TURBO EQUALIZATION OF CONVOLUTIONAL CODED AND CONCATENATED SPACE TIME TRELLIS CODED SYSTEMS USING RADIAL BASIS FUNCTION AIDED EQUALIZERS

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Abstract - In this contribution a reduced-complexity radial basis function (RBF) aided neural-network based turbo equalization (TEQ) scheme is proposed for employment in a serially concatenated convolutional coded and systematic space time trellis coded (CC-SSTTC) arrangement. A two-path Rayleigh fading channel having a normalised Doppler frequency of 3.3615×10^{-5} was used. The BER performance of the RBF-CC-SSTTC(4,4) scheme employing a transmission burst consisting of 100 symbols using a space-time-trellis (STT) interleaver of at least 400 symbols and eight turbo equalization iterations was found to be similar to that of the CC-SSTTC system using a trellis-based TEQ, which attains the optimum performance. However, the Jacobian RBF based TEQ provided a complexity reduction factor of 14.

1. INTRODUCTION

The third-generation proposals aim for guaranteeing low-cost, high-capacity mobile communications offering data rates of up to 2Mbps [1]. However, there are a number of problems associated with high data rate transmissions, especially when aiming for creating spectrally efficient systems. Systems transmitting at high bit rates, such as 2 Mbps, experience a high grade of channel-induced dispersion and suffer from Inter Symbol Interference (ISI). Therefore, typically channel equalizers are employed for mitigating the effects of ISI. In addition to channel equalization, channel coding can also be invoked for further improving the performance of the system. Powerful error correction schemes, such as turbo codes [2], have been shown to yield a performance close to Shannonian performance limits. The discovery of turbo codes and turbo decoding led to the development of turbo equalization [3]. Turbo equalization is a technique that performs chan-nel equalization and channel decoding jointly and iteratively. This scheme has been shown to successfully mitigate the effects of channel-induced ISI, resulting in a Bit Error Rate (BER) performance close to that recorded for transmission over non-dispersive Gaussian channels.

The family of transmission diversity techniques referred to as Space Time Trellis (STT) coding [4] provides a substantial diversity gain for mobile stations by upgrading the base stations, hence potentiallly increasing the achievable user capacity of the system. STT coding [4] jointly designs the channel coding, modulation, transmit diversity and the optional receiver diversity schemes invoked. Following the research by Tarokh *et al.* [4], Bauch *et al.* [5] proposed a joint equalization and STT decoding scheme, which yielded an improved performance with the advent of exploiting the soft-decision based feedback from the STT decoder's output to the channel equalizer's input. In [6] the performance of the STT encoded system was further improved by employing additional channel encoding in conjunction with turbo equalization. We refer to this turbo equalizer as the TEQ-STTC scheme. However, due to the associated computational complexity, the employment of this scheme was limited to low-order modulation modes, such as for example 4-level Quadrature Amplitude Modulation (4QAM).

Motivated by these trends, in this contribution we aim for reducing the complexity associated with the channel-coded and concatenated STT encoded system by using a reduced-complexity Jacobian Radial Basis Function (RBF) equalizer [7], which we will refer to as the RBF-TEQ-STTC scheme. We will investigate the BER performance achieved by the RBF-TEQ-STTC scheme and evaluate the achievable computational complexity reduction compared to the conventional trellis-based TEQ-STTC (CT-TEQ-STTC) arrangement of [6].
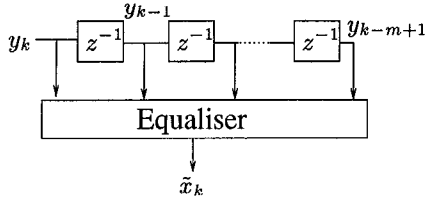
Figure 1: Schematic of m-tap equalizer

