguration in the power network; iii) Supplied safe zone: parts that are safe from damages and still energized. Formation of these zones may differ depending on the nature of the event and the type of the network. A reasonable assumption is made here about the ability of opened lines to interrupt the spread of failures, meaning that only propagation in closed lines is considered. Still, whether lines are overhead or underground affects the expanse of the respective zones. Underground grids have the advantage of reducing outage exposure, maintenance cost, and transmission losses [2], mainly at a cost of repair difficulty and increased expenses compared to overhead networks.

III. System Model

The MV level of the SDG is modeled in this work. A graph representation is adopted, where *nodes* are the MV buses and the HV/MV SS, while *edges* are the power lines. Connected grid assets include HV/MV substations, circuit breakers, autoreclosers, and RCSs. Fault detectors are considered perfect in this study as the focus is on the contribution of RCSs to service restoration. Without loss of generality, failures are only considered in power lines, and can propagate to buses and other lines. The operation of the distribution grid is captured by the LinDistFlow model, describing power flow from HV/MV substations to low voltage (LV) loads connected at MV buses.

A. Optimization Objective

The *supplied power* is used to evaluate the performance of restoration efforts, and set as the objective function of the formulated mixed integer linear programming (MILP) problem, alongside a second term to reduce switching cost.

$$\min_{\boldsymbol{p},\boldsymbol{d},\boldsymbol{sw},\boldsymbol{a},\boldsymbol{y},\boldsymbol{w}} \Bigg[\alpha \sum_{\forall t}\Bigg(\sum_{\forall i\in N} C_i^{ns}\cdot p_{i,t}^{ns} + \sum_{\forall i\in N} C_i^{e}\cdot a_{i,t}\Bigg) + \beta \sum_{\forall t}\sum_{\forall l\in L} C^{sw}\cdot w_{l,t}\Bigg] \tag{2}$$

Equation (2) is the objective function with $\boldsymbol{p}$ a vector of electrical quantities (line active/reactive power, node voltages, non-supplied load), $\boldsymbol{d}$ the directions of power flow in power lines, $\boldsymbol{sw}$ the statuses of line switches, $\boldsymbol{a}$ the availability of power buses, and $\boldsymbol{y}$ the connectivity of power buses. The first double summation term represents the total cost of not supplying a portion of the system load, where each load has its associated criticality-based cost C_i^{ns}. The next term expresses the cumulative cost induced by the extent of the damaged zone, where C_i^e is the cost of losing each electrical node. The final term is designed to include the cost of switching, as no change on the configuration is desired unless there is a gain in restored power. C^{sw} is considered the same for all operated switches, and variable $w_{ij,t}$ results from the linearization of the absolute value of $sw_{l,t} - sw_{l,t-1}$. For the constants, $\alpha \gg \beta$ as from the standpoint of a DSO during an extreme event, restoring power to clients is given priority and costs are only considered when equivalently performing strategies are obtained.

B. Radiality Constraints

We propose in this work a simplified model that gets around the changing parent and child sets [16, 7], as well as bypasses both the need for the ST polytope [12] and identification of sub-network roots [10] as illustrated in equation (1).

$$d_{ij,t} + d_{ji,t} \leq sw_{l,t}, \forall l = (i,j) \in L, \forall t \tag{3}$$

$$sw_{l,t} - (2 - a_{i,t} - a_{j,t}) \leq d_{ij,t} + d_{ji,t}, \forall l = (i,j) \in L, t = 3 \tag{4}$$

$$d_{ij,t} + d_{ji,t} - (2 - a_{i,t} - a_{j,t}) \leq sw_{l,t}, \forall l = (i,j) \in L, t = 3 \tag{5}$$

$$\sum_{\forall j \in n(i)} d_{ji,t} \leq a_{i,t} - s_i - y_{i,t}^{dg}, \forall i \in N, \forall t \tag{6}$$

