# Not All Frequencies Are Created Equal: Towards a Dynamic Fusion of Frequencies in Time-Series Forecasting

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# ABSTRACT

Long-term time series forecasting is a long-standing challenge in various applications. A central issue in time series forecasting is that methods should expressively capture long-term dependency. Furthermore, time series forecasting methods should be flexible when applied to different scenarios. Although Fourier analysis offers an alternative to effectively capture reusable and periodic patterns to achieve long-term forecasting in different scenarios, existing methods often assume high-frequency components represent noise and should be discarded in time series forecasting. However, we conduct a series of motivation experiments and discover that the role of certain frequencies varies depending on the scenarios. In some scenarios, removing high-frequency components from the original time series can improve the forecasting performance, while in others scenarios, removing them is harmful to forecasting performance. Therefore, it is necessary to treat the frequencies differently

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MM '24, October 28–November 1, 2024, Melbourne, VIC, Australia. © 2024 Copyright held by the owner/author(s). ACM ISBN 979-8-4007-0686-8/24/10 https://doi.org/10.1145/3664647.3681210 according to specific scenarios. To achieve this, we first reformulate the time series forecasting problem as learning a transfer function of each frequency in the Fourier domain. Further, we design Frequency Dynamic Fusion (FreDF), which individually predicts each Fourier component, and dynamically fuses the output of different frequencies. Moreover, we provide a novel insight into the generalization ability of time series forecasting and propose the generalization bound of time series forecasting. Then we prove FreDF has a lower bound, indicating that FreDF has better generalization ability. Extensive experiments conducted on multiple benchmark datasets and ablation studies demonstrate the effectiveness of FreDF. The code is available at https://github.com/Zh-XY22/FreDF.

# **CCS CONCEPTS**

• **Computing methodologies** → Spectral methods; Time series forecasting;

# **KEYWORDS**

Time series forecasting, Fourier analysis, Dynamic fusion, Generalization analysis

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## **1 INTRODUCTION**

Time series forecasting is a well-established problem in various fields including energy usage [6], economic planning [22], weather alerts [17], and traffic forecasting [32]. With the development of deep learning [29], numerous methods have emerged for this forecasting tasks [4, 19, 24, 57, 81]. A central issue in time series forecasting is that existing methods could not expressively capture long-term dependency, which is often characterized as periodicity and trends [7, 13, 20, 30, 58, 74]. However, Fourier analysis has the strong potential to deal with long-term dependency, thereby making related methods more flexible when adapted to different sequence prediction scenarios [55, 59].

In the realm of TFS, an effective approach to addressing longterm dependency is to utilize Fourier analysis [37, 59, 60, 79, 80]. Fourier analysis is a powerful method that represents complex time series as a series of cosine functions, each with its unique frequency [9]. This capability to represent infinitely long-term trends with a finite set of frequency components makes it efficient when applied to long-term time series forecasting.

Existing methods based on Fourier analysis often assume that high-frequency components represent noise and should be discarded during forecasting tasks [63, 79]. Additionally, some methods treat all frequencies equally, without considering their varying importance across different scenarios [68, 69, 73]. However, we argue that the role of certain frequencies varies in different scenarios. To validate this assumption, we conduct experiments on three datasets, eliminating low, middle, and high-frequency components respectively from the input of the training set to train a vanilla Transformer [54]. The results, depicted in Figure 1, suggest that eliminating certain frequencies may improve performance in specific datasets while decreasing in others. In Exchange-rate(Figure 1(e)), we get more accurate prediction results after eliminating high frequencies. But it is less precise in Figure 1(b). The same phenomenon occurs at other frequencies. More detailed experimental setup and analysis are provided in Section 3.

These findings emphasize that simply marking high-frequency components as noise is undesirable. Without prior knowledge, determining which frequencies compose noise remains uncertain [20]. Consequently, it is necessary to utilize different frequencies for forecasting and assign more rational weights to these forecasting results to improve the final prediction.

To separately assess the impact of different frequencies, it is necessary to predict each frequency individually. To begin with, we propose a mathematical reformulation of the TFS task in the Fourier domain. Then we propose Frequency Dynamic Fusion (FreDF), a novel framework to process time series datasets in decomposition, forecasting, and dynamic fusion, which individually forecasts each Fourier component, and dynamically fuses the output of different frequencies. The advantage of dynamic fusion lies in its capacity to flexibly adjust the weights of each frequency component, leading to more precise predictions. Additionally, we propose the generalization bound of TFS based on Rademacher complexity [5], and we prove that dynamic fusion improves the model's generalization ability. Experimental results on long-term forecasting datasets confirm the superiority of FreDF. Overall, the main contributions of this paper can be summarized as the following:

- We conduct a series of experiments to explore the role of different frequencies in prediction. Based on experimental phenomena we discover that the role of certain frequencies varies depending on the scenarios.
- We reformulate the TFS problem as learning a transfer function in the Fourier domain. Further, we design FreDF, which individually forecasts each Fourier component, and dynamically fuses the output of different frequencies.
- We propose the generalization bound of TFS. Then we prove FreDF has a lower bound, indicating that FreDF has better generalization ability.
- Extensive experiments conducted on various benchmark datasets demonstrate the effectiveness of FreDF.

## 2 RELATED WORK

With the advancement of deep learning, various methods, including CNN [4, 72], RNN [21, 52], and Transformer-based approaches [54], have been developed for time series forecasting tasks. While most previous works focus on learning models in the time domain (e.g., Informer [31], PeriodFormer [34], GCformer [77], Preformer [16], and Infomaxformer [51]), the core of these methods lies in utilizing correlations in the time domain to forecast future data.

In the Fourier domain, FEDformer [80] applies Transformer using Frequency Enhanced Blocks and Attention modules, and CoST [59] explores learning seasonal representations. TimesNet [60] utilizes frequency for analysis and period calculation, mapping one-dimensional series to two-dimensional. FiLM [79] retains lowfrequency Fourier components. FreTS [69] uses Frequency-domain MLPs to predict. FITS uses Low Pass Filter to filter frequency. However, these methods, involving Fourier analysis, do not explicitly model TFS problems in the Fourier domain. In contrast, we reformulate the TFS problem as learning a transfer function of each frequency in the Fourier domain.

Classical time series decomposition techniques [7] have been utilized to decompose time series into seasonal and trend components for interpretability. For instance, Autoformer [61] decomposes the data into trend and seasonal components, then employs the Transformer architecture for independent forecasts. Similarly, CoST [59] decomposes series into trend and seasonal components, carrying out separate forecasts in both time and Fourier domains. Different from these methods, our approach introduces a novel framework for dynamic decomposition, prediction, and fusion.

# **3 EMPIRICAL ANALYSIS**

Several studies suggest that high-frequency signals often represent noise and therefore should be discarded [79]. However, we argue that the role of certain frequencies is not universal and can be varied across different scenarios. In some cases, high-frequency signals may indeed be noise, while in others, they may hold valuable information. To confirm this idea, we conduct experiments on three datasets. The experimental settings and analysis are detailed below.



Figure 1: The comparison of prediction results using different frequencies on different datasets. (a) ETTm1; (b)ETTm2; (c)ETTh1; (d)ETTh2; (e) Exchange-rate; (f) Weather. Eliminating certain frequencies may improve performance in specific datasets while decreasing in others.

#### 3.1 Experimental Setup

We conduct experiments on six datasets: ETT(ETTm1, ETTm2, ETTh1, ETTh2), Weather, ECL, and Exchange-rate. For each dataset, we conduct a set of four forecasting tasks with the lookback length and prediction length both fixed to 96. The first task is the regular forecasting task. For the other three tasks, we transform the input series from the training set to the Fourier domain using Fast Fourier Transform (FFT) [45] and divide the frequency spectra into three subsets: the first third of the spectrum as low-frequency, the second third as middle-frequency, and the final third as high-frequency. We randomly set the Fourier coefficients corresponding to different subsets of the frequency spectrum to zero respectively in different experiments, and convert it back into the time domain as the input series. This step is to eliminate the influence of a certain subset of frequencies when training the model. We train an individual vanilla Transformer [54] following the standard setting [36] for each task in all three datasets. We visualize the prediction and ground truth of future series for all tasks and all datasets in Figure 1.