2. RBF AIDED CHANNEL EQUALIZER FOR SPACE-TIME-CODING

In this section we will show that the channel equalization problem encountered in a space-time coded system can be considered as a geometric classification problem [8], namely that of classifying an $\mathcal{M}$-ary received phasor into one of $\mathcal{M}$ classes. Figure 1 shows an m-tap equalizer schematic, where the channel output observed by the equalizer can be written in vectorial form as

$$\mathbf{y}_k = \begin{bmatrix} y_k & y_{k-1} & \cdots & y_{k-m+1} \end{bmatrix}. \qquad (1)$$

The baseband representation of the p-transmitter space-time coded system is shown in Figure 2, which transmits a sequence of p symbols $\mathbf{x}_k^p = \begin{bmatrix} x_{1,k} & \cdots & x_{p,k} \end{bmatrix}$ during each signalling instant k. The channel output at instant k is given by:

$$y_k = \sum_{i=1}^{p} \mathbf{h}_i \mathbf{x}_{i,k}^T + \eta_k, \qquad (2)$$

where the i-th channel impulse response (CIR) $\mathbf{h}_i = \begin{bmatrix} h_{0,i} & h_{1,i} & \cdots & h_L \end{bmatrix}$, having a memory of L symbols, is convolved with a sequence of $L+1$ transmitted symbols, namely with $\mathbf{x}_{i,k} = \begin{bmatrix} x_{i,k} & x_{i,k-1} & \cdots \end{bmatrix}$ $x_{i,k-L}]$ and η_k is the additive Gaussian noise term having a variance of σ_η. For a p-transmitter system using an m-tap equalizer and communicating over a channel having a CIR memory of L (assumming that all of the p CIRs have the same memory), there are $n_s = \mathcal{M}^{(m+L)\cdot p}$ number of possible received phasor combinations due to the transmitted sequence, hence producing n_s number of different possible channel output vectors in the absence of channel noise:

$$\tilde{\mathbf{y}}_k = \begin{bmatrix} \tilde{y}_k & \tilde{y}_{k-1} & \cdots & \tilde{y}_{k-m+1} \end{bmatrix}, \qquad (3)$$

where m is the length of the equalizer in Figure 1. Upon adding the noise we have: $\mathbf{y}_k = \tilde{\mathbf{y}}_k + \eta_k$. Expounding further, we denote each of the n_s number of different possible combinations of the channel's input sequence $\bar{\mathbf{x}}_k = \begin{bmatrix} \mathbf{x}_k^p & \cdots & \mathbf{x}_{k-m+1}^p \end{bmatrix}$ of length $(L+m) \times p$ symbols as $\mathbf{s}_i, i = 1, \ldots, n_s$, where the channel's input state $\mathbf{s}_i$ determines the desired channel output state $\mathbf{r}_i, i = 1, \ldots, n_s$. This is formulated as:

$$\tilde{\mathbf{y}}_\mathbf{k} = \mathbf{r}_i, \quad \text{if } \bar{\mathbf{x}}_k = \mathbf{s}_i, \quad i = 1, \ldots, n_s. \qquad (4)$$
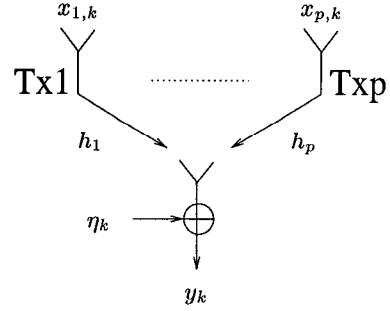

Figure 2: Baseband representation of p-transmitter space-time coded system using one receiver.

