Degrees-of-Freedom of the MIMO Three-Way Channel with Node-Intermittency

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Abstract—The characterization of fundamental performance bounds of many-to-many communication systems in which participating nodes are active in an intermittent way is one of the major challenges in communication theory. In order to address this issue, we introduce the multiple-input multiple-output (MIMO) three-way channel (3WC) with an intermittent node and study its degrees-of-freedom (DoF) region and sum-DoF. We devise a non-adaptive encoding scheme based on zero-forcing, interference alignment and erasure coding, and show its DoF region (and thus sum-DoF) optimality for non-intermittent 3WCs and its sum-DoF optimality for (node-)intermittent 3WCs. However, we show by example that in general some DoF tuples in the intermittent 3WC can only be achieved by adaptive schemes, such as decodeforward relaying. This shows that non-adaptive encoding is sufficient for the non-intermittent 3WC and for the sum-DoF of intermittent 3WCs, but adaptive encoding is necessary for the DoF region of intermittent 3WCs. Our work contributes to a better understanding of the fundamental limits of multi-way communication systems with intermittency and the impact of adaptation therein.

Index Terms—Degrees-of-freedom, three-way channel, MIMO, intermittent connectivity, interference alignment, zero forcing, relay networks, interference channel.

I. INTRODUCTION

I N multi-way communication scenarios multiple nodes communicate with each other, each acting as a source, a destination, and possibly a relay at the same time. This mode of communication is especially important for future systems

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employing full-duplex and device-to-device (D2D) communication, [2]–[4]. It is an important technique for efficient resource utilization that is expected to gain more prominence in future communication systems, especially with the rise of mesh networks, e.g., in industrial and vehicular networks.

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Several multi-way communication scenarios have been studied in the literature, [5]. It started with the two-way channel which was first studied by Shannon in [6] and subsequently in [7]–[10] for instance. Then, the scope was extended to two-way networks where two groups of nodes communicate with each other in a two-way fashion [11]–[13], and two-way relay networks where two nodes communicate with each other in a two-way fashion via a relay node [14]–[17]. This line of research has been further extended to multi-way networks where multiple nodes communicate with each other in a multiway fashion, each node being a source and a destination at the same time [18]–[22], and to multi-way relay networks where the same communication as in multi-way networks takes place via a relay node [23]–[34].

A. The Three-Way Channel with Intermittency

In this work, we focus on the multiple-input multiple-output (MIMO) three-way channel (3WC) which can be described as follows: Consider a system consisting of three terminals communicating with each other in a multi-way fashion, e.g., two D2D user terminals and a base station (BS) where the D2D users communicate with each other while exchanging signals with the BS (control signals or data). This 3WC is an extension of Shannon's two-way channel [6] and has been studied in [20]–[22].

Therein, it is assumed that the three nodes are connected all the time (Fig. 1a). This forms a *non-intermittent* 3WC, which, although subsumed by the work in this paper, is not its main focus. The main focus of this paper is the intermittent 3WC instead. There are several reasons which motivate studying the intermittent 3WC channel, some of which are discussed next.

One motivation stems from practice, where connectivity can be intermittent. For instance, a pair of D2D users, commonly chosen to be nearby users [3], [4], might be both disconnected from the BS due to shadowing (Fig. 1b). Future generations of mobile communication systems (e.g., using mmWaves, where line-of-sight (LOS) propagation will become dominant) are expected to be heavily susceptible to this type of shadowing. In another scenario, the D2D users might operate in an underlay mode over a resource block used by the BS to

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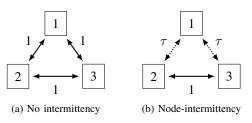


Fig. 1. MIMO 3WC with no intermittency such that all nodes are always connected (a) and node-intermittency where node 1 is available only τ fraction of the time (b)

communicate to cellular users (CUs). The BS connects to the D2D users whenever it does not communicate with a CU, and disconnects from the D2D users otherwise (see Fig. 2). Note that D2D users are chosen so that they cause/receive negligible interference (relative to the desired signals) to/from other nodes using the same resource block [35], [36]. In both cases the two links from the D2D users to the BS are jointly intermittent, i.e., both are available or blocked at the same time. We call this *node intermittency* and call the BS an *intermittent node* (from the D2D users' perspective).

Another motivation stems from theory. A channel with intermittency is a special 'extreme' case of a channel with state. Point-to-point channels, multiple-access channels (MAC), and broadcast channels (BC) with state have been studied in the past, see [37, ch. 7] and references therein. The impact of intermittency on these channels can be studied based on these results. There are two approaches to extend results on channels with intermittency. The first consists of considering larger networks, such as the X channel [38]. The other is to consider networks with bidirectional links (feedback) such as [39]-[41]. We take the second approach in this paper by focusing on multi-way communication. In such networks, feedback links enable relaying and hence provide additional paths for information flow that might be interrupted by intermittency. This gives intermittency a more 'global' impact since it affects all nodes indirectly by interrupting useful paths through nodes that act as relays. Thus it is important to study networks with feedback and intermittency. The smallest multi-way network (in terms of number of nodes) that one could study in this context is the two-way channel (TWC) [6]. However, analyzing the impact of intermittency in the Gaussian TWC is straightforward as we shall see in Section II-A. Therefore, the 3WC is the smallest viable example of a multi-way network with non-trivial behavior under intermittency. Moreover, the 3WC subsumes other channels of interest such as the twoway MAC and BC [11], and MAC and BC with cooperation [42], [43].

B. Degrees-of-Freedom and Intermittency

In many previously mentioned works, the focus is generally on the capacity of the studied network. Since finding the capacity is elusive in most cases, some works (and also this work) focus on the degrees-of-freedom (DoF) which provide a good capacity approximation at high signal-to-noise ratio (SNR) [44], thereby highlighting the interaction between the signals of the different nodes while diminishing the impact

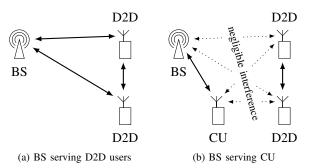


Fig. 2. A D2D pair sharing the same resources with a cellular user (CU), where the BS communicates with the D2D pair part of the time (a) and with the CU the rest of the time (b). The D2D pair is far enough from both the BS and the CU and thus causes/receives negligible interference (dotted).

of noise. This is of interest since state-of-the-art wireless communication systems operate in a regime where they are essentially interference-limited rather than noise-limited.

The concept of intermittency as a form of channel impairment also fits well with the philosophy of DoFs: With increasing SNR, signal components 'harden' in the sense that some allow (almost) noise- and interference-free communication at a rate that scales with SNR on a logarithmic scale, while others are hopelessly burried in uncancelable interference and are therefore useless. This effect is exactly what the DoF perspective captures as it essentially counts available Shannon-Hartley communication 'units'. Intermittency is the channel impairment that goes well together with this 'all or nothing' perspective, capturing the notion that some signal dimensions might become useless due to stochastic processes in the channel, such as shadowing of dominant LOS components.

C. Scope of this Work

Here, we study the impact of node-intermittency on the DoF region and sum-DoF, i.e., the capacity scaling versus SNR in a dB scale, of a full-duplex MIMO 3WC. In this MIMO network, each node generally has two independent messages, each intended for one of the remaining two nodes. We pay particular attention to the necessity (or the lack thereof) of *adaptive encoding* for reaching DoF region/sum-DoF. Adaptive encoding enables cooperative communication schemes by allowing the transmit signal of a node to depend on its previously received signals, which can be interpreted as a form of feedback. In contrast, with *non-adaptive encoding* transmit signals depend only on the messages to be sent, and cooperation (e.g., in the form of relaying) is excluded. The issue of (non-)adaptive encoding has been studied for other networks earlier in [9]–[11], [45] for instance.

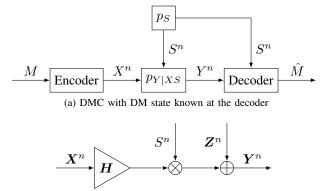
We assume that nodes have strictly causal knowledge of the intermittency state of adjacent links. This can be obtained by estimating the connectivity from the receive signals (e.g., signal strength). Note that in some cases the intermittency state can be known without the need for estimation, occasionally even ahead of time, e.g., in the scenario of D2D/CU/BS users (Fig. 2), where the BS knows its scheduling of users in advance and can anticipate when it will not be able to receive the D2D user signals. The reader might ask how our findings relate to previous results on channels with state and whether the intermittent 3WC is just a channel with state to which known techniques [37, ch. 7] can be applied. In fact, we make use of the known results for point-to-point channels with random state known to the receiver [37, eq. (7.2)] throughout this paper. However, the *multi-way* intermittent 3WC asks for a holistic analysis of its fundamental limits and cannot be exhaustively studied as a mere collection of independent *one-way* one-hop multi-user channels with state, e.g., MACs with state [37, p. 175].

D. Outline and Overview of Results

After introducing the details of the system model in Section II-B and highlighting the main results in Section III, we examine the (node-)intermittent 3WC in Section IV. We devise a non-adaptive encoding scheme based on zero-forcing (ZF), interference alignment (IA) and erasure coding (EC) and derive its achievable DoF region and sum-DoF. We present the genie-aided converse techniques used to derive DoF region and sum-DoF upper bounds, both under nonadaptive and adaptive encoding. We conclude that for the intermittent 3WC the presented non-adaptive scheme is sum-DoF optimal, so adaptation is not necessary to achieve sum-DoF. Then, we provide examples of adaptive relaying schemes that can achieve a DoF region point that no non-adaptive scheme can achieve. This shows that adaptive schemes can achieve strictly larger DoF regions, and therefore adaptation is required to achieve the DoF region of intermittent 3WCs. To complete the picture, we examine the non-intermittent 3WC as special case of intermittent 3WCs in Section V. We show that for the non-intermittent 3WC the presented scheme is DoF region optimal (and thus also sum-DoF optimal) and therefore adaptive encoding is not required. This reveals an interesting interplay between intermittency and adaptation. In Section VI we provide conclusive remarks and directions for future research.

E. Notation

Throughout the paper, we use $x_i^n \triangleq (x_{i,1}, \ldots, x_{i,n})$ for some index i, and $x_{i,\ell}^n \triangleq (x_{i,\ell}, \ldots, x_{i,n})$. We use regular letters to denote scalar-valued quantities, boldface letters to denote vector- und matrix-valued quantities; lowercase letters for scalar and vector values (e.g., realizations of random variables), uppercase letters for matrix values and for random variables. The $N \times N$ identity matrix is denoted I_N , the $N \times N$ all-zero matrix is $\mathbf{0}_N$. We write $\mathbf{X} \sim \mathcal{CN}(\mathbf{0}, \mathbf{Q})$ to indicate that X is a multivariate complex Gaussian random variable with zero mean and covariance matrix Q, and $S \sim \text{Bern}(\tau)$ to indicate that S is Bernoulli distributed with $\Pr[S=1] = \tau$ and $\Pr[S=0] = 1 - \tau =: \overline{\tau}$. We write x^+ to denote $\max\{0, x\}$ for some $x \in \mathbb{R}$, and H^{\dagger} , H^{T} , H^{H} , and span(H) to denote the Moore-Penrose pseudo-inverse, the transpose, the Hermitian transpose, and the subspace spanned by the columns of the matrix **H**. By $\log(x)$ we denote the logarithm of x to base 2, by $i \rightarrow j$ the communication from node i to node j one-way, and by $i \leftrightarrow j$ both $i \rightarrow j$ and $j \rightarrow i$. By $p_X(x)$ we denote the probability density function (PDF) of random variable X.



(b) Intermittent Gaussian MIMO channel

Fig. 3. The capacity of the DMC with state known at the decoder (a) is wellknown, and used to derive the capacity of the intermittent Gaussian MIMO channel (b)

II. PREREQUISITES AND SYSTEM MODEL

In this section, we briefly recite the basics of channels with state, a fundamental building block of multi-way communication scenarios with intermittency. We then present the system model of the intermittent 3WC.

A. Channels with State and Intermittency

Recall the definition of a discrete memoryless channel (DMC) with discrete memoryless (DM) state known at the decoder (Fig. 3a): Its state sequence S^n is independent and identically distributed (i.i.d.) $S_{\ell} \sim p_S$ and independent of the input X^n , where n is the number of channel uses. Since both output Y^n and state sequence S^n are known at the decoder, it can be viewed as a DMC with channel law

$$p_{Y^n S^n | X^n}(y^n, s^n | x^n) = \prod_{\ell=1}^n p_{Y | XS}(y_\ell | x_\ell, s_\ell) p_S(s_\ell).$$

The capacity of this channel is [37, eq. (7.2)]

$$C = \max_{p_X} I(X; YS) = \max_{p_X} I(X; Y \mid S).$$

We use this result to derive the capacity of the intermittent Gaussian MIMO channel (Fig. 3b) which will be useful in the sequel. Here, the input X^n is a sequence of vectors x_ℓ of length M with average power constraint

$$\sum_{\ell=1}^{n} \mathbb{E}[\|\boldsymbol{X}_{\ell}\|_{2}^{2}] \leq nP.$$

 $S_\ell \sim \operatorname{Bern}(\tau)$ models the intermittency. The output \boldsymbol{Y}^n of dimension N is given as

$$oldsymbol{y}_\ell = s_\ell oldsymbol{H} oldsymbol{x}_\ell + oldsymbol{z}_\ell, \quad orall \ell,$$

where $\boldsymbol{H} \in \mathbb{C}^{N \times M}$ is the channel matrix, and \boldsymbol{Z}^n is a noise sequence, i.i.d., with $\boldsymbol{Z}_{\ell} \sim \mathcal{CN}(\boldsymbol{0}, \sigma^2 \boldsymbol{I}_N)$.

Lemma 1. The capacity of the intermittent Gaussian MIMO point-to-point (P2P) channel is

$$C_{P2P} = I(\boldsymbol{X}; \boldsymbol{Y} \mid S)$$

= $\tau I(\boldsymbol{X}; \boldsymbol{Y} \mid S = 1) + \overline{\tau} I(\boldsymbol{X}; \boldsymbol{Y} \mid S = 0)$

$$= \tau \log \det \left(\boldsymbol{I}_N + \frac{P}{\sigma^2} \boldsymbol{H} \boldsymbol{H}^{\mathsf{H}} \right).$$

Now we turn to the intermittent Gaussian MIMO TWC, where two nodes communicate full-duplex over a bidirectional intermittent Gaussian MIMO channel and the intermittency state is known strictly causally at the encoders and decoders, i.e., only intermittency states $s^{\ell-1}$ can be used in encoding. For simplicity, let the channel be reciprocal, such that the channel matrices for the two directions are H and H^{H} , respectively. As the outputs (Y_1^n, S^n) and (Y_2^n, S^n) at nodes 1 and 2, with

$$egin{aligned} oldsymbol{y}_{1,\ell} &= s_\ell oldsymbol{H}^{\mathsf{H}} oldsymbol{x}_{2,\ell} + oldsymbol{z}_{1,\ell}, \quad oldsymbol{Z}_{1,\ell} &\sim \mathcal{CN}(oldsymbol{0},\sigma^2 oldsymbol{I}_M), \quad orall \ell, \ oldsymbol{y}_{2,\ell} &= s_\ell oldsymbol{H} \quad oldsymbol{x}_{1,\ell} + oldsymbol{z}_{2,\ell}, \quad oldsymbol{Z}_{2,\ell} &\sim \mathcal{CN}(oldsymbol{0},\sigma^2 oldsymbol{I}_N), \quad orall \ell, \ oldsymbol{y}_{2,\ell} &= s_\ell oldsymbol{H} \quad oldsymbol{x}_{1,\ell} + oldsymbol{z}_{2,\ell}, \quad oldsymbol{Z}_{2,\ell} &\sim \mathcal{CN}(oldsymbol{0},\sigma^2 oldsymbol{I}_N), \quad orall \ell, \ oldsymbol{y}_{\ell} &= s_\ell oldsymbol{H} \quad oldsymbol{X}_{1,\ell} + oldsymbol{z}_{2,\ell}, \quad oldsymbol{Z}_{2,\ell} &\sim \mathcal{CN}(oldsymbol{0},\sigma^2 oldsymbol{I}_N), \quad orall \ell, \ oldsymbol{y}_{\ell} &= s_\ell oldsymbol{H} \quad oldsymbol{X}_{1,\ell} + oldsymbol{z}_{2,\ell}, \quad oldsymbol{Z}_{2,\ell} &\sim \mathcal{CN}(oldsymbol{0},\sigma^2 oldsymbol{I}_N), \quad orall \ell, \ oldsymbol{y}_{\ell} &= s_\ell oldsymbol{H} \quad oldsymbol{X}_{1,\ell} + oldsymbol{z}_{2,\ell}, \quad oldsymbol{Z}_{2,\ell} &\sim \mathcal{CN}(oldsymbol{0},\sigma^2 oldsymbol{I}_N), \quad orall \ell, \ oldsymbol{Y}_{\ell} &= s_\ell oldsymbol{H} \quad oldsymbol{X}_{2,\ell} + oldsymbol{Z}_{2,\ell}, \quad oldsymbol{Z}_{2,\ell} &\sim \mathcal{CN}(oldsymbol{0},\sigma^2 oldsymbol{I}_N), \quad orall \ell, \ oldsymbol{Y}_{\ell} &= s_\ell oldsymbol{H} \quad oldsymbol{X}_{2,\ell} + oldsymbol{Z}_{2,\ell}, \quad oldsymbol{Z}_{2,\ell} &\sim \mathcal{CN}(oldsymbol{0},\sigma^2 oldsymbol{I}_N), \quad orall \ell, \ oldsymbol{Y}_{\ell} &= s_\ell oldsymbol{H} \quad oldsymbol{X}_{2,\ell} + oldsymbol{Z}_{2,\ell}, \quad oldsymbol{Z}_{2,\ell} &\sim \mathcal{CN}(oldsymbol{0},\sigma^2 oldsymbol{I}_N), \quad oldsymbol{U}_{\ell} &= s_\ell oldsymbol{U}_{\ell} \quad oldsymbol{U}_{\ell} \quad oldsymbol{U}_{\ell} &= s_\ell oldsymbol{H} \quad oldsymbol{X}_{\ell} \quad oldsymbol{Z}_{\ell} &= s_\ell oldsymbol{U}_{\ell} \quad oldsymbol{U}_$$

and $Z_{1,\ell}$ and $Z_{2,\ell}$ independent, depend only on the inputs X_2^n and X_1^n of the respective other node, the channel law distributes as

$$p_{\mathbf{Y}_{1}^{n}\mathbf{Y}_{2}^{n}S^{n}|\mathbf{X}_{1}^{n}\mathbf{X}_{2}^{n}}(\mathbf{y}_{1}^{n}, \mathbf{y}_{2}^{n}, s^{n}|\mathbf{x}_{1}^{n}\mathbf{x}_{2}^{n}) \\ = \prod_{\ell=1}^{n} p_{\mathbf{Y}_{1}|\mathbf{X}_{2}S}(\mathbf{y}_{1,\ell}|\mathbf{x}_{2,\ell}, s_{\ell}) p_{\mathbf{Y}_{2}|\mathbf{X}_{1}S}(\mathbf{y}_{2,\ell}|\mathbf{x}_{1,\ell}, s_{\ell}) p_{S}(s_{\ell}).$$

