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Expectations-Based Loss Aversion in Auctions with Interdependent Values: Extensive vs. Intensive Risk

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Abstract. We analyze the bidding behavior of expectations-based loss-averse bidders in auctions with interdependent values. We emphasize the difference between the risk bidders face over whether they win the auction (extensive risk) and the risk they face over the value of the prize conditional on winning (intensive risk). The extensive risk creates an "attachment" effect, whereas the intensive risk operates via a "comparison" effect. How loss-averse bidders react to these different risks depends on whether they incorporate their bid into their reference point. Under "unacclimating personal equilibrium" (UPE), where bidders keep their expectations fixed when choosing their bids, both the extensive and intensive risks induce them to bid more aggressively. Moreover, bidders are exposed to the "winner's curse" and a seller can attain higher revenue by hiding information in order to leverage the intensive risk. By contrast, under "choice-acclimating personal equilibrium" (CPE), where a bid determines both the reference lottery and the outcome lottery, the intensive risk creates a "precautionary bidding" effect that pushes bidders to behave less aggressively; whether this effect is reinforced or undermined by the extensive risk depends on a bidder's likelihood of winning the auction. Furthermore, bidders are less aggressive than under UPE and can be subject to a "loser's curse." Yet, by committing to bidding less aggressively, such as by engaging in proxy bidding, loss-averse bidders are better off under CPE than UPE.

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I hate to lose more than I love to win. —Jimmy Connors

1. Introduction

In many auctions, the value of the good for sale is subject to ex post risk, and bidders will learn its true value only after the auction is over. A preeminent example of such auctions is the so-called common-value auction, where bidders share the same value for the good up for sale, but at the time of the auction, each bidder is only partially informed about this value. More generally, bidders' valuations are often interdependent, as a bidder's value also depends on other bidders' private information. This is the case, for instance, in auctions for wine, antiques, artworks, or real estate. For all of these goods, their future resale value, authenticity, or quality cannot be perfectly known at the time of sale, and so they are stochastic from the buyers' point of view. Therefore, in these auctions, bidders' attitudes toward risk play a crucial role in determining their bidding strategies and, in turn, the auction's performance in terms of revenue.

In this paper, we analyze first-price auctions (FPAs) and second-price auctions (SPA) in which bidders have interdependent values and are expectations-based loss-averse, à la Kőszegi and Rabin (2006). For each auction format, we derive the unique symmetric equilibrium and characterize the impact of loss aversion on bidding, highlighting how the equilibrium strategy of loss-averse bidders differs from that of risk-neutral and risk-averse bidders. Moreover, we show that revenue equivalence between the FPA and the SPA might fail even when bidders have independent signals about the good's unknown value.

In auctions with interdependent values, bidders are exposed to two kinds of risk. First, there is the uncertainty regarding losing and winning the auction. We call this risk, which stems from the uncertainty about whether a bidder submitted the highest bid, the "extensive risk." Second, different from the case of private values, uncertainty does not fully resolve for the winner because the value of the good is unknown ex ante and is affected by the information held by the other bidders; moreover, depending on the auction format, a winning bidder is also exposed to uncertainty over the price to pay. We call this risk, to which a bidder is exposed even conditional on the fact that he or she submitted the highest bid, the "intensive risk." Table 1 summarizes the different types of risk that arise in private-value and interdependent-value auctions, depending on the auction format. The SPA always entails intensive risk in the payment for both private and interdependent values; yet, if values are interdependent, both the SPA and FPA expose bidders to additional intensive risk in consumption.

While both the extensive and intensive risk in money unambiguously lower equilibrium bids, in the consumption dimension, the effect of these two risks depends on the extent to which loss-averse bidders incorporate their bid into their reference point. We consider two alternative formulations introduced by Kőszegi and Rabin (2007). In a "choice-acclimating personal equilibrium" (CPE), bidders choose the strategy that maximizes their expected payoff given that the strategy determines both the distribution of the reference point and the distribution of outcomes; hence, when contemplating whether to deviate from their equilibrium bid, bidders adjust their reference point accordingly. In an "unacclimating personal equilibrium" (UPE), instead, bidders choose the strategy that maximizes their expected payoff conditional on expectations, and the distribution of outcomes so generated must coincide with the expectations; hence, when contemplating whether to deviate from their equilibrium bid, bidders hold their reference point fixed.[1] Both specifications are sensible from a theoretical perspective and have been applied in various economic settings. We refrain from taking a stand on which specification is more appropriate but point out that the two yield very different predictions in terms of bidders' behavior and welfare. Under UPE, both the intensive and extensive risks in consumption push bidders to bid more aggressively. Under CPE, instead, the intensive risk pushes bidders to bid less aggressively; whether this effect is reinforced or undermined by the extensive risk depends on a bidder's likelihood of winning the auction.

Section 2 introduces bidders' preferences and solution concepts. We analyze an environment with interdependent values that encompasses pure private and pure common values as special cases and in which bidders receive independent private signals. This formulation preserves revenue equivalence under risk neutrality; hence, in our model, any difference in the expected revenue between auction formats is driven by bidders' preferences and not by correlation in bidders' signals or values. Following Kőszegi and Rabin (2006), we posit that, in addition to classical material utility, bidders also experience "gain–loss utility" when comparing their material outcomes to a reference point equal to their expectations regarding the same outcomes, with losses being more painful than equal-sized gains are pleasant. We focus on symmetric equilibria in increasing strategies; thus, the bidder with the highest signal wins the auction.

Section 3 analyzes the FPA and SPA under UPE. In both formats, the extensive risk in consumption creates an "attachment" effect that pushes loss-averse bidders to bid more aggressively compared with the risk-neutral benchmark. Indeed, because bidders treat their reference point as fixed when choosing their bids, they are willing to pay more in order to win the auction and reduce the chances of experiencing a loss. Furthermore, the intensive risk over the value of the good creates a "comparison" effect that also pushes bidders to bid more aggressively. Intuitively, in a symmetric equilibrium, winning with a high bid makes it more likely that a high-value good will be obtained; this, in turn, pushes bidders to bid more in order to reduce the likelihood of experiencing a loss in the event that the realized value of the good falls short of their expectations. Notice that this comparison effect does not arise in private-value auctions where the value of the object is perfectly known in advance. Thus, under UPE, the uncertainty in the good's value makes loss-averse bidders worse off compared with a comparable environment with private values; this is in stark contrast to the result by Eső and White (2004) that bidders displaying decreasing absolute risk aversion (DARA) are made better off by the uncertainty in the good's value. Therefore, an implication of our model is that, under UPE, a revenue-maximizing seller facing loss-averse bidders has a strong incentive to purposely introduce risk into their values.[2] Moreover, as both the extensive and the intensive risks induce bidders to

Table 1. Extensive and Intensive Risk with Private and Interdependent Values

	Private values		Interdependent values	
	Consumption	Money	Consumption	Money
FPA	Extensive risk	Extensive risk	Extensive + intensive risk	Extensive risk
SPA	Extensive risk	Extensive + intensive risk	Extensive + intensive risk	Extensive + intensive risk

bid rather aggressively, expectations-based loss aversion also provides a novel explanation for the "winner's curse" often observed in interdependent-value auctions.[3] Our explanation for the winner's curse, however, differs from traditional ones based on limited rationality, whereby bidders fail to realize that, conditional on winning, they have the most optimistic signal. In fact, loss-averse bidders hold correct expectations about the good's value; yet, they bid aggressively in order to win more often and against opponents with relatively high signals, thereby reducing their chances of experiencing a loss. Hence, under UPE, both the extensive and intensive risks in the consumption dimension push loss-averse bidders to bid aggressively. Bidding aggressively, however, also exposes a bidder to losses in the money dimension. In particular, in the FPA, raising your bid by $1 leads to a $1 increase in the expected payment conditional on winning. In the SPA, instead, raising your bid by $1 leads to a less than $1 increase in the expected payment conditional on winning; yet, in the SPA, a bidder is also exposed to intensive risk in money. We show that, in equilibrium, the losses in the money dimension implied by raising one's bid are the same in both formats; hence, the FPA and SPA are revenue equivalent under UPE.

Section 4 analyzes the FPA and SPA under CPE. In both formats, the extensive risk has a "bifurcating" effect whereby loss-averse bidders with low signals underbid relative to risk-neutral bidders, whereas those with high signals might overbid. Hence, in equilibrium, bidders with high signals might suffer from the winner's curse. By contrast, the intensive risk creates a precautionary bidding effect—akin to the one identified by Eső and White (2004) for DARA bidders—that pushes loss-averse bidders to bid less aggressively.[4] Yet, the mechanism behind the precautionary bidding effect in our model differs from the one in Eső and White (2004). As DARA bidders prefer to have a higher income when winning the auction, they reduce their bids by more than the appropriate risk premium; in other words, DARA bidders are "prudent"; see Kimball (1990) and Eeckhoudt et al. (1996). Instead, our precautionary bidding effect under CPE has a first-order nature in the sense that it does not depend on the curvature of the utility function. Indeed, as shown by Cerreia-Vioglio et al. (2017), loss aversion and CPE are incompatible with prudence.[5] Moreover, we show that the FPA fetches a higher expected revenue than the SPA. The reason is that, under CPE, when raising their bid, bidders immediately adjust their reference point (i.e., bidding more aggressively does not lead to surprise losses in the money dimension). However, in the SPA, bidders are also exposed to intensive risk in their monetary outcomes. As loss-averse bidders dislike the additional intensive risk ingrained in the SPA, they bid more aggressively in the FPA than in the SPA.

Section 5 compares UPE and CPE from the bidders' perspective. Although bidders might suffer from the winner's curse in both cases, under CPE, ex post a bidder regrets not bidding more. This happens because, under CPE, bidders commit in advance to their bids, and this weakens the attachment effect. Thus, under CPE, bidders also suffer from a "loser's curse" as in Holt and Sherman (1994) and Pesendorfer and Swinkels (1997). Yet, by committing to bidding less aggressively—for instance, by using a bidding proxy—bidders are better off under CPE than under UPE.

Section 6 compares the behavior of loss-averse bidders with that of risk-averse ones under either DARA or constant absolute risk aversion (CARA). We show that loss aversion delivers different implications than both. Indeed, whereas DARA bidders prefer bidding in an interdependent-value environment rather than in a (comparable) private-value one, loss-averse bidders attain the same utility in both environments under CPE, but they strictly prefer bidding in a private-value environment under UPE. Moreover, whereas CARA bidders always bid less aggressively than risk-neutral ones, loss-averse bidders might bid more aggressively than risk-neutral ones under both CPE and UPE.

Section 7 concludes the paper. Proofs are relegated to the appendix, whereas the web appendix gathers extensions omitted from the main text. The remainder of this section discusses the related literature.

1.1. Literature Review

Next to expected utility, Kahneman and Tversky's (1979, 1991) has arguably become the most prominent approach for modeling risk preferences. Together with probability weighting and diminishing sensitivity, the central building blocks of prospect theory are reference dependence and loss aversion. In a series of influential papers, Kőszegi and Rabin (2006, 2007, 2009) developed a model of reference-dependent preferences and loss aversion where "gain–loss utility" is derived from standard "consumption utility," and the reference point is determined endogenously by rational expectations. Their model has found many fruitful applications in different areas of economics, finance, and decision analysis, including firms' pricing and advertising strategies (Heidhues and Kőszegi 2008, 2014; Spiegler 2012; Herweg and Mierendorff 2013; Karle and Peitz 2014, 2017; Rosato 2016; Karle and Schumacher 2017), incentives provision (Herweg et al. 2010, Eliaz and Spiegler 2015, Daido and Murooka, 2016, Macera 2018), tournaments and contests (Gill and Stone 2010, Gül Mermer 2017, Dato et al. 2018, Fu et al. 2019), asset pricing (Pagel 2016), life cycle consumption (Pagel 2017), and bilateral negotiations (Benkert 2017, Rosato 2017, Herweg et al. 2018). In particular, there have been several studies on the implications of expectations-based loss aversion in auctions. Lange and Ratan (2010) study private-value auctions under CPE and show that the FPA outperforms the SPA in terms of revenue. Belica and Ehrhart (2014) also compare FPA and SPA under both UPE and CPE (but without loss aversion in money); like us, they also find more aggressive bidding under UPE than under CPE. Eisenhuth (2019) shows that under CPE, the all-pay auction yields the highest revenue among sealed-bid formats. Ehrhart and Ott (2017) show that under UPE, the Dutch auction yields a higher expected revenue than the English auction. von Wangenheim (2017) compares the English auction with the second-price one under UPE, showing that the latter yields a higher expected revenue. Rosato (2019) studies sequential sealed-bid auctions of multiple objects under CPE and shows that expectations-based loss aversion can explain the "afternoon effect"—the puzzling yet robust empirical phenomenon whereby prices of identical goods tend to decline between rounds. Yet, our paper is the first to study the implications of expectations-based loss aversion in auctions with interdependent values.

2. The Model

In this section, we introduce the auction environment and bidders' preferences, and provide formal definitions of the solution concepts—CPE and UPE—in the context of sealed-bid auctions.