For an $\mathcal{M}$-level modulation scheme, the noisy channel output states $\mathbf{y}_k$ can be partitioned into $\mathcal{M}^p$ classes according to the sequence of p number of τ-delayed transmitted symbols, $\mathbf{x}_{k-\tau}^p$. The equalizer has to provide the associated non-linear decision boundaries for the classification strategy. The optimum equalizer is the so-called Bayesian equalizer [8], which has an excessive complexity. Hence here we advocate the reduced-complexity, but suboptimum Jacobian RBF equalizer, introduced in [7], which has N hidden nodes. The output of this Jacobian RBF equalizer can be represented mathematically as [7]:

$$
\begin{aligned}
f_{RBF}^{\ln}(\mathbf{y}_k) &= \ln\left(\sum_{i=1}^{N} w_i \exp(-\|\mathbf{y}_k - \mathbf{c}_i\|^2/\lambda)\right) \quad (5) \\
&= \ln\left(\sum_{i=1}^{N} \exp(\ln(w_i) - \|\mathbf{y}_k - \mathbf{c}_i\|^2/\lambda)\right) \\
&= J(\delta_{N,k}, J(\delta_{N-1,k}, \ldots J(\delta_{2,k}, \delta_{1,k}) \ldots)),
\end{aligned}
$$

where the terms w_i, $\mathbf{c}_i$ and λ are the weights, centers and width of the RBF nodes, respectively. Furthermore, we have $\delta_{i,k} = \exp(\ln(w_i) - \|\mathbf{y}_k - \mathbf{c}_i\|^2/\lambda)$ and $J(\delta_1, \delta_2)$ is the Jacobian logarithmic relationship defined in [9] as $J(\delta_1, \delta_2) \approx \max(\delta_1, \delta_2) + f_c(\|\delta_1 - \delta_2\|)$. The correction function $f_c(x) = \ln(1 + \exp(-x))$ is tabulated in a look-up table, in order to reduce the computational complexity [9].

The full-complexity RBF equalizer provides the so-called optimal Bayesian equalization solution [8] and generates the conditional probability density functions of $\mathcal{M}^p$ number of possible transmitted symbols $\mathbf{x}_{k-\tau}^p$ emitted by the transmitters at instant $k - \tau$ in the form of:

$$
\begin{aligned}
P(\mathbf{y}_k|\mathbf{x}_{k-\tau}^p = \mathbf{I}_j) &= \sum_{i=1}^{n_s^j} p_{i,j}(2\pi\sigma_\eta^2)^{-m/2} \cdot \\
&\quad \exp\left\{-\frac{1}{2\sigma_\eta^2}\|\mathbf{y}_k - r_{i,j}\|^2\right\}, \\
&\qquad j = 1, \ldots, \mathcal{M}^p, \qquad (6)
\end{aligned}
$$

where the RBF parameters defined in the context of Equation 6 are assigned the values of $\mathbf{w}_i = p_{i,j}(2\pi\sigma_\eta^2)^{-m/2}$, $\mathbf{c}_i = r_{i,j}$, $N = n_s^j$ and $\lambda = 2\sigma_\eta^2$. The term n_s^j is the number of possible channel states $\mathbf{r}_{i,j}$ corresponding to the jth transmitted symbol sequence $\mathbf{I}_j$ of the p-antenna SSTTC scheme that consist of p symbols, where we have $j = 1, \ldots, \mathcal{M}^p$. The term $p_{i,j}$ is the a priori probability of occurance of the channel state $r_{i,j}$. The a posteriori probability of the transmitted symbols $\mathbf{x}_{k-\tau}^p$ in Equation 6 provides the a posteriori Log-Likelihood Ratio (LLR) values of the convolutionally coded symbols, which can then be fed to the STT decoder, as shown in Figure 4. The a priori probability of occurance of the ith channel state $r_{i,j}$ corresponding to the transmitted symbol sequence $\mathbf{I}_j$, $p_{i,j}$, can be evaluated from the LLRs generated by the STT decoder as described in Section 3.
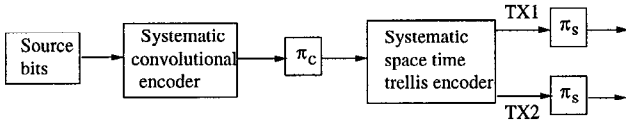
3. SYSTEM OVERVIEW

Figure 3: Transmitter of the serially concatenated systematic convolutional coded and systematic STT coded system.

