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Corrections and Comments on "An Efficient Algorithm for Maneuvering Target Tracking"

This article contributes comments and corrections to the technical content of "An Efficient Algorithm for Maneuvering Target Tracking," by Arman Kheirati Roonizi [1]. In the "The Singer Acceleration Model" section of [1], Roonizi wrote:

> It is worth noting that as the maneuver time constant τ increases, the acceleration becomes a white noise jerk model, and the Singer model reduces to the CV model. In cases where the maneuver time constant decreases, the acceleration becomes white noise, and the Singer model reduces to the CA model.

The correct version has the words *constant velocity (CV)* and *constant acceleration (CA)* interchanged in these sentences [2, Sec. 4(D)]. In addition, although the maneuver time constant is usually denoted with τ (and its reciprocal with α) in the literature, τ had already been used in the article to denote the argument of the autocorrelation function of the target acceleration. So, in the preceding sentence, τ should either be omitted or replaced with $1/\alpha$. The correct version can be written as:

> It is worth noting that as the maneuver time constant ($1/\alpha$) increases, the acceleration becomes a white noise jerk model, and the Singer model reduces to the CA model. In cases where the maneuver time constant decreases, the acceleration becomes white noise, and the Singer model reduces to the CV model.

In the same section, the author wrote, "The target acceleration is described by the following linear time-invariant model:

$$a_{k+1} = -\beta a_k + w_k,$$

where w_k is zero-mean white Gaussian noise with variance $\sigma^2(1-\beta^2), \beta = e^{-\alpha T_s}$ and T_s is the sampling period." Under this parametrization, the Singer acceleration model is correctly written without the minus sign, as follows [2, Eq. 27]:

$$a_{k+1} = \beta a_k + w_k.$$

The expression $T_s((1-\beta)/\alpha^2)$, which appears in the third row of (6a), in the definition of ζ_3 following (7), and at the (first row, third column) position of the matrix given inside the solid rectangle on [1, p. 124], needs to be corrected as $T_s((1-\beta)/\alpha)$. A coefficient of two appears erroneously at the beginning of [1, Eq. (6a)], and a parenthesis is missing at the end of [1, Eq. (6a)]. For clarity, the correct form of (6a) is

$$\begin{aligned}(z-1)^2(z-\beta)x_{1,k} &= \Big(m_1(z^2-(\beta+1)z+\beta) \\ &\quad + m_2 T_s(z-\beta) + m_3\Big(T_s\frac{1-\beta}{\alpha} \\ &\quad + (z-1)\frac{\alpha T_s - 1 + \beta}{\alpha^2}\Big)\Big)w_k,\end{aligned}$$

the correct form of ζ_3 is

$$\begin{aligned}\zeta_3 &= m_1\beta - m_2 T_s \beta \\ &\quad + m_3\Big(T_s\frac{1-\beta}{\alpha} - \frac{\alpha T_s - 1 + \beta}{\alpha^2}\Big),\end{aligned}$$

and the correct form of the matrix equation in the box at the bottom of [1, p. 124] is given in (1).

In [1, Eq. 9], the author wrote, "The optimal solution is [7]

$$\hat{x}_1 = (\Gamma^T\Gamma + \lambda\Upsilon^T\Upsilon)^{-1}\Gamma^T\Gamma y."$$

The matrix sum $\Gamma^T\Gamma + \lambda\Upsilon^T\Upsilon$ that appears in the preceding may not be invertible.

$$(z-1)^2(z-\beta)\begin{bmatrix}x_{1,k}\\ x_{2,k}\\ x_{3,k}\end{bmatrix} = \begin{bmatrix}(z-1)(z-\beta) & T_s(z-\beta) & T_s\dfrac{1-\beta}{\alpha} + (z-1)\dfrac{\alpha T_s - 1 + \beta}{\alpha^2} \\ 0 & (z-1)(z-\beta) & (z-1)\dfrac{1-\beta}{\alpha} \\ 0 & 0 & (z-1)^2\end{bmatrix}\begin{bmatrix}m_1\\ m_2\\ m_2\end{bmatrix}w_k. \quad (1)$$

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As seen from their definitions preceding [1, Eq. 9], Γ and Υ are rectangular matrices with sizes $(N-2) \times N$ and $(N-3) \times N$, respectively. Therefore, $\Gamma^T\Gamma$ can be of rank $(N-2)$ at most. Likewise, $\Upsilon^T\Upsilon$ can be of rank $(N-3)$ at most.

At the beginning of the "A Causal Filter" section in [1], the author wrote, "The matrix $M = (\Gamma^T\Gamma + \lambda\Upsilon^T\Upsilon)^{-1}\Gamma^T\Gamma$ is a symmetric positive definite real matrix." As mentioned, Γ is an $(N-2) \times N$ rectangular matrix with at least a 2D null space. This means there exists $y \neq 0$ such that $y^T My = 0$. Hence, M is not positive definite. In addition, for a sufficiently small sampling time T_s, the author argues that $\Gamma^T\Gamma$ can be replaced by the identity matrix (see [1, Eq. 10]). However, a rigorous proof of this statement is not provided. As noted, $\Gamma^T\Gamma$ can be of rank $(N-2)$ at most. Hence, the claim about its approximation with the full-rank $N \times N$ identity matrix needs to be substantiated.

The claims following [1, Eq. 9] are based on the assumption that the sampling time T_s is sufficiently small, characterized by the author with the condition $T_s \ll 1$. However, it is customary in the literature to compare the sampling time T_s with the maneuver time constant $1/\alpha$ instead of simply letting T_s go to arbitrarily small values. For example, in [3, Eqs. 2.9-9, 2.9-11, and 2.9-12], the state transition matrix and maneuver excitation covariance matrix are explicitly specified for $\alpha T_s \ll 1/2$ and $\alpha T_s \gg 1$. Setting T_s too small leads to a trivial constant acceleration model according to a Newtonian matrix with vanishing noise. Finally, the trick employed in the article to convert the problem into an independent difference model is based on the z transform, which is a standard technique in the analysis of discrete-time linear dynamical systems. This connection is not explicitly mentioned in the article.

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References

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