2.1. Environment

A seller auctions off an item to $N \geq 2$ bidders via a sealed-bid auction. Each bidder, $i \in \{1, 2, \ldots, N\}$, observes a private signal t_i independently and identically distributed on the support $[\underline{t}, \bar{t}]$, with $\underline{t} \geq 0$ and $\bar{t} > \underline{t}$, according to the cumulative distribution function F. We assume that F is continuously differentiable, with strictly positive density f on its support. Bidder i's value for the object is $V_i = v_i(t_1, t_2, \ldots, t_N)$, where the function $v_i : \mathbb{R}_+^N \to \mathbb{R}_+$ is increasing in all of its arguments and is twice continuously differentiable. Moreover, v_i is strictly increasing in t_i. Notice that this formulation nests private values ($v_i(t_1, t_2, \ldots, t_N) = t_i$) and pure common values ($V = v(t_1, t_2, \ldots, t_N)$) as special cases. We focus on symmetric environments. Let $\mathbf{t}_{-i} := (t_1, \ldots, t_{i-1}, t_{i+1}, \ldots, t_N)$. Then, $v_i(t_1, t_2, \ldots, t_N) = v(t_i, \mathbf{t}_{-i})$, and the function v, which is the same for all bidders, is symmetric in its last $N-1$ arguments. We consider two canonical selling mechanisms: the FPA and the SPA. We restrict attention to symmetric equilibria in increasing strategies and abstract from reserve prices.

2.2. Bidders' Preferences

Bidders have reference-dependent preferences as formulated by Kőszegi and Rabin (2006). Bidder i's utility function has two components. First, if they win the auction at price p, bidders experience *consumption utility* $V_i - p$, which represents the classical notion of outcome-based utility. Second, bidders also derive utility from comparing their actual consumption to a reference consumption outcome given by their recent expectations (probabilistic beliefs).[6] Hence, for a deterministic outcome (V_i, p) and deterministic reference point (r^V, r^p), a bidder's total utility is

$$U\left[(V_i, p) \middle| \left(r^V, r^p\right)\right] = V_i - p + \mu^g\left(V_i - r^V\right) + \mu^m\left(r^p - p\right), \tag{1}$$

where

$$\mu^l(x) = \begin{cases} \eta^l x & \text{if} \quad x \geq 0 \\ \eta^l \lambda^l x & \text{if} \quad x < 0 \end{cases}$$

is the *gain–loss utility*.[7] The parameter $\eta^l \geq 0$ captures the relative weight a consumer attaches to gain–loss utility, whereas $\lambda^l > 1$ is the coefficient of loss aversion, with $l \in \{m, g\}$.[8] Moreover, according to (1), a bidder assesses gains and losses separately over each dimension of consumption utility; this is consistent with much of the experimental evidence commonly interpreted in terms of loss aversion.

Because expectations are stochastic in many situations, Kőszegi and Rabin (2006) allow for the reference point to be a pair of probability distributions, $\mathbf{H} = (H^V, H^p)$, over the two dimensions of consumption utility; then, a bidder's total utility from the outcome (V_i, p) can be written as

$$U\left[(V_i, p) \middle| \left(H^V, H^p\right)\right] = V_i - p + \int_{r^V} \mu^g\left(V_i - r^V\right) dH^V\left(r^V\right) + \int_{r^p} \mu^m\left(r^p - p\right) dH^p\left(r^p\right).$$

In other words, for each utility dimension, a bidder compares the realized outcome to all possible outcomes in the reference lottery, each one weighted by its probability.

2.3. Solution Concepts

Bidders learn their signal (or type) before bidding and thus maximize their interim expected utility. If the distribution of the reference points is $\mathbf{H} = (H^V, H^p)$ and the distribution of consumption outcomes is $\mathbf{G} = (G^V, G^p)$, then the interim expected utility of a bidder with signal t_i is

$$EU[\mathbf{G}|\mathbf{H}, t_i] = \int_{\{V_i, p\}} \int_{\{r^V, r^p\}} U\left[(V_i, p) \middle| \left(r^V, r^p\right)\right] \times d\mathbf{H}\left(r^V, r^p\right) d\mathbf{G}\left(V_i, p\right).$$

A strategy of bidder i is a function $\beta_i : [\underline{t}, \bar{t}] \to \mathbb{R}_+$. Fixing all other bidders' strategies, $\boldsymbol{\beta}_{-i}$, the bid of bidder i with type t_i, $\beta_i(t_i)$, induces a distribution over the set of final consumption outcomes. Let $\Gamma(\beta_i(t_i), \boldsymbol{\beta}_{-i})$ denote this distribution. In a sealed-bid auction, uncertainty is resolved *after* all bids are submitted. As pointed out by Kőszegi and Rabin (2007), when a person makes a committed decision long before outcomes occur, that person affects the reference point with his or her choice so that $\mathbf{G} \equiv \mathbf{H}$. This is what Kőszegi and Rabin (2007) call CPE.

Definition 1. A strategy profile $\boldsymbol{\beta}^*$ constitutes a CPE if, for all i and for all t_i,

$$EU\left[\Gamma\left(\beta_i^*(t_i), \boldsymbol{\beta}_{-i}^*\right) | \Gamma\left(\beta_i^*(t_i), \boldsymbol{\beta}_{-i}^*\right), t_i\right] \geq EU\left[\Gamma\left(b, \boldsymbol{\beta}_{-i}^*\right) | \Gamma\left(b, \boldsymbol{\beta}_{-i}^*\right), t_i\right]$$

for any $b \in \mathbb{R}_+$.[9]

In a CPE, if a bidder deviates to a different strategy, the bidder's reference point changes accordingly. However, if a decision is made shortly before outcomes are realized, the reference point is fixed by past expectations; in this case, the bidder maximizes his or her expected utility, taking the reference point as given. Hence, a bidder can plan to submit a bid only if this is optimal given the reference point induced by the expectation to do so. This is what Kőszegi and Rabin (2007) call UPE.

Definition 2. A strategy profile $\boldsymbol{\beta}^*$ constitutes a UPE if, for all i and for all t_i,

$$EU\left[\Gamma\left(\beta_i^*(t_i), \boldsymbol{\beta}_{-i}^*\right) | \Gamma\left(\beta_i^*(t_i), \boldsymbol{\beta}_{-i}^*\right), t_i\right] \geq EU\left[\Gamma\left(b, \boldsymbol{\beta}_{-i}^*\right) | \Gamma\left(\beta_i^*(t_i), \boldsymbol{\beta}_{-i}^*\right), t_i\right]$$

for any $b \in \mathbb{R}_+$.

Therefore, different from CPE, under UPE, if a bidder deviates to a different strategy, the reference point does not change. Notice that there might exist multiple UPEs—that is, multiple plans that the bidder is willing to follow through. In this case, following Kőszegi and Rabin (2006, 2007), we assume that bidders select the UPE that yields the highest expected utility among all symmetric UPEs; that is, bidders select their (symmetric) preferred personal equilibrium (PPE). The following assumption, maintained for the remainder of the paper, guarantees the existence of an equilibrium in increasing strategies.

Assumption 1 (No Dominance of Gain–Loss Utility in the Item Dimension)**.** Let $\Lambda^g \equiv \eta^g(\lambda^g - 1) \leq 1$.

For given λ^g, Assumption 1 places an upper bound on η^g (and vice versa).[10] This bound ensures that a bidder's expected utility is increasing in his or her type by imposing that the weight on gain–loss utility does not (strictly) exceed the weight on consumption utility.[11] Finally, notice that risk neutrality is embedded in the model as a special case (for either $\eta^g = \eta^m = 0$ or $\lambda^g = \lambda^m = 1$).

2.4. An Illustrative Example

We now provide an extended example to illustrate how the intensive and extensive risks affect the behavior of loss-averse bidders under both UPE and CPE. For simplicity, we focus on the FPA with two bidders and assume $\eta^m = 0$—so that bidders are loss-averse only with respect to consumption but risk neutral over money. The value of the object for sale is the same for both bidders and equal to $v(t_i, t_{-i}) = t_i + t_{-i}$, where t_i and t_{-i} represent bidder i's and his opponent's signal, respectively. This structure is known as the "wallet game" (Klemperer 1998, Bulow and Klemperer 2002).

We begin by describing the behavior of loss-averse bidders under UPE. Consider bidder i with type $t_i = t$, who planned to bid according to his or her true type but is contemplating deviating and bid as if his or her type were $\tilde{t} > t$. Assume i's opponent follows the posited equilibrium strategy $\beta_I^*(\cdot)$, and let $F(\tilde{t})$ denote the probability that the opponent's type is lower than $\tilde{t}$. Similarly, let $F(s|\tilde{t})$ denote the probability that the opponent's type is lower than s, conditional on it being lower than $\tilde{t}$. Then, the expected utility of a bidder of type t who mimics type $\tilde{t} > t$ is

$$\begin{aligned} EU(\tilde{t}, t) = {} & F(\tilde{t})\left[t + \int_{\underline{t}}^{\tilde{t}} s dF(s|\tilde{t}) - \beta_I^*(\tilde{t})\right] \\ & - \eta^g \lambda^g [1 - F(\tilde{t})] F(t)\left[t + \int_{\underline{t}}^{t} s dF(s|t)\right] \\ & + \eta^g F(\tilde{t})[1 - F(t)]\left[t + \int_{\underline{t}}^{\tilde{t}} s dF(s|\tilde{t})\right] \\ & - \tilde{\Omega}(\tilde{t}, t), \end{aligned} \quad (2)$$

where $\tilde{\Omega}(\tilde{t}, t) := F(t)F(\tilde{t})\{\eta^g \lambda^g \int_{\underline{t}}^{t} \int_{\underline{t}}^{x} (x - y) dF(y|\tilde{t}) dF(x|t) - \eta^g \int_{\underline{t}}^{\tilde{t}} \int_{\underline{t}}^{x} (x - y) dF(y|t) dF(x|\tilde{t})\}$. The bidder wins the auction with probability $F(\tilde{t})$; conditional on winning, the expected value of the good is $[t + \int_{\underline{t}}^{\tilde{t}} s dF(s|t)]$, and the payment is $\beta_I^*(\tilde{t})$. Thus, the first term on the right-hand side of (2) represents the standard expected material payoff. The other terms represent expected gain–loss utility and are derived as follows. The first two terms arise from the extensive risk. Indeed, with probability $F(\tilde{t})$, the bidder wins the auction; comparing this outcome to the possibility of losing the auction, which, given his or her reference point, the bidder expects to happen with probability $[1 - F(t)]$, generates a gain of $\eta^g[(t + \int_{\underline{t}}^{\tilde{t}} s dF(s|\tilde{t})) - 0]$. Similarly, with probability $[1 - F(\tilde{t})]$, the bidder loses the auction; comparing this outcome to the possibility of winning the auction, which, given reference point, the bidder expects to happen with probability $F(t)$, entails a loss of $\eta^g\lambda^g[0-(t+\int_{\underline{t}}^{t} sdF(s|t))]$. Finally, the term $\tilde{\Omega}(\tilde{t},t)$ arises from the intensive risk, capturing the feelings of gain and loss the bidder experiences if he or she wins the auction, and compares the actual realized value of the opponent's signal—and hence of the good—to the other values that could have been realized.

We now show that both the extensive and intensive risk create an upward pressure on the equilibrium bid. Hence, loss-averse bidders bid more aggressively than risk-neutral ones. Indeed, in equilibrium, a bidder of type t prefers to bid $\beta_I^*(t)$ instead of mimicking type $\tilde{t}>t$ by bidding $\beta_I^*(\tilde{t})$:

$$\begin{aligned} &EU(t,t)\geq EU(\tilde{t},t)\\ &\quad\Leftrightarrow F(\tilde{t})\beta_I^*(\tilde{t})-F(t)\beta_I^*(t)\geq[F(\tilde{t})-F(t)]t+\int_t^{\tilde{t}} sdF(s)\\ &\qquad+\eta^g\lambda^g[F(\tilde{t})-F(t)]\left[F(t)t+\int_{\underline{t}}^{t} sdF(s)\right]\\ &\qquad+\eta^g[1-F(t)]\left\{[F(\tilde{t})-F(t)]t+\int_t^{\tilde{t}} sdF(s)\right\}\\ &\qquad+\underbrace{\Omega(t,t)-\Omega(\tilde{t},t)}_{>0}. \end{aligned} \tag{3}$$

The left-hand side of (3) represents the material cost for a type-t bidder to mimic a higher type: a higher expected payment. The right-hand side of (3) represents the bidder's benefits from mimicking a higher type. The terms on the first line capture the standard material benefits: a higher chance of winning the auction and winning against an opponent with a higher signal. The other terms stem from loss aversion. The first two capture gain–loss utility associated with a change in the extensive risk. Indeed, for a fixed reference point, bidding more aggressively increases the likelihood of realizing a gain and reduces the likelihood of experiencing a loss. This creates an attachment effect that pushes loss-averse bidders to bid more aggressively than risk-neutral ones. Finally, the last term on the right-hand side of (3) captures the gain–loss utility associated with a change in the intensive risk. This term is also positive, capturing a comparison effect that also pushes the bidder to bid more in order to win against opponents with relatively high signals. Indeed, conditional on winning, a bidder prefers his or her opponent's signal to be relatively high, as in this case, he or she is less likely to be disappointed by the realized value of the good. Thus, the comparison effect also pushes loss-averse bidders to bid more aggressively than risk-neutral ones. Moreover, notice that the comparison effect does not arise in an environment with private values, where the value of the object is perfectly known. Therefore, under UPE, loss-averse bidders bid more aggressively in auctions with interdependent values than in comparable auctions with private values. For concreteness, if types are uniformly distributed on the unit interval, the equilibrium bid—which satisfies (3) as an equality when $\tilde{t}\to t$—is $\beta_I^*(t)=(1+\eta^g)t+\eta^g(\lambda^g-1)t^2/2$, which is higher than the risk-neutral bid.