One can readily show that Shannon's outer bound [37, p. 447] holds despite the shared state S of $X_2 \rightsquigarrow Y_1$ and $X_1 \rightsquigarrow Y_2$. The bounds are achieved by coding independently in both directions for an intermittent Gaussian MIMO P2P channel. Hence, the capacity region of this TWC is the rectangle

$$\mathcal{C}_{\text{TWC}} = \{ (R_1, R_2) \in \mathbb{R}^2_+ \mid R_1 \le C_{\text{P2P}}, R_2 \le C_{\text{P2P}} \},\$$

and the sum-capacity of the TWC is twice the capacity of the P2P channel,

$$C_{\rm TWC} = 2C_{\rm P2P}.$$

We presented the capacity of the intermittent Gaussian MIMO P2P channel and TWC, which follow from established results for channels with state. In particular, intermittency affects these channels in that it linearly scales the capacity of the non-intermittent channel with the fraction τ of time in which the channel is non-intermittent. While the impact of intermittency is straightforward in the TWC, we shall see that this is not the case in larger multi-way communication channels. The smallest (in terms of number of nodes) scenario larger than the TWC where this can be demonstrated is the 3WC. Hence, the impact of intermittency on the 3WC is studied in this paper. In the following, we introduce the system model of the intermittent 3WC.

B. The MIMO Three-Way Channel with Node-Intermittency

Throughout this section we assume $i, j, k \in \{1, 2, 3\}$ and mutually distinct, and n is the number of channel accesses. The MIMO 3WC with node-intermittency is comprised of three terminals 1, 2 and 3 communicating with each other in full-duplex mode over a shared medium. Each node i has two messages w_{ij} and w_{ik} ($w_i \triangleq (w_{ij}, w_{ik})$) to be delivered to the remaining nodes j and k, and two messages \hat{w}_{ji} and \hat{w}_{ki} to be decoded from the received signals (Fig. 4). Each message w_{ij} is a realization of the random variable W_{ij} . All random variables W_{ij} are independent. In the (node-)intermittent 3WC one link is always available and two links are jointly intermittent with probability of being available τ (Fig. 1b and 4).

Our objects under investigation (sum-DoF and DoF region of the intermittent 3WC – both are introduced in the sequel) depend on how the numbers of antennas at each node relate to each other, i.e., whether the intermittent node has most, second most, or least antennas. To reduce the number of cases one has to analyze, yet study the system without loss of generality (w.l.o.g.), two symmetries come to mind that can be exploited: Either a) fix a certain node to be intermittent (e.g., node 1 is intermittent), and investigate all possible relations among the numbers of antennas, or b) fix a relation between the numbers of antennas, and allow any one of the three nodes to be intermittent. The respective remaining cases follow by renaming. Preliminary work [1] fixed node 1 to be intermittent and further assumed a relation on the numbers of antennas, which is not w.l.o.g. We fix node 1 to be intermittent, but allow for any combination of numbers of antennas, which is approach a) and w.l.o.g.

Node *i* is equipped with M_i antennas that are used for reception and transmission simultaneously. We assume channel accesses of the nodes are synchronized and time-discretized: At time instance ℓ the transmit signal $\boldsymbol{x}_{i,\ell} \in \mathbb{C}^{M_i}$ is a realization of a random vector $\boldsymbol{X}_{i,\ell}$ satisfying the power constraint

$$\sum_{\ell=1}^{n} \mathbb{E}[\|\boldsymbol{X}_{i,\ell}\|_{2}^{2}] \leq nP.$$

The receive signals $y_{i,\ell} \in \mathbb{C}^{M_i}$ are

$$\begin{array}{ll} { { \boldsymbol{y}}_{1,\ell} = & s_\ell { \boldsymbol{H}}_{21} { \boldsymbol{x}}_{2,\ell} + s_\ell { \boldsymbol{H}}_{31} { \boldsymbol{x}}_{3,\ell} + { \boldsymbol{z}}_{1,\ell}, \\ { \boldsymbol{y}}_{2,\ell} = s_\ell { \boldsymbol{H}}_{12} { \boldsymbol{x}}_{1,\ell} & + & { \boldsymbol{H}}_{32} { \boldsymbol{x}}_{3,\ell} + { \boldsymbol{z}}_{2,\ell}, \\ { \boldsymbol{y}}_{3,\ell} = s_\ell { \boldsymbol{H}}_{13} { \boldsymbol{x}}_{1,\ell} + & { \boldsymbol{H}}_{23} { \boldsymbol{x}}_{2,\ell} & + { \boldsymbol{z}}_{3,\ell}. \end{array}$$

 $H_{ij} \in \mathbb{C}^{M_j \times M_i}$ represents the channel matrix from node i to node j and is constant over time and known to all nodes in advance. The elements of these matrices are drawn independently from the same continuous distribution, such that rank $(\boldsymbol{H}_{ij}) = \min(M_i, M_i)$ almost surely. $\boldsymbol{z}_{i,\ell} \in \mathbb{C}^{M_i}$ is a realization of the noise process $m{Z}_{i,\ell} \sim \mathcal{CN}(m{0},\sigma^2m{I}_{M_i})$ independent and identically distributed (i.i.d.) with respect to (w.r.t.) ℓ , and $s_{\ell} \in \{0, 1\}$ is a realization of the intermittency state process S_{ℓ} . $S_{\ell} \sim \text{Bern}(\tau)$ is assumed to be i.i.d. w.r.t. ℓ . The state sequence s^n is known strictly causally at all nodes (i.e., in time instance ℓ all nodes know $s^{\ell-1}$), because every node can correctly estimate s_{ℓ} from its $y_{i,\ell}$ with very high probability, e.g., based on the received signal strength. Due to the physical properties of the channel and the distinct receivers, all random variables H_{ii} , Z_i^n and S^n are assumed to be mutually independent. We remark that our analysis continues to hold if the noise at different receivers is correlated. We denote $\rho \triangleq \frac{P}{\sigma^2}$ and call it SNR (signal-tonoise-power-ratio) throughout the paper.

The messages W_{ij} are each uniformly distributed over $W_{ij} = \{1, \ldots, |W_{ij}(\rho)|\}$. Using an encoding function $\mathcal{E}_{i,\ell}$ node *i* constructs $\boldsymbol{x}_{i,\ell}$ either from (w_{ij}, w_{ik}) (non-adaptive encoding) or from $(w_{ij}, w_{ik}, \boldsymbol{y}_i^{\ell-1}, s^{\ell-1})$ (adaptive encoding). After *n* transmissions (where *n* is the code length), node *i*

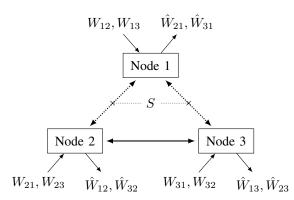


Fig. 4. Every node *i* in the MIMO 3WC with node-intermittency sends messages w_{ij} and w_{ik} to nodes *j* and *k*, respectively (with *i*, *j*, *k* \in {1, 2, 3} mutually distinct). W.l.o.g. node 1 is assumed to be intermittent.

decodes its desired messages using a decoding function \mathcal{F}_i to obtain $(\hat{w}_{ji}, \hat{w}_{ki}) = \mathcal{F}_i(s^n, y_i^n, w_{ij}, w_{ik})$. Transmission is considered successful if all messages are recovered successfully $(w_{ij} = \hat{w}_{ij})$, otherwise an error is reported. The average over all messages of the error probability is denoted by $P_{e,n}$. A rate tuple

$$\boldsymbol{R}(\rho) \triangleq \Big(R_{12}(\rho), R_{13}(\rho), R_{21}(\rho),$$

$$R_{23}(
ho), R_{31}(
ho), R_{32}(
ho) \Big) \in \mathbb{R}^6_+$$

with $R_{ij}(\rho) = \frac{\log(|\mathcal{W}_{ij}(\rho)|)}{n}$ is said to be achievable if there exists a sequence of encoder-decoder pairs for increasing code length n, where $P_{e,n} \to 0$ as $n \to \infty$. The capacity region $\mathcal{C}(\rho)$ is the set of all achievable rate tuples.

The DoF region \mathcal{D} is the set of achievable DoF tuples

$$d \triangleq (d_{12}, d_{13}, d_{21}, d_{23}, d_{31}, d_{32}) \in \mathbb{R}^{0}_{+}$$

defined as in [46], i.e.,

$$\mathcal{D} \triangleq \left\{ (d_{12}, ..., d_{32}) \in \mathbb{R}^6_+ \mid \forall (\beta_{12}, ..., \beta_{32}) \in \mathbb{R}^6_+ : \\ \sum_{i,j} \beta_{ij} d_{ij} \leq \limsup_{\rho \to \infty} \sup_{\mathbf{R}(\rho) \in \mathcal{C}(\rho)} \sum_{i,j} \beta_{ij} \frac{R_{ij}(\rho)}{\log(\rho)} \right\}.$$

Roughly speaking, if a rate tuple $\mathbf{R}(\rho)$ as a function of ρ is achievable, i.e., $\mathbf{R}(\rho) \in \mathcal{C}(\rho)$ for all $\rho > 0$, then the DoF tuple d with $d_{ij} = \limsup_{\rho \to \infty} \frac{R_{ij}(\rho)}{\log(\rho)}$ is achievable. We denote DoF regions with \mathcal{D} and use suitable subscripts when further restricting assumptions apply (e.g., under non-adaptive encoding). We define the corresponding sum-DoF as $d_{\text{sum}} = \max_{d \in \mathcal{D}} \sum_{i,j \in \{1,2,3\}, i \neq j} d_{ij}$. The DoF perspective only captures rate contributions that are non-vanishing relative to $\log(\rho)$ as we let $\rho \to \infty$. It neglects vanishing rate portions $f(\rho)$ that grow sublinear in $\log(\rho)$ and are therefore $o [\log(\rho)]$, i.e., where $\lim_{\rho \to \infty} \frac{f(\rho)}{\log(\rho)} = 0$.

III. MAIN RESULTS

In this section, we summarize and discuss the main results of this paper, listed in Table I. We denote the DoF region of the intermittent 3WC under adaptive encoding by D^{I} , the DoF region of the intermittent 3WC under non-adaptive encoding by $\mathcal{D}_{\overline{A}}^{I}$, and the sum-DoF by d_{sum}^{I} . Obviously, $\mathcal{D}_{\overline{A}}^{I} \subseteq \mathcal{D}^{I}$. We denote the DoF region of the non-intermittent 3WC by \mathcal{D}^{N} and the sum-DoF by d_{sum}^{N} . In Section IV-A we devise a nonadaptive encoding scheme whose achievable DoF region (DoF region inner bound) we denote by $\mathcal{D}_{IB,\overline{A}}^{I}$. We start with this achievable DoF region given in the following theorem.

Theorem 1 (DoF Region Inner Bound for Node-Intermittent 3WC). All DoF tuples $d \in D^{I}_{IB,\overline{A}}$ satisfying the following set of inequalities are achievable in the node-intermittent 3WC using non-adaptive encoding:

$$\max\{d_{12} + d_{13}, d_{21} + d_{31}\} \le \tau M_1$$
$$\max\{d_{21} + \tau d_{23}, d_{12} + \tau d_{32}\} \le \tau M_2$$
$$\max\{d_{31} + \tau d_{32}, d_{13} + \tau d_{23}\} \le \tau M_3$$

 $\max\{d_{12} + d_{13} + \tau d_{23}, d_{21} + d_{31} + \tau d_{32}\} \le \tau \max\{M_1, M_3\} \\ \max\{d_{12} + d_{13} + \tau d_{32}, d_{21} + d_{31} + \tau d_{23}\} \le \tau \max\{M_1, M_2\} \\ \max\{d_{12} + d_{31} + \tau d_{32}, d_{21} + d_{13} + \tau d_{23}\} \le \tau \max\{M_2, M_3\} \\ \min\{d_{12}, d_{13}, d_{21}, d_{23}, d_{31}, d_{32}\} \ge 0$

Therefore, $\mathcal{D}^{I}_{IB,\overline{A}}$ constitutes an inner bound on the DoF region of the node-intermittent 3WC, such that

$$\mathcal{D}^{\mathrm{I}}_{\mathrm{IB},\overline{\mathrm{A}}} \subseteq \mathcal{D}^{\mathrm{I}}_{\overline{\mathrm{A}}} \subseteq \mathcal{D}^{\mathrm{I}}$$

Proof. This theorem follows by the ZF/IA/EC-based construction in Section IV-A. \Box

Subsequently we investigate whether this non-adaptive scheme is optimal. For the intermittent 3WC it turns out to be sum-DoF optimal, establishing the sum-DoF of the intermittent 3WC and the fact that adaptive encoding is not necessary to achieve it.

Theorem 2 (Sum-DoF of Node-Intermittent 3WC). A nonadaptive encoding scheme achieves the sum-DoF of the nodeintermittent 3WC given by

$$d_{\text{sum}}^{\text{I}} = 2\overline{\tau}\min\{M_2, M_3\} + 2\tau \Big(M_1 + M_2 + M_3 \\ -\min\{M_1, M_2, M_3\} - \max\{M_1, M_2, M_3\}\Big).$$

Proof. The theorem follows from achievability and converse results developed in Sections IV-B1 and IV-B2 (Lemmas 2 and 3).

Theorem 2 is based on an instrumental genie-aided upper bound, which we prove to be tighter than cut-set bounds in Section IV-C. Hence, cut-set bounds alone can not describe the DoF of this network comprehensively.

The sum-DoF optimality of non-adaptive schemes, i.e., of schemes that dispense with relaying, agrees with the following intuition: Assume relaying was used for (any part of) any message, say w_{ik} was relayed via node j. Then this message occupies communication resources on two links, $i \rightarrow j$ and $j \rightarrow k$, introducing redundancy and thus waste of resources. Since the sum-DoF criterion allows to trade DoFs among messages, one could instead use the resources used by the one relayed message w_{ik} to increase the DoFs of the two nonrelayed messages w_{ij} and w_{ik} . This is possible since every

TABLE I Overview of Main Results

Channel	Criterion	Necessity of Adaptation	Coding Scheme
Node-intermittent 3WC	Sum-DoF	Non-adaptive encoding suffices	ZF/IA/EC-based
		(Theorem 2)	(Section IV-A, Theorems 1 and 2)
	DoF region	Adaptive encoding required	Counterexample based on decode-forward relaying
		(Theorem 3)	(Section IV-D2)
Non-intermittent 3WC	Sum-DoF	Non-adaptive encoding suffices	ZF/IA-based
		(Theorem 4, Corollary 1)	(Sections IV-A, V-A and V-C)
	DoF region	Non-adaptive encoding suffices	ZF/IA-based
	-	(Theorem 4)	(Sections IV-A and V-A)

node has messages for the two other nodes in the 3WC. Such a reassignment could improve resource utilization and thus sum-DoF, rendering relaying dispensable.

From a DoF region perspective however it turns out that non-adaptive schemes cannot be optimal, and therefore the presented scheme is not DoF region optimal in the intermittent 3WC.