π_c : channel bit interleaver
π_c^{-1} : channel bit deinterleaver

π_s : space-time symbol interleaver
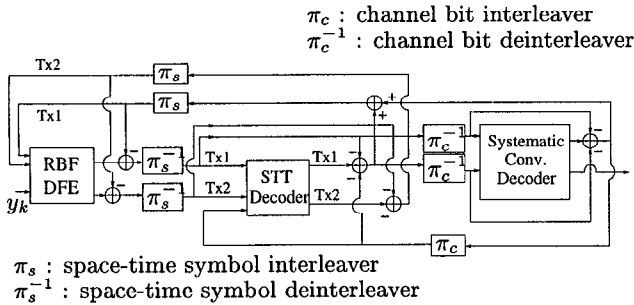π_s^{-1} : space-time symbol deinterleaver

Figure 4: Receiver of the serially concatenated systematic convolutional coded and systematic STTC system using RBF DFE assisted turbo equalization.

In an effort to create a low-complexity, high-performance system, we employ the Jacobian RBF equalizer [7] in the context of a turbo equalizer in conjunction with a convolutional coded systematic STTC (CC-SSTTC) system employing two transmitters. Specifically, we use the decision feedback assisted Jacobian RBF equalizer (Jacobian RBF DFE) [7] for the sake of attaining a reduced computational complexity, where the detected symbol is fed back to the equalizer for selecting a reduced-size subset of RBF centers, which

are then used for evaluating the a posteriori probability of the transmitted signal [8]. The schematic of the CC-SSTTC transmitter consists of a serially concatenated systematic convolutional encoder and a systematic STT encoder, as shown in Figure 3. The transmitted source bits are convolutionally encoded and directed to a random channel bit interleaver π_c. The convolutional encoder denoted as CC(2,1,3) is a $\frac{1}{2}$-rate Recursive Systematic Convolutional (RSC) coding scheme having a constraint length of $K = 3$ and octal generator polynomials of $G_0 = 7$ and $G_1 = 5$. The RSC codeword consists of a systematic bit and a parity bit.

Subsequently, the encoded bits are passed to a systematic STT encoder using two transmit antennas, as illustrated in Figure 3. We denote the systematic STT encoder used as the SSTTC($n = 4, m = 4$) scheme, since it is an $n = 4$-state, $m = 4$-PSK based STT code [4]. Upon receiving an input symbol, the SSTTC produces a symbol in each transmitter arm of Figure 3. Note that we have employed the simple SSTTC(4,4) code instead of more complex systematic STT codes using a higher number of encoder states, since our aim was to invoke the turbo equalization principle and 'invest' the affordable implementational complexity in a number of consecutive iterations, rather than in a high-complexity non-iterative decoder. The STT encoded symbols are interleaved by a random STT symbol interleaver represented as π_s in Figure 3.

The schematic of the receiver is shown in Figure 4. The channel equalizer of Figure 4 computes the a posteriori LLR values for the systematic STT coded symbols of both transmitter TX1 and TX2. Subsequently, these LLR values are deinterleaved by the STT deinterleaver π_s^{-1} of Figure 4 and passed to the SSTTC(4,4) decoder. In the first iteration, the channel equalizer only evaluates the received signal y_k, since there is no a priori feedback information from the output of the RSC decoder. However, in subsequent iterations the channel equalizer will receive additional a priori information concerning the STT codeword from the other decoding stages. In order to avoid passing the a priori information contributed by the other concatenated decoding states back to these stages in Figure 4, we subtract the a priori LLRs fed back to the input of the equalizer from the corresponding a posteriori LLRs output by the equalizer, in order to derive the combined channel and extrinsic information. Similar LLR subtraction stages can be seen at the output of the STT decoder and that of the convolutional decoder, again providing the extrinsic information for the next component of the receiver, as detailed in [6].