# 3.2 Experimental Observations and Analysis

Figure 1(a) shows that in the ETTm1 dataset, after eliminating high-frequency signals, the prediction results are closer to the ground truth compared to using all frequencies. However, the prediction

results are further from the ground truth after eliminating lowfrequency or mid-frequency signals. Figure 1(b) shows that in the ETTm2 dataset, we will get more accurate prediction results after eliminating low-frequency from historical time series. On the contrary, we will get worse prediction results after eliminating low-frequency form historical time series, shown in Figure 1(c). In Figure 1(d), no matter which subsets of the frequency we eliminate in the ETTh2 dataset, the prediction results are more accurate than those obtained using the original frequency for prediction, among which, eliminating high frequency has a better effect. As shown in Figure 1(e), in the Exchange-rate dataset, the prediction results are more accurate in the long term after eliminating mid or highfrequency signals. Conversely, the results predicted by eliminating low-frequency signals or using all signals are closer to ground truth in the mid-term. In the weather dataset, which is shown in Figure 1(f), the results predicted by eliminating low-frequency signals are more accurate in the short-term, predicted by eliminating highfrequency are more accurate in the mid-term, and predicted by eliminating mid-frequency are more accurate in the long term.

Yet, with the absence of prior knowledge, it remains challenging to distinguish noise from vital features. Therefore, we cannot merely mark high-frequency signals as noise. Considering this, it is necessary to utilize different frequencies for forecasting and subsequently adopt a more rational method to weight these forecasting results, thus attaining the final prediction.



Figure 2: Overall structure of FreDF, which consists of an Embedding module f for embedding the feature dimension C to D, a Projection module g for projection back to C, and L FDBlocks. In FDBlock, we decompose and forecast in the frequency domain, and dynamically fuse the prediction results for each Fourier component.

# 4 METHOD

In this section, we begin by reformulating the TFS problem in the Fourier domain. Subsequently, we propose FreDF, a model designed to predict the output of each frequency component respectively, then combine each output using a dynamic fusion strategy. We also present theoretical evidence supporting the idea that this dynamic fusion strategy enhances the generalization ability of FreDF.

#### 4.1 Time series forecasting in Fourier domain

To achieve effective long-term forecasting, the model must go beyond merely memorizing past data points; it needs to grasp the underlying physical rules or inherent dynamics of the observed phenomena [42]. These dynamics governing the behavior of the time series, are presumed to be independent and unchanging over time [47]. In Fourier analysis, any time series can be represented by a set of orthogonal bases, i.e., the Fourier components; this orthogonal characteristic helps represent each rule with the dynamic of a single Fourier component [28]. In this section, we assume that the time series forecasting task is under a Linear Time-invariant (LTI) condition for the independent and time-invariant property of the inherent dynamics without loss of generality.

Specifically, from [47], let  $x(t) \in I$  be the input function and  $y(t) \in O$  be the output function, they are both functions of time *t* defined in Banach space *I* and *O*. The output of the LTI system can be defined as:

$$y(t) = \int_0^\infty h(t-\tau)x(\tau)d\tau.$$
 (1)

The goal of time-series forecasting can be regarded as finding a suitable transfer function  $h: \mathcal{I} \to O$ .

In discrete case, the Equation 1 can be express as:

$$Y[n] = h[n] * X = \sum_{m=0}^{\infty} h[n-m]X[m],$$
 (2)

X[n] and Y[n] is the discrete form of x(t) and y(t), respectively,  $n \in [0, 1, ..., N]$ , N is the length of time series, and \* is the convolution operator. The output series Y = [Y[0], Y[1], ..., Y[N]] are obtained by applying the convolution operator between h and X.

The Discrete Fourier Transform (DFT)  $\mathcal{F}$  [46] can transform *X* from a function of discrete time to a function of Fourier component:

$$\mathcal{X}[k] = \mathcal{F}(X)[k] = \sum_{n=0}^{N-1} X[n] \cdot e^{-j\frac{2\pi}{N}kn},$$
(3)

where *j* is the imaginary unit, X[k] is the *k*-th Fourier components,  $k \in [0, 1, ..., K]$  and *K* is the total number of Fourier components.

THEOREM 4.1. (*The convolution theorem [25]*). The convolution theorem states that the Fourier transform of a convolution of two functions equals the point-wise product of their Fourier transform:

$$\mathcal{F}(h * X)[k] = \mathcal{F}(h)[k] \cdot \mathcal{F}(X)[k].$$
(4)

Applying DFT to the output sequence *Y* according to Theorem 4.1 can convert the convolution in Equation 2 into a multiplication in the Fourier domain as:

$$\mathcal{Y}[k] = \mathcal{F}(h * X)[k] = \mathcal{F}(h)[k] \cdot \mathcal{X}[k].$$
(5)

Note that *h* is an unknown operator in the aforementioned analysis. Therefore we propose to estimate  $\mathcal{F}(h)$  directly with a learnable matrix  $H_{\theta}$ , where  $\theta$  is the parameter. The transfer process is:

$$\hat{\mathcal{Y}}[k] = H_{\theta} \cdot \mathcal{X}[k], \tag{6}$$

where  $\hat{\mathcal{Y}}$  is the estimated output in Fourier domain.

Applying inverse Discrete Fourier transform  $\mathcal{F}^{-1}$  (iDFT) can convert the estimated output back to the time domain with:

$$\hat{Y}[n] = \mathcal{F}^{-1}(\hat{\mathcal{Y}}) = \frac{1}{N} \sum_{k=1}^{K} \hat{\mathcal{Y}}[k] \cdot e^{j\frac{2\pi}{N}kn}.$$
(7)

The learning objective for the learnable matrix is then to minimize the Mean Square Error (MSE) between the estimated output and Not All Frequencies Are Created Equal: Towards a Dynamic Fusion of Frequencies in Time-Series Forecasting

the ground truth of the output:

$$\min_{\theta} \frac{1}{N} \sum_{n=0}^{N-1} (Y[n] - \hat{Y}[n])^2.$$
(8)

So far, the TFS problem in the time domain has been reformulated as learning a transfer function  $H_{\theta}$  in the Fourier domain.

#### 4.2 Frequency Dynamic Fusion

Based on the findings in Section 3, there is no universal criteria to determine the importance of a specific frequency in different situations, for the role of certain frequency changes across various scenarios. For instance, a frequency may be crucial in one scenario but negatively impact performance in another. To address this variability, we propose FreDF (Frequency Dynamic Fusion), which dynamically calculates the weights for the estimated prediction of each frequency, taking their importance into account. Our proposed FreDF consists of the Embedding, the FDBlock, and the Projection layers. We provide the pseudo-code of FreDF in algorithm 1.

To predict the future S timestamps, we padding X[n] in time dimension with S zeros as unknown data.

*4.2.1 Embedding.* In the Embedding module, we lift the input time series into an embedding space:

$$M^{1}[n] = f(X[n]),$$
 (9)

here,  $M^1[n]$  is the embedded representation of the input time series,  $f : \mathbb{R}^C \to \mathbb{R}^D$  is a multi-layer perceptron (MLP) used for the embedding, C is the number of variables in the input time series, and D is the dimension of the embedding space. It's crucial to note that we are embedding the feature dimensions, not the time dimensions. This means that the transformation does not affect the temporal characteristics of the data. Therefore, subsequent operations, such as Fourier transformations that target the time dimensions, remain unaffected by the embedding process.