TABLE I
COMPARISON OF VARIABLES AND CONSTRAINTS NUMBER

	[12]	[16]	Proposed DLF
Variables	$2 \cdot \lvert N \rvert \cdot \lvert L \rvert + \lvert L \rvert$	$\lvert N \rvert + 3 \cdot \lvert L \rvert$	$2 \cdot \lvert N \rvert + 3 \lvert L \rvert$
Constraints	$\lvert N \rvert^2 + 2 \cdot \lvert N \rvert \cdot \lvert L \rvert - \lvert N \rvert - \lvert L \rvert + 1$	$\lvert N \rvert + \lvert L \rvert$	$2 \cdot \lvert N \rvert + 3 \cdot \lvert L \rvert$

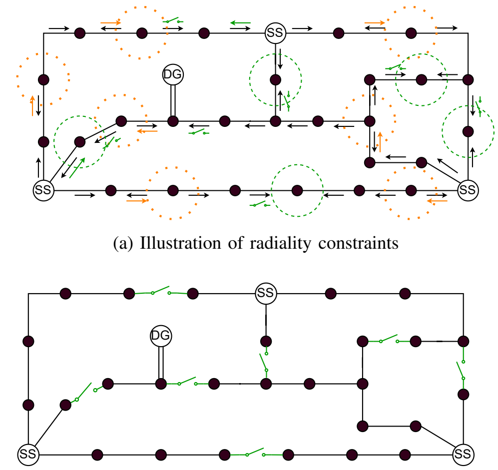
$$\sum_{\forall j \in n(i)} d_{ij,t} \leq M \cdot \left(\sum_{\forall j \in n(i)} d_{ji,t} + s_i + y_{i,t}^{dg} \right), \forall i \in N, \forall t \tag{7}$$

Constraint (3) imposes unidirectional flow of power, while capturing the existence of *unsupplied* closed lines in damaged zones. This fact is missed in all reviewed works as an equality sign in (1) would force energizing (de-energizing) a line to be equivalent to closing (opening) it. Then, (4) and (5) state that for the reconfiguration phase ($t = 3$), a line out of damaged zones is safely energized as soon as closed. The damage in a line is represented by directly failing the two connected nodes, meaning that both *failed-open* and *failed-closed* events can be considered. Constraint (6) prohibits power from flowing into HV/MV substations or nodes with a DG source, while indicating that any other bus has at most one parent node. If this parent node is not supplying power to the considered node i, or i is neither a substation nor a DG, no downstream power supply is possible from i as encoded in (7). This construction admits the formation of out-of-service islands affected either by the failure event or a shortage of power supply. Our directed local flow (DLF) approach relies on basic local rules that ensure the systemic validity of the radiality requirement, without the need for *restricting* global constraints like (1).

Table I summarizes the number of variables and constraints in the proposed radiality formulation alongside two recent works [12] and [16]. The least number of constraints is presented in [16], where radiality conditions need the implicit contribution of power flow equations to avoid disconnected graphs with loops [6] (power flow constraints are not counted in Table I). This can be argued to be more compact (*compact* in the sense of less constraints to achieve the same goal), but when solving the MILP problem, infeasible configurations are considered due to insufficient radiality constraints, causing a larger number of iterations [17]. Besides, unlike DLF, out-of-service zones are not seized, imposing the energization of all nodes out of damaged zone.

The number of variables $a_{i,t}$ can be subtracted from our model when compared to [16] because the availability status of power nodes is defined anyway in the global distribution service restoration model, making the two models equivalent in terms of the number of variables. A tight construction is presented in [12] with the expense of an increased number of variables and constraints. The same level of tightness is achieved by applying constraints (3), (7), and (6) sequentially, to result in a directed spanning forest polytope as a solution set.