Next, we describe the behavior of loss-averse bidders under CPE. Similar to UPE, with CPE the bidder's reference point must be consistent with rational expectations in equilibrium. Under CPE, however, when choosing how much to bid, the bidder does not keep the reference point fixed; instead, the bidder immediately adjusts the reference point to the new bid. Hence, under CPE a bidder is able to commit to a particular bid in advance. Then, the expected utility of a bidder of type t who mimics type $\tilde{t}>t$ is

$$\begin{aligned} EU(\tilde{t},t) &= F(\tilde{t})\left[t+\int_{\underline{t}}^{\tilde{t}} sdF(s|\tilde{t})-\beta_I^*(\tilde{t})\right]\\ &\quad-\eta^g\lambda^g[1-F(\tilde{t})]F(\tilde{t})\left[t+\int_{\underline{t}}^{\tilde{t}} sdF(s|\tilde{t})\right]\\ &\quad+\eta^g[1-F(\tilde{t})]F(\tilde{t})\left[t+\int_{\underline{t}}^{\tilde{t}} sdF(s|\tilde{t})\right]\\ &\quad-\tilde{\Omega}(\tilde{t},\tilde{t}). \end{aligned} \tag{4}$$

Notice the crucial difference between (2) and (4): under UPE, to evaluate gains and losses, the bidder uses the lottery induced by the equilibrium bid $\beta_I^*(t)$ as the reference point; under CPE, instead, the bidder uses the lottery induced by the deviation $\beta_I^*(\tilde{t})$ as the reference point. To see how the extensive and intensive risks affect the bidding strategy under CPE, it is again useful to consider the costs and benefits for a type-t bidder to mimic type $\tilde{t}>t$:

$$\begin{aligned} &EU(t,t)\geq EU(\tilde{t},t)\\ &\quad\Leftrightarrow F(\tilde{t})\beta_I^*(\tilde{t})-F(t)\beta_I^*(t)\geq[F(\tilde{t})-F(t)]t+\int_t^{\tilde{t}} sdF(s)\\ &\qquad-\eta^g(\lambda^g-1)[(1-F(\tilde{t}))F(\tilde{t})-(1-F(t))F(t)]t\\ &\qquad-\eta^g(\lambda^g-1)\left[(1-F(\tilde{t}))\int_{\underline{t}}^{\tilde{t}} sdF(s)\right.\\ &\qquad\left.-(1-F(t))\int_{\underline{t}}^{t} sdF(s)\right]+\underbrace{\Omega(t,t)-\Omega(\tilde{t},t)}_{<0}. \end{aligned} \tag{5}$$

The term on the left-hand side of (5) represents the material cost for a type-t bidder to mimick a higher type, whereas the first two terms on the right-hand side represent the material benefits. The other terms on the right-hand side of (5) stem from loss aversion. The first two represent the gain–loss utility associated with a change in the extensive risk. Under CPE, when deviating and bidding more aggressively, a bidder is not only more likely to win but also expects to win with a higher probability. Hence, under CPE, bidding more aggressively does not always reduce the likelihood of experiencing a loss; in particular, this depends on whether the bidder's chances of winning are higher or lower than 50%, which is the point at which the extensive risk is the highest. Thus, under CPE, the extensive risk pushes bidders with relatively high (resp. low) signals to overbid (resp. underbid) compared with the risk-neutral benchmark. Moreover, bidding more aggressively also increases the intensive risk, as captured by the last term on the right-hand side of (5). Indeed, when bidding more aggressively, the bidder also expects to win against opponents with higher signals; this, in turn, makes it more likely for the bidder to be ex post disappointed by the realized value of the good. Therefore, different from UPE, with CPE the intensive risk creates a precautionary bidding effect that pushes bidders to bid less aggressively. For concreteness, if types are uniformly distributed on the unit interval, the equilibrium bid is $\beta_I^*(t) = [1-\eta^g(\lambda^g-1)]t + \eta^g(\lambda^g-1)t^2$, which is lower than the bid under UPE.

The example in this section already illustrates many of the key insights of the general model. The next sections formalize and further generalize these insights by considering a more general valuation structure and additional auction formats with more than two bidders.

3. Unacclimating Personal Equilibrium

In this section, we analyze the bidding behavior of loss-averse bidders under UPE for the FPA and SPA with interdependent values and compare it with the risk-neutral and private-value benchmarks.

3.1. First-Price Auctions

We focus on symmetric pure-strategy equilibria which feature bidding functions that are increasing in the bidders' types. To begin, we take the point of view of bidder i with type t_i and consider the order statistics associated with the types of the other bidders. Let τ_1 be the highest of $N-1$ values. Also, let F_1 be the distribution of τ_1 with corresponding density f_1. We claim the existence of a symmetric equilibrium and then verify the claim. Consider bidder i with type $t_i = t$ who bids as if his or her type were $\tilde{t}$ when all other $N-1$ bidders follow the posited equilibrium strategy $\beta_I^*(\cdot)$. This bidder faces a lottery $X_{t_i}^{\tilde{t}} = (V_i, p) \in \mathbb{R}^2$, which realizes as $(v_i(t_1,t_2,\dots,t_N), \beta_I^*(\tilde{t}))$ if $\tau_1 < \tilde{t}$ and as $(0,0)$ otherwise. Let $V(\tilde{t},t) := \mathbb{E}[v_i(t_1,t_2,\dots,t_N)|t_i = t, \tau_1 \le \tilde{t}]$ denote the expected value of the prize for such a bidder, conditional on winning the auction. Moreover, let $\tilde{F}(\cdot|\tilde{t},t)$ denote the distribution of $v_i(t_1,t_2,\dots,t_N)$, conditional on $t_i = t$ and $\tau_1 \le \tilde{t}$, with $\tilde{f}(\cdot|\tilde{t},t)$ denoting the corresponding density. Then, in a symmetric equilibrium, a type-t bidder's direct utility, when he or she expects to bid $\beta_I^*(t)$ but deviates and bids $\beta_I^*(\tilde{t})$, as if the bidder's type were $\tilde{t} > t$, has the following representation:

$$EU(\tilde{t},t) = F_1(\tilde{t})V(\tilde{t},t) - \tilde{\Psi}(\tilde{t},t) - \tilde{\Omega}(\tilde{t},t) - \tilde{\mathcal{T}}_I(\tilde{t},t), \quad (6)$$

with

$$\begin{aligned}\tilde{\mathcal{T}}_I(\tilde{t},t) := {} & F_1(\tilde{t})\beta_I^*(\tilde{t}) + \eta^m\{\lambda^m[1-F_1(t)]F_1(\tilde{t})\beta_I^*(\tilde{t}) \\ & - F_1(t)[1-F_1(\tilde{t})]\beta_I^*(t) \\ & + \lambda^m F_1(\tilde{t})F_1(t)[\beta_I^*(\tilde{t}) - \beta_I^*(t)]\},\end{aligned}$$

$$\begin{aligned}\tilde{\Omega}(\tilde{t},t) := {} & F_1(t)F_1(\tilde{t})\eta^g\Bigg\{\lambda^g\int_{v(t,\underline{t})}^{v(t,t)}\int_{v(t,\underline{t})}^{x}(x-y)\tilde{f}(y|\tilde{t},t)dy \\ & \times\tilde{f}(x|t,t)dx - \int_{v(t,\underline{t})}^{v(t,\tilde{t})}\int_{v(t,\underline{t})}^{x}(x-y)\tilde{f}(y|t,t)dy \\ & \times\tilde{f}(x|\tilde{t},t)dx\Bigg\},\end{aligned}$$

and

$$\begin{aligned}\tilde{\Psi}(\tilde{t},t) := {} & \eta^g\{\lambda^g[1-F_1(\tilde{t})]F_1(t)V(t,t) \\ & -[1-F_1(t)]F_1(\tilde{t})V(\tilde{t},t)\}.\end{aligned}$$

Note that $\tilde{\Psi}$ captures the extensive risk, whereas $\tilde{\Omega}$ captures the intensive one and $\tilde{\mathcal{T}}_I$ is the bidder's disutility from deviating to a larger bid. Indeed, if type t mimics type $\tilde{t} > t$, then type t, conditional on winning, always experiences a loss in the monetary dimension as $\beta_I^*(\tilde{t}) > \beta_I^*(t)$.[12] Taking the first-order condition and evaluating it at $\tilde{t} = t$ yields a differential equation with a solution that provides us with the equilibrium bidding strategy.

Proposition 1. *The symmetric PPE strategies in the FPA are given by*

$$\beta_I^*(t) = \int_{\underline{t}}^{t}\frac{f_1(s)V(s,s) + F_1(s)V_1(s,s) - \left[\tilde{\Psi}_1(s,s) + \tilde{\Omega}_1(s,s)\right]}{F_1(t)\{1+\eta^m\lambda^m\}} \times e^{\frac{\eta^m(\lambda^m-1)[F_1(t)-F_1(s)]}{1+\eta^m\lambda^m}}ds. \quad (7)$$

Next, we compare (7) with the risk-neutral bid. Suppose first that $\eta^m = 0$. Then, it is easy to see that $\beta_I^*(t)$ is equal to the difference between the risk-neutral bid (the terms that involve $V(s,s)$) and a term capturing how a loss-averse bidder reacts to the intensive and extensive risks, $-[\tilde{\Psi}_1(s,s) + \tilde{\Omega}_1(s,s)] > 0$. Thus, we have the following result.

Proposition 2. *For $\eta^m = 0$, loss-averse bidders always overbid relative to risk-neutral bidders.*

Hence, under UPE, both the extensive and intensive risks exert an upward pressure on bids of all types.

We focus first on the extensive risk, captured by the term $-\tilde{\Psi}_1(s,s)$. Under UPE, in equilibrium, bidders take their reference point as given. This creates an attachment effect that drives up the equilibrium bid. Indeed, by increasing their bids, bidders both reduce the probability of realizing a loss and, at the same time, increase the probability of realizing a gain.

Next, to see how the intensive risk affects the equilibrium bidding strategy, we compare (7) with its private-value analogue, where bidders are exposed only to the extensive risk. It is easy to see that the private-value bid has the same structure as in (7) but without the term $-\tilde{\Omega}_1(s,s) > 0$. This term captures the intensive risk, which increases bids via a comparison effect: bidders have a lower incentive to reduce their bids, as doing so would imply winning against opponents with lower signals and obtaining an item with a lower value. Therefore, loss-averse bidders bid more aggressively in an auction with interdependent values than in one with private values; thus, they may fall prey to the winner's curse. Yet, in our framework, the winner's curse does not operate via a form of limited rationality as in, for instance, the Eyster and Rabin (2005) model of "cursedness." In that model, "cursed" bidders overbid because they overestimate the good's value by failing to realize that, conditional on winning, they have the highest signal. By contrast, loss-averse bidders hold correct expectations about the good's value but bid aggressively in order to reduce the chance of being ex post disappointed by its realized value. Hence, as the intensive risk pushes them to bid more aggressively, loss-averse bidders are worse off in an interdependent-value environment than in a comparable private-value one.[13]

Proposition 3. *For values of η^m that are sufficiently small, loss-averse bidders receive lower utility in an interdependent-value environment than in a comparable private-value one.*

This result is in stark contrast to the Eső and White (2004) finding that DARA bidders are made better off by the riskiness of the good's value. Moreover, an interesting implication of Proposition 3 is that, when facing loss-averse bidders (and under UPE), a revenue-maximizing seller has a strong incentive to purposely introduce risk into bidders' values. This is indeed consistent with actual practice in many real-world auctions. For instance, in commercial auctions for fish in Japan and Australia, sellers usually auction off crates of fish without disclosing their weight in advance. Participants in these auctions are usually restaurant owners whose main goal is to buy as much fish as possible at a reasonable price. Auctioneers could easily weigh the crates before the auction and provide this information publicly to all bidders; yet, they choose not to. This creates a nonnegligible amount of risk for bidders who have to come up with an estimate of how much fish is in a crate; moreover, bidders are likely to differ in experience and skills, leading them to come up with different estimates for the weight. Another example of auctions in which sellers choose not to disclose readily available information is the "name your own price" selling mechanism used, for instance, by internet platforms such as Priceline.com. In this type of auction, a potential buyer makes an offer for a particular service—such as a one-way flight from Sydney to Dubai. The platform then searches within its database for deals that fit only some minimum requirements (e.g., the flight must depart on the date requested by the buyer) but without committing to a particular standard of service (e.g., the trip from Sydney to Dubai might require two or more layovers). Crucially, the buyer must accept in advance to pay the offered amount if the platform finds a deal that only meets the minimum requirements. Again, the seller (the platform in this case) could easily provide more detailed information about the deals, but chooses not to. Yet another example are real estate auctions in Australia, where sellers often use secret reserve prices; indeed, it is not uncommon for prospective home buyers to beat the competitors in the auction only to find out that their bid is below the seller's reserve; insofar as reserve prices contain some valuable information about a house's quality or its resale value, by choosing not to disclose them, sellers are purposely subjecting bidders to additional risk over the value of the house.

So far, we have highlighted that, under UPE, both the intensive risk and extensive risk in the item dimension lead to more aggressive bidding. The following proposition characterizes the effect of the extensive risk in money on the equilibrium bid.

Proposition 4. *Suppose $\eta^m > 0$. Then, an increase in λ^m reduces every type's bid. Moreover, for values of λ^m that are sufficiently large, all types underbid compared with risk neutrality. However, for $\eta^g = \eta^m$ and a given λ^g, there is a nongeneric set of values of λ^m such that every type bids more aggressively under loss aversion than under risk neutrality.*

Hence, if loss aversion over money is not too strong, loss-averse bidders continue to bid more than risk-neutral ones.