4.2.2 *FDBlocks.* Within each FDBlock, we apply Fast Fourier Transform (FFT)[45] to the input embedding  $M^{l}[n]$ , transforming it into the Fourier domain, which is an efficient algorithm to perform DFT:

$$\mathcal{M}^{l}[k] = \mathcal{F}(M^{l})[k], \tag{10}$$

here  $\mathcal{M}^{l}[k]$  is the Fourier components, l = 1, 2, ..., L denotes the *l*-th FDBlock and *L* is the total number of FDBlocks.

To facilitate independent processing, we propose a decoupling strategy. Instead of treating the Fourier components  $\mathcal{M}^{l}[k] \in \mathbb{C}^{K \times D}$  as a whole, we create *K* copies of each frequency component and retain the *m*-th frequency in *m*-th copy, denoted as  $\mathcal{M}_{in}^{l,m}[k]$ :

$$\mathcal{M}_{in}^{l,m}(k) = \begin{cases} 0 & \text{if } k \neq m \\ \mathcal{M}^{l}(k) & \text{if } k = m \end{cases}, k = 0, 1, ..., K.$$
(11)

This strategy allows us to maintain the original dimensionality of the data while enabling independent processing of each frequency. Next, based on subsection 4.1, we aim to learn transfer functions  $H^{l,m} \in \mathbb{C}^{D \times D}, m \in [1, ..., K]$  for each independent component  $\mathcal{M}_{in}^{l,m} = \mathcal{M}^{l}[m], m \in [1, ..., K]$ , and obtain the estimated output  $\mathcal{M}_{out}^{l,m}$  in the Fourier domain with:

$$\mathcal{M}_{out}^{l,m} = \mathcal{M}_{in}^{l,m} \cdot H^{l,m}.$$
 (12)

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#### Algorithm 1 Pseudo-Code of FreDF

**Input:** Time series data  $X \in \mathbb{R}^{T \times C}$ , lookback length *T*, predict length *S*, variables number *C*, FDBlock number *L*, token dimension *D*, *K* is computed as  $K = \frac{T+S}{2} + 1$ , frequency spectrum length *K*.

1:	X' = ZeroPadTimeSeries(X, S)	$\triangleright \mathbf{X} \in \mathbb{R}^{(T+S) \times C}$
2:	$M^1 = \mathrm{MLP}(X')$	$\triangleright M^1 \in \mathbb{R}^{(T+S) \times D}$
3:	for $l = 1$ to $L$ do	
4:	for $m = 1$ to $K$ do	
5:	$\mathcal{M}^{l}[k] = \mathcal{F}(M^{l})[k]$	$\triangleright \mathcal{M}^l \in \mathbb{R}^{K  imes D}$
6:	for $k = 1$ to $K$ do	
7:	if $k \neq m$ then	
8:	$\mathcal{M}_{in}^{l,m}[k] = 0$	
9:	else	
10:	$\mathcal{M}_{in}^{l,m}[k] = \mathcal{M}^{l}[k]$	
11:	end if	
12:	end for	
13:	Learn transfer function $H^{l,m}$	$\triangleright H^{l,m} \in \mathbb{R}^{D \times D}$
14:	$\mathcal{M}_{out}^{l,m}[k] = \mathcal{M}_{in}^{l,m}[k] \cdot H^{l,m}$	
15:	$Z^{l,m} = \mathcal{F}^{-1}(\mathcal{M}_{out}^{l,m})$	
16:	end for	
17:	$M^{l+1} = \hat{M}^{l} = \sum_{m=0}^{K} Z^{l,m} \cdot W_{m},$	$\triangleright  \hat{M}^l \in \mathbb{R}^{(T+S) \times D}$
18:	end for	
19:	$\hat{Y} = MLP(\hat{M}^L)[T:T+S,:]$	
20:	return $\hat{Y} > F$	Return the prediction results

The estimated output for frequency *m* in the time domain  $Z^{l,m}[n]$  can be obtained by applying inverse Fast Fourier Transform (iFFT) to  $\mathcal{M}_{out}^{l,m}$ . The result of this operation is represented as:

$$Z^{l,m}[n] = \mathcal{F}^{-1}(\mathcal{M}_{out}^{l,m})[n].$$
<sup>(13)</sup>

So far, we have decomposed the prediction process of each individual frequency m.

Next, we apply a trainable weight vector  $W \in \mathbb{R}^K$ , where each component  $W_m$  represents the importance of the *m*-th frequency when predicting the output embedding. The estimated output  $\hat{M}^l[n]$  is then represented as a weighted sum of all the individual frequency predictions  $Z^{l,m}[n]$ , with each prediction multiplied by its corresponding weight  $W_m$ . The estimated output  $\hat{M}^l[n]$  is represented as a weighted sum of all the individual frequency predictions  $Z^{l,m}[n]$ , with each prediction multiplied by its corresponding weight  $W_m$ . The estimated output  $\hat{M}^l[n]$  is represented as a weighted sum of all the individual frequency predictions  $Z^{l,m}[n]$ , as given by the following equation:

$$\hat{M}^{l}[n] = \sum_{m=0}^{K} Z^{l,m}[n] \cdot W_{m},$$
(14)

where prediction  $Z^{l,m}[n]$  is multiplied by its corresponding weight  $W_m$  and  $W_m$  can be either static or dynamic, i.e. fixed or learnable.

The FDBlock is formulated as an iterative architecture, where output  $\hat{M}^{l}[n]$  of the *l*-th layer serves as input of the (*l* + 1)-th layer.

During the training process, we aim to learn the transfer functions  $H^{l,m} \in \mathbb{C}^{D \times D}$  and the weight vector  $W \in \mathbb{R}^{K}$  by minimizing a loss function, which measures the difference between the estimated output  $\hat{Y}[n]$  and the true output Y[n].

Table 1: Long-term multivariate forecasting results with prediction lengths  $S \in \{96, 192, 336, 720\}$  and fixed lookback length T = 96. The best Forecasting results in bold and the second <u>underlined</u>. The lower MSE/MAE indicates the more accurate prediction result.