In our investigations the transmission burst structure consists of 100 data symbols. A two-path, symbol-spaced fading Channel Impulse Response (CIR) of equal weights was used, where the Rayleigh fading statis-

tics obeyed a normalised Doppler frequency of 3.3615×10^{-5}. The fading magnitude and phase was kept constant for the duration of a transmission burst, a condition which we refer to as employing burst-invariant fading. Furthermore, in order to investigate the best-case performance of these systems, we have assumed that the CIR was perfectly estimated at the receiver. Our future research will characterise the ability of the proposed turbo scheme to compensate for the effects of CIR estimation errors. At the receiver, the systematic STT decoder and the RSC decoder employed the Log-MAP algorithm [9]. The Jacobian RBF DFE has a feedforward order of $m = 2$, feedback order of $n = 1$ and decision delay of $\tau = 1$.
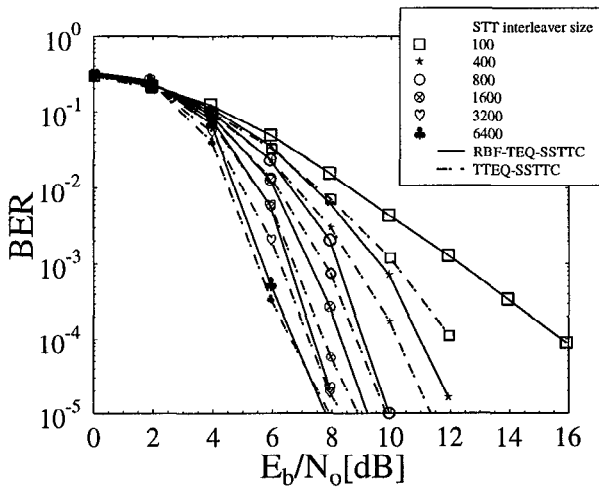
4. RESULTS AND DISCUSSION

Figure 5: BER performance of the RBF DFE ($m = 2$, $n = 1$, $\tau = 1$) assisted turbo-equalized serially concatenated convolutional coded and STTC system using various STTC interleaver sizes, namely 100, 400, 800, 1600, 3200 and 6400 symbols, after eight turbo equalization iterations. The performance of the CT-TEQ-SSTTC system is also shown as a benchmarker.

Figure 5 shows the performance of the proposed RBF-TEQ-STTC and that of the CT-TEQ-STTC scheme [6], using various STTC interleaving sizes, namely 100, 400, 800, 1600, 3200 and 6400 symbols after eight turbo equalization iterations. It was observed in Figure 5 that by increasing the STTC interleaving size from 100 to 6400, the performance degradation of the RBF-TEQ-STTC scheme compared to the CT-TEQ-STTC arrangement expressed in terms of the excess SNR required for attaining a BER of 10^{-4} decreases from 3.8dB recorded for an STTC interleaver size of 100 symbols to 0dB, as observed for the STTC interleaver size of 6400 symbols. This is because the error

propagation of the RBF DFE component decreases, as the BER performance improves, when using a longer STTC interleaver. The performance difference of the two schemes is less than 1dB at a STTC interleaver length of 400 symbols, although the RBF-TEQ-STTC scheme has a lower computational complexity, when the feedforward order m and feedback order n are set to $m = L + 1, n = L$. The interleaving gain attained by the RBF-TEQ-STTC scheme was approximately 9dB at a BER of 10^{-4}. Although higher interleaving gains can be achieved using longer STTC interleavers, the interleaver gain gradually saturates, when the STTC interleaver size is in excess of 1600 symbols.

	trellis-based	Jacobian RBF
sub. and add.	$n_{s,f}(6\mathcal{M}^p + 2)$ -3	$n_{s,f}+$ $\mathcal{M}^p n_s^i(m + 2) - 4$
mult. and div.	$2n_{s,f}$	$2n_{s,f}$

