ISSN 0005-1179 (print), ISSN 1608-3032 (online), Automation and Remote Control, 2024, Vol. 85, No. 1, pp. 73–84.

CONTROL IN TECHNICAL SYSTEMS

On the Determination of the Region Border Prior to the Limit Steady Modes of Electric Power Systems by the Analysis Method of the Tropical Geometry of the Power Balance Equations

M. I. Danilov[*,a] **and I. G. Romanenko**[*,b]

[*]*North Caucasus Federal University, Stavropol, Russia*
e-mail: [a]*mdanilov@ncfu.ru,* [b]*irina_romanenko_@mail.ru*

Received September 28, 2023
Revised October 30, 2023
Accepted December 21, 2023

Abstract—The analysis of the known [8] approach in which tropical geometry over complex multifields of active power balances is used to estimate the region of existence of the electric power system mode. Its limitations are shown and a new approach is proposed, a criterion is also represented for determining the boundary that precedes the violation of the stability of the energy system due to the restructuring of the tropical set of solutions. The developed approach allows to determine the approach of the power system mode to the limit by the known parameters of the lines and the dynamics of changes of the nodes voltage modules and the nodes load.

Keywords: electric power system, static stability, regime existence subdomains, tropical geometry

DOI: 10.31857/S0005117924010066

1. INTRODUCTION

Calculation of steady-state modes (states) of electric power systems is necessary to verify the actual possibility of transmitting of the required power to consumers from existing generators [1–3]. In addition, it is important for control problems [4–6] to be able to estimate the design parameters of the power system state under study relative to the maximum possible mode [7–9]. The limiting mode is determined by increasing the energy consumption of nodes (buses) to critical values, at which the power balance in the system is still maintained. In the event of a further increase of the nodes load, the balance in power system will not be maintained due to a violation of static stability, and such a mode is impossible to actually implement. The procedure of finding limiting modes, called weighting, is performed by selecting nodes and the step of incrementing their power, which can be determined by empirical considerations based on an analysis of the network topology or other necessary criteria. There are various approaches to searching of limiting modes and the criteria used in this case. The traditional [3] method includes the Zhdanov approach and criterion; in this case, weighting is performed and the Jacobian of the linearized equations of the power system steady state is controlled to be equal zero [1, 2]. An optimization model of electrical power systems limiting modes is proposed and the importance of method of balancing node setting is shown in [3], which is also noted in [7]. It is proposed to use an approach based on tropical geometry over complex multipoles of active power balance equations in [8]. In [9], measurement data performed by PMU (Phasor Measurement Unit) devices are used to estimate the voltage safety margin. In [10], the limiting states of the power system are found by incrementing of the nodes load at each step of the iterative calculation of the steady state. In [11, 12] it is proposed to use a modal approach to solve the problem of ensuring static stability, which consists in analyzing of the eigenvalues of the Jacobian matrix. In [13, 14], the limiting steady states of the power system are found by the Holomorphic Embedding Load-flow Method (HELM), which guarantees that the found solution, when it exists, always corresponds to the correct one, and otherwise signals about the absence of solutions. In [15], an approach to approximating of region boundaries of the regime existence is proposed, taking into account the limitation of generator reactive power. In [16], emergency modes and the using of group lines switching to control and ensure static voltage stability were considered. In [17], an algorithmic procedure of adjusting of the known (previously found) base point of the network limit mode according to the occurrence of changes in node loads is proposed. In [18, 19], methods of estimating of the voltage stability margin for power supply systems with distributed and [20] renewable energy sources are proposed. The authors of [21] proposed a method using the holomorphic load embedding approach, as well as arc length parameterization and piecewise approximations to determine the boundary of the mode existence and to track the entire PV curve of power system nodes. The shape of the boundary of the energy network limiting modes (in the multidimensional space of parameters), which has a complex topology with non-intersecting isolated sections, is studied in [22, 23]. In [24], the conditions of the mode existence of are given only for linear circuits of four-terminal networks operating on alternating current with constant power loads. It must be noted that such schemes correspond to simulated power lines when calculating critical modes.

Determination and research of the stability boundary of the power system and its power lines and associated mode calculation methods [25, 26] are also important when optimizing of the operation of both the distribution network [27–29] and power plants [30].

The presented article analyzes the results of work [8], notes its shortcomings, and proposes an original approach and criterion for determining the boundary of the region of permissible modes preceding instability.