Ν	lodels	FreDF	F(Ours)	iTrans	former	Patcl	hTST	Cross	former	Til	DE	Time	esNet	DLi	near	SCI	Net	FEDfo	ormer	Statio	onary	Auto	former
Ν	<i>letric</i>	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE
_	96	0.324	0.367	0.334	0.368	$\frac{0.329}{0.329}$	0.367	0.404	0.426	0.364	0.387	0.338	0.375	0.345	0.372	0.418	0.438	0.379	0.419	0.386	0.398	0.505	0.475
ETTm	192	0.365	$\frac{0.387}{0.405}$	0.377	0.391	$\frac{0.367}{0.300}$	0.385	0.450	0.451	0.398	0.404	0.374	0.387	0.380	0.389	0.439	0.450	0.426	0.441	0.459	0.444	0.553	0.496
	720	0.391	0.405	0.420	0.420	0.355	$\frac{0.410}{0.439}$	0.552	0.515	0.428	0.425	0.410	0.411	0.415	0.413	0.595	0.485	0.543	0.439	0.585	0.404	0.671	0.561
	Avø	0.384	0.398	0.407	0 4 1 0	0.387	0.400	0.513	0.496	0 419	0.419	0.400	0 406	0 403	0 407	0 485	0 481	0 4 4 8	0.452	0.481	0 456	0.588	0.517
	06	0 175	0.257	0 180	0.264	0 175	0.250	0 287	0.366	0 207	0 305	0 1 8 7	0.267	0 103	0.202	0.286	0 377	0.203	0.287	0 102	0.274	0 255	0.330
n2	192	0.175	0.299	0.250	0.204	$\frac{0.173}{0.241}$	$\frac{0.239}{0.302}$	0.287	0.300	0.207	0.364	0.187	0.207	0.195	0.292	0.399	0.377	0.269	0.328	0.192	0.274	0.235	0.339
Ë	336	0.303	0.341	0.311	0.348	0.305	0.343	0.597	0.542	0.377	0.422	0.321	0.351	0.369	0.427	0.637	0.591	0.325	0.366	0.334	0.361	0.339	0.372
Ξ	720	0.405	0.396	0.412	0.407	0.402	0.400	1.730	1.042	0.558	0.524	0.408	0.403	0.554	0.522	0.960	0.735	0.421	0.415	0.417	0.413	0.433	0.432
	Avg	0.281	0.323	0.288	0.332	0.281	0.326	0.757	0.610	0.358	0.404	0.291	0.333	0.350	0.401	0.571	0.537	0.305	0.349	0.306	0.347	0.327	0.371
	96	0.367	0.397	0.386	0.405	0.414	0.419	0.423	0.448	0.479	0.464	0.384	0.402	0.386	0.400	0.654	0.599	0.376	0.419	0.513	0.491	0.449	0.459
ĥ1	192	0.416	0.424	0.441	0.436	0.460	0.445	0.471	0.474	0.525	0.492	0.436	0.429	0.437	0.432	0.719	0.631	0.420	0.448	0.534	0.504	0.500	0.482
L	336	0.477	0.443	0.487	$\frac{0.458}{0.404}$	0.501	0.466	0.570	0.546	0.565	0.515	0.491	0.469	0.481	0.459	0.778	0.659	0.459	0.465	0.588	0.535	0.521	0.496
щ	720	0.478	0.458	0.503	0.491	0.500	0.488	0.653	0.621	0.594	0.558	0.521	0.500	0.519	0.516	0.836	0.699	0.506	0.507	0.643	0.616	0.514	0.512
	Avg	0.435	0.431	0.454	<u>0.447</u>	0.469	0.454	0.529	0.522	0.541	0.507	0.458	0.450	0.456	0.452	0.747	0.647	<u>0.440</u>	0.460	0.570	0.537	0.496	0.487
	96	0.292	0.341	0.297	0.349	0.302	0.348	0.745	0.584	0.400	0.440	0.340	0.374	0.333	0.387	0.707	0.621	0.358	0.397	0.476	0.458	0.346	0.388
$h_2$	192	0.376	0.391	0.380	0.400	0.388	0.400	0.877	0.656	0.528	0.509	0.402	0.414	0.477	0.476	0.860	0.689	0.429	0.439	0.512	0.493	0.456	0.452
ΤT	336	0.415	0.426	0.428	$\frac{0.432}{0.445}$	$\frac{0.426}{0.421}$	0.433	1.043	0.731	0.643	0.571	0.452	0.452	0.594	0.541	1.000	0.744	0.496	0.487	0.552	0.551	0.482	0.486
щ	720	0.420	0.439	0.427	0.445	0.431	0.446	1.104	0.763	0.874	0.679	0.462	0.468	0.831	0.657	1.249	0.838	0.463	0.474	0.562	0.560	0.515	0.511
	Avg	0.376	0.399	0.383	0.407	0.387	0.407	0.942	0.684	0.611	0.550	0.414	0.427	0.559	0.515	0.954	0.723	0.437	0.449	0.526	0.516	0.450	0.459
e	96	0.082	0.199	0.086	0.206	0.088	0.205	0.256	0.367	0.094	0.218	0.107	0.234	0.088	0.218	0.267	0.396	0.148	0.278	0.111	0.237	0.197	0.323
ang	192	0.172	0.294	0.177	0.299	0.176	0.299	0.470	0.509	0.184	0.307	0.226	0.344	0.176	0.315	0.351	0.459	0.271	0.315	0.219	0.335	0.300	0.369
śch	330 720	0.310	$\frac{0.405}{0.687}$	0.331	0.417	0.301	0.39/	1.208	0.883	0.349	0.431	0.367	0.448	0.313	0.427	1.324	0.855	0.460	0.427	0.421	0.4/6	0.509	0.524
Ĥ	<u>720</u>	0.055	0.007	0.047	0.402	0.301	0.714	0.040	0.707	0.032	0.070	0.704	0.740	0.057	0.093	0 750	0.67	0.510	0.095	0.461	0.709	0 6 1 2	0.541
	Avg	0.551	0.396	0.360	0.405	0.307	0.404	0.940	0.707	0.570	0.415	0.410	0.445	0.554	0.414	0.750	0.020	0.519	0.429	0.461	0.454	0.015	0.559
er	96 102	0.157	0.208	0.1/4	0.214	0.1//	$\frac{0.218}{0.250}$	$\frac{0.158}{0.206}$	0.230	0.202	0.261	0.1/2	0.220	0.196	0.255	0.221	0.306	0.21/	0.296	0.1/3	0.223	0.266	0.336
ath	336	0.203	0.240	0.221	$\frac{0.234}{0.296}$	0.223	0.239	0.200	0.277	0.242	0.290	0.219	0.201	0.237	0.290	0.201	0.340	0.270	0.330	0.245	0.203	0.307	0.307
We	720	0.341	0.339	0.358	$\frac{0.270}{0.349}$	0.354	0.348	0.398	0.418	0.351	0.386	0.365	0.359	0.345	0.381	0.377	0.427	0.403	0.428	0.414	0.410	0.419	0.428
	Avg	0.241	0.270	0.258	0.279	0.259	0.281	0.259	0.315	0.271	0.320	0.259	0.287	0.265	0.317	0.292	0.363	0.309	0.360	0.288	0.314	0.338	0.382
	96	0.150	0.242	0.148	0.240	0.181	0.270	0.219	0.314	0.237	0.329	0.168	0.272	0.197	0.282	0.247	0.345	0.193	0.308	0.169	0.273	0.201	0.317
. 1	192	0.161	0.253	0.162	0.253	0.188	0.274	0.231	0.322	0.236	0.330	0.184	0.289	0.196	0.285	0.257	0.355	0.201	0.315	0.182	0.286	0.222	0.334
ũ	336	0.176	0.268	0.178	0.269	0.204	0.293	0.246	0.337	0.249	0.344	0.198	0.300	0.209	0.301	0.269	0.369	0.214	0.329	0.200	0.304	0.231	0.338
-	720	0.217	0.311	0.225	0.317	0.246	0.324	0.280	0.363	0.284	0.373	0.220	0.320	0.245	0.333	0.299	0.390	0.246	0.355	0.222	0.321	0.254	0.361
	Avg	0.176	0.268	0.178	0.270	0.205	0.290	0.244	0.334	0.251	0.344	0.192	0.295	0.212	0.300	0.268	0.365	0.214	0.327	0.193	0.296	0.227	0.338
gy	96	0.214	0.247	0.203	0.237	0.234	0.286	0.310	0.331	0.312	0.399	0.250	0.292	0.290	0.378	0.237	0.344	0.242	0.342	0.215	0.249	0.884	0.711
lerg	192	0.230	0.255	0.233	0.261	0.267	0.310	0.734	0.725	0.339	0.416	0.296	0.318	0.320	0.398	0.280	0.380	0.285	0.380	0.254	0.272	0.834	0.692
Ξ	336	0.242	0.266	$\frac{0.248}{0.248}$	0.273	0.290	0.315	0.750	0.735	0.368	0.430	0.319	0.330	0.353	0.415	0.304	0.389	0.282	0.376	0.290	0.296	0.941	0.723
olar	720	0.245	0.271	0.249	0.275	0.289	0.317	0.769	0.765	0.370	0.425	0.338	0.337	0.356	0.413	0.308	0.388	0.357	0.427	0.285	0.295	0.882	0.717
S	Avg	0.232	0.259	0.233	0.262	0.270	0.307	0.641	0.639	0.347	0.417	0.301	0.319	0.330	0.401	0.282	0.375	0.291	0.381	0.261	0.381	0.885	0.711
1 <sup>st</sup>	Count	t <b>34</b>	36	<u>2</u>	<u>2</u>	3	2	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0

4.2.3 Projection. After *L* FDBlocks, we apply another MLP  $g : \mathbb{R}^D \to \mathbb{R}^C$  to the final estimated output  $\hat{M}^L[n]$ , projecting it back to the variable space. The result of this operation is represented as:

$$\hat{Y}[n] = g(\hat{M}^{L}[n])[T:T+S,:].$$
(15)

# 4.3 Theoretical Analysis

In this subsection, we provide a theoretical analysis to demonstrate the effectiveness of our dynamic fusion method. Without loss of generality, time series forecasting methods could be regarded as auto-regressive models [7], from this perspective, we indicate that the generalization ability of time series prediction models could be reflected in the following two aspects: the capacity to capture the long-term dependency of time series, as well as the capacity to achieve good prediction results in different scenarios.