Figure 1a shows an example of applying constraints (3)–(7) in a multi-source distribution grid. The DG is assumed to



(b) Obtained spanning forest configuration

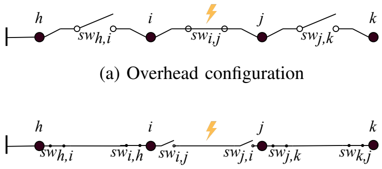
Fig. 1. Example of sequential application of the proposed DLF

be connected at the associated bus, and a general objective can be defined as maximizing the number of connected buses. First, only one power flow direction is allowed, and the dark arrows show a possible configuration that satisfies (3). Then, close inspection of this configuration reveals three issues: i) No power ingress to a bus that is supplying other buses (dotted orange circles); ii) More than one power ingress to a bus node (dashed green circles); iii) Power flow into substations or DGs. Constraint (7) solves issue i) by imposing the existence of a path from a source to any energized node, and the initial randomly-chosen configuration is updated with the orange arrows. Remaining issues are resolved using (6) (update with green arrows and opened switches) to yield a directed spanning forest with four tree-like islands interconnected with normally-open switches (Figure 1b).

C. Cascade Constraints

Figures 2a and 2b show two widely used topologies in overhead and underground SDGs, respectively. An electric bus in overhead lines allows downstream power flow to other buses, and supplies any load directly connected to it (e.g. MV/LV substation is the load in MV distribution grid). A single line in Figure 2a corresponds to many successive poles that join line segments in any large scale distribution grid. A switch is generally present in one of the poles, so it is fairly representative to model this by a switch for each line (that includes many poles and line segments). This representation is less valid in case of underground networks with less derivations as MV/LV substations are powered in series. A straightforward consequence of Figure 2b is that MV/LV SSs can be considered electrical buses with switches at any interface with a power line. Despite the expensive deployment of additional switches, the series configuration can achieve the narrowest isolation in case of a failure, which contributes to maintain more connected loads.

$$a_{i,t} + sw_{l,t-1} \cdot (1 - l_l^{cb}) \cdot (1 - l_l^{ar}) - 1 \leq a_{j,t}, \quad \forall l = (i,j) \in L, t = 1 \tag{8}$$



(a) Overhead configuration

(b) Underground configuration

Fig. 2. Failure propagation in power distribution networks

$$a_{i,t} + sw_{l,t} - 1 \leq a_{j,t}, \forall l = (i,j) \in L, t \in \{2,3\} \quad (9)$$

$$a_{i,t} \leq 1 - f_l \cdot sw_{l,0} + s_i, \forall l = (i,j) \in L^o, \forall t \quad (10)$$

$$a_{i,t} \leq 1 - f_l \cdot sw_{ij,t} + s_i, \forall l = (i,j) \in L^u, \forall t \quad (11)$$

$$sw_{l,t} = sw_{ij,t} \cdot sw_{ji,t}, \forall l = (i,j) \in L^u, \forall t \quad (12)$$

Constraint (8) represents the automatic response of the grid ($t = 1$), where the presence of a circuit breaker or an automatic recloser ($l_{ij}^{cb} = 1$ or $l_{ij}^{ar} = 1$) at line (i,j) stops the propagation of the failure. Each line is visited twice in this expression, with $a_{i,t}$ and $a_{j,t}$ commuting positions, to yield an equality in case of an automatic response in the line. Constraint (9) ensures that damaged zones are not connected to safe (supplied or out-of-service) zones. This is guaranteed by requiring open lines between safe and damaged zones. Connecting out-of-service zones to supplied zones is possible.

Damages in power lines are fed to the model through parameter f_{ij}. Using (10), both connected nodes to an overhead line (i,j) become unavailable if the line was initially closed (failed-closed event). To include the failed-open case, (10) can be easily adapted by removing the operand $sw_{ij,0}$. Similarly, underground lines propagate the initial failure in (11), with the subtlety that $sw_{ij,t}$ is no longer indirected, because it represents the switch associated to node i and $sw_{ji,t}$ is the switch closest to node j. The underground case triples thereby the number of variables for switch states as the indirected variable $sw_{l,t}$ (switch status for line $l = (i,j)$) is kept. Constraint (12) determines that an underground line is closed only when both switches are closed. The non-linear quadratic component therein can be easily linearized to an equivalent set of constraints as the involved variables are integers.