The material of the article is structured as follows. In Section 2, the problem of determining the boundary of the region preceding the limiting steady states of a power system with an arbitrary number of buses is formulated. Section 3 discusses the problem of load power supply through a line from an infinite power bus and the application of analysis over a complex tropical multipole to this system. The main properties of their solutions are noted. Section 4 demonstrates the results obtained using two examples of power system calculations. The first example: a power system with four nodes, in one of which the active power is increased. The second is a standard IEEE 5-bus scheme. The possibility of determining of the boundary preceding the loss of the power system mode stability is shown. The appendix provides the derivation of the theoretical expressions used in the article.

2. STATEMENT OF THE PROBLEM OF DETERMINING OF THE REGION BOUNDARY PRECEDING THE LIMITING POWER SYSTEM STEADY STATES WITH AN ARBITRARY NUMBER OF BUSES

Let's consider a power system with a known electrical network topology, nominal voltage classes, and a number of buses n. Let's introduce the variable v ($v = \overline{0, n}$) to indicate the node numbers. Electricity consumption in load nodes is specified by complex power values $\dot{p}_v^{\text{load}} = {p_{\text{load}}}_v^{\text{Re}} + j{p_{\text{load}}}_v^{\text{Im}}$ ($v \in PQ$), where PQ is the set of load nodes with active ${p_{\text{load}}}_v^{\text{Re}}$ and reactive ${p_{\text{load}}}_v^{\text{Im}}$ powers. Electricity enters the system through generating units. Nodes with given complex values of powers $\dot{p}_v^{\text{gen}} = {p_{\text{gen}}}_v^{\text{Re}} + j{p_{\text{gen}}}_v^{\text{Im}}$ are called generator nodes of PQ-type ($v \in PQ$). Nodes with given values of active ${p_{\text{gen}}}_v^{\text{Re}}$ powers and adjustable voltage modules U_v are called PV-type nodes ($v \in PV$).

Also, when calculating modes in the power system, one of the nodes is assumed to be balancing ($v = 0$), it represents an infinite power bus with a given complex voltage $\dot{U}_0$.

Next, we make the following assumptions.

1. Electricity transmission is carried out using lines, which are represented by π-shaped equivalent circuits with lumped parameters.

2. The parameters of all electrical network lines are known and are represented by longitudinal active $R_{v,k}^{\text{line}}$ and reactive $X_{v,k}^{\text{line}}$ resistances, transverse capacitive $B_{v,k}^{\text{line}}$ conductivities to ground, where v and k are the numbers of nodes between which the line is connected.

The equations necessary to calculate the steady state of the power system are written in complex form according to the nodal voltage method, taking into account the presence of PV-type nodes:

$$\begin{gathered}\dot{U}_v^* \dot{\hat{I}}_v = \dot{p}_v^{*\text{ gen}} - \dot{p}_v^{*\text{ load}},\ v \in PQ,\quad v = \overline{1,n}, \\ \operatorname{Re}\left[\dot{U}_v^* \dot{\hat{I}}_v\right] = \dot{p}_v^{\text{Re}},\quad \left|\dot{U}_v\right| = \text{const},\quad v \in PV,\quad v = \overline{1,n},\end{gathered} \tag{1}$$

in which

$$\dot{\hat{I}}_v = \dot{U}_v \underline{Y}_{v,v} - \sum_{k \in A_v^g} \dot{U}_k \underline{Y}_{v,k} - \dot{U}_0 \underline{Y}_{0,v},$$

where $\dot{U}_v^*$ is the conjugate voltage complex $\dot{U}_v$; $\underline{Y}_{v,v}$ – self-conductance of all branches connected to node v; A_v^g – nodes directly connected to node v; $\dot{U}_k$ – voltage of node k; $\underline{Y}_{v,k}$ – mutual conductance between nodes v and k; $\dot{U}_0$ – the known complex voltage of the balancing ($v = 0$) node; $\underline{Y}_{0,v}$ – conductivity of all branches directly connecting node v and balancing; $\dot{p}_v^{*\text{ gen}}$, $\dot{p}_v^{*\text{ load}}$ – conjugate complexes $\dot{p}_v^{\text{gen}}$ and $\dot{p}_v^{\text{load}}$.