For simplicity, consider the fusion strategy in a regression setting using a mean squared loss function. Firstly, we propose to characterize the generalization error bound using Rademacher complexity

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[5] and separate the bound into three components (Theorem 4.2). Meanwhile, we also give further proof based on the above separation to illustrate that the dynamic fusion method achieves a better ability to capture long-term dependency under certain conditions (Theorem 4.3). Secondly, we demonstrate that the quantity of parameters in our method is fewer than compared methods, which indicates that our method is more flexible to apply to more scenarios, experiment results in Section 5.2 also validate our illustration as well. See Appendix A for details.

Specifically, we use X,  $\mathcal{Y}$ , and  $\mathcal{Z}$  to denote the input space (historical sequence), target space (prediction sequence), and latent space. Define  $u: X \to \mathcal{Z}$  is a fusion mapping from the input space to the latent space,  $g: \mathcal{Z} \to \mathcal{Y}$  is a task mapping. Our goal is to learn the fusion operator  $f = g \circ u(x)$ , which is essentially a regression model. Under an H frequency components scenario,  $f^h$  is the frequency-specific composite function of frequency component  $x^h$ . The final prediction of the dynamic fusion method is calculated by:  $f(x) = \sum_{h=1}^{H} w^h \cdot f^h(x^h)$ , where f(x) denotes the final prediction. In contrast to static fusion, i.e., every frequency is given a predefined weight which is a constant, dynamic fusion calculates the weights of every frequency dynamically. To distinguish them, denotes  $w^h_{static}$  the weight of frequency h in static situation and  $w^h_{dynamic}$  the weight of frequency h in dynamic situation. The generalization error of regression model f is defined as:

$$GError = \mathbb{E}_{(x,y)\sim\mathcal{D}}[l(f(x),y)], \tag{16}$$

where  $\mathcal{D}$  is the unknown joint distribution, l is mean squared loss function. Conveniently, we simplify the regression loss  $l(f^h(x^h), y)$  as  $l^h$ . Now we present the first main result of frequency fusion.

THEOREM 4.2. Given the historical sequence  $X_T \in \mathbb{R}^{T \times C}$  and the ground truth of prediction sequence  $Y_{T'} \in \mathbb{R}^{T' \times C}$ ,  $\hat{E}(f^h)$  is the empirical error of  $f^h$  on frequency h. Then for any hypothesis fin the finite set F and  $1 > \delta > 0$ , with probability at least  $1 - \delta$ , it holds that

$$GError \leq \sum_{h=1}^{H} \mathbb{E}(w^{h})\hat{E}(f^{h}) + \sum_{h=1}^{H} \mathbb{E}(w^{h})\mathfrak{R}_{h}(f^{h}) + \sum_{h=1}^{H} Cov(w^{h}, l^{h}) + M\sqrt{\frac{ln(1/\delta)}{2H}},$$
(17)

where  $\mathbb{E}(w^h)$  represents the expectations of fusion weights on joint distribution  $\mathcal{D}$ ,  $\mathfrak{R}_h(f^h)$  represents Rademacher complexity, and  $Cov(w^h, l^h)$  represents covariance between fusion weight and loss.

Theorem 4.2 demonstrates that the generalization error of the regression model is bounded by the weighted average performances of all regression operators for each frequency in terms of empirical loss, model complexity, and the covariance between fusion weight and regression loss of all frequencies. Next, we aim to confirm whether dynamic fusion offers a tighter general error bound than static fusion. Informally, in Equation 17, the covariance term measures the joint variability of  $w^h$  and  $l^h$ . However, in static fusion,  $w^h_{static}$  is a constant, which means that the covariance is equal to zero for any static fusion method. Thus the generalization error

bound of static fusion methods is reduced to:

$$GError(f_{static}) \leq \sum_{h=1}^{H} (w^{h}_{static}) \hat{E}(f^{h})$$

$$+ \sum_{h=1}^{H} (w^{h}_{static}) \Re_{h}(f^{h}) + M \sqrt{\frac{\ln(1/\delta)}{2H}}.$$

$$(18)$$

So when the summation of the average empirical loss, the average complexity is invariant or smaller in dynamic fusion and the covariance is no greater than zero, we can ensure that dynamic fusion provably outperforms static fusion. This theorem is formally presented as:

THEOREM 4.3. Let  $\overline{\operatorname{GError}}(f_{dynamic})$ ,  $\overline{\operatorname{GError}}(f_{static})$  be the upper bound of generalization regression error of dynamic and static fusion method respectively.  $\hat{E}(f^h)$  is the empirical error defined in Theorem 4.2. Then for any hypothesis  $f_{dynamic}$ ,  $f_{static}$  in finite set F and  $1 > \delta > 0$ , it holds that

$$\overline{\text{GError}}(f_{dynamic}) \le \overline{\text{GError}}(f_{static}) \tag{19}$$

with probability at least  $1 - \delta$ , if we have

$$\mathbb{E}(w^{h}_{dynamic}) = w^{h}_{static}$$
(20)

and

$$\cdot (w^h_{dunamic}, l^h) \le 0 \tag{21}$$

for all frequencies, where *r* is the Pearson correlation coefficient which measures the correlation between fusion weights  $w^h_{dynamic}$  and the loss of each frequency  $l^h$ .

Theorem 4.2 and Theorem 4.3 verify that the dynamic fusion method has a lower generalization bound, which indicates the capacity to capture the long-term dependency of our method. Furthermore, suppose for each frequency, the regression operator used in dynamic and static fusion are of the same architecture, then the intrinsic complexity  $\Re_h(f^h)$  can be invariant. Thus, it holds that

$$\sum_{h=1}^{H} \mathbb{E}(w_{dynamic}^{h}) \Re_{h}(f^{h}) \leq \sum_{h=1}^{H} (w_{static}^{h}) \Re_{h}(f^{h}).$$
(22)

In Equation 22, it is easy to derive the conclusion that our model has a lower average complexity, corresponding to a lower quantity of parameters during the training process. Experiment results in Section 5.2 also validate this conclusion.

## **5 EXPERIMENTS**

In this section, we first provide the details of the implementation and datasets. Next, we present the comparison results on eight benchmark datasets. Lastly, we conduct ablation studies to evaluate the effectiveness of each module in our method.

#### 5.1 Main Results

We thoroughly evaluate the proposed FreDF on various long-term time series forecasting benchmarks. For better comparison, we follow the experiment settings of iTransformer in [36] the prediction lengths for both training and evaluation vary within the set  $S \in \{96, 192, 336, 720\}$ , with a fixed lookback length of T = 96.

Mathada	Matuia	ETTh1				ETTm1				Exchange-rate			
Methods	Metric	96	192	336	720	96	192	336	720	96	192	336	720
W Transfer function	MSE MAE	0.367 0.397	0.416 0.424	0.477 0.443	0.478 0.458	0.324 0.367	0.365 0.387	0.391 0.405	0.459 0.436	0.082 0.199	0.172 0.294	0.316 0.405	0.835 0.687
W/O Transfer function	MSE MAE	0.439 0.444	0.492 0.505	0.529 0.561	0.522 0.541	0.378	0.421 0.432	0.441 0.439	0.518 0.487	0.129 0.251	0.218 0.344	0.254 0.312	0.897 0.709

Table 2: Ablation on the influence of transfer function.