$$sw_{l,t} \leq sw_{ij,t}, \forall l = (i,j) \in L^u, \forall t \quad (13)$$

$$sw_{l,t} \leq sw_{ji,t}, \forall l = (i,j) \in L^u, \forall t \quad (14)$$

$$sw_{ij,t} + sw_{ji,t} - 1 \leq sw_{l,t}, \forall l = (i,j) \in L^u, \forall t \quad (15)$$

The two variables $sw_{l,t}$ and $sw_{ij,t}$ are different. Other than constraints (11)-(15), the undirected variable $sw_{l,t}$ is used throughout the model to represent the state of line $l = (i,j)$.

IV. Case Study

A case study of 315 MV buses with 4 HV/MV substations (SS) is extracted from a french MV distribution network to demonstrate the effectiveness of the proposed approach. Capacitors, transformers, and regulators are ignored in compliance with the study objectives. The analysis is conducted in the

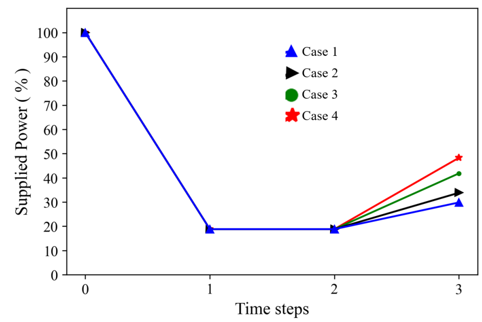


Fig. 3. Supplied power evolution during degradation, isolation, and reconfiguration

real 20 kV nominal voltage unbalanced distribution network of total 58.935 MW demand. The model is implemented in Pyomo, and solved by Cplex solver. No optimality gap is specified to the solver as the model yields the optimal solution in all cases presented below. We choose: $\alpha = 10, \beta = 0.1, C_i^{ns} = 0.5, C_i^e = 1$, and $C_i^{sw} = 0.1$. Power lines contain either remote or manual switches, and a scenario of 15 damages is considered. We set:

- Case 1: all failed lines are overhead lines;
- Case 2: 5 out of 15 failed lines are remote and underground;
- Case 3: 10 out of 15 failed lines are remote and underground;
- Case 4: all failed lines are remote and underground.

Figure 3 depicts the evolution of the percentage of supplied power over degradation, isolation, and reconfiguration phases in the SDG. Performance is the same at degradation ($t = 1$) and isolation ($t = 2$) phases for different configurations, but the degree of isolation is different. This comes to light on the remote reconfiguration phase where Case 4, having the highest number of involved underground lines, achieves the best restoration strategy (47.84%). The other cases attain less recovery due to a broader isolation, which caused some buses to remain in faulted zones and not being able to reconnect.

The changing configuration of the SDG is tracked, in order of occurrence, by figures 4b−4d for automatic isolation, remote isolation, and remote reconfiguration. Examples of switch maneuvers are shown in 4b and 4c by an opened-switch symbol. The supplied buses are reached by green lines, while unserved manual and remote lines are shown in gold and violet, respectively. Opened lines for substation protection and isolation are shown in dashed lines (violet for remote switches, and light pink for circuit breakers). Radiality is respected as all damaged zones remain isolated from safe zones. Note that, for all considered cases, a simulation of the model including data fetching, parameter initialization, solution, and result-retrieving takes 5 to 6 seconds, with less than 1 second for finding the solution by the optimizer (given the 34782 constraints and 14122 variables). This demonstrates the computational efficiency of the proposed radiality model, designed for use during crisis management situations.