Theorem 3 (Necessity of Adaptive Encoding for DoF Region of Node-Intermittent 3WC). *Adaptive encoding is required to achieve the DoF region of the node-intermittent 3WC, i.e.,*

$$\mathcal{D}^{\mathbf{I}} \setminus \mathcal{D}^{\mathbf{I}}_{\overline{\mathbf{A}}} \neq \emptyset.$$

Proof. The theorem follows from an upper bound on d_{31} under non-adaptive encoding, presented in Section IV-D1, and counterexamples of adaptive schemes exceeding this bound, devised in Section IV-D2.

Theorems 2 and 3 show that non-adaptive encoding is sufficient to achieve sum-DoF, but not sufficient to achieve the DoF region of the intermittent 3WC. Unlike the sum-DoF criterion, the DoF region criterion does not allow to trade DoFs among messages. Instead, e.g., 'extreme' DoF tuples need to be achievable as well, that make every effort (e.g., through relaying) to maximize a single message's DoF, usually at the cost of low sum-DoF. The result is particularly interesting in light of the fact, that non-adaptive encoding is sufficient to achieve the DoF region of the non-intermittent 3WC, which the presented non-adaptive scheme does.

Theorem 4 (DoF Region of Non-Intermittent 3WC). *The DoF* region of the non-intermittent 3WC \mathcal{D}^{N} (with $M_1 \ge M_2 \ge M_3$ w.l.o.g.) is given by

$$\max\{d_{12} + d_{13} + d_{23}, d_{12} + d_{13} + d_{32}\} \le M_1 \\ \max\{d_{21} + d_{31} + d_{32}, d_{21} + d_{31} + d_{23}\} \le M_1 \\ \max\{d_{21} + d_{13} + d_{23}, d_{12} + d_{31} + d_{32}\} \le M_2 \\ \max\{d_{31} + d_{32}, d_{13} + d_{23}\} \le M_3 \\ \min\{d_{12}, d_{13}, d_{21}, d_{23}, d_{31}, d_{32}\} \ge 0$$

and achievable using a non-adaptive encoding scheme.

Proof. The theorem follows from the ZF/IA-based achievability results in Section V-A ((55) to (63)) and the converse results in Section V-B ((72) to (79)). \Box

This complements earlier work [18] that characterized the sum-DoF of the non-intermittent 3WC and showed its achievability using non-adaptive encoding. $M_1 = 10, M_2 = 7$

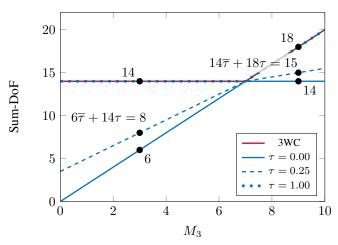


Fig. 5. Sum-DoF of 3WC (red) and intermittent 3WC (blue) with varying τ (black marks show the case $\tau = 0.25$ as convex combination of the extreme cases $\tau = 0$ and $\tau = 1$)

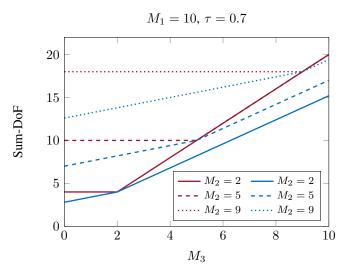


Fig. 6. Sum-DoF of 3WC (red) and intermittent 3WC (blue) for varying M_2

Our results show that intermittency does not decrease sum-DoF as long as node 1 has the smallest number of antennas. Otherwise, intermittency does affect sum-DoF, which is affinelinearly increasing in τ . Fig. 5 and 6 plot sum-DoF of nonintermittent and node-intermittent 3WC against M_3 , for different values of τ and M_2 , respectively, in a scenario where node 1 has the largest number of antennas. The resulting graphs are piecewise linear with a change in slope at $M_3 = M_2$. For $M_3 < M_2$ the sum-DoF of the 3WC is a constant depending only on M_2 , for $M_3 > M_2$ it is linear in M_3 . The slope of the sum-DoF of the intermittent 3WC is proportional to τ for $M_3 < M_2$ and proportional to $\overline{\tau}$ for $M_3 > M_2$.

The sum-DoF of the intermittent 3WC with $0 < \tau < 1$ is a convex time-sharing combination of the two extreme cases $\tau = 0$ and $\tau = 1$ (Fig. 5). Note, that such a time-sharing combination is the best any non-adaptive coding scheme can achieve, when each node knows the intermittency state ahead of time and codes optimally for the respective state in each time instance ℓ . The achievability scheme presented in this work does not use intermittency state information at the encoder, yet achieves the same sum-DoF, averaging out intermittency state through erasure coding. This shows that intermittency state information is dispensable at the transmitter for the non-adaptive scheme in this case. Furthermore, note that the sum-DoF of the 3WC is larger than the sum-DoF of the intermittent 3WC (Fig. 6).

Intermittency impacts the DoF region of the 3WC in a pivotal way: While non-adaptive encoding schemes, if suitably designed, are optimal for the non-intermittent 3WC, adaptive encoding techniques are indispensable for the intermittent 3WC. For sum-DoF this is not the case; instead, non-adaptive encoding suffices for both intermittent and non-intermittent 3WC. This reinforces that changes in fundamental qualitative channel properties might not be recognizable from the sum-DoF perspective, corroborating the necessity to study the full DoF region of multi-way communication scenarios.

Considering multi-way communication networks, recall that adaptive encoding opens new paths for information flow which are otherwise unavailable to non-adaptive schemes. As intermittency impairs parts of the network, the ability of adaptive schemes to exploit path diversity and steer clear of the impairment becomes crucial to achieve 'extreme' DoF tuples (e.g., tuples where all resources are used to maximize a certain DoF). It is in line with this informal reasoning to find that adaptive encoding is required to achieve the DoF region of a multi-way communication network with intermittency.

In the sequel we provide detailed derivations and proofs of our main results highlighted in this section.

IV. NODE INTERMITTENCY

In this section we first introduce a non-adaptive ZF/IA/ECbased scheme and derive its achievable sum-DoF and DoF region. We then show, using enhanced genie-aided bounds for both adaptive and non-adaptive encoding, that this scheme is sum-DoF optimal, but not DoF region optimal. Furthermore, no non-adaptive scheme can be DoF region optimal, since tighter outer bounds hold for non-adaptive schemes, that however can be exceeded by adaptive schemes, as we show by the example of decode-forward relaying. This establishes that adaptive encoding is necessary from a DoF region perspective, but non-adaptive encoding is sufficient to achieve sum-DoF.

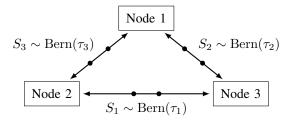


Fig. 7. Rationale behind the aliases $s_{1,\ell} \triangleq 1, s_{2,\ell} \triangleq s_{3,\ell} \triangleq s_{\ell}, \tau_1 \triangleq 1, \tau_2 \triangleq \tau_3 \triangleq \tau$ introduced in Section IV-A: treating every link $j \leftrightarrow k$ as potentially intermittent with intermittency state variable $s_{i,\ell}$ (marginally distributed as Bern (τ_i)) allows to derive general expressions for, e.g., $y_{i,\ell}$ based on $x_{j,\ell}$, $x_{k,\ell}, s_{j,\ell}$ and $s_{k,\ell}$, independent of whether i = 1, i = 2 or i = 3

A. A Non-Adaptive Scheme Based on ZF, IA and EC

In this subsection we present a non-adaptive transmission scheme based on ZF, IA and EC that provides an inner bound on sum-DoF and DoF region of 3WCs. Previous works [18], [19], [21], [47] developed similar ZF/IA-based schemes only to the limited extent necessary to analyze the sum-DoF of various 3WCs. We take EC as additional technique to mitigate intermittency and develop the resulting ZF/IA/ECbased scheme in full generality, i.e., for arbitrary numbers of antennas and flexible allocation of DoFs to the available transmission techniques ZF, IA and EC. The resulting DoF region is optimal for the non-intermittent 3WC. Furthermore, the DoF region/sum-DoF constitutes inner/lower bounds for 3WCs with arbitrary intermittency models beyond nodeintermittency, some of which are mentioned in Section VI.

Throughout this section we continue to assume $i, j, k \in \{1, 2, 3\}$ and mutually distinct. For notational simplicity (capturing the symmetries inherent in the model thereby avoiding case distinctions), we introduce the following aliases that will be resubstituted towards the end of the section,

$$s_{1,\ell} \triangleq 1, \quad s_{2,\ell} \triangleq s_{3,\ell} \triangleq s_\ell, \quad \tau_1 \triangleq 1, \quad \tau_2 \triangleq \tau_3 \triangleq \tau.$$
 (1)

The idea behind these aliases is visualized in Fig. 7: Using these aliases we can, for instance, find a general expression for the receive signal $y_{i,\ell}$ of node *i* in terms of the transmit signals $x_{j,\ell}$ and $x_{k,\ell}$ of nodes *j* and *k*, and the intermittency states $s_{k,\ell}$ and $s_{j,\ell}$ (marginally distributed as $\text{Bern}(\tau_k)$ and $\text{Bern}(\tau_j)$, respectively) of the links $j \leftrightarrow i$ and $k \leftrightarrow i$, respectively. The fact that $2 \leftrightarrow 3$ is not intermittent, and $1 \leftrightarrow 2$ and $1 \leftrightarrow 3$ are jointly intermittent, is accounted for by the appropriate resubstitution at due time.

1) Encoding: Each node splits each message w_{ij} into $w_{ij}^{[\text{ZF}]}$ and $w_{ij}^{[\text{IA}]}$ to be sent via zero-forcing and interference alignment, respectively. At node *i*, the four messages $w_{ij}^{[q]}$ ($q \in \{\text{ZF}, \text{IA}\}$) are encoded into codewords $x_{ij}^{[q]n}$ with symbols $x_{ij,\ell}^{[q]} \in \mathbb{C}^{a_{ij}^{[q]}}$ each, for some vector lengths $a_{ij}^{[q]} \in \mathbb{N}_0$. The symbols of these codewords are chosen i.i.d. $\mathcal{CN}(\mathbf{0}, p_i \boldsymbol{I}_{a_{ij}^{[q]}})$ respectively, where p_i is the power. The power constraints on x_i^n stated in Section II-B are satisfied by choosing

$$p_i = \frac{P}{a_{ij}^{[\text{ZF}]} + a_{ij}^{[\text{IA}]} + a_{ik}^{[\text{ZF}]} + a_{ik}^{[\text{IA}]}}.$$
 (2)

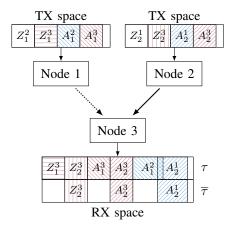


Fig. 8. Visualization of transmit (TX) signal spaces at nodes 1 and 2 and receive (RX) signal space at node 3 depending on intermittency state of $1 \leftrightarrow 3$ where the first and second row are received with probability τ and $\overline{\tau}$, respectively $(Z_i^j \text{ denotes } w_{ij}^{[\text{ZF}]}, A_i^j \text{ denotes } w_{ij}^{[\text{IA}]}$, desired signals in red, interfering signals in blue)

For encoding, the codes are designed to employ EC to be able to tolerate a certain number of symbol erasures (e.g., caused by intermittency), by not using all codeword symbols for net user data, but deliberately adding some redundancy. The rates and DoFs are thereby reduced accordingly. While for the non-intermittent 3WC this additional layer of EC is not required, it is made use of for intermittent 3WCs to cope with intermittency.

2) Transmission: At time ℓ , node i sends

$$oldsymbol{x}_{i,\ell} = \sum_{q \in \{ ext{ZF,IA}\}} \left[oldsymbol{V}_{ij}^{[q]} oldsymbol{x}_{ij,\ell}^{[q]} + oldsymbol{V}_{ik}^{[q]} oldsymbol{x}_{ik,\ell}^{[q]}
ight]$$

where $V_{ij}^{[q]} \in \mathbb{C}^{M_i \times a_{ij}^{[q]}}$ are pre-coding matrices with unitnorm column vectors. Zero-forcing is achieved by choosing the $V_{ij}^{[\text{ZF}]}$ such that

$$\boldsymbol{H}_{ik}\boldsymbol{V}_{ij}^{[\mathrm{ZF}]} = \boldsymbol{0}.$$
 (3)

Such matrices $V_{ij}^{[\text{ZF}]}$ exist if node *i* has enough antennas to send $a_{ij}^{[\text{ZF}]}$ streams to node *j* without interfering with node *k*, i.e.

$$a_{ij}^{[\text{ZF}]} \le (M_i - M_k)^+.$$
 (4)

To avoid any overlap of the different transmit signal subspaces, we require furthermore that

$$a_{ij}^{[\text{ZF}]} + a_{ij}^{[\text{IA}]} + a_{ik}^{[\text{ZF}]} + a_{ik}^{[\text{IA}]} \le M_i.$$
 (5)

3) Decoding: Node i receives (Fig. 8)

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with $G_i = [s_{k,\ell}H_{ji}V_{jk}^{[IA]} s_{j,\ell}H_{ki}V_{kj}^{[IA]}]$. Note that the terms $H_{ji}V_{jk}^{[ZF]}x_{jk,\ell}^{[ZF]}$ and $H_{ki}V_{kj}^{[ZF]}x_{kj,\ell}^{[ZF]}$ vanished due to the zero-forcing condition (3). The first summand represents the four desired signals from $j \to i$ and $k \to i$, the second summand represents occasional interference from $j \leftrightarrow k$, and the third summand is noise.

To decode a desired signal $x_{ji}^{[q]n}$, node *i* zero-forces the remaining signals by multiplying y_i^n with a suitable post-coder $T_{ji}^{[q]} \in \mathbb{C}^{a_{ji}^{[q]} \times M_i}$ with unit-norm row vectors satisfying:

$$\boldsymbol{T}_{ji}^{[q]} \boldsymbol{T}_{ji}^{[q]\mathsf{H}} = \boldsymbol{I}_{a_{ji}^{[q]}} \quad (6)$$
$$\boldsymbol{T}_{ji}^{[q]} \begin{bmatrix} \boldsymbol{H}_{ji} \boldsymbol{V}_{ji}^{[\overline{q}]} & \boldsymbol{H}_{ki} \boldsymbol{V}_{ki}^{[\mathrm{ZF}]} & \boldsymbol{H}_{ki} \boldsymbol{V}_{ki}^{[\mathrm{IA}]} & \boldsymbol{G}_{i} \end{bmatrix} = \boldsymbol{0}$$
$$\text{with } \overline{q} \in \{\mathrm{ZF}, \mathrm{IA}\} \setminus \{q\} (7)$$
$$\text{rank}(\boldsymbol{T}_{ji}^{[q]} \boldsymbol{H}_{ji} \boldsymbol{V}_{ji}^{[q]}) = a_{ji}^{[q]} \quad (8)$$

Here, (7) ensures zero-forcing of the remaining three messages and the interference and (8) ensures post-coding without loss of meaningful signal dimensions. The existence of such postcoders $T_{ji}^{[q]}$ is guaranteed as long as the columns of

$$\begin{bmatrix} \boldsymbol{H}_{ji} \boldsymbol{V}_{ji}^{[\text{ZF}]} & \boldsymbol{H}_{ji} \boldsymbol{V}_{ji}^{[\text{IA}]} & \boldsymbol{H}_{ki} \boldsymbol{V}_{ki}^{[\text{ZF}]} & \boldsymbol{H}_{ki} \boldsymbol{V}_{ki}^{[\text{IA}]} & \boldsymbol{G}_i \end{bmatrix}$$

are linearly independent. Let γ_i be the dimension of $\operatorname{span}(\boldsymbol{H}_{ji}\boldsymbol{V}_{jk}^{[\operatorname{IA}]}) \cap \operatorname{span}(\boldsymbol{H}_{ki}\boldsymbol{V}_{kj}^{[\operatorname{IA}]})$. Then, the dimension of $\operatorname{span}(\boldsymbol{G}_i)$ is $a_{jk}^{[\operatorname{IA}]} + a_{kj}^{[\operatorname{IA}]} - \gamma_i$, and the above linear independence is possible almost surely if we choose

$$a_{ji}^{[\text{ZF}]} + a_{ji}^{[\text{IA}]} + a_{ki}^{[\text{ZF}]} + a_{ki}^{[\text{IA}]} + (a_{jk}^{[\text{IA}]} + a_{kj}^{[\text{IA}]} - \gamma_i) \le M_i.$$
(9)

To minimize the impact of interference, we choose the precoders $V_{ij}^{[q]}$ such that all γ_i are maximized, i.e., we 'maximally' align the interference subspaces at the receivers. The γ_i cannot be chosen arbitrarily large, the dimension of the intersection of the interference subspaces (γ_i) is upper bounded by the dimensions of the interference subspaces ($a_{jk}^{[IA]}$ and $a_{kj}^{[IA]}$). Furthermore, γ_i needs to be smaller than the dimension of span(H_{ji}) \cap span(H_{ki}), which is ($M_j + M_k - M_i$)⁺ almost surely. Therefore, we require that

$$\gamma_i \le \min\{a_{jk}^{[\text{IA}]}, a_{kj}^{[\text{IA}]}, (M_j + M_k - M_i)^+\}.$$
 (10)

After post-coding, node i is left with the signals

$$m{y}_{ji,\ell}^{[q]} = s_{k,\ell} m{T}_{ji}^{[q]} m{H}_{ji} m{V}_{ji}^{[q]} m{x}_{ji,\ell}^{[q]} + m{T}_{ji}^{[q]} m{z}_{i,\ell}.$$

The resulting channel is an erasure-Gaussian MIMO channel with erasure probability $\overline{\tau}_k$, i.e., a channel whose output is

$$Y_{ji}^{[q]} = S_k T_{ji}^{[q]} H_{ji} V_{ji}^{[q]} X_{ji}^{[q]} + T_{ji}^{[q]} Z_i$$

with random variables $S_k \sim \text{Bern}(\tau_k)$, $\mathbf{X}_{ji}^{[q]} \sim \mathcal{CN}(\mathbf{0}, p_j \mathbf{I}_{a_{ji}^{[q]}})$ and $\mathbf{Z}_i \sim \mathcal{CN}(\mathbf{0}, \sigma^2 \mathbf{I}_{M_i})$. We treat this as a channel with state known causally to the receiver, cf. Section II-A, Lemma 1, such that for large n, the achievable rate over this channel is the mutual information between $\mathbf{X}_{ji}^{[q]}$ and $(\mathbf{Y}_{ji}^{[q]}, S_k)$:

$$I(\mathbf{X}_{ji}^{[q]}; \mathbf{Y}_{ji}^{[q]} S_k) \\= I(\mathbf{X}_{ji}^{[q]}; \mathbf{Y}_{ji}^{[q]} \mid S_k)$$

$$= \tau_k \log \det \left(\boldsymbol{I}_{a_{ji}^{[q]}} + \frac{p_j}{\sigma^2} \boldsymbol{T}_{ji}^{[q]} \boldsymbol{H}_{ji} \boldsymbol{V}_{ji}^{[q]} \boldsymbol{V}_{ji}^{[q]\mathsf{H}} \boldsymbol{H}_{ji}^{\mathsf{H}} \boldsymbol{T}_{ji}^{[q]\mathsf{H}} \right)$$

Due to (2) and (8), this leads to a DoF of $\tau_k a_{ji}^{[q]}$, and the code that achieves it is an EC.