Table 1: Computational complexity of generating the a posteriori LLRs for the trellis-based equalizer and for the Jacobian RBF equalizer [10]. The RBF equalizer's feedforward and feedback order are denoted by m and n, respectively, and the number of RBF nodes is $n_s^i = \mathcal{M}^{(m+L-n)\cdot p}/\mathcal{M}, i = 1, \ldots, \mathcal{M}^p$, where L is the CIR memory and p is the number of STTC transmitters. The notation $n_{s,f} = \mathcal{M}^{(L+1)\cdot p}$ indicates the number of trellis transitions encountered in the trellis-based equalizer and also the number of possible different noise-free channel outputs $\tilde{y}_k$ of the Jacobian RBF equalizer.

Following the approach of our computational complexity study in [10], Table 1 summarises the computational complexity of generating the a posteriori LLRs for each received signal at instant k in the context of a p-transmitter space-time coded system. Figure 6 demonstrates the complexity reduction achieved by the Jacobian RBF DFE for various feedforward orders m, over the trellis-based equalizer. The feedback order n and decision delay τ of the RBF DFE was set to $n = L$ and $\tau = m$ for the sake of attaining the optimum performance, as stated in [8]. The performance of the RBF DFE improves, when increasing the feedforward order [8]. However, Figure 6 shows that the Jacobian RBF DFE only provides a significant complexity reduction compared to the trellis based equalizer, when the feedforward order is less than $L + 2$ and imposes a higher computational complexity for $m > L+2$. Therefore, as a rule of thumb, the feedforward order of the Jacobian RBF DFE must not exceed $L + 1$ in order to achieve a computational complexity improvement relative to the trellis-based equalizer. The complexity imposed by the RBF-TEQ-STTC scheme using an equalizer feedforward order of 2 and a feedback order of 1 was found
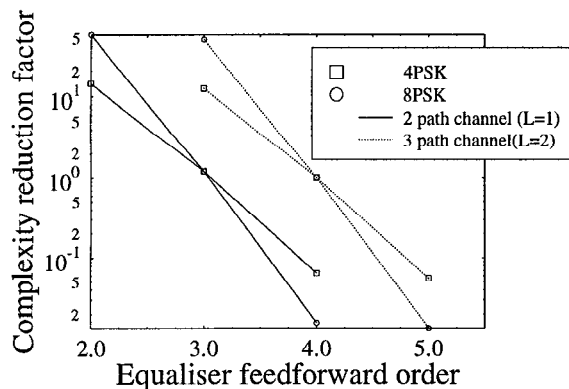
Figure 6: Complexity reduction factor achieved by the RBF DFE equalizer over the trellis based equalizer according to Table 1. The feedback order n was set to L and the number of transmitters was two.

to be a factor of 14 lower, than that of the CT-TEQ-STTC scheme in the context of a two transmitter, one receiver system, based on the general complexity expressions of Table 1. For example, if we used a higher order modulation mode, such as 8PSK used in [4] along with the same number of transmitters, as well as equalizer and channel parameters, the achievable computational complexity reduction is a factor of 55, as shown in Figure 6.

5. CONCLUSIONS

A turbo equalization scheme using the Jacobian RBF equalizer principle of [7] was invoked in a serially concatenated systematic convolutional coded and systematic STT coded system. It was observed in Figure 5 that the BER performance degradation compared to the CT-TEQ-STTC system [6] was less than 1dB for a STTC interleaver length of 400 symbols, while achieving a computational complexity reduction factor 14. Hence, the Jacobian RBF equalizer based TEQ constitutes a better design choice in STTC systems, especially in the context of complex STTC schemes, having a high number of encoder states. Near-optimum performance was achieved, provided that a sufficiently high STTC interleaver length was affordable.

6. REFERENCES

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