3.2. Second-Price Auctions

In a symmetric equilibrium, a type-t bidder's direct utility when he or she expects to bid $\beta^*_{II}(t)$ but deviates and bids $\beta^*_{II}(\tilde{t})$, as if his or her type were $\tilde{t} > t$, has the following representation:[14]

$$EU(\tilde{t},t) = F_1(\tilde{t})V(\tilde{t},t) - \tilde{\Psi}(\tilde{t},t) - \tilde{\Omega}(\tilde{t},t) - \tilde{\mathcal{T}}_{II}(\hat{t},t), \quad (8)$$

where $\tilde{\Psi}$ and $\tilde{\Omega}$ are defined as in Section 3.1 and

$$\begin{aligned}\tilde{\mathcal{T}}_{II}(\tilde{t},t) := &\int_{\underline{t}}^{\tilde{t}} \beta^*_{II}(\tau_1)f_1(\tau_1)d\tau_1 + \eta^m\Big\{\lambda^m[1 - F_1(t)] \\ &\times \int_{\underline{t}}^{\tilde{t}} \beta^*_{II}(\tau_1)f_1(\tau_1)d\tau_1 - [1 - F_1(\tilde{t})] \\ &\times \int_{\underline{t}}^{t} \beta^*_{II}(\tau_1)f_1(\tau_1)d\tau_1\Big\} \\ &+ \eta^m\Big\{\lambda^m \int_{\underline{t}}^{\tilde{t}} \int_{\underline{t}}^{\min\{x,t\}} [\beta^*_{II}(x) - \beta^*_{II}(y)] \\ &\times f_1(y)dyf_1(x)dx - \int_{\underline{t}}^{t} \int_{\underline{t}}^{x} [\beta^*_{II}(x) - \beta^*_{II}(y)] \\ &\times f_1(y)dyf_1(x)dx\Big\}.\end{aligned}$$

Comparing (8) with (6), we see that the two differ only in those terms related to the bidder's payment, $\tilde{\mathcal{T}}_I$ and $\tilde{\mathcal{T}}_{II}$. Indeed, the SPA exposes bidders to additional intensive risk in their payment in the event of winning the auction. The following proposition characterizes the symmetric equilibrium bidding strategies for the SPA under UPE.

Proposition 5. *The symmetric PPE strategies in the SPA are given by*

$$\begin{aligned}\beta^*_{II}(t) = &\frac{1}{1+\eta^m\lambda^m} \\ &\times \left\{\frac{f_1(t)V(t,t) + F_1(t)V_1(t,t) - [\tilde{\Psi}_1(t,t) + \tilde{\Omega}_1(t,t)]}{f_1(t)}\right\} \\ &+ \frac{\eta^m(\lambda^m - 1)}{(1+\eta^m\lambda^m)^2}\int_{\underline{t}}^{t}\{f_1(s)V(s,s) + F_1(s)V_1(s,s) \\ &- [\tilde{\Psi}_1(s,s) + \tilde{\Omega}_1(s,s)]\}e^{\frac{\eta^m(\lambda^m-1)[F_1(t)-F_1(s)]}{1+\eta^m\lambda^m}}ds. \quad (9)\end{aligned}$$

It is easy to verify that results analogous to those in Propositions 2, 3, and 4 also apply to the SPA. Thus, $\beta^*_{II}(t)$ is increasing in both the extensive and intensive risk, and if $\eta^m = 0$, loss-averse buyers always bid more aggressively than risk-neutral ones (and they continue to do so for $\eta^m > 0$ as long as λ^m is not too large). Moreover, the difference between the common-value bidding strategy and the private-value one is given by the term related to the intensive risk in consumption, which exerts an additional upward pressure on the bids. Hence, as in the FPA, the extensive risk and intensive risk work in the same direction and induce loss-averse bidders to bid more aggressively compared with the case of private values.

3.3. FPA vs. SPA

We have shown that, under UPE, loss-averse bidders react to both the intensive and extensive risks in the good dimension by increasing their bids. The extensive risk increases bids via the attachment effect—fixing the reference point, bidders have a higher willingness to pay and thus bid more in order to reduce the chance of experiencing a loss in the item dimension when failing to win the auction. The intensive risk instead increases bids via the comparison effect—bidders have a lower incentive to reduce their bids, as doing so would imply winning against opponents with lower signals and obtaining an item of lower value. Bidding more aggressively, however, also exposes bidders to losses in the money dimension. The next proposition shows that the implications for the seller's revenue are the same in both auction formats.

Proposition 6. *If bidders bid according to the PPE, then the FPA is revenue equivalent to the SPA.*

The intuition is that, in both formats, equilibrium behavior is driven by a bidder's desire to bid more aggressively in order to reduce potential losses. Bidding more aggressively in the FPA entails a loss in the money dimension—conditional on winning—exactly equal to the increase in the bid; this, in turn, partially mitigates the desire to bid aggressively. In contrast, bidding more aggressively in the SPA leads to a monetary loss only if the bidder is tied with his or her strongest opponent. Yet, in the SPA, the payment conditional on winning is uncertain; this additional intensive risk also partially mitigates the desire to bid aggressively. In the PPE, these two negative effects have exactly the same magnitude, so that the two auction formats raise the same expected revenue.

4. Choice-Acclimating Personal Equilibrium

In this section, we analyze the FPA and SPA under CPE and show that the effects of the extensive and intensive risks on the bidding strategies are remarkably different than under UPE.

4.1. First-Price Auctions

Let F_1, f_1, $V(\tilde{t},t)$, $\tilde{F}(\cdot|\tilde{t},t)$, and $\tilde{f}(\cdot|\tilde{t},t)$ be defined as in Section 3, and recall that $\Lambda^l := \eta^l(\lambda^l - 1)$ for $l \in \{g,m\}$. We claim the existence of a symmetric equilibrium and then verify the claim. Consider bidder i with type $t_i = t$ who bids as if his or her type were $\tilde{t}$ when all other $N-1$ bidders follow the posited equilibrium strategy $\beta^*_I(\cdot)$. Then, in a symmetric equilibrium, a type-t bidder's direct utility when he or she expects to bid $\beta^*_I(t)$

but deviates and bids $\beta_I^*(\tilde{t})$, as if the bidder's type were $\tilde{t} > t$, has the following representation:

$$EU(\tilde{t},t) = F_1(\tilde{t})V(\tilde{t},t) - \Psi(\tilde{t},t) - \Omega(\tilde{t},t) - \mathcal{T}_I(\tilde{t}), \quad (10)$$

where

$$\Omega(\tilde{t},t) := F_1(t)F_1(\tilde{t})\Lambda^g \int_{v(t,\underline{t})}^{v(t,\bar{t})} \int_{v(t,\underline{t})}^{x} (x-y)\tilde{f}(y|\tilde{t},t)dy \times \tilde{f}(x|\tilde{t},t)dx$$

captures the intensive risk whereas $\Psi(\tilde{t},t) := \Lambda^g[1 - F_1(\tilde{t})]F_1(\tilde{t})V(\tilde{t},t)$ captures the extensive risk, and $\mathcal{T}_I(\tilde{t}) := F_1(\tilde{t})\beta_I^*(\tilde{t})\{1+ \Lambda^m[1 - F_1(\tilde{t})]\}$. Notice the difference between (10) and (6). Under CPE, the reference point adjusts to the deviation, and the extensive and intensive risks depend only on $\tilde{t}$, the type the bidder pretends to be. Under UPE, instead, these risks depend also on t, the bidder's true type. Taking the first-order condition and evaluating it at $\tilde{t} = t$ yields a differential equation whose solution provides us with the equilibrium bidding strategy.

Proposition 7. *Under Assumption 1, symmetric equilibrium strategies in the FPA are given by*

$$\beta_I^*(t) = \frac{\int_{\underline{t}}^{t}\{f_1(s)V(s,s)+F_1(s)V_1(s,s)-[\Psi_1(s,s)+\Omega_1(s,s)]\}ds}{F_1(t)\{1+\Lambda^m[1-F_1(t)]\}}. \quad (11)$$

To highlight the effect of the risk in the good's value on the behavior of a loss-averse bidder, it is useful to compare the bidding function in (11) with its private-value analogue in Lange and Ratan (2010). It is easy to see that the private-value bid has the same structure as in (11) but without the term $-\Omega_1(s,s) < 0$. Hence, this term captures the impact of the intensive risk on the bidder's equilibrium strategy. Different from UPE, when deviating to a higher bid, the bidder's reference point adjusts immediately; hence, bidding more aggressively raises the expected value of the good conditional on winning and thus the probability of experiencing a loss. Loss-averse bidders dislike risk in consumption outcomes. Thus, the intensive risk creates a precautionary bidding effect that reduces bids compared with the case of private values.

Next, we compare $\beta_I^*(t)$ to the risk-neutral bid $\beta_I^{RN}(t)$. We start with the following observation.

Observation 1. Notice that $\frac{\partial \beta_I^*(t)}{\partial \Lambda^m} \leq 0\; \forall t$ and the inequality is strict if $t \in (0,\bar{t})$.

Intuitively, loss aversion over money lowers equilibrium bids compared with the risk-neutral benchmark, as loss-averse bidders dislike the extensive risk in monetary outcomes. Yet, the strategy of the bidder with the highest signal is not affected by Λ^m, as in equilibrium, the bidder expects to win the auction and pay his or her bid for sure. The same applies to the bidder with the lowest signal who expects to never win the auction and hence to never pay. The following proposition compares $\beta_I^*(t)$ to $\beta_I^{RN}(t)$ for any Λ^g and Λ^m.

Proposition 8. *Let t^m be such that $F_1(t^m) = 0.5$. Comparing $\beta_I^*(t)$ to $\beta_I^{RN}(t)$, we have the following:*

(i) *If $t \leq t^m$, then $\beta_I^*(t) < \beta_I^{RN}(t)$.*

(ii) *There is a t' such that, for $t \geq t'$, $\beta_I^*(t) \geq \beta_I^{RN}(t)$ if and only if $\int_{\underline{t}}^{\bar{t}} \Omega_1(x,x)+ \Psi_1(x,x)dx \leq 0$.*

Proposition 8 characterizes how the behavior of loss-averse bidders differs from their risk-neutral counterparts. Whether a loss-averse bidder behaves more or less aggressively than a risk-neutral one depends on the magnitudes of the extensive and intensive risks. Whereas the intensive risk unambiguously pushes loss-averse bidders to bid less aggressively compared with the risk-neutral benchmark, the effect of the extensive risk depends on the bidder's type. First, consider those bidders whose type is (weakly) lower than t^m. These bidders have less than a 50% chance of winning the auction and bid less than their risk-neutral counterparts. The intuition is as follows. When comparing the outcome of winning the auction to its counterfactual, a loss-averse bidder with type t experiences expected gain–loss (dis)utility proportional to $-\Psi(t,t)$. Notice that, fixing $V(\tilde{t},t)$, $\Psi(\tilde{t},t)$ is maximized at $F_1(\tilde{t}) = 0.5$, which is the point with the highest extensive risk. Bidders who expect to win with less than 50% probability do not feel attached to the good and therefore bid less aggressively to keep their expectations low and mitigate their disappointment if they lose. Hence, for these bidders, both the intensive risk and the extensive risk have a negative effect on bids. In contrast, bidders who expect to win with more than 50% probability have an incentive to increase their bids in order to reduce their extensive risk. Thus, the effect of the extensive risk must outweigh that of the intensive risk for these bidders to bid more aggressively than their risk-neutral counterparts. The condition in part ii reveals that this depends on the comparison between the extensive and intensive risks for the bidder with the highest type. If the condition holds as a strict inequality, then a bidder with type $\bar{t}$ bids strictly more than under risk neutrality, and so do bidders with types sufficiently close to $\bar{t}$. Hence, high-type bidders can be exposed to the winner's curse in equilibrium. By contrast, the Eyster and Rabin (2005) model of cursedness predicts that low-type bidders overbid and high-type bidders underbid compared with the risk-neutral benchmark. Therefore, although cursedness implies that those bidders who are more pessimistic about the good's value will overbid, expectations-based loss aversion yields the exact opposite prediction.

4.2. Second-Price Auctions

Consider bidder i with type $t_i = t$ who plans to bid as if his or her type were $\tilde{t} > t$ when all other $N-1$ bidders follow the posited symmetric equilibrium strategy $\beta^*_{II}(\cdot)$. The bidder's expected utility is

$$EU(\tilde{t},t) = F_1(\tilde{t})V(\tilde{t},t) - \Psi(\tilde{t},t) - \Omega(\tilde{t},t) - \mathcal{T}_{II}(\tilde{t}), \quad (12)$$

with

$$\mathcal{T}_{II}(\tilde{t}) := \int_{\underline{t}}^{\tilde{t}} \beta^*_{II}(\tau_1) f_1(\tau_1) d\tau_1 \{1 + \Lambda^m[1 - F_1(\tilde{t})]\} + \Lambda^m \int_{\underline{t}}^{\tilde{t}} \left(\int_{\underline{t}}^{x} (\beta^*_{II}(x) - \beta^*_{II}(v)) f_1(v) dv \right) f_1(x) dx.$$

Comparing (12) with (10), it is easy to see that the two expressions differ only in those terms that are related to the bidder's payment. Intuitively, as we are focusing on equilibria in increasing strategies, the two auction formats lead to the same allocation of the good. Yet, in the SPA, bidders face risk regarding their monetary payment when winning, whereas this risk is not present in the FPA. In particular, $\mathcal{T}_{II}(\tilde{t})$ contains an additional term, $-\Lambda^m \int_{\underline{t}}^{\tilde{t}} (\int_{\underline{t}}^{x} (\beta^*_{II}(x) - \beta^*_{II}(v)) f_1(v) dv) \times f_1(x) dx$, which captures the expected gain–loss (dis) utility in the money dimension arising from the intensive risk in the payment. The following proposition describes the symmetric equilibrium strategies for the SPA.