4) Achievable DoF Region: For n large, $i \to j$ has a total of $\tau_k a_{ij}^{[\text{ZF}]} + \tau_k a_{ij}^{[\text{IA}]}$ DoFs per channel use,

$$d_{ij} = \tau_k a_{ij}^{[\text{ZF}]} + \tau_k a_{ij}^{[\text{IA}]}.$$
 (11)

Collecting (4), (5), (9) and (10) as well as non-negativity of every $a_{ij}^{[q]}$, we obtain:

$$a_{ij}^{[\text{ZF}]} + a_{ij}^{[\text{IA}]} + a_{ik}^{[\text{ZF}]} + a_{ik}^{[\text{IA}]} \le M_i$$
(12)
$$a_{ji}^{[\text{ZF}]} + a_{ji}^{[\text{IA}]} + a_{ki}^{[\text{IA}]} + (a_{jk}^{[\text{IA}]} + a_{kj}^{[\text{IA}]} - \gamma_i) \le M_i$$
(13)

$$a_{ij}^{[\mathrm{ZF}]} \le (M_i - M_k)^+ \tag{14}$$

$$\gamma_i \le \min\{a_{jk}^{[\text{IA}]}, a_{kj}^{[\text{IA}]}, (M_j + M_k - M_i)^+\}$$
 (15)

$$0 \le a_{ij}^{[q]} \tag{16}$$

Using (11), we obtain:

С

$$\tau_j d_{ij} + \tau_k d_{ik} \le \tau_j \tau_k M_i \qquad (17)$$

$$\tau_j \tau_i d_{ji} + \tau_k \tau_i d_{ki} + \tau_j \tau_k d_{jk} + \tau_k \tau_j d_{kj} - \tau_i \tau_j \tau_k a_{jk}^{[\text{ZF}]} - \tau_i \tau_k \tau_j a_{kj}^{[\text{ZF}]} - \tau_i \tau_j \tau_k \gamma_i \le \tau_i \tau_j \tau_k M_i$$
(18)

$$\tau_i(\gamma_i + a_{jk}^{[\text{ZF}]}) \le d_{jk} \tag{19}$$

$$\gamma_i \le (M_j + M_k - M_i)^+ \tag{20}$$

$$a_{ij}^{[2IF]} \le (M_i - M_k)^+$$
 (21)

$$\min\{a_{ij}^{[\text{ZF}]}, d_{ij}, \gamma_i\} \ge 0 \tag{22}$$

Instantiating these constraints for every possible combination of $i, j, k \in \{1, 2, 3\}$ mutually distinct, resubstituting all τ_i from (1), collecting the resulting bounds and eliminating redundant bounds, finally yields (for both $(i, \bar{i}) \in \{(2, 3), (3, 2)\}$):

$$d_{12} + d_{13} \le \tau M_1 \tag{23}$$

$$d_{i1} + \tau d_{i\bar{i}} \le \tau M_i \tag{24}$$

$$d_{21} + d_{31} + \tau d_{23} + \tau d_{32} - \tau a_{23}^{[\text{ZF}]} - \tau a_{32}^{[\text{ZF}]} - \tau \gamma_1 \le \tau M_1$$

$$d_{25} + \tau d_{23} + d_{23} + d_{32} = 0$$
(25)

$$-\tau a_{1\bar{i}}^{[\text{ZF}]} - \tau a_{\bar{i}1}^{[\text{ZF}]} - \tau a_{\bar{i}1}^{[\text{ZF}]} - \tau \gamma_i \le \tau M_i$$
(26)

$$0 \le (\gamma_1 + a_{i\bar{i}}^{[\text{ZF}]}) \le d_{i\bar{i}}$$

$$(27)$$

$$0 \le \tau(\gamma_i + a_{1\bar{i}}^{[\text{III}]}) \le d_{1\bar{i}}$$
(28)
$$0 \le \tau(\gamma_i + a_{\bar{i}}^{[\text{ZF}]}) \le d_{\bar{i}1}$$
(29)

$$(\gamma_i + u_{\bar{i}1}) \le u_{i1}$$

$$0 < \gamma_1 < (M_2 + M_3 - M_1)^+$$

$$(30)$$

$$0 \le \gamma_i \le (M_1 + M_{\overline{i}} - M_i)^+$$
 (31)

$$0 \le a_{1i}^{[\text{ZF}]} \le (M_1 - M_{\overline{i}})^+$$
 (32)

$$0 \le a_{i1}^{[\text{ZF}]} \le (M_i - M_{\bar{i}})^+ \tag{33}$$

$$0 \le a_{i\bar{i}}^{[\text{ZF}]} \le (M_i - M_1)^+ \tag{34}$$

In order to resolve the $(.)^+$ expressions, we do the following for each of the twelve cases of $M_i \ge M_j + M_k \ge M_j \ge$ M_k and $M_j + M_k \ge M_i \ge M_j \ge M_k$ (for all possible combinations $i, j, k \in \{1, 2, 3\}$ mutually distinct):

- 1) Instantiate the $(.)^+$ expressions under the respective assumption on the numbers of antennas, therefore some of the $a_{ij}^{[\text{ZF}]}$ and γ_i will be forced to zero.
- 2) Perform Fourier-Motzkin's elimination to remove all remaining $a_{ij}^{[\text{ZF}]}$ and γ_i and obtain the achievable DoF region.

We then combine the resulting achievable DoF regions into the following compact formulation:

$$\max\{d_{12} + d_{13}, d_{21} + d_{31}\} \le \tau M_1 \tag{35}$$

$$\max\{d_{21} + \tau d_{23}, d_{12} + \tau d_{32}\} \le \tau M_2 \tag{36}$$

$$\max\{d_{31} + \tau d_{32}, d_{13} + \tau d_{23}\} \le \tau M_3 \tag{37}$$

$$\max\{d_{12} + d_{13} + \tau d_{23}, \\ d_{24} + d_{24} + \tau d_{26}\} \le \tau \max\{M_1, M_2\} \quad (7)$$

$$d_{21} + d_{31} + \tau d_{32} \le \tau \max\{M_1, M_3\} \quad (38)$$
$$\max\{d_{12} + d_{13} + \tau d_{32},$$

$$d_{21} + d_{31} + \tau d_{23} \le \tau \max\{M_1, M_2\} \quad (39)$$

$$\max\{d_{12} + d_{31} + \tau d_{32},\,$$

$$d_{21} + d_{13} + \tau d_{23} \le \tau \max\{M_2, M_3\} \quad (40)$$

$$\min\{d_{12}, d_{13}, d_{21}, d_{23}, d_{31}, d_{32}\} \ge 0 \tag{41}$$

This region is achievable for tuples d with non-negative integer entries. Tuples with non-negative real entries (such as the corner points of the region) are first approximated by nonnegative rationals which then can be achieved using symbol extension, as in [48].

The set of all DoF tuples d satisfying constraints (35) to (41) is denoted by $\mathcal{D}_{IB,\overline{A}}^{I}$. This proves Theorem 1. What is the rationale behind the DoF region inner bound (35) to (41)?

The first three inequalities constrain the sum-DoF of outbound and inbound streams at each node, similar to cut-set bounds. The next three inequalities follow this rule: For each of the three links $1 \leftrightarrow 2$, $1 \leftrightarrow 3$ and $2 \leftrightarrow 3$ there are two DoF variables, one for each direction (i.e., d_{12} and d_{21} , etc.). For each link pick one direction. There are eight such combinations. Whenever a combination contains both DoF variables that occur in a node's outbound or inbound sum-DoF constraint, the number of antennas at this node appears in the max{.} operator at the right side of the inequality. This means that whenever a node's index appears two times as left or two times as right index, this node's index appears also in the max{.} on the right side.

As can be seen above, there are six cases where each case applies to two nodes each, while for the third node it does not, because the third node's index appears once as left and once as right index. There are two cases missing altogether, $d_{12} + d_{31} + \tau d_{23}$ and $d_{21} + d_{13} + \tau d_{32}$, where the indices of all three nodes appear once as left and once as right index. Depending on numbers of antennas, this achievable DoF region yields four bounds for the largest and two bounds for the second largest node from the inequalities (38) to (40), and two bounds for the third largest node from (35) to (37). The remaining bounds from (35) to (37) are inactive due to the tighter bounds from (38) to (40).

Although we proved that the above region is achievable, it is still useful to provide a 'recipe' which describes how a specific DoF tuple can be achieved. To obtain the actual allocation of signal dimensions $a_{ii}^{[q]}$ for a DoF tuple *d* satisfying (35) to (41), proceed as follows: First, use as many ZF resources as possible. Only once the ZF dimensions are exhausted, assign IA resources and align as much of the resulting interference as possible. Throughout the process, account for redundancy required by EC to be able to tolerate intermittency. As an example (Fig. 9), assume $(M_1, M_2, M_3, \tau) = (5, 7, 4, 0.5)$. There is one ZF dimension $1 \rightarrow 2$, three ZF dimensions $2 \rightarrow 1$ and two ZF dimensions $2 \rightarrow 3$, all other communication cannot be zero-forced. We try to achieve d =(0.5, 0, 0.5, 4, 0, 4). To transmit on average half a symbol per channel access full-duplex over the intermittent $1 \leftrightarrow 2$, we use a rate $\frac{1}{2}$ EC over one available ZF dimension in we use a rate $\frac{1}{2}$ EC over one available ZF dimension in each direction $(a_{12}^{[ZF]} = a_{21}^{[ZF]} = 1)$, the remaining two ZF dimensions $2 \rightarrow 1$ remain unused. IA is not required for $1 \leftrightarrow 2$ $(a_{12}^{[IA]} = a_{21}^{[IA]} = 0)$. No communication $1 \leftrightarrow 3$ takes place $(a_{13}^{[ZF]} = a_{13}^{[IA]} = a_{31}^{[ZF]} = a_{31}^{[IA]} = 0)$. To transmit four symbols per channel access $2 \rightarrow 3$, we use the two available ZF dimensions for two of them $(a_{23}^{[ZF]} = 2)$, and two IA dimensions $(a_{23}^{[IA]} = 2)$ that occupy a two-dimensional interference subspace at node 1. To transmit four symbols per interference subspace at node 1. To transmit four symbols per channel access $3 \rightarrow 2$, we use four IA dimensions $(a_{32}^{[IA]} = 4)$, since ZF is not possible $(a_{32}^{[ZF]} = 0)$. All of them create interference at node 1, but this four-dimensional interference subspace can be aligned with the two-dimensional interference subspace caused by $2 \rightarrow 3$ ($\gamma_1 = 2$). As a result, four of the five receive dimensions at node 1 are interference of $2 \leftrightarrow 3$ communication, while the remaining was used for zero-forced and erasure-coded communication $2 \rightarrow 1$. At nodes 2 and 3 no interference is caused, such that no interference alignment takes place ($\gamma_2 = \gamma_3 = 0$).

B. Sum-DoF

The devised non-adaptive encoding scheme based on ZF, IA and EC is sum-DoF optimal in the intermittent 3WC, as we will prove in this section.

1) Lower Bounds: Using (23) to (34), we derive a lower bound on the sum-DoF of the intermittent 3WC with intermittent node 1. For each of the six cases of $M_i \ge M_j \ge M_k$ (for all possible combinations $i, j, k \in \{1, 2, 3\}$ mutually distinct), we solve the linear program maximizing sum-DoF using, e.g., the simplex algorithm.

Definition 1. We denote

$$d_{\text{sum},\text{LB},\overline{\text{A}}}^{\text{I}} \triangleq \max_{d \in \mathcal{D}_{\text{IB},\overline{\text{A}}}^{\text{I}}} \left[d_{12} + d_{13} + d_{21} + d_{23} + d_{31} + d_{32} \right].$$

The resulting lower bounds for each of the different cases are listed in Table II, and are condensed into a single expression in the following lemma:

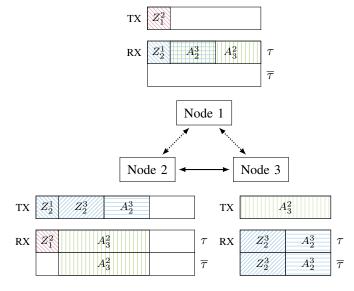


Fig. 9. Visualization of transmit (TX) and receive (RX) signal spaces achieving $\boldsymbol{d} = (0.5, 0, 0.5, 4, 0, 4)$ under the assumption of $(M_1, M_2, M_3, \tau) = (5, 7, 4, 0.5)$ (depending on intermittency state, where the first and second row are received with probability τ and $\overline{\tau}$, respectively; Z_i^j denotes $w_{ij}^{[ZF]}$, A_i^j denotes $w_{ij}^{[IA]}$)

TABLE II Achievable Sum-DoFs of Intermittent 3WC

Case	Sum-DoF Lower Bound	
$M_1 \ge M_2 \ge M_3$	$d^{\mathrm{I}}_{\mathrm{sum,LB},\overline{\mathrm{A}}} = 2M_3 + 2\tau M_2 - 2\tau M_3$	
$M_2 \ge M_1 \ge M_3$	$ \begin{aligned} & d^{\mathrm{I}}_{\mathrm{sum, LB}, \overline{\mathrm{A}}} = 2M_3 + 2\tau M_2 - 2\tau M_3 \\ & d^{\mathrm{I}}_{\mathrm{sum, LB}, \overline{\mathrm{A}}} = 2M_3 + 2\tau M_1 - 2\tau M_3 \end{aligned} $	
$M_2 \ge M_3 \ge M_1$	$d^{\mathrm{I}}_{\mathrm{sum LB}\overline{\mathrm{A}}} = 2M_3$	
$M_1 \ge M_3 \ge M_2$	$d_{\text{sum,LB},\overline{A}}^{\text{I}} = 2M_2 + 2\tau M_3 - 2\tau M_2$	
$M_3 \ge M_1 \ge M_2$	$ \begin{aligned} d^{\mathrm{I}}_{\mathrm{sum,LB},\overline{\mathrm{A}}} &= 2M_2 + 2\tau M_3 - 2\tau M_2 \\ d^{\mathrm{I}}_{\mathrm{sum,LB},\overline{\mathrm{A}}} &= 2M_2 + 2\tau M_1 - 2\tau M_2 \end{aligned} $	
$M_3 \ge M_2 \ge M_1$	$d_{\mathrm{sum,LB},\overline{\mathrm{A}}}^{\mathrm{I}} = 2M_2$	

Lemma 2 (Sum-DoF Lower Bound for Node-Intermittent 3WC). *The sum-DoF*

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$$d_{\text{sum,LB},\overline{A}}^{I} = 2\overline{\tau}\min\{M_{2}, M_{3}\} + 2\tau \Big(M_{1} + M_{2} + M_{3} \\ - \min\{M_{1}, M_{2}, M_{3}\} - \max\{M_{1}, M_{2}, M_{3}\}\Big) \\ \leq d_{\text{sum}}^{\text{I}}$$

is achievable in the node-intermittent 3WC and therefore constitutes a lower bound on the sum-DoF of the node-intermittent 3WC.