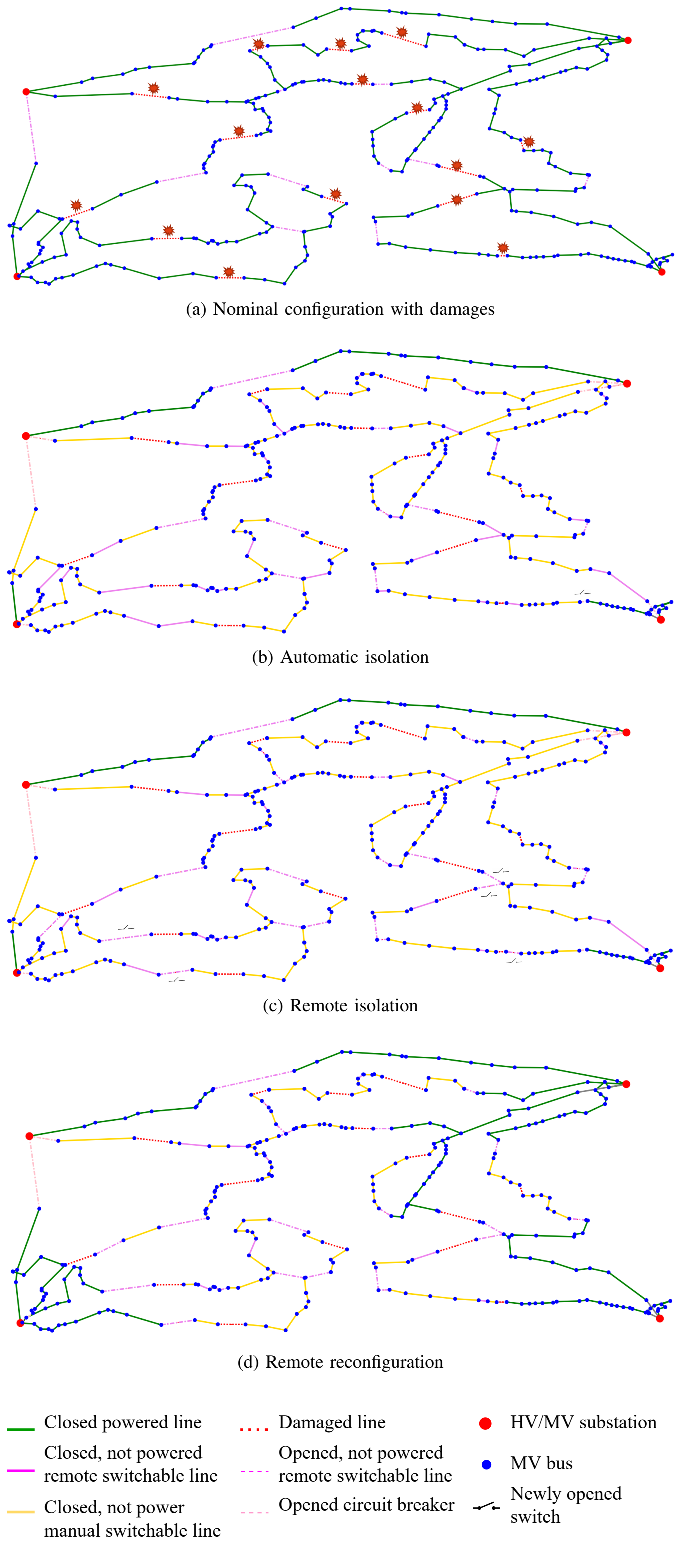


Fig. 4. Multi-feeder 315-bus distribution network in different phases

V. Conclusion

A modeling approach using a MILP is adopted in this work to represent the variety of configurations in SDGs in terms of conductor types. Thus, underground and overhead lines are well captured by proposed constraints, while radiality of the distribution system is guaranteed through a novel formulation. The model is applied to solve the remote reconfiguration problem that seeks a combination of switch states to maximize supplied load with minimal costs. Simulation results corroborate the effectiveness of the proposed approach. A single deterministic failure scenario is evaluated, based on the assumption of accurate damage assessment. This assumption need to be investigated in coming work by considering a damage impact model or a stochastic approach for damage scenario generation. Other future extensions feature the study of restoration operations after the first remote response. In such case, better performance of underground networks can be decreased due to associated long repair times.

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