Proposition 9. *Under Assumption 1, symmetric equilibrium strategies in the SPA are given by*

$$\beta^*_{II}(t) = \frac{f_1(t)V(t,t) + F_1(t)V_1(t,t) - [\Psi_1(t,t) + \Omega_1(t,t)]}{(1+\Lambda^m) f_1(t)} + \frac{2\Lambda^m \int_{\underline{t}}^{t} \{f_1(s)V(s,s) + F_1(s)V_1(s,s) - [\Psi_1(s,s) + \Omega_1(s,s)]\} e^{\frac{2\Lambda^m [F_1(t) - F_1(s)]}{1+\Lambda^m}} ds}{(1+\Lambda^m)^2}. \quad (13)$$

It is easy to verify that $\beta^*_{II}(t)$ has the same structure as its private-value analogue in Lange and Ratan (2010) but with an additional term, $-\Omega_1(t,t) < 0$, which comes from the intensive risk. The following proposition compares $\beta^*_{II}(t)$ to the risk-neutral bid, β^{RN}_{II}, for any Λ^g and Λ^m.

Proposition 10. *Let t^m be such that $F_1(t^m) = 0.5$. Comparing $\beta^*_{II}(t)$ to $\beta^{RN}_{II}(t)$, we have the following:*

(i) *If $t \le t^m$, then $\beta^*_{II}(t) < \beta^{RN}_{II}(t)$ for any Λ^m.*

(ii) *There are $\hat{\Lambda}^m > 0$ and t' such that, for $\Lambda^m < \hat{\Lambda}^m$ and $t \ge t'$, $\beta^*_{II}(t) \ge \beta^{RN}_{II}(t)$ if and only if $\Psi_1(\bar{t},\bar{t}) + \Omega_1(\bar{t},\bar{t}) \le 0$.*

As in the FPA, loss aversion has a bifurcating effect. First, loss-averse bidders who have less than a 50% chance of winning the auction bid less than their risk-neutral counterparts; this holds true irrespective of the strength of loss aversion over money. Second, when loss aversion over money is not too strong, those bidders with relatively high types might overbid compared with the risk-neutral benchmark. This happens if and only if the condition in part ii is satisfied. Notice that this condition is a differential version of the condition in part ii of Proposition 8; indeed, different from $\beta^*_I(t)$, $\beta^*_{II}(t)$ depends on the derivative of the net utility. In both the FPA and the SPA, the difference in the expected payments between type t and a slightly lower type t' is the difference in the net utility type t receives from not imitating t'. In the SPA, however, a change in his or her bid affects a bidder's payment only when he or she is tied with the strongest competitor. Finally, the reason why loss aversion over money cannot be too strong for the result in part ii to hold is that the SPA exposes bidders to intensive risk also in the money dimension. Hence, if loss aversion over money is strong enough, bidders will reduce their bids compared with the risk-neutral benchmark, irrespective of their signals.

4.3. FPA vs. SPA

In equilibrium, the FPA and SPA lead to the same allocation of the good and thus expose the bidders to the same extensive risk. Yet, the payment rule of the SPA exposes bidders to additional intensive risk in money. If $\Lambda^m > 0$, this implies that bidders have a lower direct utility in the SPA than in the FPA. In equilibrium, however, bidders react by appropriately shading down their bids in the SPA, and as a result, they enjoy the same equilibrium utility in both auction formats. Hence, we have the following result.

Proposition 11. *In equilibrium, bidders attain the same utility in both auction formats. However, the expected payment of the type-t bidder, for $t > \underline{t}$, is strictly larger in the FPA than in the SPA if $\Lambda^m > 0$.*

The following corollary is an immediate consequence of Proposition 11.

Corollary 1. *The expected revenue in the SPA is the same as in the FPA if $\Lambda^m = 0$, and it is strictly lower if $\Lambda^m > 0$.*

Corollary 1 extends the Lange and Ratan (2010) revenue-ranking result for auctions with independent private values to the case of interdependent values. Recall that with risk-neutral bidders, the revenue equivalence theorem applies to our model because bidders' signals are independent. Moreover, in the Milgrom and Weber (1982) general symmetric model with risk-neutral bidders and affiliated signals, the SPA yields a higher expected revenue than the FPA.[15] Our analysis instead shows that if bidders are loss-averse and signals are independent, revenue-maximizing sellers should favor the FPA over the SPA.

5. UPE vs. CPE

In this section we compare the different implications of UPE and CPE for bidders' behavior and welfare. Under UPE, both the attachment effect and the comparison effect make it attractive for bidders to raise their bids, independently of their signal. In particular, because of the comparison effect, a bidder wants to bid aggressively in order to win against opponents with high signals. Indeed, by doing so, the bidder is less likely to be disappointed about the value of the good when winning the auction; that is, the intensive risk in the good dimension pushes bidders to bid aggressively and exposes them to the winner's curse.

By contrast, under CPE, the attachment effect makes it attractive for bidders to raise their bid only if, by doing so, they can reduce the extensive risk; this, however, only happens if the bidder's signal is sufficiently high. Moreover, and in stark contrast to UPE, the comparison effect stemming from the intensive risk decreases the bids. This is because a bidder's reference point immediately adjusts to the new bid he or she chooses. Yet, after having formed a reference point and having submitted a bid, under CPE, a loss-averse bidder regrets not having bid more because, by doing so, he or she could have reduced the likelihood of experiencing a loss. Thus, under CPE, loss-averse bidders suffer from a loser's curse similar to the one identified by Holt and Sherman (1994) and Pesendorfer and Swinkels (1997). If η^m is small, then, in each auction format, the bids in the PPE are larger than those under CPE.

Proposition 12. *There exists $\hat{\eta}^m > 0$ such that, if $\eta^m \leq \hat{\eta}^m$, bids are larger in the PPE than under CPE for both the FPA and SPA.*

An immediate corollary of Proposition 12 is that, although bidders suffer from a loser's curse under CPE, they are better off than in the PPE.

Corollary 2. *For $\eta^m \leq \hat{\eta}^m$, bidders are better off under CPE than in the PPE.*

By Proposition 12, for $\eta^m \leq \hat{\eta}^m$, a bidder's expected payment in each auction format is lower under CPE than under UPE.[16] On the equilibrium path, however, bidders hold the same reference points in both auction formats and face the same lottery over their material outcomes. Thus, bidders must receive a larger utility under CPE than in the PPE. This result implies that loss-averse bidders have an incentive to commit in advance to their bids, which can rationalize several tactics often employed by bidders in many real-world auctions. For instance, in internet auctions such as eBay, bidders often use a proxy that submits bids on their behalf and thereby allows them to precommit to a maximum price. Similarly, another form of commitment, often used in auctions for real estate or collectable items, is hiring an agent who submits bids on behalf of the actual buyer and is instructed not to bid above a prespecified price.

Finally, the next proposition shows that none of these results depend on the assumption that bidders' utility is linear in the material payoffs.

Proposition 13. *The results of Proposition 12 and Corollary 2 continue to hold for any monotone transformation of the material payoffs.*

6. Loss Aversion vs. Risk Aversion

In this section, we compare the behavior of loss-averse bidders with that of risk-averse ones for the case where the bidder's value is additively separable in his or her own signal. First, we compare the behavior of loss-averse bidders with that of DARA bidders. Eső and White (2004) showed that, in interdependent-value auctions, DARA bidders display a precautionary bidding behavior: they respond to the risk in the good's value by decreasing their equilibrium bid by more than the appropriate increase in the risk premium. Thus, DARA bidders prefer participating in an interdependent-value setting to participating in a private-value one. Under CPE, loss-averse bidders also reduce their bid in response to the intensive risk. Yet, the reduction in the bid is exactly equal to the disutility bidders suffer from the uncertainty in the good's value; hence, they are as well off as in a comparable private-value environment without intensive risk. Under UPE, instead, bidders react to the intensive risk in the good's value by actually increasing their bid. Hence, they not only fully internalize the disutility from the uncertainty in the good's value but also suffer from a larger expected payment resulting from their attempt to minimize losses. Therefore, for moderate degrees of money-loss aversion, under UPE, bidders are worse off in an interdependent-value environment than in a comparable private-value one; on the other hand, the seller is strictly better off in an interdependent-value environment. These different implications stem from the difference in how the intensive risk affects the incentives of DARA bidders and loss-averse bidders under UPE. Under DARA, bidders' degree of risk aversion decreases in their wealth, which is their private type. Bidders with more optimistic signals are therefore less risk-averse and enjoy an additional information rent from this heterogeneity in the degree of risk aversion. Under UPE, in equilibrium, bidders' (stochastic) reference point is correlated with their type so that bidders with more optimistic signals also have a more optimistic reference point, which makes it more likely for them to be disappointed compared with bidders with less optimistic signals. By contrast, under CPE (as with risk neutrality), the direct utility only depends on the action taken, as when deviating from their equilibrium strategy, bidders immediately adjust their reference point. Hence, under CPE, bidders have fewer incentives to bid aggressively, as this increases their reference point, thereby exposing them to a higher probability of realizing a loss.

Next, we compare the behavior of loss-averse bidders with that of CARA bidders. With CARA preferences, bidders are exposed to the same extensive risk in the FPA as in the SPA; thus, it is the intensive risk that determines the auction's revenue. Indeed, we have the following result.

Proposition 14. *With CARA bidders, the auction format that induces less variance in bidders' payoffs conditional on winning raises the highest revenue. Moreover, CARA bidders behave less aggressively than risk-neutral bidders.*

The first part of Proposition 14 shows that, for CARA bidders, as for loss-averse bidders under CPE, it is the intensive risk that determines the performance of a selling mechanism. Yet, under CARA, the SPA might entail a smaller intensive risk than the FPA. In this case, the winner's expected payment—and thus, the seller's revenue—is larger in the SPA than in the FPA, as shown for instance by Murto and Valimäki (2015) for the case of large auctions. With loss-averse bidders under CPE, instead, the opposite result holds. Moreover, the second part of Proposition 14 states that, in both auction formats, CARA bidders bid less than risk-neutral ones, as Menicucci (2004) shows for the FPA. In contrast, loss-averse bidders might bid more than risk-neutral ones under CPE as well as UPE.

7. Conclusion

Ample evidence, gathered from both the field and the laboratory, indicates that people evaluate outcomes not (only) in absolute terms but (also) relative to a reference point, and that losses (relative to this reference point) loom larger than equal-size gains; see, for instance, Kahneman et al. (1990, 1991) on the endowment effect in laboratory trade experiments; Odean (1998), Genesove and Mayer (2001), and Meng and Weng (2017) on the disposition effect in the stock and housing market; and Crawford and Meng (2011) on cabdrivers' labor supply decisions. In particular, as shown by Lange and Ratan (2010), Banerji and Gupta (2014), Eisenhuth (2019), Rosato (2019), Rosato and Tymula (2019), and von Wangenheim (2017), expectations-based loss aversion has important implications for auction design.

Whereas these previous contributions have focused solely on auctions with private values, our paper is the first to study the role of expectations-based loss aversion in auctions with interdependent values. Our analysis highlights how the behavior of loss-averse bidders depends on how they react to the extensive and intensive risk, which in turn depends on the extent to which bidders incorporate their bid into their reference point. In particular, the intensive risk in the good's value—which represents the main difference with respect to private-value environments—affects the bidding behavior of loss-averse bidders in opposite ways under UPE compared to CPE.

Our findings have important implications for the welfare of bidders and the auctioneer's revenue. Concerning the bidders, we have shown that, under both UPE and CPE, loss aversion exposes them to the winner's curse; yet, the testable predictions of our model differ from those of models based on bounded rationality, such as cursedness à la Eyster and Rabin (2005) and "level-k" bidding à la Crawford and Iriberri (2007). Moreover, under CPE, bidders also suffer from a loser's curse as they commit to their bid in advance, but afterward regret not having bid higher; however, exactly because of this ability to commit, loss-averse bidders attain a higher utility under CPE than UPE. Hence, our model provides a new explanation for the use of commitment devices in auctions, such as proxy bidding in online auctions.

With respect to the auctioneer, we have shown that revenue equivalence might fail even if bidders have independent signals. Indeed, under CPE, the FPA yields a higher revenue than the SPA; moreover, this ranking might continue to hold also when bidders' signals are affiliated (see the web appendix for details). Finally, our results identify the intensive risk as a crucial determinant of the auctioneer's revenue. In particular, risk in the good's value leads to more (resp. less) aggressive bidding under UPE (resp. CPE). Hence, revenue decreases (resp. increases) if the auctioneer provides bidders with additional information about the good's value under UPE (resp. CPE).

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Appendix. Proofs

Definition (∗). The following two expressions play an important role in some of the proofs:

$$\mathcal{V}(\tau_1,t) := \int_{v(t,\tau_1,\underline{t},\dots,\underline{t})}^{v(t,\tau_1,\dots,\tau_1)} x d\tilde{F}_{\tau_1,t}(x) \text{ and}$$

$$\mathcal{L}(\tau_1,y_1,t) := \int_{\max\{v(t,\tau_1,\underline{t},\dots,\underline{t}),v(t,y_1,\underline{t},\dots,\underline{t})\}}^{v(t,\tau_1,\dots,\tau_1)} \times \int_{v(t,y_1,\underline{t},\dots,\underline{t})}^{x} (x-y) d\tilde{F}_{y_1,t}(y) d\tilde{F}_{\tau_1,t}(x),$$

where $\tilde{F}_{\tau_1,t}$ is the cumulative distribution function (cdf) of $v(t,\tau_1,\dots)$. In words, $\mathcal{V}(\tau_1,t)$ is the expected value of the prize to a type-t bidder conditional on winning and on his or her strongest competitor having type τ_1; similarly, $\mathcal{L}(\tau_1,y_1,t)$ represents the expected gain (resp. loss) of a type-t bidder conditional on (resp. expecting to) winning against a bidder with type τ_1 while expecting to win (resp. winning) against a bidder with type y_1.