*5.1.1 Baselines.* We carefully choose 10 well-acknowledged forecasting models as our benchmark, including (1) Transformer-based methods: iTransformer [36], Autoformer [61], FEDformer [80], Stationary [38], Crossformer [76], PatchTST [44]; (2) Linear-based methods: DLinear [71], TiDE [15]; and (3) TCN-based methods: SCINet [35], TimesNet [60].

5.1.2 Forecasting Results. Table 1 presents the results of FreDF in long-term multivariate forecasting with the best in **bold** and the second <u>underlined</u>. The lower MSE/MAE indicates the more accurate prediction result. Results demonstrate that our model performs optimally in 70 out of 80 benchmarks. Compared to FEDformer [80], FreDF shows an average improvement of 13% in terms of MSE and MAE, reaching up to 33% improvement on the Exchangerate dataset. Compared to the best-performing Transformer-based model:iTransformer [36], FreDF consistently achieves superior performance across almost all datasets.

Table 3: Ablation between static fusion and dynamic fusion.

Models		Fre	DF	FreSF			
Me	etric	MSE	MAE	MSE	MAE		
r	96	0.153	0.199	0.175	0.239		
the	192	0.205	0.246	0.215	0.276		
Vea	336	0.259	0.587	0.263	0.312		
~	720	0.341	0.339	0.343	0.377		
ge	96	0.082	0.199	0.129	0.239		
an	192	0.172	0.294	0.231	0.332		
xch	336	0.316	0.405	0.360	0.451		
щ	720	0.835	0.687	0.891	0.741		
_	96	0.367	0.397	0.428	0.437		
Th	192	0.416	0.424	0.475	0.456		
ΕŢ	336	0.477	0.443	0.509	0.477		
	720	0.478	0.458	0.509	0.490		
1	96	0.324	0.367	0.369	0.401		
Γm	192	0.365	0.387	0.419	0.430		
ĒŢ	336	0.391	0.405	0.440	0.438		
	720	0.459	0.436	0.497	0.468		

## 5.2 Ablation Study

In this section, we conduct ablation studies to examine the influence of transfer functions, dynamic fusion mechanisms, and the number of parameters in the proposed FreDF.

*5.2.1 Influence of transfer function.* We conduct an ablation study about the influence of the transfer function. We remove the transfer function in FreDF as the control group, follow the setup in Section

5.1, and carry out predictions on the ETTh1, ETTm1, and Exchangerate dataset. We present the results in Table 2, which illustrates the crucial role of the transfer function and confirms the correctness of our analysis in Section 4.1.

5.2.2 Influence of dynamic fusion. We conduct an ablation study to investigate the influence of dynamic fusion. We replace the learnable weight vector with a fixed weight vector and name this modified model FreSF. Predictions are carried out on the ETT(ETTh1, ETTm1), Weather, and Exchang-rate datasets using the setup outlined in Section 5.1. The results are presented in Table 3. Additionally, we visualize the prediction results (with a prediction length S = 96) for both FreSF and FreDF in Appendix D. The experimental results demonstrate the effectiveness of the dynamic fusion strategy.

#### Table 4: Comparison of the number of parameters.

Models	Ours	iTransformer	PatchTST	FEDformer	FiLM
params	151.4K	3.1M	3.5M	14.0M	12.0M

5.2.3 Number of parameters. We use iTransformer [36], patchTST [44], FEDformer [80] and FiLM [79] for comparison, and calculate the number of model parameters when forecasting the same task, present the results in Table 4. Our model despite using a relatively small number of parameters, can achieve good accuracy in prediction tasks. This also validates the superiority of our model, which is consistent with the theoretical analysis in Section 4.3.

#### 6 CONCLUSION

In this paper, we experimentally explore the different roles of frequency in various scenarios. To better utilize these distinctions, we reformulate the problem of time series forecasting as learning transfer functions in the Fourier domain and design the FreDF model, which can independently forecast each Fourier component and dynamically fuse outputs from different frequencies. Then, we provide a novel understanding of the generalization ability of time series forecasting. Further, we also propose the generalization bound for time series forecasting and demonstrate that FreDF has a lower generalization bound, indicating its better generalization ability. Extensive experiments validate the effectiveness of FreDF on multiple benchmark datasets.

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## REFERENCES

- [1] Michael D Adams. [n. d.]. Signals and Systems. ([n. d.]).
- [2] Fadi Al-Turjman, Mohsin Nawaz, Utku Ulusar, and Raed I. Saeed. 2023. Artificial intelligence-based traffic flow prediction: a comprehensive review. Journal of Electrical Systems and Information Technology (2023). https://doi.org/10.1186/ s43067-023-00081-6
- [3] Adebiyi A. Ariyo, Adewumi O. Adewumi, and Charles K. Ayo. 2014. Stock Price Prediction Using the ARIMA Model. In 2014 UKSim-AMSS 16th International Conference on Computer Modelling and Simulation. 106-112. https://doi.org/10. 1109/UKSim.2014.67
- [4] Shaojie Bai, J. Zico Kolter, and Vladlen Koltun. 2018. An Empirical Evaluation of Generic Convolutional and Recurrent Networks for Sequence Modeling. arXiv:1803.01271 [cs.LG]
- [5] Bartlett, Peter, L., Mendelson, and Shahar. 2003. Rademacher and Gaussian Complexities: Risk Bounds and Structural Results. Journal of Machine Learning Research 3, 3 (2003), 463-463.
- [6] Muhammad Bilal, Hyeok Kim, Muhammad Fayaz, and Pravin Pawar. 2022. Comparative Analysis of Time Series Forecasting Approaches for Household Electricity Consumption Prediction. arXiv:2207.01019 [cs.LG]
- [7] George EP Box, Gwilym M Jenkins, Gregory C Reinsel, and Greta M Ljung. 2015. Time series analysis: forecasting and control.
- [8] George E P Box. [n. d.]. Time Series Analysis. ([n. d.]).
  [9] R.N. Bracewell. 2000. The Fourier Transform and Its Applications. https: //books.google.com.ph/books?id=ecH2KgAACAAJ
- [10] R. N. Bracewell. 1986. The Fourier Transform and its Applications. McGraw Hill.
- [11] David R. Brillinger. 1981. Time Series: Data Analysis and Theory. Holden-Day.
- [12] Peng Chen, Yingying ZHANG, Yunyao Cheng, Yang Shu, Yihang Wang, Qingsong Wen, Bin Yang, and Chenjuan Guo. 2024. Pathformer: Multi-scale Transformers with Adaptive Pathways for Time Series Forecasting.
- [13] Yue Cheng, Weiwei Xing, Witold Pedrycz, Sidong Xian, and Weibin Liu. 2023. NFIG-X: Non-linear fuzzy information granule series for long-term traffic flow time series forecasting.
- [14] Giacomo Chiarot and Claudio Silvestri. 2023. Time Series Compression Survey. Comput. Surveys 55, 10 (Feb. 2023), 1-32. https://doi.org/10.1145/3560814
- [15] Abhimanyu Das, Weihao Kong, Andrew Leach, Rajat Sen, and Rose Yu. 2023. Long-term Forecasting with TiDE: Time-series Dense Encoder.
- [16] Dazhao Du, Bing Su, and Zhewei Wei. 2023. Preformer: Predictive Transformer with Multi-Scale Segment-wise Correlations for Long-Term Time Series Forecasting.
- [17] Claude Duchon and Robert Hale. 2012. Time Series Analysis in Meteorology and Climatology: An Introduction.
- [18] Lawrence C. Evans. 2022. Partial differential equations. Vol. 19. American https://books.google.com/books?hl=zh-CN&lr= Mathematical Society. & id = Ott 1 EAAAQBAJ& oi = fnd&pg = PP1&dq = Partial + Differential + Equations + Constraints + Cevans&ots=cUNzAH2MyK&sig=yAAqmzvrLLXVgW7JzKTZ3qzvlqo
- [19] Jinkang Guo, Zhibo Wan, and Zhihan Lv. 2023. Digital Twins Fuzzy System Based on Time Series Forecasting Model LFTformer. In Proceedings of the 31st ACM International Conference on Multimedia (Ottawa ON, Canada) (MM '23). Association for Computing Machinery, New York, NY, USA, 7094-7100. https: //doi.org/10.1145/3581783.3612936
- [20] James D Hamilton. 2020. Time series analysis.
- [21] Hansika Hewamalage, Christoph Bergmeir, and Kasun Bandara. 2021. Recurrent Neural Networks for Time Series Forecasting: Current status and future directions. International Journal of Forecasting 37, 1 (Jan. 2021), 388-427. https://doi.org/10. 1016/j.ijforecast.2020.06.008
- [22] Javier Hidalgo. 2009. Journal of Time Series Econometrics. https://www. degruyter.com/journal/key/jtse/html
- [23] Maximilian Hoffmann, Leander Kotzur, Detlef Stolten, and Martin Robinius. 2020. A Review on Time Series Aggregation Methods for Energy System Models. Energies 13, 3 (2020), 641. https://doi.org/10.3390/en13030641
- [24] Yuntong Hu and Fuyuan Xiao. 2022. Time-Series Forecasting Based on Fuzzy Cognitive Visibility Graph and Weighted Multisubgraph Similarity. IEEE Transactions on Fuzzy Systems 31, 4 (2022), 1281-1293.
- [25] Yitzhak Katznelson. 2004. An introduction to harmonic analysis.
- [26] H. Kim, S. Kim, S. Min, and B. Lee. 5555. Contrastive Time-Series Anomaly Detection. 01 (nov 5555), 1-14. https://doi.org/10.1109/TKDE.2023.3335317