2) Upper Bounds: We first motivate the converse techniques used throughout this section, then summarize the resulting upper bounds in Lemma 3, and in the sequel provide rigorous proofs. The general approach for upper bounding the sum-DoF of the intermittent 3WC is as follows: Partition the DoF sum $d_{ij}+d_{ik}+d_{ji}+d_{jk}+d_{ki}+d_{kj}$ into two partial sums $d_{ij}+d_{kj}+d_{ki}$ and $d_{ik}+d_{jk}+d_{ji}$ (Fig. 10), where w_{ij}, w_{kj}, w_{ki} are to be decoded by node j and w_{ik}, w_{jk}, w_{ji} are to be decoded by node k. There are three such partitions, and the partition is fully determined by choosing which node takes the

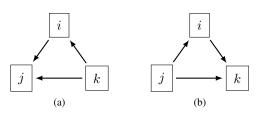


Fig. 10. Partition the DoF sum $d_{ij} + d_{ik} + d_{ji} + d_{jk} + d_{ki} + d_{kj}$ into two partial sums $d_{ij} + d_{kj} + d_{ki}$ to be decoded at node *j* (a) and $d_{ik} + d_{jk} + d_{ji}$ to be decoded at node *k* (b), where nodes *j* and *k* are provided enough side information such that they can recover the receive signal y_i^n of node *i* and from it decode the one message not originally intended for them (additional d_{ki} and d_{ji} DoFs, respectively).

role of node i. While nodes j and k function exclusively as source and sink in any one of the partial sums, node i is an intermediary node in both.

Have an imaginary genie provide just enough side information to node j and k (hence the name 'genie-aided' bound), respectively, such that they can recover the receive signal y_i^n of node i, then (assuming existence of a suitable coding scheme) nodes j and k can decode the additional messages w_{ki} and w_{ji} , respectively. The details of this decoding process and the required side information will be presented in due course. Here we only remark, that the side information can serve for the following four purposes (some cases might not require some of the types of side information):

- 1) An additional message is required such that node j (or k) can decode the additional message w_{ki} (or w_{ji}) from the recovered y_i^n , because decoding requires knowledge of w_i , and node j (or k) only knows w_{ij} (or w_{ik}) from decoding its own receive signal y_i^n (or y_k^n).
- 2) Node j (or k) might be incapable of capturing enough information about y_i^n because M_j (or M_k) is small. In this case, additional measurements about y_i^n (or alternatively about the unknown 'ingredient' of interest, x_k^n or x_j^n , respectively) need to be provided by side information.
- 3) Node j (or k) might be incapable of capturing enough information about y_i^n because of intermittency. In this case, additional measurements about y_i^n for those time instances ℓ where $s_\ell = 0$ need to be provided by side information.
- For rather technical reasons a noise correction signal is required to accurately recover yⁿ_i. However, the information contained in this signal about the three desired messages in question scales only as o [log(ρ)].

Assuming reliable communication, the partial DoF sum of each three messages is necessarily upper bounded by a mutual information expression, using Fano's inequality. Adding the two resulting bounds yields an upper bound on the sum-DoF. The challenge with this approach is two-fold: a) Provide as little side information as possible to the respective nodes. Otherwise the genie can be 'mis-used' for information exchange, resulting in a larger DoF for the genie-enhanced system and thus loose bounds for the non-enhanced system. b) Use tight bounding when expanding the mutual information expression from Fano's inequality. For the intermittent 3WC the partition that yields the tightest upper bound on the sum-DoF depends on the numbers of antennas. The node with largest number of antennas should take the role of the intermediary node i (Fig. 10). The order among the remaining two nodes decides about which side information to give to which node, to compensate for insufficient number of antennas or intermittency. We prove upper bounds for the three cases $M_1 \ge M_2 \ge M_3$, $M_2 \ge M_1 \ge M_3$ and $M_2 \ge M_3 \ge M_1$, the remaining three cases go by renaming node 2 and 3. We obtain:

Lemma 3 (Sum-DoF Upper Bound for Node-Intermittent 3WC).

$$d_{\text{sum}}^{\text{I}} \le 2\overline{\tau} \min\{M_2, M_3\} + 2\tau \Big(M_1 + M_2 + M_3 \\ - \min\{M_1, M_2, M_3\} - \max\{M_1, M_2, M_3\}\Big)$$

Proof. The lemma follows from (44), (47), (50) below, and symmetry of node 2 and 3. \Box

Using the achievability and converse results developed in Sections IV-B1 and IV-B2, we establish the sum-DoF of the intermittent 3WC for which non-adaptive encoding is sufficient, i.e.

$$d_{\text{sum,LB},\overline{A}}^{\text{I}} = 2\overline{\tau}\min\{M_2, M_3\} + 2\tau \left(M_1 + M_2 + M_3 - \min\{M_1, M_2, M_3\} - \max\{M_1, M_2, M_3\}\right)$$
$$= d_{\text{sum}}^{\text{I}}.$$

This proves Theorem 2. We proceed to present rigorous proofs for the aforementioned three cases.

a) Case 1: $M_1 \ge M_2 \ge M_3$: For the case where M_1 is the largest number of antennas, we develop two partial sums around nodes 2 and 3 (Fig. 10a and 10b, with (i, j, k) =(1, 2, 3)), respectively. We start with the bound around node 2 (Fig. 10a, with (i, j, k) = (1, 2, 3)), as it requires less side information and is therefore simpler to argue, and in the sequel extend the basic technique to develop the bound around node 3 (Fig. 10b, with (i, j, k) = (1, 2, 3)), which requires more side information and is therefore slightly more involved.

Which side information does node 2 need to be able to recover y_1^n and decode w_{31} from it, assuming a scheme allowing every node to decode its desired messages with high probability? We present a suitable iterative multi-step process depicted in Fig. 11. Along the way, side information is introduced (*highlighted in italic*) as found necessary for the reconstruction process. At the end of the transmission, node 2 has w_2 , y_2^n , s^n and x_2^n , as shown on the top left of the figure. Using the decoder \mathcal{F}_2 it can decode ($\hat{w}_{12}, \hat{w}_{32}$) = (w_{12}, w_{32}) with high probability. Assume we provide w_{13} as side information, so that node 2 can decode messages intended for node 1 using \mathcal{F}_1 as soon as it obtains y_1^n , as shown on the bottom left of the figure. Node 2 can now obtain $x_{1,1}$ from w_1 using $\mathcal{E}_{1,1}$. From ($s_1, y_{2,1}$) it can obtain $H_{32}x_{3,1} + z_{2,1}$ using $x_{1,1}$. Since H_{32} is a tall matrix, a noisy version of $x_{3,1}$ can be

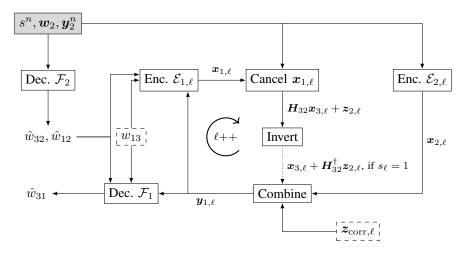


Fig. 11. Decoding of w_{31} at node 2 by iterative reconstruction of y_1^n from a-priori knowledge w_2 and observations (y_2^n, s^n) using side information (w_{13}, z_{corr}^n) (grey box: a-priori knowledge and channel output, dashed boxes: side information as introduced in the running text): successively obtain $x_{1,\ell}$, cancel its effect from $y_{2,\ell}$ to obtain a noisy version of $x_{3,\ell}$, and combine this with $x_{2,\ell}$ and $z_{corr,\ell}$ to finally obtain $y_{1,\ell}$; repeat for next ℓ .

obtained from $H_{32}x_{3,1} + z_{2,1}$ using the pseudo-inverse H_{32}^{\dagger} , i.e.,

$$H_{32}^{\dagger}(H_{32}x_{3,1}+z_{2,1})=x_{3,1}+H_{32}^{\dagger}z_{2,1}.$$

Using the noisy version of $x_{3,1}$, node 2 can obtain a noisy version of $y_{1,1}$, i.e.,

$$s_1 \left(oldsymbol{H}_{21} oldsymbol{x}_{2,1} + oldsymbol{H}_{31} \left(oldsymbol{x}_{3,1} + oldsymbol{H}_{32}^{\dagger} oldsymbol{z}_{2,1}
ight)
ight) \ = s_1 oldsymbol{H}_{21} oldsymbol{x}_{2,1} + s_1 oldsymbol{H}_{31} oldsymbol{x}_{3,1} + s_1 oldsymbol{H}_{31} oldsymbol{H}_{32}^{\dagger} oldsymbol{z}_{2,1}.$$

Given a suitably formed noise correction term $\mathbf{z}_{\text{corr},1} \triangleq \mathbf{z}_{1,1} - s_1 \mathbf{H}_{31} \mathbf{H}_{32}^{\dagger} \mathbf{z}_{2,1}$ as side information, node 2 can finally obtain $\mathbf{y}_{1,1}$, i.e.

$$y_{1,1} = s_1 H_{21} x_{2,1} + s_1 H_{31} x_{3,1} + s_1 H_{31} H_{32}^{\dagger} z_{2,1} + z_{\text{corr},1}.$$

Then, this reconstruction cycle repeats for the next $\ell = 2, ..., n$, where all previous $(y_1^{\ell-1}, s^{\ell-1})$ are used to obtain $x_{1,\ell}$ using $\mathcal{E}_{1,\ell}$. After completing reconstruction of y_1^n , node 2 uses \mathcal{F}_1 and (\hat{w}_{12}, w_{13}) to decode $\hat{w}_{31} = w_{31}$ with high probability. From the four abstract types of side information introduced before, only two are required for the reconstruction and subsequently for this bound: a message and a noise correction term. *No side information to compensate for insufficient number of antennas or intermittency is required*.

In a nutshell, side information w_{13} and z_{corr}^n is provided to node 2 by a genie, defined as

$$\boldsymbol{Z}_{\text{corr},\ell} \triangleq \boldsymbol{Z}_{1,\ell} - S_{\ell} \boldsymbol{H}_{31} \boldsymbol{H}_{32}^{\dagger} \boldsymbol{Z}_{2,\ell}.$$

Since the scheme ought to be reliable, we bound the sum rate of w_{12} , w_{32} and w_{31} using Fano's inequality:

$$n(R_{12} + R_{32} + R_{31} - \varepsilon_n^{(1)}) \\ \leq I(W_{12}W_{32}W_{31}; \mathbf{W}_2\mathbf{Y}_2^n S^n \underbrace{\widetilde{W_{13}\mathbf{Z}_{corr}^n}}_{W_{13}\mathbf{Z}_{corr}^n}) \\ \stackrel{\text{(a)}}{=} I(W_{12}W_{32}W_{31}; \mathbf{Y}_2^n \mid W_{13}\mathbf{W}_2S^n\mathbf{Z}_{corr}^n) \\ \stackrel{\text{(b)}}{=} \sum_{\ell=1}^n I(W_{12}\mathbf{W}_3; \mathbf{Y}_{2,\ell} \mid \mathbf{Y}_2^{\ell-1}S^nW_{13}\mathbf{W}_2\mathbf{Z}_{corr}^n)$$

$$\begin{split} &= \sum_{\ell=1}^{n} \left[h(\mathbf{Y}_{2,\ell} \mid \mathbf{Y}_{2}^{\ell-1} S^{n} W_{13} \mathbf{W}_{2} \mathbf{Z}_{\text{corr}}^{n}) \\ &\quad -h(\mathbf{Y}_{2,\ell} \mid \mathbf{Y}_{2}^{\ell-1} S^{n} \mathbf{W}_{1} \mathbf{W}_{2} \mathbf{W}_{3} \mathbf{Z}_{\text{corr}}^{n}) \right] \\ \stackrel{\text{(c)}}{\leq} \sum_{\ell=1}^{n} \left[h(\mathbf{Y}_{2,\ell} \mid S_{\ell}) \\ &\quad -h(\mathbf{Y}_{2,\ell} \mid \mathbf{Y}_{2}^{\ell-1} S^{n} \mathbf{W}_{1} \mathbf{W}_{2} \mathbf{W}_{3} \mathbf{Z}_{\text{corr}}^{n} \mathbf{X}_{1,\ell} \mathbf{X}_{3,\ell}) \right] \\ \stackrel{\text{(d)}}{=} \sum_{\ell=1}^{n} \left[h(\mathbf{Y}_{2,\ell} \mid S_{\ell}) - h(\mathbf{Y}_{2,\ell} \mid S_{\ell} \mathbf{Z}_{\text{corr},\ell} \mathbf{X}_{1,\ell} \mathbf{X}_{3,\ell}) \right] \\ &= \sum_{\ell=1}^{n} I(\mathbf{Z}_{\text{corr},\ell} \mathbf{X}_{1,\ell} \mathbf{X}_{3,\ell}; \mathbf{Y}_{2,\ell} \mid S_{\ell}) \\ \stackrel{\text{(b)}}{=} \sum_{\ell=1}^{n} \left[I(\mathbf{X}_{1,\ell} \mathbf{X}_{3,\ell}; \mathbf{Y}_{2,\ell} \mid S_{\ell}) \\ &\quad + I(\mathbf{Z}_{\text{corr},\ell}; \mathbf{Y}_{2,\ell} \mid S_{\ell}) + \widetilde{I(\mathbf{Z}_{\text{corr},\ell}; \mathbf{Z}_{2,\ell} \mid S_{\ell})} \right] \\ \stackrel{\text{(e)}}{=} \sum_{\ell=1}^{n} \left[I(\mathbf{X}_{1,\ell} \mathbf{X}_{3,\ell}; \mathbf{Y}_{2,\ell} \mid S_{\ell}) + \widetilde{I(\mathbf{Z}_{\text{corr},\ell}; \mathbf{Z}_{2,\ell} \mid S_{\ell})} \right] \\ \stackrel{\text{(f)}}{=} n \left[\tau M_{2} + \overline{\tau} M_{3} \right] \log(\rho) + no \left[\log(\rho) \right] \end{split}$$

These steps are justified as follows:

- (a) (W_{12}, W_3) is independent of $(W_{13}, W_2, S^n, \mathbb{Z}_{corr}^n)$
- (b) Chain rule for mutual information
- (c) Conditioning reduces entropy
- (d) $\boldsymbol{Y}_{2,\ell}$ is independent of $(\boldsymbol{Y}_2^{\ell-1}, S^{\ell-1}, S_{\ell+1}^n, \boldsymbol{Z}_{\text{corr}}^{\ell-1}, \boldsymbol{Z}_{1,\ell}^n, \boldsymbol{W}_1, \boldsymbol{W}_2, \boldsymbol{W}_3)$ given $(S_\ell, \boldsymbol{Z}_{\text{corr},\ell}, \boldsymbol{X}_{1,\ell}, \boldsymbol{X}_{3,\ell})$
- (e) $I(\mathbf{Z}_{\text{corr},\ell}; \mathbf{Y}_{2,\ell} \mid S_{\ell} \mathbf{X}_{1,\ell} \mathbf{X}_{3,\ell}) = I(\mathbf{Z}_{\text{corr},\ell}; \mathbf{Z}_{2,\ell} \mid S_{\ell} \mathbf{X}_{1,\ell} \mathbf{X}_{3,\ell})$, and $(\mathbf{Z}_{\text{corr},\ell}, \mathbf{Z}_{2,\ell})$ is independent of $(\mathbf{X}_{1,\ell}, \mathbf{X}_{3,\ell})$ given S_{ℓ}
- (f) $(\mathbf{X}_{1,\ell}, \mathbf{X}_{3,\ell}) \rightsquigarrow \mathbf{Y}_{2,\ell}$ is a MIMO channel with $\min\{M_1 + M_3, M_2\} = M_2$ DoFs if $s_\ell = 1$, and $\min\{0+M_3, M_2\} = M_3$ DoFs if $s_\ell = 0$