Proof of Proposition 1. Differentiating the direct utility function, (6), with respect to $\tilde{t}$ and evaluating the first-order condition at $\tilde{t}=t$ yields the following differential equation:

$$\begin{aligned}&(F_1(t)\beta_I^*(t))' + \eta^m\{\lambda^m(1-F_1(t))(F_1(t)\beta_I^*(t))' + f_1(t)\beta_I^*(t)F_1(t)\\&\quad + \lambda^m(\beta_I^*(t))'F_1(t)F_1(t)\}\\&\quad = f_1(t)V(t,t) + F_1(t)V_1(t,t) - \tilde{\Psi}_1(t,t) - \tilde{\Omega}_1(t,t).\end{aligned}$$

Solving this equation yields expression (7) in the text. Next, we verify sufficiency. To do so, we show that the cross-partial derivative of the direct utility function is positive. Under UPE, the direct utility reads as follows:

$$\begin{aligned}EU(\tilde{t},t) &= \int_{\underline{t}}^{\tilde{t}} \mathcal{V}(\tau_1,t) f_1(\tau_1) d\tau_1 - \eta^g\lambda^g(1-F_1(\tilde{t}))\\&\quad \times \int_{\underline{t}}^{t} \mathcal{V}(\tau_1,t) f_1(\tau_1) d\tau_1 + \eta^g(1-F_1(t))\\&\quad \times \int_{\underline{t}}^{\tilde{t}} \mathcal{V}(\tau_1,t) f_1(\tau_1) d\tau_1\\&\quad - \eta^g\lambda^g \int_{\underline{t}}^{t}\int_{\underline{t}}^{\tilde{t}} \mathcal{L}(\tau_1,y_1,t) f_1(\tau_1) f_1(y_1) dy_1 d\tau_1\\&\quad + \eta^g \int_{\underline{t}}^{\tilde{t}}\int_{\underline{t}}^{t} \mathcal{L}(\tau_1,y_1,t) f_1(\tau_1) f_1(y_1) dy_1 d\tau_1\\&\quad - \mathcal{T}_I(\tilde{t},t),\end{aligned} \tag{A.1}$$

where $\mathcal{V}$ and $\mathcal{L}$ are defined as in (∗). Differentiating (A.1) with respect to $\tilde{t}$ yields

$$f_1(\tilde{t})\left[\mathcal{V}(\tilde{t},t)(1+\eta^g) + \eta^g\lambda^g \int_{\underline{t}}^{t}[\mathcal{V}(\tau_1,t) - \mathcal{L}(\tau_1,\tilde{t},t)] f_1(\tau_1) d\tau_1 - \eta^g \int_{\underline{t}}^{t}[\mathcal{V}(\tilde{t},t) - \mathcal{L}(\tilde{t},y_1,t)] f_1(y_1) dy_1\right] - \frac{\partial \mathcal{T}_I(\tilde{t},t)}{\partial \tilde{t}}.$$

In Lemma 1 in the web appendix, we show that the derivative of the aforementioned expression with respect to t is positive. Hence, $EU(\tilde{t},t)$ satisfies single crossing, which implies sufficiency of the first-order condition. Next, notice that single crossing of $EU(\tilde{t},t)$ also implies that the objective function satisfies the strict Spence-Mirrlees condition as defined by Milgrom and Shannon (1994). Hence, $\beta_I^*(t)$ is increasing by Theorem 4 in Milgrom and Shannon (1994). Finally, we show that the bidding function presented in (7) represents the PPE. To see this, we derive the set of all personal equilibrium (PE) bids. Take any increasing function β_I. If this is an equilibrium bid, then bidder t has no incentive to mimic a lower type $\tilde{t} < t$. The bidder's payoff from mimicking a lower type is

$$\begin{aligned}EU(\tilde{t},t) &= F_1(\tilde{t})V(\tilde{t},t) - \tilde{\Psi}(\tilde{t},t) - \tilde{\Omega}(\tilde{t},t) - F_1(\tilde{t})\beta_I(\tilde{t})\\&\quad - \eta^m\{\lambda^m[1-F_1(t)]F_1(\tilde{t})\beta_I(\tilde{t})\\&\quad - F_1(t)[1-F_1(\tilde{t})]\beta_I(t) - F_1(\tilde{t})F_1(t)[\beta_I(\tilde{t}) - \beta_I(t)]\}.\end{aligned}$$

In equilibrium, $\lim_{\tilde{t}\to t}(EU(t,t) - EU(\tilde{t},t)) \geq 0$, which is equivalent to

$$\begin{aligned}A_1(t,t) &\geq (F_1(t)\beta_I(t))' + \eta^m\{\lambda^m(1-F_1(t))(F_1(t)\beta_I(t))'\\&\quad + f_1(t)\beta_I(t)F_1(t) + \beta_I(t)'F_1(t)^2\},\end{aligned} \tag{A.2}$$

where $A(\tilde{t},t) = F_1(\tilde{t})V(\tilde{t},t) - \tilde{\Psi}(\tilde{t},t) - \tilde{\Omega}(\tilde{t},t)$. Making (A.2) bind and solving the resulting differential equation provides us with an upper bound, $\overline{\beta}$, on any PE bidding strategy. Similarly, to derive a lower bound on any PE bidding strategy, we need to ensure that a bidder of type t does not want to deviate and mimic a higher type $\tilde{t} > t$. The bidder's payoff from mimicking a higher type is given by (6) in the main text. In equilibrium, $\lim_{\tilde{t}\to t}(EU(t,t) - EU(\tilde{t},t)) \geq 0$, which is equivalent to

$$\begin{aligned}A_1(t,t) &\leq (F_1(t)\beta_I(t))' + \eta^m\{\lambda^m(1-F_1(t))(F_1(t)\beta_I(t))'\\&\quad + f_1(t)\beta_I(t)F_1(t) + \lambda^m(\beta_I(t))'F_1(t)F_1(t)\}.\end{aligned} \tag{A.3}$$

Making (A.3) bind and solving the resulting differential equation provides us with a lower bound, $\underline{\beta}$, on any PE bidding strategy. Thus, PE bidding strategies must lie in the set $[\underline{\beta},\overline{\beta}]$. Finally, notice that the ex ante preferred strategy is the one where bidders pay the least; that is, $\underline{\beta}$ is the PPE. Hence, $\beta_I^* = \underline{\beta}$. □

Proof of Proposition 2. Let $\eta^m = 0$ and observe that $F_1(t)\times[\beta_I^*(t) - b_I^*(t)] = -\int_{\underline{t}}^{t}(\tilde{\Psi}_1(s,s) + \tilde{\Omega}_1(s,s))ds$. Then, it suffices to show that $-\tilde{\Psi}_1(t,t) - \tilde{\Omega}_1(t,t) > 0$. Using the representation in (A.1), we can identify $-\tilde{\Psi}_1(t,t) - \tilde{\Omega}_1(t,t)$ as

$$f_1(t)\left\{\mathcal{V}(t,t)\eta^g + \eta^g\lambda^g \int_{\underline{t}}^{t}[\mathcal{V}(\tau_1,t) - \mathcal{L}(\tau_1,t,t)] f_1(\tau_1) d\tau_1 - \eta^g \int_{\underline{t}}^{t}[\mathcal{V}(t,t) - \mathcal{L}(t,y_1,t)] f_1(y_1) dy_1\right\},$$

where $\mathcal{V}$ and $\mathcal{L}$ are defined as in (∗). This term is positive because $(1-F_1(t))\mathcal{V}(t,t) > 0$, $\mathcal{V}(\tau_1,t) - \mathcal{L}(\tau_1,t,t) > 0$, and $\mathcal{L}(t,y_1,t) > 0$. □

Proof of Proposition 3. Using (6), we have

$$\begin{aligned}EU(t) &= F_1(t)V(t,t) - \tilde{\Psi}(t,t) - \tilde{\Omega}(t,t) - F_1(t)\beta_I^*(t)\\&\quad \times\{1 + \eta^m(\lambda^m - 1)[1 - F_1(t)]\}.\end{aligned}$$

Similarly, under private values (i.e., when $V(t,t)=h(t)$), the equilibrium utility, EU^{PV}, reads as

$$\begin{aligned} EU^{PV}(t) = F_1(t)h(t) - \eta^g\big\{\lambda^g[1-F_1(t)]F_1(\tilde{t})h(t) \\ -\big[1-F_1(\tilde{t})\big]F_1(t)h(t)\big\} - F_1(t)b_I^*(t) \\ \times\big\{1+\eta^m(\lambda^m-1)[1-F_1(t)]\big\}, \end{aligned}$$

where $b_I^*(t)$ is the private-value bid. Recall that $v(t_i,\mathbf{t}_{-i}) = h(t_i)+g(\mathbf{t}_{-i})$; then, substituting for the bids, the difference in the equilibrium utility between the common-value and the private-value environments is

$$\begin{aligned} F_1(t)q(t) - \int_{\underline{t}}^{t}\{F_1(s)q'(s)+f_1(s)q(s)\}w(s,t)ds \\ -\eta^g\{\lambda^g-1\}[1-F_1(t)]F_1(t)q(t) \\ -\int_{\underline{t}}^{t}\{\eta^g\lambda^g f_1(s)q(s)+\eta^g f_1(s)q(s)[1-F_1(s)] \\ +\eta^g[1-F_1(s)]F_1(s)q'(s)\}w(s,t)ds \\ -\tilde{\Omega}(t,t)+\int_{\underline{t}}^{t}\tilde{\Omega}_1(s,s)w(s,t)ds, \end{aligned} \quad (A.4)$$

where $q(\tilde{t}) := \mathbb{E}[g(\mathbf{t}_{-i})|\tau_1\le\tilde{t}]$ and $w(s,t) := \frac{1+\eta^m(\lambda^m-1)[1-F_1(t)]}{1+\eta^m\lambda^m}\times e^{\frac{\eta^m(\lambda^m-1)}{1+\eta^m\lambda^m}(F_1(t)-F_1(s))}$. Observe first that the third and fourth terms of (A.4) are always negative. Moreover, if $\eta^m=0$, then $w(s,t)=1$ and the first two terms of (A.4) add up to zero. If $\eta^m=0$, the last term is negative because $-\tilde{\Omega}(t,t)+\int_{\underline{t}}^{t}\tilde{\Omega}_1(s,s)ds = -\int_{\underline{t}}^{t}\tilde{\Omega}_2(s,s)ds<0$. Thus, if $\eta^g>0$, $\lambda^g>1$, and $\eta^m=0$, then the claim holds true for any λ^m. By continuity, we can find a sufficiently small η^m such that the payoff difference is negative. □

Proof of Proposition 4. The first statement immediately follows from the equilibrium bid. Moreover, if λ^m grows large, the denominator grows without bound, which proves the second statement. To prove the last statement, we evaluate the bidding function at $\eta^g=\eta^m=\eta$ and $\lambda^m=1$. Then, $F_1(t)\beta_I^*(t) = \int_{\underline{t}}^{t}\frac{f_1(s)V(s,s)+F_1(s)V_1(s,s)-[\tilde{\Psi}_1(s,s)+\tilde{\Omega}_1(s,s)]}{1+\eta}ds$. The last step in the proof of Proposition 2 implies that $-[\tilde{\Psi}_1(s,s)+\tilde{\Omega}_1(s,s)]$ is strictly increasing in λ^g. Applying partial integration, it is easy to see that

$$\begin{aligned} \tilde{\Omega}(\tilde{t},t) = F_1(t)F_1(\tilde{t})\eta^g\Bigg[\lambda^g\int_{v(t,\underline{\mathbf{t}})}^{v(t,\mathbf{t})}\tilde{F}\Big(y\Big|\tilde{t},t\Big)\Big(1-\tilde{F}(y|t,t)\Big)dy \\ -\int_{v(t,\underline{\mathbf{t}})}^{v(t,\tilde{\mathbf{t}})}\tilde{F}(y|t,t)\Big(1-\tilde{F}\Big(y\Big|\tilde{t},t\Big)\Big)dy\Bigg]. \end{aligned}$$

Thus, at $\lambda^g=1$, we have

$$\tilde{\Omega}_1(t,t) = F_1(t)^2\int_{v(t,\underline{\mathbf{t}})}^{v(t,\mathbf{t})}\frac{\partial\tilde{F}(y|\tilde{t})}{\partial\tilde{t}}\bigg|_{\tilde{t}=t}dy.$$

Moreover, note that $V(\tilde{t},t)$ has the following representation:

$$V(\tilde{t},t) = \int_{v(t,\underline{\mathbf{t}})}^{v(t,\tilde{\mathbf{t}})}x\tilde{f}(x|\tilde{t},t)dx = v(t,\tilde{\mathbf{t}}) - \int_{v(t,\underline{\mathbf{t}})}^{v(t,\tilde{\mathbf{t}})}F(x|\tilde{t},t)dx.$$