- [27] Diederik P. Kingma and Jimmy Ba. 2015. Adam: A Method for Stochastic Optimization. ICLR (2015).
- [28] Henning Lange, Steven L. Brunton, and J. Nathan Kutz. 2021. From Fourier to Koopman: Spectral methods for long-term time series prediction. The Journal of Machine Learning Research 22, 1 (2021), 1881-1918. Publisher: JMLRORG.
- [29] Yann LeCun, Yoshua Bengio, and Geoffrey Hinton. 2015. Deep learning. Nature 521, 7553 (May 2015), 436-444. https://doi.org/10.1038/nature14539 Number: 7553 Publisher: Nature Publishing Group.
- [30] Fang Li, Yuqing Tang, Fusheng Yu, Witold Pedrycz, Yuming Liu, and Wenyi Zeng. 2021. Multilinear-trend fuzzy information granule-based short-term forecasting for time series. IEEE Transactions on Fuzzy Systems 30, 8 (2021), 3360-3372.
- [31] Jianxin Li, Xiong Hui, and Wancai Zhang. 2021. Informer: Beyond efficient transformer for long sequence time-series forecasting.
- [32] Li Li, Xiaonan Su, Yi Zhang, Yuetong Lin, and Zhiheng Li. 2015. Trend modeling for traffic time series analysis: An integrated study. IEEE Transactions on Intelligent Transportation Systems 16, 6 (2015), 3430-3439.
- [33] Zongyi Li, Nikola Kovachki, Kamyar Azizzadenesheli, Burigede Liu, Kaushik Bhattacharya, Andrew Stuart, and Anima Anandkumar. 2021. Fourier Neural Operator for Parametric Partial Differential Equations. https://doi.org/10.48550/ arXiv.2010.08895 arXiv:2010.08895 [cs, math].
- [34] Daojun Liang, Haixia Zhang, Dongfeng Yuan, Xiaoyan Ma, Dongyang Li, and Minggao Zhang. 2023. Does Long-Term Series Forecasting Need Complex Attention and Extra Long Inputs?
- [35] Minhao Liu, Ailing Zeng, Muxi Chen, Zhijian Xu, Qiuxia Lai, Lingna Ma, and Qiang Xu. 2022. SCINet: time series modeling and forecasting with sample convolution and interaction.
- Yong Liu, Tengge Hu, Haoran Zhang, Haixu Wu, Shiyu Wang, Lintao Ma, and [36] Mingsheng Long. 2023. iTransformer: Inverted Transformers Are Effective for Time Series Forecasting. arXiv:arXiv:2310.06625
- Yong Liu, Chenyu Li, Jianmin Wang, and Mingsheng Long. 2023. Koopa: Learning [37] Non-stationary Time Series Dynamics with Koopman Predictors.
- [38] Yong Liu, Haixu Wu, Jianmin Wang, and Mingsheng Long. 2022. Non-stationary Transformers: Rethinking the Stationarity in Time Series Forecasting
- [39] Jerry Lonlac, Arnaud Doniec, Marin Lujak, and Stephane Lecoeuche. 2020. Extracting Seasonal Gradual Patterns from Temporal Sequence Data Using Periodic Patterns Mining. (2020). https://arxiv.org/abs/2010.10289
- Edward N Lorenz. 1956. Empirical orthogonal functions and statistical weather pre-[40] diction. Vol. 1. Massachusetts Institute of Technology, Department of Meteorology Cambridge.
- [41] Edward N Lorenz. 1956. Empirical orthogonal functions and statistical weather prediction. Vol. 1. Massachusetts Institute of Technology, Department of Meteorology Cambridge.
- David G Luenberger. 1979. Dynamic Systems. [42]
- Jovana Mitrovic, Brian McWilliams, Jacob Walker, Lars Buesing, and Charles [43] Blundell. 2020. Representation learning via invariant causal mechanisms. arXiv preprint arXiv:2010.07922 (2020).
- Yuqi Nie, Nam H. Nguyen, Phanwadee Sinthong, and Jayant Kalagnanam. 2023. [44] A Time Series is Worth 64 Words: Long-term Forecasting with Transformers.
- [45] Henri I Nussbaumer and Henri I Nussbaumer, 1982. The fast Fourier transform. [46] Alan V. Oppenheim, Ronald W. Schafer, and John R. Buck. 1999. Discrete-Time
- Signal Processing.
- Alan V Oppenheim, Alan S Willsky, Syed Hamid Nawab, and Jian-Jiun Ding. [47] 1997. Signals and systems.
- [48] Adam Paszke, S. Gross, Francisco Massa, A. Lerer, James Bradbury, Gregory Chanan, Trevor Killeen, Z. Lin, N. Gimelshein, L. Antiga, Alban Desmaison, Andreas Köpf, Edward Yang, Zach DeVito, Martin Raison, Alykhan Tejani, Sasank Chilamkurthy, Benoit Steiner, Lu Fang, Junjie Bai, and Soumith Chintala. 2019. PyTorch: An Imperative Style, High-Performance Deep Learning Library. NeurIPS (2019)
- [49] Zequn Qin, Pengyi Zhang, Fei Wu, and Xi Li. 2021. FcaNet: Frequency Channel Attention Networks. arXiv:2012.11879 [cs.CV]
- H.C. So. 2024. Discrete-Time Fourier Transform. City University of Hong Kong [50] (2024). [^1^][5]
- Peiwang Tang and Xianchao Zhang. 2023. Infomaxformer: Maximum Entropy [51] Transformer for Long Time-Series Forecasting Problem.
- Yuqing Tang, Fusheng Yu, Witold Pedrycz, Xiyang Yang, Jiayin Wang, and Shihu [52] Liu. 2021. Building trend fuzzy granulation-based LSTM recurrent neural network for long-term time-series forecasting. IEEE transactions on fuzzy systems 30, 6 (2021), 1599-1613.
- [53] Ilya O Tolstikhin, Neil Houlsby, Alexander Kolesnikov, Lucas Beyer, Xiaohua Zhai, Thomas Unterthiner, Jessica Yung, Andreas Steiner, Daniel Keysers, Jakob Uszkoreit, et al. 2021. Mlp-mixer: An all-mlp architecture for vision. NeurIPS (2021)
- [54] Ashish Vaswani, Noam Shazeer, Niki Parmar, Jakob Uszkoreit, Llion Jones, Aidan N Gomez, Łukasz Kaiser, and Illia Polosukhin. 2017. Attention is all vou need.
- [55] Jiexin Wang, Yujie Zhou, Wenwen Qiang, Ying Ba, Bing Su, and Ji-Rong Wen. 2023. Spatio-temporal branching for motion prediction using motion increments.