The conductivities $\underline{Y}_{v,v}$, $\underline{Y}_{v,k}$ and $\underline{Y}_{0,v}$ are defined as follows:

$$\begin{gathered}\underline{Y}_{v,v} = \sum_{m \in A_v^b} \frac{1}{R_m + jX_m} + \sum_{m \in A_v^b} jB_m,\quad v = \overline{1,n}, \\ \underline{Y}_{v,k} = \sum_{k \in A_v^g} \frac{1}{R_{v,k} + jX_{v,k}},\quad v \neq k,\quad \underline{Y}_{0,v} = \frac{1}{R_{0,v} + jX_{0,v}},\end{gathered} \tag{2}$$

where R_m, X_m – active and inductive resistance; B_m – the capacitive conductivity of one branch m from the set of nodes A_v^b connected to node v; $R_{v,k}$, $X_{v,k}$ – active and inductive resistance of the branch between nodes v and k; $R_{0,v}$, $X_{0,v}$ – active and inductive resistance of the branch between the balancing node and node v.

The solution of nonlinear equations system (1) is the complex voltages of all nodes $\dot{U}_v$ ($v = \overline{1,n}$), which can be found by various numerical iterative methods.

The procedure of searching of limiting modes is performed by selecting nodes and incremental step of their power to critical values, beyond which leads to an imbalance of power in the system.

The problem solved in the presented article is to determine the boundary within the region of existence of the steady state of the power system that precedes the emergence of the limiting mode in terms of static stability.

3. CONSUMPTION EQUATIONS FOR A LOAD FED THROUGH A LINE FROM AN INFINITE POWER BUS AND APPLICATION OF TROPICAL MULTIPOLE ANALYSIS TO THIS SYSTEM

First let's consider consumption and the appearance of the limiting (critical) mode in the simplest scheme (Fig. 1). The load $\dot{p} = p^{\text{Re}} + jp^{\text{Im}}$, where R, X – active and inductive resistance of the line,

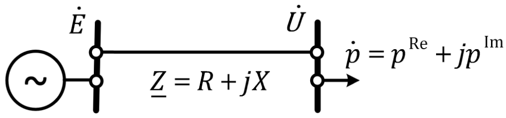

Fig. 1. Power supply circuit through the load line from the infinite power bus.

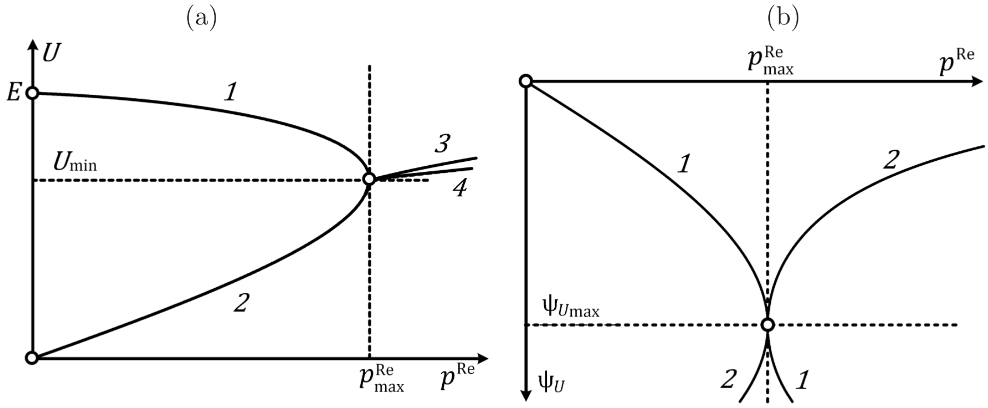

Fig. 2. Dependences of the (a) modulus and (b) angle of the load node voltage on p^{Re}: *1* and *2* – respectively, the first $U_1(p^{\mathrm{Re}})$, $\Psi_{U_1}(p^{\mathrm{Re}})$ and the second $U_2(p^{\mathrm{Re}})$, $\Psi_{U_2}(p^{\mathrm{Re}})$ roots (4) and (5); *3* – $\left|U_{1,2}(p^{\mathrm{Re}})\right|$; *4* – $\mathrm{Re}\left[U_{1,2}(p^{\mathrm{Re}})\right]$.

p^{Re}, p^{Im} – active and reactive power consumption, is connected to a generator (infinite power bus) with a known complex voltage $\dot{E} = Ee^{j\Psi_E}$ through a power line $\underline{Z} = R + jX$.

The equation for determining the steady state of the presented circuit is written in the following form:

$$\dot{U} + \frac{p^{\mathrm{Re}} - jp^{\mathrm{Im}}}{\dot{U}^*}(R + jX) = \dot{E}, \tag{3}$$

in which $\dot{U} = Ue^{j\Psi_U}$, $\dot{U}^* = Ue^{-j\Psi_U}$, where U is the module voltage of the node (bus) with load $\dot{p}$, Ψ_U is the angle of complex voltage $\dot{U}$, measured from the generator voltage vector $\dot{E}$.