Hence, it follows that

$$V_1(t,t) = v_2(t,\ \mathbf{t}) - v_2(t,\ \mathbf{t}) - \int_{v(t,\underline{\mathbf{t}})}^{v(t,\ \mathbf{t})}\frac{\partial\tilde{F}(y|\tilde{t},t)}{\partial\tilde{t}}\bigg|_{\tilde{t}=t}dy.$$

Thus, for $\lambda^g=1$, we have that $-\tilde{\Omega}_1(s,s)=F_1(s)^2V_1(s,s)$; hence, the risk-neutral and the loss-averse bid coincide. By continuity, therefore, for every $\lambda^g>1$, we can find an upper bound on λ^m such that the claim holds. □

Proof of Proposition 5. Differentiating the direct utility (8) with respect to $\tilde{t}$ and evaluating the first-order condition at $\tilde{t}=t$ yields the following differential equation:

$$\beta_{II}^*(t)f_1(t)(1+\eta^m\lambda^m) - \int_{\underline{t}}^{t}\beta_{II}^*(s)f_1(s)dsf_1(t)\eta^m(\lambda^m-1) = A_1(t,t),$$

where $A(\tilde{t},t) := F_1(\tilde{t})V(\tilde{t},t)-\tilde{\Psi}(\tilde{t},t)-\tilde{\Omega}(\tilde{t},t)$. Define $\mathscr{A}(t) := A_1(t,t)/f_1(t)$. Then,

$$\beta_{II}^*(t)(1+\eta^m\lambda^m) - \int_{\underline{t}}^{t}\beta_{II}^*(s)f_1(s)ds\eta^m(\lambda^m-1) = \mathscr{A}(t).$$

Differentiating and dividing by $(1+\eta^m\lambda^m)$ on both sides, we obtain

$$(\beta_{II}^*(t))' - \beta_{II}^*(t)f_1(t)\frac{\eta^m(\lambda^m-1)}{1+\eta^m\lambda^m} = \frac{\mathscr{A}'(t)}{1+\eta^m\lambda^m}.$$

The solution of this differential equation reads

$$\begin{aligned} \beta_{II}^*(t) &= \frac{e^{\left[\frac{\eta^m(\lambda^m-1)}{1+\eta^m\lambda^m}\right]F_1(t)}}{1+\eta^m\lambda^m}\int_{\underline{t}}^{t}\mathscr{A}'(s)e^{-\left[\frac{\eta^m(\lambda^m-1)}{1+\eta^m\lambda^m}\right]F_1(s)}ds \\ &= \frac{1}{1+\eta^m\lambda^m}\Bigg\{\mathscr{A}(t)+\frac{\eta^m(\lambda^m-1)}{1+\eta^m\lambda^m} \\ &\quad\times\int_{\underline{t}}^{t}\mathscr{A}(s)f_1(s)e^{\frac{\eta^m(\lambda^m-1)[F_1(t)-F_1(s)]}{1+\eta^m\lambda^m}}ds\Bigg\}. \end{aligned}$$

Substituting for $\mathscr{A}(t)$ leads to (9). Sufficiency of the first-order condition and monotonicity of $\beta_{II}^*(t)$ follow from arguments similar to those in the proof of Proposition 1. □

Proof of Proposition 6. Let $A(\tilde{t},t)$ be defined as in the proof of Proposition 5. We have that

$$\begin{aligned} & F_1(t)\beta_I^*(t) - \int_{\underline{t}}^{t}\beta_{II}^*(\theta_1)f_1(\theta_1)d\theta_1 = 0 \Leftrightarrow \\ & \int_{\underline{t}}^{t}A_1(s,s)\left[e^{\frac{\eta^m(\lambda^m-1)[F_1(t)-F_1(s)]}{1+\eta^m\lambda^m}}-1\right]ds \\ & = \frac{\eta^m(\lambda^m-1)}{1+\eta^m\lambda^m}\int_{\underline{t}}^{t}\int_{\underline{t}}^{s}A_1(v,v)e^{\frac{\eta^m(\lambda^m-1)[F_1(s)-F_1(v)]}{1+\eta^m\lambda^m}}dvf_1(s)ds. \end{aligned} \quad (A.5)$$

Notice that

$$\begin{aligned} & \int_{\underline{t}}^{t}\int_{\underline{t}}^{s}A_1(v,v)e^{\frac{\eta^m(\lambda^m-1)[F_1(s)-F_1(v)]}{1+\eta^m\lambda^m}}dvf_1(s)ds \\ & = \frac{1+\eta^m\lambda^m}{\eta^m(\lambda^m-1)}\int_{\underline{t}}^{t}\left(e^{\frac{\eta^m(\lambda^m-1)F_1(s)}{1+\eta^m\lambda^m}}\right)'\int_{\underline{t}}^{s}A_1(v,v)e^{\frac{-\eta^m(\lambda^m-1)F_1(v)}{1+\eta^m\lambda^m}}dvds \\ & = \frac{1+\eta^m\lambda^m}{\eta^m(\lambda^m-1)}\Bigg[e^{\frac{\eta^m(\lambda^m-1)F_1(t)}{1+\eta^m\lambda^m}}\int_{\underline{t}}^{t}A_1(s,s)e^{\frac{-\eta^m(\lambda^m-1)F_1(s)}{1+\eta^m\lambda^m}}ds \\ & \quad -\int_{\underline{t}}^{t}A_1(s,s)ds\Bigg], \end{aligned}$$

where the second equality follows by applying integration by parts. Thus, (A.5) holds with equality. □

Proof of Proposition 8. Claim (i) Observe first that

$$\beta_I^*(t) \leq \int_{\underline{t}}^{t} \{f_1(s)V(s,s) + F_1(s)V_1(s,s) - [\Psi_1(s,s) + \Omega_1(s,s)]\}ds/F_1(t), \tag{A.6}$$

because $\beta_I^*(t)$ is decreasing in Λ^m. We show that (A.6) is lower than the risk-neutral bid, $\int_{\underline{t}}^{t}\{f_1(s)V(s,s)+F_1(s)V_1(s,s)\}ds/F_1(t)$, for $t \leq t^m$. Indeed, it is easily verified that $\Omega_1(s,s) > 0$, whereas $-\Psi_1(s,s) = -\Lambda^g[1-F_1(s)]F_1(s)V_1(s,s) + \Lambda^g[2F_1(s)-1] f_1(s)V(s,s)$ can only be positive if $F_1(t) > 0.5$.

Claim (ii) We derive a condition for when the highest type bids more aggressively under loss aversion than under risk neutrality. Because the difference in these two bids is continuous in the type, whenever this condition is satisfied for the highest type, there exists a threshold type, t', such that every $t \in [t', \bar{t}]$ bids more aggressively than under risk neutrality.

Note that for $t = \bar{t}$, (A.6) is satisfied with equality for any Λ^m. Thus, the highest type overbids relative to risk neutrality if and only if $\int_{\underline{t}}^{\bar{t}}[\Psi_1(s,s) + \Omega_1(s,s)]ds < 0$. □

Proof of Proposition 7. Differentiating the direct utility function with respect to $\tilde{t}$ and evaluating the first-order condition at $\tilde{t} = t$ yields the following differential equation:

$$\frac{F_1(t)V_1(t,t) + f_1(t)V(t,t) - \Psi_1(t,t) - \Omega_1(t,t)}{F_1(t)\{1 + \Lambda^m[1 - F_1(t)]\}} = \beta_I^*(t)\frac{f_1(t)\{1 + \Lambda^m[1 - 2F_1(t)]\}}{F_1(t)\{1 + \Lambda^m[1 - F_1(t)]\}} + (\beta_I^*(t))'.$$

Solving this differential equation yields the bidding function in the text. Next, as in Proposition 1, sufficiency of the first-order conditions and monotonicity of the bidding function hold if the direct utility satisfies single crossing—that is, if $\frac{\partial^2 EU(\tilde{t},t)}{\partial \tilde{t}\partial t} > 0$. Under CPE, the direct utility reads as follows:

$$\begin{aligned} EU(\tilde{t},t) = &\int_{\underline{t}}^{\tilde{t}} \mathcal{V}(\tau_1,t)f_1(\tau_1)d\tau_1 - \eta^g\lambda^g(1 - F_1(\tilde{t}))\\ &\times \int_{\underline{t}}^{\tilde{t}} \mathcal{V}(\tau_1,t)f_1(\tau_1)d\tau_1 + \eta^g(1 - F_1(\tilde{t}))\\ &\times \int_{\underline{t}}^{\tilde{t}} \mathcal{V}(\tau_1,t)f_1(\tau_1)d\tau_1\\ &- \lambda^g\eta^g \int_{\underline{t}}^{\tilde{t}}\int_{\underline{t}}^{\tilde{t}} \mathcal{L}(\tau_1,y_1,t)f_1(\tau_1)f_1(y_1)dy_1d\tau_1\\ &+ \eta^g \int_{\underline{t}}^{\tilde{t}}\int_{\underline{t}}^{\tilde{t}} \mathcal{L}(\tau_1,y_1,t)f_1(\tau_1)f_1(y_1)dy_1d\tau_1\\ &- \mathcal{T}_I(\tilde{t}), \end{aligned} \tag{A.7}$$

where $\mathcal{V}$ and $\mathcal{L}$ are defined as in (∗). Differentiating (A.7) with respect to $\tilde{t}$ yields

$$f_1(\tilde{t})\left\{(1 - \Lambda^g)\mathcal{V}(\tilde{t},t) + \Lambda^g \int_{\underline{t}}^{\tilde{t}}\left[\mathcal{V}(\tau_1,t) - \mathcal{L}(\tau_1,\tilde{t},t)\right]f_1(\tau_1)d\tau_1 + \Lambda^g \int_{\underline{t}}^{\tilde{t}}\left[\mathcal{V}(\tilde{t},t) - \mathcal{L}(\tilde{t},y_1,t)\right]f_1(y_1)\,dy_1\right\} - \mathcal{T}_I'(\tilde{t}).$$

Differentiating the aforementioned expression with respect to t, we obtain

$$f_1(\tilde{t})\left\{(1 - \Lambda^g)\mathcal{V}_2(\tilde{t},t) + \Lambda^g \int_{\underline{t}}^{\tilde{t}}\left[\mathcal{V}_2(\tau_1,t) - \mathcal{L}_3(\tau_1,\tilde{t},t)\right]f_1(\tau_1)d\tau_1 + \Lambda^g \int_{\underline{t}}^{\tilde{t}}\left[\mathcal{V}_2(\tilde{t},t) - \mathcal{L}_3(\tilde{t},y_1,t)\right]f_1(y_1)dy_1\right\}.$$

Notice that $\mathcal{V}_2(\tilde{t},t) > 0$ and $\Lambda^g \leq 1$; moreover, Lemma 2 in the web appendix shows that $\int_{\underline{t}}^{\tilde{t}}[\mathcal{V}_2(\tau_1,t) - \mathcal{L}_3(\tau_1,\tilde{t},t)] \times f_1(\tau_1)d\tau_1 > 0$ and $\int_{\underline{t}}^{\tilde{t}}[\mathcal{V}_2(\tilde{t},t) - \mathcal{L}_3(\tilde{t},y_1,t)]f_1(y_1)dy_1 > 0$. Hence, $\frac{\partial^2 EU(\tilde{t},t)}{\partial \tilde{t}\partial t} > 0$. □

Proof of Proposition 9. Define $\mathcal{A}(t,t) := f_1(t)V(t,t) + F_1(t) \times V_1(t,t) - \Psi_1(t,t) - \Omega_1(t,t)$. Differentiating the direct utility function with respect to $\tilde{t}$, equating it with zero, and evaluating it at $\tilde{t} = t$ yields the following equation:

$$\frac{A_1(t,t)}{f_1(t)} = \beta_{II}^*(t)[1 + \Lambda^m] - 2\Lambda^m \int_{\underline{t}}^{t} \beta_{II}^*(v)f_1(v)dv. \tag{A.8}$$

Differentiating (A.8) with respect to t and rearranging yields

$$\frac{1}{1+\Lambda^m}\left(\frac{A_1(t,t)}{f_1(t)}\right)' = (\beta_{II}^*(t))' - \frac{2\Lambda^m f_1(t)}{1+\Lambda^m}\beta_{II}^*(t).$$

Solving the aforementioned differential equation yields

$$\begin{aligned}\beta_{II}^*(t) &= \frac{e^{\frac{2\Lambda^m}{1+\Lambda^m}F_1(t)}}{1+\Lambda^m}\int_{\underline{t}}^{t}\left(\frac{A_1(s,s)}{f_1(s)}\right)' e^{-\frac{2\Lambda^m}{1+\Lambda^m}F_1(s)}ds\\ &= \frac{1}{1+\Lambda^m}\left(\frac{A_1(t,t)}{f_1(t)} - \int_{\underline{t}}^{t}\frac{A_1(s,s)}{f_1(s)}\left(-\frac{2\Lambda^m f_1(s)}{1+\Lambda^m}\right)e^{\frac{2\Lambda^m[F_1(t)-F_1(s)]}{1+\Lambda^m}}ds\right),\end{aligned}$$

where the second equality follows from partial integration and $A_1(\underline{t},\underline{t})/f_1(\underline{t}) = 0$. Sufficiency and monotonicity of β_{II}^* follow from arguments similar to those in the proof of Proposition 7. □

Proof of Proposition 10. Claim (i) Observe first that

$$\beta_{II}^*(t) \leq \{f_1(t)V(t,t) + F_1(t)V_1(t,t)[\Psi_1(t,t) + \Omega_1(t,t)]\}/f_1(t), \tag{A.9}$$

because β_{II}^* is decreasing in Λ^m. We show that (A.9) is lower than the risk-neutral bid, $\{f_1(t)V(t,t) + F_1(t)V_1(t,t)\}/f_1(t)$, for $t \leq t^m$. Indeed, it is easily verified that $\Omega_1(t,t) > 0$, whereas $-\Psi_1(t,t) = -\Lambda^g[1-F_1(t)]F_1(t)V_1(t,t) + \Lambda^g[2F_1(t)-1]f_1(t)V(t,t)$ can only be positive if $F_1(t) > 0.5$.