MM '24, October 28-November 1, 2024, Melbourne, VIC, Australia.

Xingyu Zhang et al.

, 4290-4299 pages.

- [56] Shiyu Wang, Haixu Wu, Xiaoming Shi, Tengge Hu, Huakun Luo, Lintao Ma, James Y. Zhang, and JUN ZHOU. 2024. TimeMixer: Decomposable Multiscale Mixing for Time Series Forecasting. In *The Twelfth International Conference on Learning Representations*. https://openreview.net/forum?id=70LshfEIC2
- [57] Qingsong Wen, Tian Zhou, Chaoli Zhang, Weiqi Chen, Ziqing Ma, Junchi Yan, and Liang Sun. 2023. Transformers in Time Series: A Survey. arXiv:2202.07125 [cs.LG]
- [58] Peter R Winters. 1960. Forecasting sales by exponentially weighted moving averages. *Management science* 6, 3 (1960), 324–342.
- [59] Gerald Woo, Chenghao Liu, Doyen Sahoo, Akshat Kumar, and Steven Hoi. 2022. CoST: Contrastive learning of disentangled seasonal-trend representations for time series forecasting.
- [60] Haixu Wu, Tengge Hu, Yong Liu, Hang Zhou, Jianmin Wang, and Mingsheng Long. 2023. TimesNet: Temporal 2D-Variation Modeling for General Time Series Analysis.
- [61] Haixu Wu, Jiehui Xu, Jianmin Wang, and Mingsheng Long. 2021. Autoformer: Decomposition Transformers with Auto-Correlation for Long-Term Series Forecasting.
- [62] Kai Xu, Minghai Qin, Fei Sun, Yuhao Wang, Yen-Kuang Chen, and Fengbo Ren. 2020. Learning in the Frequency Domain. arXiv:2002.12416 [cs.CV]
- [63] Zhijian Xu, Alling Zeng, and Qiang Xu. 2024. FITS: Modeling Time Series with \$10k\$ Parameters. https://openreview.net/forum?id=bWcnvZ3qMb
- [64] Zhi-Qin John Xu. 2020. Frequency Principle: Fourier Analysis Sheds Light on Deep Neural Networks. *Communications in Computational Physics* 28, 5 (June 2020), 1746–1767. https://doi.org/10.4208/cicp.oa-2020-0085
- [65] Chunwei Yang, Xiaoxu Chen, Lijun Sun, and Hongyu Yang. 2023. Enhancing Representation Learning for Periodic Time Series with Floss: A Frequency Domain Regularization Approach. (2023). https://arxiv.org/pdf/2308.01011.pdf
- [66] Zhangjing Yang, Weiwu Yan, Xiaolin Huang, and Lin Mei. 2020. Adaptive temporal-frequency network for time-series forecasting. *IEEE Transactions on Knowledge and Data Engineering* 34, 4 (2020), 1576–1587.
- [67] Kun Yi, Qi Zhang, Longbing Cao, Shoujin Wang, Guodong Long, Liang Hu, Hui He, Zhendong Niu, Wei Fan, and Hui Xiong. 2023. A Survey on Deep Learning based Time Series Analysis with Frequency Transformation. arXiv:2302.02173 [cs.LG]
- [68] Kun Yi, Qi Zhang, Wei Fan, Hui He, Liang Hu, Pengyang Wang, Ning An, Longbing Cao, and Zhendong Niu. 2023. FourierGNN: Rethinking Multivariate Time Series Forecasting from a Pure Graph Perspective. arXiv:2311.06190 [cs.LG]

- [69] Kun Yi, Qi Zhang, Wei Fan, Shoujin Wang, Pengyang Wang, Hui He, Defu Lian, Ning An, Longbing Cao, and Zhendong Niu. 2023. Frequency-domain MLPs are More Effective Learners in Time Series Forecasting. arXiv:2311.06184 [cs.LG]
- [70] Peter C Young, Diego J Pedregal, and Wlodek Tych. 1999. Dynamic harmonic regression. *Journal of forecasting* 18, 6 (1999), 369–394.
- [71] Ailing Zeng, Muxi Chen, Lei Zhang, and Qiang Xu. 2023. Are Transformers Effective for Time Series Forecasting?
- [72] Tianxiang Zhan, Yuanpeng He, Yong Deng, and Zhen Li. 2023. Differential Convolutional Fuzzy Time Series Forecasting.
- [73] Jinxin Zhang, Wei Li, Weidong Sun, Yuxiang Zhang, and Ran Tao. 2024. Locality Robust Domain Adaptation for cross-scene hyperspectral image classification. https://doi.org/10.1016/j.eswa.2023.121822
- [74] Jianqi Zhang, Jingyao Wang, Wenwen Qiang, Fanjiang Xu, Changwen Zheng, Fuchun Sun, and Hui Xiong. 2024. Intriguing Properties of Positional Encoding in Time Series Forecasting.
- [75] Xiang Zhang, Ziyuan Zhao, Theodoros Tsiligkaridis, and Marinka Zitnik. 2022. Self-Supervised Contrastive Pre-Training For Time Series via Time-Frequency Consistency. arXiv:2206.08496 [cs.LG]
- [76] Yunhao Zhang and Junchi Yan. 2023. Crossformer: Transformer Utilizing Cross-Dimension Dependency for Multivariate Time Series Forecasting.
- [77] Yanjun Zhao, Ziqing' Ma, Tian Zhou, Mengni Ye, Liang Sun, and Yi Qian. 2023. GCformer: An Efficient Solution for Accurate and Scalable Long-Term Multivariate Time Series Forecasting. In Proceedings of the 32nd ACM International Conference on Information and Knowledge Management (Birmingham, United Kingdom) (CIKM '23). Association for Computing Machinery, New York, NY, USA, 3464–3473. https://doi.org/10.1145/3583780.3615136
- [78] Kun Zhou, Hui Yu, Wayne Xin Zhao, and Ji-Rong Wen. 2022. Filter-enhanced MLP is All You Need for Sequential Recommendation. In Proceedings of the ACM Web Conference 2022 (WWW '22). ACM. https://doi.org/10.1145/3485447.3512111
- [79] Tian Zhou, Ziqing Ma, Qingsong Wen, Liang Sun, Tao Yao, Wotao Yin, Rong Jin, et al. 2022. Film: Frequency improved legendre memory model for long-term time series forecasting., 12677–12690 pages.
- [80] Tian Zhou, Ziqing Ma, Qingsong Wen, Xue Wang, Liang Sun, and Rong Jin. 2022. FEDformer: Frequency enhanced decomposed transformer for long-term series forecasting.
- [81] Chenglong Zhu, Xueling Ma, Weiping Ding, and Jianming Zhan. 2024. Long-Term Time Series Forecasting With Multilinear Trend Fuzzy Information Granules for LSTM in a Periodic Framework. *IEEE Transactions on Fuzzy Systems* 32, 1 (2024), 322–336. https://doi.org/10.1109/TFUZZ.2023.3298970