Nonlinear equation (3) has an analytical solution, which roots (modules $U_{1,2}(p^{\mathrm{Re}}, p^{\mathrm{Im}})$ and angles (arguments) $\Psi_{U_{1,2}}(p^{\mathrm{Re}}, p^{\mathrm{Im}})$ voltages) are determined according to the expressions:

$$(U_{1,2})^2 = \frac{E^2}{2} - p^{\mathrm{Re}}R - p^{\mathrm{Im}}X \pm \frac{\sqrt{(2(p^{\mathrm{Re}}R + p^{\mathrm{Im}}X) - E^2)^2 - 4(R^2 + X^2)((p^{\mathrm{Re}})^2 + (p^{\mathrm{Im}})^2)}}{2}, \tag{4}$$

$$\Psi_{U_{1,2}} = -\arg\left[(U_{1,2})^2 + p^{\mathrm{Re}}R + p^{\mathrm{Im}}X + j(p^{\mathrm{Re}}X - p^{\mathrm{Im}}R)\right]. \tag{5}$$

Assuming as known and constant power factor ($\cos\phi$) of the node load with an increase in active power p^{Re}, we determine the component $p^{\mathrm{Im}} = p^{\mathrm{Re}}\tan\phi$. Let's build (Fig. 2) dependencies (*2*) and (*3*). The angle $\Psi_{U_{\max}}$ is determined by the expression

$$\Psi_{U_{\max}} = -\mathrm{atan}\left[\frac{X - R\tan\phi}{R + X\tan\phi}\right]. \tag{6}$$

At $p^{\mathrm{Re}} > p^{\mathrm{Re}}_{\max}$ the voltage modulus U takes complex values, and therefore, in the specified range the curves *1* and *2* are missing. The value $p^{\mathrm{Re}}_{\max}$ corresponds to the regime existence boundary. Dependencies *3* and *4* are the modulus and real part of the voltage U at $p^{\mathrm{Re}} > p^{\mathrm{Re}}_{\max}$, respectively.

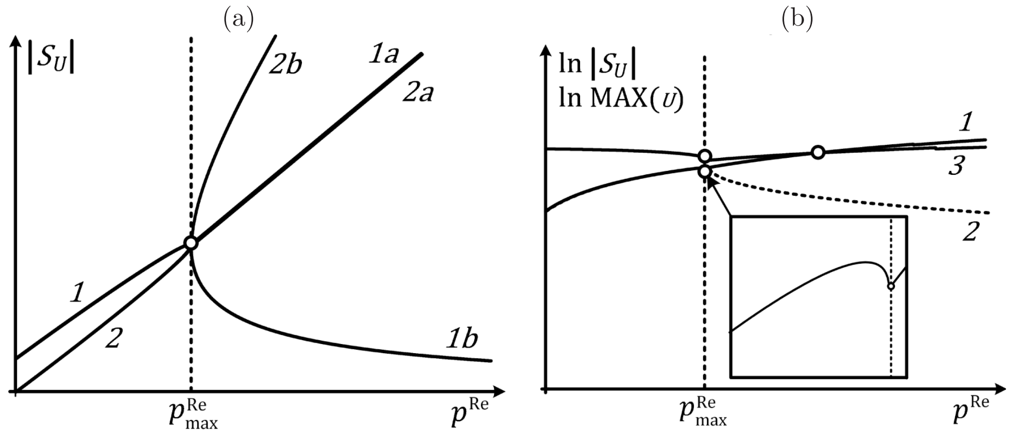

Fig. 3. Dependencies of parameters S_U, $\ln|S_U|$ and $\ln \mathrm{MAX}(U)$ from p^{Re}: on (a) *1* and *2* – $S_U(p^{\mathrm{Re}})$ for the first and second roots, respectively, while *1*, *1a* and *2*, *2a* according to the left side of (7), and *1*, *1b* and *2*, *2b* according to the right side (7); on (b) – for the first roots *1* and *2* – $\ln|S_U|$ and $\ln \mathrm{MAX}(U)$, respectively, according to the data of the left and right sides of (7), *3* – $\ln \mathrm{MAX}(U)$.