Dividing both sides by $n \log(\rho)$ and letting $\rho, n \to \infty$ we

obtain

$$d_{12} + d_{32} + d_{31} \le \tau M_2 + \overline{\tau} M_3. \tag{42}$$

We turn to the second partial sum, developed around node 3 (Fig. 10b, with (i, j, k) = (1, 2, 3)). This bound is slightly more involved, as an additional type of side information is required which compensates for the small number of antennas M_3 . Which side information does node 3 need to be able to recover y_1^n and decode w_{21} from it, assuming a scheme allowing every node to decode its desired messages with high probability? A suitable process is depicted in Fig. 12. At the end of the transmission, node 3 has w_3 , y_3^n , s^n and x_3^n , as shown on the top left of the figure. Using the decoder \mathcal{F}_3 it can decode $(\hat{w}_{13}, \hat{w}_{23}) = (w_{13}, w_{23})$ with high probability. Assume we provide w_{12} as side information, so that node 3 can decode messages intended for node 1 using \mathcal{F}_1 as soon as it obtains y_1^n , as shown on the bottom left of the figure. Node 3 can now obtain $x_{1,1}$ from w_1 using $\mathcal{E}_{1,1}$. From $(s_1, y_{3,1})$ it can obtain $H_{23}x_{2,1} + z_{3,1}$ using $x_{1,1}$. Assume we 'virtually' increase the number of antennas at node 3 so that it can fully observe $x_{2,1}$ whenever node 1 can, by providing $\tilde{y}_{3,1} = \tilde{H}_{23}x_{2,1} + \tilde{z}_{3,1}$ as side information if $s_1 = 1$, with $\tilde{H}_{23} \in \mathbb{C}^{(M_2 - M_3) \times M_2}$ such that rank $\begin{pmatrix} H_{23} \\ \tilde{H}_{23} \end{pmatrix} = M_2$. Note that if $s_1 = 0$ then $x_{2,1}$ does not contribute to $y_{1,1}$. In this case, $x_{2,1}$ does not need to be reconstructed, and therefore no side information to compensate for insufficient number of antennas is required. We define shortcuts to group receive signal y_3^n and side information \tilde{y}_3^n into a joint signal \hat{y}_3^n , i.e.,

$$\hat{\boldsymbol{y}}_{3,1} \triangleq \begin{bmatrix} \boldsymbol{y}_{3,1} \\ \tilde{\boldsymbol{y}}_{3,1} \end{bmatrix}, \qquad \hat{\boldsymbol{H}}_{23} \triangleq \begin{bmatrix} \boldsymbol{H}_{23} \\ \tilde{\boldsymbol{H}}_{23} \end{bmatrix}, \qquad \hat{\boldsymbol{z}}_{3,1} \triangleq \begin{bmatrix} \boldsymbol{z}_{3,1} \\ \tilde{\boldsymbol{z}}_{3,1} \end{bmatrix}.$$

A matrix \hat{H}_{23} satisfying rank $(\hat{H}_{23}) = M_2$ exists almost surely and it allows to obtain $x_{2,1} + \hat{H}_{23}^{-1}\hat{z}_{3,1}$ if $s_1 = 1$. Assume we provide $z_{\text{corr},1} = z_{1,1} - s_1H_{21}\hat{H}_{23}^{-1}\hat{z}_{3,1}$ as side information. Then node 3 can obtain $y_{1,1}$ from $z_{\text{corr},1}$ if $s_1 = 0$, and from $x_{3,1}, x_{2,1} + \hat{H}_{23}^{-1}\hat{z}_{3,1}$ and $z_{\text{corr},1}$ if $s_1 = 1$, i.e.,

$$egin{aligned} m{y}_{1,1} &= s_1 m{H}_{21}(m{x}_{2,1} + m{H}_{23}^{-1} \hat{m{z}}_{3,1}) + s_1 m{H}_{31} m{x}_{3,1} + m{z}_{ ext{corr},1} \ &= s_1 m{H}_{21} m{x}_{2,1} + s_1 m{H}_{31} m{x}_{3,1} + m{z}_{1,1}. \end{aligned}$$

Using $(y_{1,1}, s_1, w_1)$ and the encoder $\mathcal{E}_{1,2}$ node 3 can obtain $x_{1,2}$ and the cycle repeats, for $\ell = 2, ..., n$. Finally, node 3 obtains y_1^n , and decodes w_{21} from (y_1^n, w_1, s^n) . Side information to compensate for intermittency is not required.

In a nutshell, side information w_{12} , \tilde{y}_3^n and z_{corr}^n is provided to node 3 by a genie, following the definitions

$$egin{aligned} & extbf{Y}_{3,\ell} \triangleq S_\ell(extbf{H}_{23} oldsymbol{X}_{2,\ell} + oldsymbol{\hat{Z}}_{3,\ell}), \ & extbf{Z}_{ ext{corr},\ell} \triangleq oldsymbol{Z}_{1,\ell} - S_\ell(oldsymbol{H}_{21} oldsymbol{\hat{H}}_{23}^{-1} oldsymbol{\hat{Z}}_{3,\ell}) \end{aligned}$$

where

$$\begin{split} \hat{\boldsymbol{H}}_{23} &\triangleq \begin{bmatrix} \boldsymbol{H}_{23} \\ \tilde{\boldsymbol{H}}_{23} \end{bmatrix}, \\ \tilde{\boldsymbol{H}}_{23} &\in \mathbb{C}^{(M_2 - M_3) \times M_2} \quad \text{such that } \operatorname{rank}(\hat{\boldsymbol{H}}_{23}) = M_2, \\ \tilde{\boldsymbol{Z}}_{3,\ell} &\sim \mathcal{CN}(\boldsymbol{0}, \sigma^2 \boldsymbol{I}_{M_2 - M_3}), \\ \hat{\boldsymbol{Z}}_{3,\ell} &\triangleq \begin{bmatrix} \boldsymbol{Z}_{3,\ell} \\ \tilde{\boldsymbol{Z}}_{3,\ell} \end{bmatrix}, \quad \hat{\boldsymbol{Y}}_{3,\ell} \triangleq \begin{bmatrix} \boldsymbol{Y}_{3,\ell} \\ \tilde{\boldsymbol{Y}}_{3,\ell} \end{bmatrix}. \end{split}$$

Since the scheme ought to be reliable, we again bound the sum rate of w_{13} , w_{23} and w_{21} using Fano's inequality, and following similar steps as before (for details see Appendix B) we obtain

$$d_{13} + d_{23} + d_{21} \le \tau M_2 + \overline{\tau} M_3. \tag{43}$$

Adding (42) and (43) yields a sum-DoF upper bound for the case $M_1 \ge M_2 \ge M_3$,

$$d_{\rm sum}^{\rm I} \le 2\tau M_2 + 2\overline{\tau} M_3. \tag{44}$$

b) Case 2: $M_2 \ge M_1 \ge M_3$: Since M_2 is the largest number of antennas, we develop two partial sums around nodes 3 and 1 (Fig. 10a and 10b, with (i, j, k) = (2, 3, 1)), respectively.

The reasoning around node 3 in this case proceeds in close analogy to the bound around node 3 in the previous case, just with 1 and 2 interchanged. We provide as side information w_{21} (to allow for decoding using \mathcal{F}_2), \tilde{y}_3^n (to compensate for small number of antennas M_3) and z_{corr}^n (a noise correction), defined as

$$\begin{split} \tilde{\mathbf{Y}}_{3,\ell} &\triangleq S_{\ell}(\hat{\mathbf{H}}_{13}\mathbf{X}_{1,\ell} + \hat{\mathbf{Z}}_{3,\ell}), \\ \mathbf{Z}_{\text{corr},\ell} &\triangleq \mathbf{Z}_{2,\ell} - S_{\ell}(\mathbf{H}_{12}\hat{\mathbf{H}}_{13}^{-1}\hat{\mathbf{Z}}_{3,\ell}). \end{split}$$

where

$$\begin{split} \hat{\boldsymbol{H}}_{13} &\triangleq \begin{bmatrix} \boldsymbol{H}_{13} \\ \tilde{\boldsymbol{H}}_{13} \end{bmatrix}, \\ \tilde{\boldsymbol{H}}_{13} &\in \mathbb{C}^{(M_1 - M_3) \times M_1} \quad \text{such that } \operatorname{rank}(\hat{\boldsymbol{H}}_{13}) = M_1, \\ \tilde{\boldsymbol{Z}}_{3,\ell} &\sim \mathcal{CN}(\boldsymbol{0}, \sigma^2 \boldsymbol{I}_{M_1 - M_3}), \\ \hat{\boldsymbol{Z}}_{3,\ell} &\triangleq \begin{bmatrix} \boldsymbol{Z}_{3,\ell} \\ \tilde{\boldsymbol{Z}}_{3,\ell} \end{bmatrix}, \qquad \hat{\boldsymbol{Y}}_{3,\ell} \triangleq \begin{bmatrix} \boldsymbol{Y}_{3,\ell} \\ \tilde{\boldsymbol{Y}}_{3,\ell} \end{bmatrix}. \end{split}$$

With this information, node 3 can construct $x_{2,1}$ from w_2 using $\mathcal{E}_{2,1}$ (since w_{23} is a message intended for node 3 and assumed to have been decoded from y_3^n using \mathcal{F}_3 , and w_{21} is side information), obtain a noisy version of $x_{1,1}$ from channel output $y_{3,1}$ and side information $\tilde{y}_{3,1}$ as necessary for $y_{2,1}$ (i.e., if $s_1 = 1$), combine all relevant signals into $y_{2,1}$, encode w_2 using $\mathcal{E}_{2,2}$ and $(y_{2,1}, s_1)$ to obtain $x_{2,2}$, and continue this cycle for the next $\ell = 2, ..., n$ until y_2^n is complete, from which w_{12} can be decoded with the help of w_2 using \mathcal{F}_2 .

After similar steps as before (see Appendix C for details) we obtain

$$d_{13} + d_{23} + d_{12} \le \tau M_1 + \overline{\tau} M_3. \tag{45}$$

We turn to the second partial sum, developed around node 1 (Fig. 10b, with (i, j, k) = (2, 3, 1)), where node 1 should be enabled to decode w_{32} . The main difference to the previous cases is that the link $3 \leftrightarrow 2$ is always available, while the link $3 \leftrightarrow 1$ is intermittent. Therefore, $y_{1,\ell}$ does not contain information about $x_{3,\ell}$ if $s_{\ell} = 0$; this needs to be compensated for by side information, here \tilde{y}_1^n defined as

$$\tilde{\boldsymbol{y}}_{1,\ell} \triangleq \overline{s}_{\ell}(\boldsymbol{H}_{31}\boldsymbol{x}_{3,\ell} + \tilde{\boldsymbol{z}}_{1,\ell}),$$

which provides measurements of x_3^n for those time instances where node 1 is intermittent, i.e., $s_\ell = 0$. This is an instance of the fourth type of side information, that for previous bounds was not necessary, namely side information that compensates

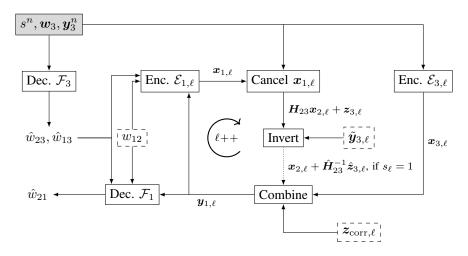


Fig. 12. Decoding of w_{21} at node 3 by iterative reconstruction of y_1^n from a-priori knowledge w_3 and observations (y_3^n, s^n) using side information $(w_{12}, \tilde{y}_3^n, \boldsymbol{z}_{corr}^n)$ (grey box: a-priori knowledge and channel output, dashed boxes: side information): successively obtain $\boldsymbol{x}_{1,\ell}$, cancel its effect from $\boldsymbol{y}_{3,\ell}$ (using side information $\tilde{\boldsymbol{y}}_{3,\ell}$) to obtain a noisy version of $\boldsymbol{x}_{2,\ell}$, and combine this with $\boldsymbol{x}_{3,\ell}$ and $\boldsymbol{z}_{corr,\ell}$ to finally obtain $\boldsymbol{y}_{1,\ell}$; repeat for next ℓ .

for intermittency. Given this side information and the customary side information (a message, to allow for decoding, and a noise correction signal), the reconstruction proceeds in analogy to the previous cases. In a nutshell, the genie provides w_{23} , \tilde{y}_1^n and z_{corr}^n to node 1, defined as

$$\begin{split} \tilde{\mathbf{Y}}_{1,\ell} &\triangleq \overline{S}_{\ell}(\mathbf{H}_{31}\mathbf{X}_{3,\ell} + \tilde{\mathbf{Z}}_{1,\ell}), \\ \mathbf{Z}_{\text{corr},\ell} &\triangleq \mathbf{Z}_{2,\ell} - \mathbf{H}_{32}\mathbf{H}_{31}^{\dagger}(S_{\ell}\mathbf{Z}_{1,\ell} + \overline{S}_{\ell}\tilde{\mathbf{Z}}_{1,\ell}), \end{split}$$

with

$$egin{aligned} oldsymbol{Z}_{1,\ell} &\sim \mathcal{CN}(oldsymbol{0}, \sigma^2 oldsymbol{I}_{M_1}) \ oldsymbol{\hat{Z}}_{1,\ell} &\triangleq (oldsymbol{Z}_{1,\ell}, \overline{S}_\ell oldsymbol{ ilde{Z}}_{1,\ell}), \ oldsymbol{\hat{Y}}_{1,\ell} &\triangleq (oldsymbol{Y}_{1,\ell}, oldsymbol{ ilde{Y}}_{1,\ell}). \end{aligned}$$

At the end of the transmission, node 1 has w_1 , y_1^n , s^n and x_1^n . It decodes (w_{21}, w_{31}) from y_1^n using its decoder \mathcal{F}_1 , and gets w_{23} from side information. It generates $x_{2,1}$, then uses its channel output $y_{1,1}$ (if $s_1 = 1$) or side information $\tilde{y}_{1,1}$ (if $s_1 = 0$) to obtain a noisy version of $x_{3,1}$, and with it $y_{2,1}$. From there the cycle repeats, until y_2^n is obtained and w_{32} can be decoded.

After similar steps as before (see Appendix D for details) we obtain

$$d_{21} + d_{31} + d_{32} \le \tau M_1 + \overline{\tau} M_3. \tag{46}$$

Adding (45) and (46) yields a sum-DoF upper bound for the case $M_2 \ge M_1 \ge M_3$,

$$d_{\rm sum}^{\rm I} \le 2\tau M_1 + 2\overline{\tau} M_3. \tag{47}$$

c) Case 3: $M_2 \ge M_3 \ge M_1$: Since M_2 is still the largest number of antennas, we again develop two partial sums around nodes 3 and 1 (Fig. 10a and 10b, with (i, j, k) = (2, 3, 1)), respectively. The only difference to the previous case is that this time $M_3 \ge M_1$, therefore the number of antennas at node 1 needs to be augmented 'virtually' to obtain sufficient measurements of x_3^n , while node 3 remains unchanged. We turn to the partial sum around node 3 and provide as side information w_{21} and z_{corr}^n with

$$\boldsymbol{Z}_{\mathrm{corr},\ell} \triangleq \boldsymbol{Z}_{2,\ell} - S_{\ell}(\boldsymbol{H}_{12}\boldsymbol{H}_{13}^{\dagger}\boldsymbol{Z}_{3,\ell})$$

Note that $x_{1,\ell}$ contributes to $y_{2,\ell}$ only if $s_{\ell} = 1$. In these instances, node 3 has sufficient information about $x_{1,\ell}$ from $y_{3,\ell}$. If $s_{\ell} = 0$, node 3 does not have information about $x_{1,\ell}$, but $x_{1,\ell}$ does not contribute to $y_{2,\ell}$ anyhow, so node 3 does not need additional side information in these cases. Therefore, with the given side information, node 3 can construct $x_{2,1}$ from w_2 , obtain a noisy version of $x_{1,1}$ from channel output $y_{3,1}$ as necessary for $y_{2,1}$ (i.e., if $s_1 = 1$), generate $y_{2,1}$, and continue this cycle for the next $\ell = 2, ..., n$ until y_2^n is complete, from which w_{12} can be decoded with the help of w_2 .

After similar steps as before (see Appendix E for details) we obtain

$$d_{13} + d_{23} + d_{12} \le M_3. \tag{48}$$

We turn to the second partial sum, developed around node 1 (Fig. 10b, with (i, j, k) = (2, 3, 1)), where node 1 should be enabled to decode w_{32} . Again the main difference to the previous cases is that the link $3 \leftrightarrow 2$ is always available, while the link $3 \leftrightarrow 1$ is intermittent. Therefore, $y_{1,\ell}$ does not contain information about $x_{3,\ell}$ if $s_{\ell} = 0$; this effect of intermittency needs to be compensated for by side information, here \tilde{y}_1^n (a formal definition follows). Furthermore, the number of antennas at node 1 needs to be increased to fully capture $x_{3,\ell}$, here accomplished by side information \check{y}_1^n (a formal definition follows). In addition, the genie provides the message w_{23} and noise correction z_{corr}^n to node 1. In a nutshell, side information variables are defined as

$$\begin{split} \tilde{\boldsymbol{Y}}_{1,\ell} &\triangleq \overline{S}_{\ell}(\boldsymbol{H}_{31}\boldsymbol{X}_{3,\ell} + \tilde{\boldsymbol{Z}}_{1,\ell}), \\ \tilde{\boldsymbol{Y}}_{1,\ell} &\triangleq \boldsymbol{\check{H}}_{31}\boldsymbol{X}_{3,\ell} + \boldsymbol{\check{Z}}_{1,\ell}, \\ \boldsymbol{Z}_{\text{corr},\ell} &\triangleq \boldsymbol{Z}_{2,\ell} - \boldsymbol{H}_{32}\hat{\boldsymbol{H}}_{31}^{-1} \begin{bmatrix} S_{\ell}\boldsymbol{Z}_{1,\ell} + \overline{S}_{\ell}\tilde{\boldsymbol{Z}}_{1,\ell} \\ \boldsymbol{\check{Z}}_{1,\ell} \end{bmatrix} \end{split}$$

with auxiliary variables

$$\tilde{Z}_{1,\ell} \sim \mathcal{CN}(\mathbf{0}, \sigma^2 I_{M_1}),$$

$$\begin{split} \check{\boldsymbol{Z}}_{1,\ell} &\sim \mathcal{CN}(\boldsymbol{0}, \sigma^2 \boldsymbol{I}_{M_3-M_1}), \\ \hat{\boldsymbol{H}}_{31} \triangleq \begin{bmatrix} \boldsymbol{H}_{31} \\ \check{\boldsymbol{H}}_{31} \end{bmatrix}, \\ \check{\boldsymbol{H}}_{31} \in \mathbb{C}^{(M_3-M_1)\times M_3} \quad \text{such that } \operatorname{rank}(\hat{\boldsymbol{H}}_{31}) = M_3, \\ \hat{\boldsymbol{Y}}_{1,\ell} \triangleq (\boldsymbol{Y}_{1,\ell}, \tilde{\boldsymbol{Y}}_{1,\ell}, \check{\boldsymbol{Y}}_{1,\ell}), \\ \hat{\boldsymbol{Z}}_{1,\ell} \triangleq (\boldsymbol{Z}_{1,\ell}, \overline{S}_\ell \tilde{\boldsymbol{Z}}_{1,\ell}, \check{\boldsymbol{Z}}_{1,\ell}). \end{split}$$

At the end of the transmission, node 1 has w_1 , y_1^n , s^n and x_1^n . It decodes (w_{21}, w_{31}) using \mathcal{F}_1 , and gets w_{23} from side information. It generates $x_{2,1}$ using $\mathcal{E}_{2,1}$, then uses side information $\breve{y}_{1,1}$ and channel output $y_{1,1}$ (if $s_1 = 1$) or side information $\breve{y}_{1,1}$ and $\tilde{y}_{1,1}$ (if $s_1 = 0$) to obtain a noisy version of $x_{3,1}$, and with it $y_{2,1}$ using $z_{\text{corr},1}$. From there the cycle repeats, until y_2^n is obtained and w_{32} can be decoded using \mathcal{F}_2 .