Claim (ii) We derive a condition for when the highest type bids more aggressively under loss aversion with $\Lambda^m = 0$ than under risk neutrality. Because the difference in these two bids is continuous in the type and in Λ^m, whenever this condition is satisfied for the highest type, there exist two thresholds, t' and $\hat{\Lambda}^m > 0$, such that for $\Lambda^m < \hat{\Lambda}^m$, every $t \in [t', \bar{t}]$ bids more aggressively than under risk neutrality.

Note that for $\Lambda^m = 0$, (A.9) is satisfied with equality. Thus, the highest type overbids relative to risk neutrality if and only if $-\Psi_1(\bar{t},\bar{t}) > \Omega_1(\bar{t},\bar{t})$. □

Proof of Proposition 11. The envelope theorem implies that, in both auction formats, $EU(t,t) = \int_{\underline{t}}^{t}[F_1(s)V_2(s,s) - \Psi_2(s,s) - \Omega_2(s,s)]ds$, which proves the first statement. Moreover, in equilibrium, we have the following:

$$\mathcal{T}_k(t) = F_1(t)V(t,t) - \Psi(t,t) - \Omega(t,t) - \int_{\underline{t}}^{t}[F_1(s)V_2(s,s) - \Psi_2(s,s) - \Omega_2(s,s)]ds,$$

where $k \in \{I, II\}$. Therefore, $\mathcal{T}_I = \mathcal{T}_{II} =: \mathcal{T}$. Moreover, notice that the expected payments to the seller from type t (i.e., $F_1(t)\beta_I^*$ and $\int_{\underline{t}}^{t}\beta_{II}^*(s)f_1(s)ds$) satisfy

$$\mathcal{T}(t) = \{1 + \Lambda^m[1 - F_1(t)]\} \times F_1(t)\beta_I^*(t) \text{ and}$$
$$\mathcal{T}(t) = \{1 + \Lambda^m[1 - F_1(t)]\}\int_{\underline{t}}^{t}\beta_{II}^*(s)f_1(s)ds + \Lambda^m I(t),$$

where $I(t) = \int_{\underline{t}}^{t}(\int_{\underline{t}}^{x}(\beta_{II}^*(x) - \beta_{II}^*(v))f_1(v)dv)f_1(x)dx$. Thus, $F_1(t) \times \beta_I^*(t) = \int_{\underline{t}}^{t}\beta_{II}^*(s)f_1(s)ds$ if $\Lambda^m = 0$. Next, let $\Lambda^m > 0$. As $I(t) > 0$ and $\mathcal{T}_I(t) = \mathcal{T}_{II}(t)$, we have that $F_1(t)\beta_I^*(t) > \int_{\underline{t}}^{t}\beta_{II}^*(s)f_1(s)ds$. □

Proof of Proposition 12. Suppose first that $\eta^m = 0$. Then, it is easy to see that for every $t > \underline{t}$, in each auction format, the bid in the PPE is larger than that under CPE because $-\tilde{\Psi}_1(t,t) > -\Psi_1(t,t)$ and $-\tilde{\Omega}_1(t,t) > -\Omega_1(t,t)$. This claim can be verified by using the representation of the direct utilities in (A.1) and in (A.7). Because both bidding functions are continuous in η^m, the stated result follows. □

Proof of Proposition 13. Recall that $v(t_i, t_{-i})$ is the buyer's valuation. Define $\tilde{v}(t_i, t_{-i}) := u_g(v(t_i, \mathbf{t}_{-i}))$, where $u_g : \mathbb{R}_+ \to \mathbb{R}_+$ is a monotone transformation. Similarly, let $u_m : \mathbb{R}_+ \to \mathbb{R}_+$ be a monotone transformation of the (ex post) payment. Fix an auction format $k \in \{I, II\}$. Substituting $\tilde{v}$ for v into the expression of $\beta_k^*(t)$ yields $u_m(\widetilde{\beta}_k^*(t))$, where $\widetilde{\beta}_k^*(t)$ is the equilibrium bid under the new transformation. This bid can be calculated as $u_m^{-1}(u_m(\widetilde{\beta}_k^*(t)))$ because u_m is a monotone function. Then, if $-\tilde{\Psi}_1(t,t) > -\Psi_1(t,t)$ and $-\tilde{\Omega}_1(t,t) > -\Omega_1(t,t)$ for $v(t_i, t_{-i})$, then the same inequalities also hold for $\tilde{v}(t_i, t_{-i})$. Thus, $\widetilde{\beta}_k^*(t)$ is larger in the PPE than under CPE. □

Proof of Proposition 14. We begin by proving the first statement. With a slight abuse of notation, we denote by $EU(t)$ type-t bidders' indirect equilibrium utility. We know that

$$EU_{II}^{CARA,CV}(t) = EU_{II}^{CARA,PV}(t) = EU_{I}^{CARA,PV}(t) = EU_{I}^{CARA,CV}(t),$$

where the first and last equalities follow from Eső and White (2004) and the middle one follows from Matthews (1983). Hence,

$$EU_{II}^{CARA,CV}(t) = EU_{I}^{CARA,CV}(t) \Leftrightarrow U(CE_{II}(t)) = U(CE_I(t)) \Leftrightarrow CE_{II}(t) = CE_I(t),$$

where CE is the certainty equivalent. In equilibrium—that is, for a fixed bidding strategy β_k^{CARA}, $k \in \{I, II\}$—a bidder of type t_i faces a lottery over basic outcomes. Let $X_k^{t_i}$ denote the random variable associated with this lottery in the FPA and the SPA, respectively. For the FPA, we have that $X_I^{t_i} = v(t_i, \mathbf{t}_{-i}) - \beta_I^{CARA}(t_i)$ if t_i is larger than the signal of i's strongest opponent—that is, if $t_i > \tau_1$—and $X_I^{t_i} = 0$ otherwise. Similarly, in the SPA, we have that $X_{II}^{t_i} = v(t_i, \mathbf{t}_{-i}) - \beta_{II}^{CARA}(\tau_1)$ if $t_i > \tau_1$, and $X_{II}^{t_i} = 0$ otherwise. The FPA gives rise to more intensive risk than the SPA if and only if $\mathrm{Var}(X_I^{t_i}|X_I^{t_i} > 0) \geq \mathrm{Var}(X_{II}^{t_i}|X_{II}^{t_i} > 0)$. Both auction formats give rise to the same extensive risk—that is, the probability of losing the auction is the same in both auction formats. Therefore, $\mathrm{Var}(X_I^{t_i}|X_I^{t_i} > 0) \geq \mathrm{Var}(X_{II}^{t_i}|X_{II}^{t_i} > 0)$ if and only if $\mathrm{Var}(X_I^{t_i}) \geq \mathrm{Var}(X_{II}^{t_i})$. As the bidder's utility function is concave and $CE_{II}(t) = CE_I(t)$, it must be that $\mathrm{Var}(X_I^{t_i}|X_I^{t_i} > 0) \geq \mathrm{Var}(X_{II}^{t_i}|X_{II}^{t_i} > 0) \Leftrightarrow \mathrm{Var}(X_I^{t_i}) \geq \mathrm{Var}(X_{II}^{t_i}) \Leftrightarrow \mathbb{E}[X_I^{t_i}] \geq \mathbb{E}[X_{II}^{t_i}]$; hence, $\mathbb{E}[\beta_{II}^{CARA}(\tau_1)] \geq \mathbb{E}[\beta_I^{CARA}(t)]$. Finally, as a bidder's expected payment is higher in the auction that leads to fewer intensive risks for any type, it follows that the seller's expected revenue is also higher in that auction.

Next, we turn to the last statement. For the SPA, the result follows from Milgrom and Weber (1982). For the FPA, we have

$$F_1(t)\beta_I^{RN}(t) = \int_{\underline{t}}^{t}\beta_{II}^{RN}(s)f_1(s)ds > \int_{\underline{t}}^{t}\beta_{II}^{CARA}(s)f_1(s)ds > F_1(t)\beta_I^{CARA}(t),$$

where the first inequality follows from Milgrom and Weber (1982) and the second follows from $\mathbb{E}[\beta_{II}^{CARA}(\tau_1)] \geq \mathbb{E}[\beta_I^{CARA}(t)]$. □

Endnotes

[1] The difference between CPE and UPE is reminiscent of a similar difference between the concepts of myopic loss-aversion equilibrium and nonmyopic loss-aversion equilibrium introduced by Shalev (2000).

[2] A similar implication arises in the works by Heidhues and Kőszegi (2014) and Rosato (2016); these papers, however, only consider posted-prices mechanisms with homogeneous buyers.

[3] We say that a bidder is exposed to the winner's curse if he or she overbids compared with the risk-neutral (Bayesian) Nash equilibrium. Some researchers, such as Kagel and Levin (1986) and Eyster and Rabin (2005), use a more stringent definition: that the winning bidder obtains a negative payoff. Yet, we think our weaker definition corresponds more closely to the deviations from the risk-neutral (and risk-averse) equilibrium, which are the main focus of our paper.

[4] In a laboratory experiment, Kocher et al. (2015) find strong evidence for precautionary bidding. Moreover, the authors report that, although their study was inspired by Eső and White (2004), their results can also be explained by loss aversion.

[5] The difference between our precautionary bidding effect and the one identified by Eső and White (2004) for DARA bidders is similar to the difference between the expected-utility-of-wealth theories of precautionary savings that rely on prudence and the first-order precautionary-savings motive that induces loss-averse consumers to increase their savings in response to an in increase in background risk; see also Kőszegi and Rabin (2009) and Pagel (2017).

[6] Recent experimental evidence supports the Kőszegi and Rabin (2006) expectations-based model of reference-dependent preferences and loss aversion; see, for instance, Abeler et al. (2011), Ericson Marzilli and Fuster (2011), Gill and Prowse (2012), Karle et al. (2015),

and Smith (2019). More pertinently, Banerji and Gupta (2014), Eisenhuth and Grunewald (2018), and Rosato and Tymula (2019) provide experimental support for this model in the context of sealed-bid auctions.

[7] Whereas the consumption value in Kőszegi and Rabin (2006) is private and deterministic, in our model, V_i is interdependent and ex ante unknown to the bidders. Nonetheless, we follow their model by assuming that $\mu^k(\cdot)$ is a "universal gain–loss function" so that how a person feels about gains or losses in a dimension depends, in a universal way, on the changes in material utility associated with that dimension. Thus, we want to capture the idea that, when losing an auction for an authentic *Monet*, the bidder experiences a loss in the consumption dimension similar to that arising if, after winning the auction, the bidder realizes that the painting is a cheap imitation.

[8] We allow for different parameters of gain–loss utility and loss aversion in the good and money dimensions because the two have different implications for bidding in auctions. In particular, our formulation is rich enough to capture situations in which bidders are loss-averse only regarding the consumption dimension. Such a case applies if bidders' income is subject to large background risk, as argued by Köszegi and Rabin (2009); in a similar vein, Novemsky and Kahneman (2005) argue that money given up in purchases is generally not subject to loss aversion.

[9] As shown by Dato et al. (2017), focusing on pure-strategy equilibria is without loss of generality under CPE.

[10] Assumption 1 is relevant for the derivation of the equilibrium bids under CPE, but it is not needed under UPE; nonetheless, we maintain it throughout the paper, as this makes it easier to compare equilibrium bids across the two specifications. We relax this assumption in the web appendix, where we show that most of our results continue to hold qualitatively. The main difference is that, if Assumption 1 does not hold, the equilibrium under CPE entails partial pooling at the bottom, whereby some bidders bid zero in order to lose the auction for sure and avoid any risk.

[11] Herweg et al. (2010) first introduced Assumption 1 and referred to it as "no dominance of gain–loss utility." This assumption ensures that a loss-averse agent does not select first-order stochastically dominated options; see also Masatlioglu and Raymond (2016).

[12] In the proof of Proposition 1, we verify that the PPE is the UPE where a bidder is indifferent between mimicking a higher type or bidding according to his or her true type.

[13] Following Eső and White (2004), we say that a private-value environment and an interdependent-value one are comparable if $v(t_i) = h(t_i)$ and $v(t_i, \mathbf{t}_{-i}) = h(t_i) + g(\mathbf{t}_{-i})$ for positive and increasing functions $h(\cdot)$ and $g(\cdot)$.

[14] It is straightforward to verify that downward deviations lead to the same equilibrium bid as the one presented in Proposition 5; see also the discussion following Proposition 6.

[15] In the web appendix, we show that loss aversion can reverse this ranking.

[16] Notice that equilibrium bids are decreasing in η^m under both UPE and CPE. Yet, bids decrease by more in the PPE than under CPE. To see the intuition, consider bidders in the FPA whose type is close to $\bar{t}$. If they bid as if their signal were $\bar{t}$, they are guaranteed to win the auction. Under CPE, therefore, even for large values of η^m, bidders will not suffer any losses in money, as they anticipate that they will pay their bid for sure. Under UPE, instead, the same bidders suffer a painful loss when winning because they pay more than expected. Thus, loss aversion over money decreases high-type bidders' incentives to bid aggressively more under UPE than under CPE. In turn, these high-type bidders submit larger bids under CPE than in the PPE if η^m is large enough.

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