After similar steps as before (see Appendix F for details) we obtain

$$d_{21} + d_{31} + d_{32} \le M_3. \tag{49}$$

Adding (48) and (49) yields a sum-DoF upper bound for the case $M_2 \ge M_3 \ge M_1$,

$$d_{\rm sum}^{\rm l} \le 2M_3. \tag{50}$$

Lemma 3 follows from (44), (47), (50), and symmetry of node 2 and 3. The achievability and converse results developed in Sections IV-B1 and IV-B2 establish the sum-DoF of the intermittent 3WC and prove Theorem 2.

Note that neither our achievability nor our converse result requires the noise at different receivers to be uncorrelated. This complies with the intuition that the DoF perspective captures the impact of interference rather than noise. For $\rho \to \infty$, usable DoFs become practically noise-free, hence it is also insignificant whether the noise is correlated or not.

C. Necessity of Genie-Aided Upper Bounds

To underline the necessity of the genie-aided upper bounds devised in Section IV-B2, we show that 'classic' cut-set type bounds [49] admit DoF tuples d = $(d_{12}, d_{13}, d_{21}, d_{23}, d_{31}, d_{32})$ that strictly exceed the sum-DoF of the intermittent 3WC stated in Theorem 2. For the intermittent 3WC with $M_1 \ge M_2 + M_3 \ge M_2 \ge M_3$, the cut-set bounds read

$$\max\{d_{12} + d_{13}, d_{21} + d_{31}\} \le \tau(M_2 + M_3), \quad \text{(Cut node 1)} \\ \max\{d_{21} + d_{23}, d_{12} + d_{32}\} \le \tau M_2 + \overline{\tau} M_3, \quad \text{(Cut node 2)}$$

$$\max\{d_{31} + d_{32}, d_{13} + d_{23}\} \le M_3.$$
 (Cut node 3)

Let $M_2 = M_3 = 2$, $\tau = \overline{\tau} = \frac{1}{2}$, $M_1 = M_2 + M_3 = 4$. Then, the cut-set bounds admit d = (1, 1, 1, 1, 1, 1), with a sum-DoF of 6. However, according to Theorem 2, for $M_1 \geq$ $M_2 + M_3 \ge M_2 \ge M_3,$

$$d_{\rm sum}^{\rm l} = 2\tau M_2 + 2\overline{\tau} M_3,$$

in the example at hand, $d_{sum}^{I} = 4$. Hence, cut-set type bounds are too loose to characterize the sum-DoF of the intermittent 3WC. The genie-aided upper bounds devised in Section IV-B2 fill this gap.

D. DoF Region

In this section we first derive an upper bound on d_{31} under the assumption of non-adaptive encoding. We then show that adaptive schemes can exceed this bound, e.g., decodeforward relaying. This proves that some DoF region points are only achievable by adaptive encoding schemes, hence adaptive encoding is in general required to achieve the DoF region of the intermittent 3WC. Note that we leave design and analysis of comprehensive adaptive encoding schemes for future work and prove our claim by means of minimal counterexamples. For the counterexample we may assume $M_1 \ge M_2 \ge M_3$.

1) Upper Bound on d_{31} under Non-Adaptive Encoding: Node 1 is able to decode w_{31} from its channel output (y_1^n, s^n) and a-priori knowledge w_1 with high probability. We further provide x_2^n as side information and bound the rate of w_{31} using Fano's inequality:

$$n(R_{31} - \varepsilon_n)$$
side information
$$\leq I(W_{31}; \boldsymbol{W}_1 \boldsymbol{Y}_1^n S^n \quad \boldsymbol{X}_2^n)$$

$$\stackrel{(a)}{=} I(W_{31}; \boldsymbol{Y}_1^n \mid \boldsymbol{W}_1 \boldsymbol{X}_2^n S^n)$$

$$\stackrel{(b)}{=} \sum_{\ell=1}^n I(W_{31}; \boldsymbol{Y}_{1,\ell} \mid \boldsymbol{Y}_1^{\ell-1} \boldsymbol{W}_1 \boldsymbol{X}_2^n S^n)$$

$$= \sum_{\ell=1}^n \left[h(\boldsymbol{Y}_{1,\ell} \mid \boldsymbol{Y}_1^{\ell-1} \boldsymbol{W}_1 \boldsymbol{X}_2^n S^n) - h(\boldsymbol{Y}_{1,\ell} \mid \boldsymbol{Y}_1^{\ell-1} \boldsymbol{W}_1 \boldsymbol{X}_2^n S^n \boldsymbol{W}_{31}) \right]$$

$$\stackrel{(c)}{\leq} \sum_{\ell=1}^n \left[h(\boldsymbol{Y}_{1,\ell} \mid \boldsymbol{X}_{2,\ell} S_\ell) - h(\boldsymbol{Y}_{1,\ell} \mid \boldsymbol{X}_2 S^n \boldsymbol{W}_{31} \boldsymbol{X}_{3,\ell}) \right]$$

$$\stackrel{(d)}{=} \sum_{\ell=1}^n \left[h(\boldsymbol{Y}_{1,\ell} \mid \boldsymbol{X}_{2,\ell} S_\ell) - h(\boldsymbol{Y}_{1,\ell} \mid \boldsymbol{X}_{2,\ell} S_\ell) - h(\boldsymbol{Y}_{1,\ell} \mid \boldsymbol{X}_{2,\ell} S_\ell) \right]$$

$$= \sum_{\ell=1}^n I(\boldsymbol{X}_{3,\ell}; \boldsymbol{Y}_{1,\ell} \mid \boldsymbol{X}_{2,\ell} S_\ell)$$

$$\stackrel{(e)}{\leq} n\tau M_3 \log(\rho) + n\tau o [\log(\rho)] \qquad (51)$$

These steps are justified as follows:

- (a) W_{31} is independent of $(\boldsymbol{W}_1, \boldsymbol{X}_2^n, S^n)$ due to non-adaptive encoding $X_2^n = \mathcal{E}_2(W_2)$
- (b) Chain rule for mutual information
- (c) Conditioning reduces entropy
- (d) $Y_{1,\ell}$ is independent of $(Y_1^{\ell-1}, W_1, X_{2,1}^{\ell-1}, X_{2,\ell+1}^n, S^{\ell-1}, S_{\ell+1}^{n}, W_{31})$ given $(S_\ell, X_{2,\ell}, X_{3,\ell})$
- (e) $X_{3,\ell} \rightsquigarrow Y_{1,\ell}$ given $X_{2,\ell}$ is a MIMO channel with maximum DoF M_3 and 0 for $s_\ell = 1$ and $s_\ell = 0$, respectively

Dividing both sides of (51) by $n \log(\rho)$ and taking $\rho, n \rightarrow$ ∞ , we see that the achievable DoF tuples of non-adaptive encoding schemes are constrained by

$$d_{31} \le \tau M_3. \tag{52}$$

2) Adaptive Schemes Achieving $d_{31} > \tau M_3$: We assume all messages are fixed to 0, except for w_{31} , which node 3 wants to convey to node 1, potentially with the help of node 2. Consider using decode-forward relaying at node 2. This can be used to achieve min{ $\tau M_1, \tau (M_2 + M_3), M_3$ } DoFs and outperforms any non-adaptive scheme as soon as $M_1 > M_3$. We derive the achievable DoF for d_{31} based on the well-known lower bound for decode-forward relaying [50]:

$$C \ge \max_{p_{\mathbf{X}_3\mathbf{X}_2}} \min\{I(\mathbf{X}_3\mathbf{X}_2; \mathbf{Y}_1S), I(\mathbf{X}_3; \mathbf{Y}_2S \mid \mathbf{X}_2)\}$$

$$\ge \min\{I(\mathbf{X}_3\mathbf{X}_2; \mathbf{Y}_1 \mid S), I(\mathbf{X}_3; \mathbf{Y}_2 \mid \mathbf{X}_2)\}$$

with $\mathbf{X}_2, \mathbf{X}_3$ Gaussian
min (second (M + M)), M) leg(s) + s (leg(s))

$$= \min\{\tau \min\{M_1, (M_2 + M_3)\}, M_3\} \log(\rho) + o[\log(\rho)]$$

 $= \min\{\tau M_1, \tau (M_2 + M_3), M_3\} \log(\rho) + o[\log(\rho)]$ (53)

Dividing both sides of (53) by $\log(\rho)$ and taking $\rho \to \infty$, we see that the decode-forward relaying achieves

$$d_{31} \ge \min\{\tau M_1, \tau (M_2 + M_3), M_3\}.$$
 (54)

Note that if $\tau M_2 > \overline{\tau} M_3$ and $\tau M_1 > M_3$, then we can transmit at M_3 DoF from node 3 to node 1 using this adaptive scheme, compensating all the the loss due to intermittency.

We proved in (52) that the DoF region point

$$\boldsymbol{d} = (0, 0, 0, 0, d_{31,A}, 0)$$
$$d_{31,A} \triangleq \min\{\tau M_1, \tau (M_2 + M_3), M_3\}$$

is not achievable for any non-adaptive encoding scheme if $M_1 > M_3$, while we showed in (54) that there exist adaptive schemes that achieve it. This proves Theorem 3, which states that adaptive encoding is in general required to achieve the DoF region of the intermittent 3WC.

Theorems 2 and 3 show that non-adaptive encoding is sufficient to achieve sum-DoF, but not sufficient to achieve the DoF region of the intermittent 3WC. This is particularly interesting in light of the next section, where we show that adaptive encoding is not beneficial in the non-intermittent 3WC even from a DoF region perspective.

V. NO INTERMITTENCY

The sum-DoF of the non-intermittent 3WC was investigated in [18]. We present the DoF region of the non-intermittent 3WC and show that the non-adaptive encoding scheme presented in Section IV-A is sufficient to achieve it; therefore, adaptive encoding is neither required from a sum-DoF nor from a DoF region perspective in the non-intermittent 3WC. The non-intermittent 3WC is a special case of the intermittent 3WC with $\tau = 1$. We may assume without loss of generality $M_1 \ge M_2 \ge M_3.$

A. Achievability

From (35) to (41) we obtain with $\tau = 1$:

$$d_{12} + d_{13} + d_{23} \le M_1 \tag{55}$$

$$d_{12} + d_{13} + d_{32} \le M_1 \tag{56}$$

$$d_{21} + d_{31} + d_{32} \le M_1 \tag{57}$$

$$d_{21} + d_{31} + d_{23} \le M_1 \tag{58}$$

$$d_{21} + d_{13} + d_{23} \le M_2 \tag{59}$$

$$d_{12} + d_{31} + d_{32} \le M_2 \tag{60}$$

$$\frac{d_{31} + d_{32} \le M_2}{d_{31} + d_{32} \le M_3} \tag{61}$$

$$d_{13} + d_{23} \le M_3 \tag{62}$$

(50)

$$\min\{d_{12}, d_{13}, d_{21}, d_{23}, d_{31}, d_{32}\} \ge 0 \tag{63}$$

All DoF tuples d satisfying constraints (55) to (63) are achievable in the non-intermittent 3WC. Therefore, by construction in Section IV-A, said set of inequalities constitutes an inner bound on the DoF region of the non-intermittent 3WC. We show in the following that the parametrization of the ZF/IA/EC-based scheme presented in Section IV-A is sufficiently general to capture the whole DoF region of the non-intermittent 3WC. Note that only in the generality elaborated in this paper does the ZF/IA-based scheme achieve the DoF region of the non-intermittent 3WC (instead of just its sum-DoF as in [18]).

B. Converses

Previous works studied the sum-DoF of different variants of the non-intermittent 3WC. To this end, several bounds are reported in the literature, among them cut-set outer bounds [49] on pairs of DoFs (involving two messages either intended for or originating at a certain node), and tighter genie-aided outer bounds on triplets of DoFs (involving two messages either intended for or originating at a certain node, and one message exchanged between the remaining two nodes). The latter bounding technique is due to [18], [19] and was later taken up in [21], [47].

$$R_{13} + R_{23} \le M_3 \log(\rho) + o [\log(\rho)]$$
(see [18, (7)], [49]) (64)

$$R_{31} + R_{32} \le M_3 \log(\rho) + o [\log(\rho)]$$

(see [18, (8)], [49]) (65)

 $R_{21} + R_{31} + R_{32} \le \min\{M_1, M_2 + M_3\}\log(\rho) + o\left[\log(\rho)\right]$

(see [21, (23)], [47, (25)]) (66)

 $R_{21} + R_{31} + R_{23} \le \min\{M_1, M_2 + M_3\}\log(\rho) + o\left[\log(\rho)\right]$ (see [21, (25)], [47, (27)]) (67)

$$R_{12} + R_{32} + R_{13} \le M_1 \log(\rho) + o\left[\log(\rho)\right]$$
(c22 [47 (20)]) (69)

$$(\sec [47, (29)]) \quad (68)$$
$$R_{13} + R_{23} + R_{12} \le M_1 \log(\rho) + o [\log(\rho)]$$

$$(\text{see } [47, (31)])$$
 (69)

$$R_{12} + R_{32} + R_{31} \le M_2 \log(\rho) + o \left[\log(\rho)\right]$$

(see (42), [18, (15)], [47, (28)]) (70)

$$R_{13} + R_{23} + R_{21} \le M_2 \log(\rho) + o \left[\log(\rho) \right]$$

(see (43), [18, (11)], [47, (30)]) (71)

Note that if $M_2 + M_3 \leq M_1$, then (66) and (67) are redundant given (65) and (71), therefore $\min\{M_1, M_2 + M_3\}$ can be replaced with M_1 in (66) and (67). Dividing these bounds by $\log(\rho)$ and taking $\rho \to \infty$ yields to the following DoF region outer bounds:

$$d_{21} + d_{31} + d_{32} \le M_1 \tag{72}$$

$$d_{21} + d_{31} + d_{23} \le M_1 \tag{73}$$

$$d_{12} + d_{32} + d_{13} \le M_1 \tag{74}$$

$$d_{13} + d_{23} + d_{12} \le M_1 \tag{75}$$

$$d_{12} + d_{32} + d_{31} \le M_2 \tag{76}$$

$$d_{13} + d_{23} + d_{21} \le M_2 \tag{77}$$

$$d_{13} + d_{23} \le M_3 \tag{78}$$

$$d_{31} + d_{32} \le M_3 \tag{79}$$

No DoF tuple d violating any of the constraints (72) to (79) can be achievable in the non-intermittent 3WC. Therefore, said set of inequalities constitutes an outer bound on the DoF region of the non-intermittent 3WC.

C. DoF Region and Sum-DoF of Non-Intermittent 3WC

The previous achievability and converse results establish the DoF region (and thus sum-DoF) optimality of non-adaptive schemes. In particular, the scheme introduced in Section IV-A is DoF region and sum-DoF optimal. This renders adaptive encoding dispensable for the non-intermittent 3WC and proves Theorem 4. From the DoF region of the 3WC and using the sum-DoF of the intermittent 3WC, we reproduce the sum-DoF of the 3WC given in [18]:

Corollary 1 (Sum-DoF of Non-Intermittent 3WC).

$$d_{\rm sum}^{\rm N} = 2M_2$$

Proof. The statement follows from Theorems 2 and 4.

VI. CONCLUSION

We introduced the MIMO 3WC with node-intermittency and studied its DoF region and sum-DoF. In particular, we devised a non-adaptive encoding scheme based on zero-forcing, interference alignment and erasure coding, and showed its DoF region (and thus sum-DoF) optimality for non-intermittent 3WCs and its sum-DoF optimality for node-intermittent 3WCs. This shows that adaptive encoding is not required in those cases. However, we showed by example that in general there are DoF region points in the node-intermittent 3WC that can only be achieved by adaptive schemes, such as decodeforward relaying, making adaptive encoding a necessity. Our work contributes to a better understanding of the necessity of adaptive schemes such as relaying in multi-way communications with intermittency.

As remarked in the introduction, node-intermittency is only one of a multitude of practically relevant intermittency scenarios. Links might be intermittent independently of each other, e.g., moving objects passing by only interrupt the link between the two D2D users from time to time, while the other links remain intact. Or all links being intermittent, but independently of each other, and with different probabilities. Here, we speak of link intermittency and intermittent links. Intermittent 3WCs with other intermittency models are interesting directions for future research.

APPENDIX

A. Proof Template for Sum-DoF Upper Bounds for Intermittent 3WC

Throughout the derivations of sum-DoF upper bounds for the intermittent 3WC, certain steps reappear in slight variations. To avoid repetition, we formulate the following 'proof template', where the Fraktur variables \mathfrak{W}_A , \mathfrak{W}_B , \mathfrak{X} , \mathfrak{Y} , and **3** serve as placeholders and need to be replaced by random variables as specified in the context of the template's invocation. We require that

- (a) \mathfrak{W}_{A} is independent of $(\mathfrak{W}_{B}, S^{n}, \mathbb{Z}_{corr}^{n})$, (b) \mathfrak{Y}_{ℓ} is independent of $(\mathfrak{Y}^{\ell-1}, S^{\ell-1}, S_{\ell+1}^{n}, \mathbb{Z}_{corr}^{\ell-1})$ $Z_{\operatorname{corr},\ell+1}^n, W_1, W_2, W_3)$ given $(S_\ell, Z_{\operatorname{corr},\ell}, \mathfrak{X}_\ell),$
- (c) $I(\mathbf{Z}_{\operatorname{corr},\ell}; \mathfrak{Y}_{\ell} \mid S_{\ell}\mathfrak{X}_{\ell}) = I(\mathbf{Z}_{\operatorname{corr},\ell}; \mathfrak{Z}_{\ell} \mid S_{\ell}\mathfrak{X}_{\ell})$, and $(\mathbf{Z}_{\operatorname{corr},\ell},\mathbf{\mathfrak{Z}}_{\ell})$ is independent of \mathfrak{X}_{ℓ} given S_{ℓ} ,

(d)
$$I(\mathbf{Z}_{\operatorname{corr},\ell}; \mathbf{\mathfrak{Z}}_{\ell} \mid S_{\ell}) = o [\log(\rho)].$$

Note that these preconditions are satisfied for every invocation of the template in this paper. Then we have

$$\begin{split} I(\mathfrak{W}_{A};\mathfrak{W}_{B}\mathfrak{Y}^{n}S^{n}Z_{\text{corr}}^{n}) \\ \stackrel{(a)}{=} I(\mathfrak{W}_{A};\mathfrak{Y}^{n} \mid \mathfrak{W}_{B}S^{n}Z_{\text{corr}}^{n}) \\ &= \sum_{\ell=1}^{n} I(\mathfrak{W}_{A};\mathfrak{Y}_{\ell} \mid \mathfrak{Y}^{\ell-1}\mathfrak{W}_{B}S^{n}Z_{\text{corr}}^{n}) \\ &= \sum_{\ell=1}^{n} \left[h(\mathfrak{Y}_{\ell} \mid \mathfrak{Y}^{\ell-1}\mathfrak{W}_{B}S^{n}Z_{\text{corr}}^{n}) \\ &- h(\mathfrak{Y}_{\ell} \mid \mathfrak{Y}^{\ell-1}\mathfrak{W}_{1}W_{2}W_{3}S^{n}Z_{\text{corr}}^{n})\right] \\ &\leq \sum_{\ell=1}^{n} \left[h(\mathfrak{Y}_{\ell} \mid S_{\ell}) \\ &- h(\mathfrak{Y}_{\ell} \mid \mathfrak{Y}^{\ell-1}W_{1}W_{2}W_{3}\mathfrak{X}_{\ell}S^{n}Z_{\text{corr}}^{n})\right] \\ \stackrel{(b)}{=} \sum_{\ell=1}^{n} \left[h(\mathfrak{Y}_{\ell} \mid S_{\ell}) - h(\mathfrak{Y}_{\ell} \mid S_{\ell}Z_{\text{corr},\ell}\mathfrak{X}_{\ell})\right] \\ &= \sum_{\ell=1}^{n} I(Z_{\text{corr},\ell}\mathfrak{X}_{\ell};\mathfrak{Y}_{\ell} \mid S_{\ell}) \\ &= \sum_{\ell=1}^{n} \left[I(\mathfrak{X}_{\ell};\mathfrak{Y}_{\ell} \mid S_{\ell}) + I(Z_{\text{corr},\ell};\mathfrak{Y}_{\ell} \mid S_{\ell}\mathfrak{X}_{\ell})\right] \\ \stackrel{(c)}{=} \sum_{\ell=1}^{n} \left[I(\mathfrak{X}_{\ell};\mathfrak{Y}_{\ell} \mid S_{\ell}) + I(Z_{\text{corr},\ell};\mathfrak{Z}_{\ell} \mid S_{\ell})\right] \\ \stackrel{(d)}{=} \sum_{\ell=1}^{n} I(\mathfrak{X}_{\ell};\mathfrak{Y}_{\ell} \mid S_{\ell}) + no\left[\log(\rho)\right] \end{split}$$

where the letters indicate the precondition that justifies each step.

B. Sum-DoF Upper Bound for Intermittent 3WC with $M_1 \ge$ $M_2 \geq M_3$ (Part II)

Since the scheme ought to be reliable, we bound the sum rate of w_{13} , w_{23} and w_{21} using Fano's inequality:

$$n(R_{13} + R_{23} + R_{21} - \varepsilon_n^{(2)})$$

$$\leq I(W_{13}W_{23}W_{21}; \boldsymbol{W}_{3}\boldsymbol{Y}_{3}^{n}S^{n} \overbrace{W_{12}\tilde{\boldsymbol{Y}}_{3}^{n}\boldsymbol{Z}_{corr}^{n}}^{\text{side information}})$$

$$\leq \sum_{\ell=1}^{n} I(\boldsymbol{X}_{1,\ell}\boldsymbol{X}_{2,\ell}; \hat{\boldsymbol{Y}}_{3,\ell} \mid S_{\ell}) + no\left[\log(\rho)\right]$$

$$\stackrel{\text{(b)}}{\leq} n\left[\tau M_{2} + \overline{\tau}M_{3}\right]\log(\rho) + no\left[\log(\rho)\right]$$

These steps are justified as follows:

- (a) Using the proof template presented in Appendix A, with $\mathfrak{W}_{A} \triangleq (W_{13}, W_{21}, W_{23}), \mathfrak{W}_{B} \triangleq (W_{12}, W_{31}, W_{32}), \mathfrak{X} \triangleq (X_1, X_2), \mathfrak{Y} \triangleq \hat{Y}_3, \mathfrak{Z} \triangleq \hat{Z}_3$
- (b) $(X_{1,\ell}, X_{2,\ell}) \rightsquigarrow (Y_{3,\ell}, Y_{3,\ell})$ is a MIMO channel with $\min\{M_1 + M_2, M_3 + (M_2 - M_3)\} = M_2$ DoFs if $s_\ell = 1$, and $\min\{M_1 + M_2, M_3 + 0\} = M_3$ DoFs if $s_\ell = 0$

Dividing both sides by $n\log(\rho)$ and letting $\rho,n\to\infty$ we obtain

$$d_{13} + d_{23} + d_{21} \le \tau M_2 + \overline{\tau} M_3.$$

C. Sum-DoF Upper Bound for Intermittent 3WC with $M_2 \ge M_1 \ge M_3$ (Part I)

Since this scheme ought to be reliable, we bound the sum rate of w_{13} , w_{23} and w_{12} using Fano's inequality:

$$n(R_{13} + R_{23} + R_{12} - \varepsilon_n^{(1)}) \leq I(W_{13}W_{23}W_{12}; W_3Y_3^n S^n W_{21}\tilde{Y}_3^n Z_{\text{corr}}^n) \\ \stackrel{\text{(a)}}{\leq} \sum_{\ell=1}^n I(X_{1,\ell}X_{2,\ell}; \hat{Y}_{3,\ell} \mid S_\ell) + no [\log(\rho)] \\ \stackrel{\text{(b)}}{\leq} n [\tau M_1 + \overline{\tau} M_3] \log(\rho) + no [\log(\rho)]$$

These steps are justified as follows:

- (a) Using the proof template presented in Appendix A, with $\mathfrak{W}_{A} \triangleq (W_{12}, W_{13}, W_{23}), \mathfrak{W}_{B} \triangleq (W_{21}, W_{31}, W_{32}), \mathfrak{X} \triangleq (X_{1}, X_{2}), \mathfrak{Y} \triangleq \hat{Y}_{3}, \mathfrak{Z} \triangleq \hat{Z}_{3}$
- (b) $(X_{1,\ell}, X_{2,\ell}) \rightsquigarrow (Y_{3,\ell}, \tilde{Y}_{3,\ell})$ is a MIMO channel with $\min\{M_1 + M_2, M_3 + (M_1 M_3)\} = M_1$ DoFs if $s_\ell = 1$, and $\min\{M_1 + M_2, M_3 + 0\} = M_3$ DoFs if $s_\ell = 0$

Dividing both sides by $n\log(\rho)$ and letting $\rho,n\to\infty$ we obtain

$$d_{13} + d_{23} + d_{12} \le \tau M_1 + \overline{\tau} M_3.$$

D. Sum-DoF Upper Bound for Intermittent 3WC with $M_2 \ge M_1 \ge M_3$ (Part II)

We bound the sum rate of w_{21} , w_{31} and w_{32} using Fano's inequality:

$$n(R_{21} + R_{31} + R_{32} - \varepsilon_n^{(2)}) \\ \leq I(W_{21}W_{31}W_{32}; \boldsymbol{W}_1\boldsymbol{Y}_1^n S^n \underbrace{W_{23}\tilde{\boldsymbol{Y}}_1^n \boldsymbol{Z}_{corr}^n}_{W_{23}\tilde{\boldsymbol{Y}}_1^n \boldsymbol{Z}_{corr}^n}) \\ \stackrel{(a)}{\leq} \sum_{\ell=1}^n I(\boldsymbol{X}_{2,\ell}\boldsymbol{X}_{3,\ell}; \hat{\boldsymbol{Y}}_{1,\ell} \mid S_\ell) + no [\log(\rho)] \\ \stackrel{(b)}{=} \sum_{\ell=1}^n \left[I(\boldsymbol{X}_{2,\ell}\boldsymbol{X}_{3,\ell}; \boldsymbol{Y}_{1,\ell} \mid S_\ell) \right]$$

$$+ I(\boldsymbol{X}_{2,\ell}\boldsymbol{X}_{3,\ell}; \tilde{\boldsymbol{Y}}_{1,\ell} \mid S_{\ell}\boldsymbol{Y}_{1,\ell}) \Big] + no\left[\log(\rho)\right]$$

$$\stackrel{\text{(c)}}{\leq} n\left[\tau M_1 + \overline{\tau} M_3\right] \log(\rho) + no\left[\log(\rho)\right]$$

These steps are justified as follows:

- (a) Using the proof template presented in Appendix A, with $\mathfrak{W}_{A} \triangleq (W_{21}, W_{31}, W_{32}), \mathfrak{W}_{B} \triangleq (W_{12}, W_{13}, W_{23}), \mathfrak{X} \triangleq (\mathbf{X}_{2}, \mathbf{X}_{3}), \mathfrak{Y} \triangleq \hat{\mathbf{Y}}_{1}, \mathfrak{Z} \triangleq \hat{\mathbf{Z}}_{1}$
- (b) Chain rule for mutual information
- (c) $(\mathbf{X}_{2,\ell}, \mathbf{X}_{3,\ell}) \rightsquigarrow \mathbf{Y}_{1,\ell}$ is a MIMO channel with $\min\{M_2 + M_3, M_1\} = M_1$ DoFs if $s_\ell = 1$, and 0 DoFs if $s_\ell = 0$; $(\mathbf{X}_{2,\ell}, \mathbf{X}_{3,\ell}) \rightsquigarrow \tilde{\mathbf{Y}}_{1,\ell}$ is a MIMO channel with 0 DoFs if $s_\ell = 1$ (because then $\tilde{\mathbf{Y}}_{1,\ell} = 0$), and $\min\{0+M_3, M_1\} = M_3$ DoFs if $s_\ell = 0$ (because then $\mathbf{Y}_{1,\ell}$ is noise, and $\tilde{\mathbf{Y}}_{1,\ell}$ is independent of $X_{2,\ell}$)

Dividing both sides by $n\log(\rho)$ and letting $\rho,n\to\infty$ we obtain

$$d_{21} + d_{31} + d_{32} \le \tau M_1 + \overline{\tau} M_3.$$

E. Sum-DoF Upper Bound for Intermittent 3WC with $M_2 \ge M_3 \ge M_1$ (Part I)

We bound the sum rate of w_{13} , w_{23} and w_{12} using Fano's inequality:

$$n(R_{13} + R_{23} + R_{12} - \varepsilon_n^{(1)})$$

$$\leq I(W_{13}W_{23}W_{12}; \boldsymbol{W}_3\boldsymbol{Y}_3^n S^n \underbrace{W_{21}\boldsymbol{Z}_{corr}^n}_{W_{21}\boldsymbol{Z}_{corr}^n})$$

$$\stackrel{(a)}{\leq} \sum_{\ell=1}^n I(\boldsymbol{X}_{1,\ell}\boldsymbol{X}_{2,\ell}; \boldsymbol{Y}_{3,\ell} \mid S_\ell) + no [\log(\rho)]$$

$$\stackrel{(b)}{\leq} n [M_3] \log(\rho) + no [\log(\rho)]$$

These steps are justified as follows:

- (a) Using the proof template presented in Appendix A, with $\mathfrak{W}_{A} \triangleq (W_{12}, W_{13}, W_{23}), \mathfrak{W}_{B} \triangleq (W_{21}, W_{31}, W_{32}), \mathfrak{X} \triangleq (X_{1}, X_{2}), \mathfrak{Y} \triangleq Y_{3}, \mathfrak{Z} \triangleq Z_{3}$
- (b) $(X_{1,\ell}, X_{2,\ell}) \rightsquigarrow Y_{3,\ell}$ is a MIMO channel with $\min\{M_1 + M_2, M_3\} = M_3$ DoFs if $s_\ell = 1$, and $\min\{M_2, M_3\} = M_3$ DoFs if $s_\ell = 0$ (because $X_{1,\ell}$ is independent of $Y_{3,\ell}$)

Dividing both sides by $n\log(\rho)$ and letting $\rho,n\to\infty$ we obtain

$$d_{13} + d_{23} + d_{12} \le M_3$$

F. Sum-DoF Upper Bound for Intermittent 3WC with $M_2 \ge M_3 \ge M_1$ (Part II)

We bound the sum rate of w_{21} , w_{31} and w_{32} using Fano's inequality:

$$n(R_{21} + R_{31} + R_{32} - \varepsilon_n^{(2)})$$

$$\leq I(W_{21}W_{31}W_{32}; \boldsymbol{W}_1\boldsymbol{Y}_1^n S^n \underbrace{W_{23}\tilde{\boldsymbol{Y}}_1^n \check{\boldsymbol{Y}}_1^n \boldsymbol{Z}_{corr}^n}_{\boldsymbol{X}_{\ell=1}^n I(\boldsymbol{X}_{2,\ell}\boldsymbol{X}_{3,\ell}; \hat{\boldsymbol{Y}}_{1,\ell} \mid S_\ell) + no\left[\log(\rho)\right]}$$

$$\stackrel{\text{(b)}}{\leq} n\left[M_3\right]\log(\rho) + no\left[\log(\rho)\right]$$

These steps are justified as follows:

- (a) Using the proof template presented in Appendix A, with $\mathfrak{W}_{A} \triangleq (W_{21}, W_{31}, W_{32}), \mathfrak{W}_{B} \triangleq (W_{12}, W_{13}, W_{23}), \mathfrak{X} \triangleq (X_{2}, X_{3}), \mathfrak{Y} \triangleq \hat{Y}_{1}, \mathfrak{Z} \triangleq \hat{Z}_{1}$
- (b) $(\mathbf{X}_{2,\ell}, \mathbf{X}_{3,\ell}) \rightsquigarrow (\mathbf{Y}_{1,\ell}, \mathbf{\tilde{Y}}_{1,\ell}, \mathbf{\tilde{Y}}_{1,\ell})$ is a MIMO channel with $\min\{M_2 + M_3, M_1 + 0 + (M_3 - M_1)\} = M_3$ DoFs if $s_{\ell} = 1$, and $\min\{M_2 + M_3, 0 + M_1 + (M_3 - M_1)\} = M_3$ DoFs if $s_{\ell} = 0$

Dividing both sides by $n\log(\rho)$ and letting $\rho,n\to\infty$ we obtain

$$d_{21} + d_{31} + d_{32} \le M_3.$$

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