For the diagram in Fig. 1, we apply the approach proposed in [8] for determining the proximity of a regime to the region of its existence. The expression for the parameter S_U, written from the condition of the balance of nodes active power, has the form

$$\frac{U^2}{2}\left(\frac{1}{R+jX}+\frac{1}{R-jX}\right)+\mathrm{Re}(\dot{p})=S_U=\frac{1}{2}\left(\frac{\dot{E}\dot{U}^*}{R+jX}+\frac{\dot{E}^*\dot{U}}{R-jX}\right). \tag{7}$$

The dependences of S_U from zero to $p^{\mathrm{Re}}_{\max}$ are the same for the left and right expressions (7) and differ for $p^{\mathrm{Re}} > p^{\mathrm{Re}}_{\max}$ (see Fig. 3). From Fig. 3 it is clear that $\ln|S_U| < \ln\left(\frac{EU}{\sqrt{R^2+X^2}}\right)$ for p^{Re} from zero to $p^{\mathrm{Re}}_{\max}$. Tropical equations used for the diagram in Fig. 1, have the form

$$\ln|S_U| \oplus \ln\left(\frac{EU}{\sqrt{R^2+X^2}}\right). \tag{8}$$

The parameter $\mathrm{MAX}(U)$ denotes the component $\frac{EU}{\sqrt{R^2+X^2}}$. It must be noted that the curve *1* in Fig. 3b (inset) has a maximum near the value $p^{\mathrm{Re}}_{\max}$, explained by the restructuring of the tropical set of solutions, which may be an additional criterion for the proximity of the limiting regime. In this case, in the entire range of p^{Re} from zero to $p^{\mathrm{Re}}_{\max}$ the condition

$$|S_U| \leqslant \max_U\left(\frac{EU}{\sqrt{R^2+X^2}}\right), \tag{9}$$

which is used in [8] to determine the proximity of a regime to the boundary of the region of its existence is not violated. Thus, it is not possible to apply (9), as proposed in [8], to the diagram in Fig. 1 to identify the region preceding the onset of the limiting regime.

4. EXAMPLES OF POWER SYSTEM CALCULATIONS

Let's consider an example of the calculation presented by the authors of [8] to demonstrate their proposed approach. A four-node power system (Fig. 4) with two generators is being studied.

The first generator is specified by an infinite power bus with the known voltage $\dot{U}_1$, the second one – by the voltage module U_3 and active power p_3^{Re}. With this formulation of the problem, it is

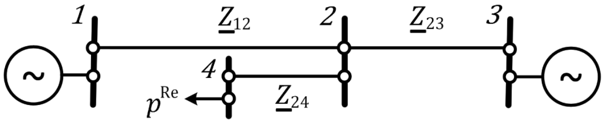

Fig. 4. Calculation scheme of a four-node electrical power system.

impossible to solve the nonlinear equations [14] of the power system state analytically:

$$\dot{U}_2\left(\frac{1}{\underline{z}_{12}}+\frac{1}{\underline{z}_{23}}+\frac{1}{\underline{z}_{24}}\right)-\dot{U}_3\left(\frac{1}{\underline{z}_{23}}\right)-\dot{U}_4\left(\frac{1}{\underline{z}_{24}}\right)=\dot{U}_1\left(\frac{1}{\underline{z}_{12}}\right), \tag{10}$$

$$\operatorname{Re}\left[\dot{U}_3^*\left(\dot{U}_3\frac{1}{\underline{z}_{23}}-\dot{U}_2\frac{1}{\underline{z}_{12}}\right)\right]=\dot{p}_3^{\mathrm{Re}},\quad \left|\dot{U}_3\right|=\mathrm{const},$$

$$\dot{U}_4\frac{1}{\underline{z}_{24}}-\dot{U}_2\frac{1}{\underline{z}_{24}}=-\frac{\dot{p}_4^*}{\dot{U}_4^*},$$

where

— branch resistances $\underline{z}_{12}=8.91+j80.91$; $\underline{z}_{23}=4.45+j40.46$; $\underline{z}_{24}=6.68+j60.68$;

— balancing node voltage $\dot{U}_1=500$;

— active power $p_3^{\mathrm{Re}}=400$ and voltage modulus $U_3=500$ in the third node;

— conjugate complex of fourth node power $\dot{p}_4^*=p_4^{\mathrm{Re}}-jp_4^{\mathrm{Im}}$ for different modes; it must be noted that the condition is accepted $p_4^{\mathrm{Im}}=0$.

The solution of system (10) was carried out in the Mathcad package by the Levenberg–Marquardt method with increasing load in the *4* node until the limiting mode was obtained (Table 1). There was no imbalance in the active power of the power system in all existing modes, i.e. it was equal to zero (with an accuracy of 10^{-307}). At $p_4^{\mathrm{Re}}>p_{4\,\max}^{\mathrm{Re}}$ the power balance in the power system was disturbed and it could be simply identified by the calculated value of the voltage modulus U_3, which became different from the specified value $U_3=500$ in the indicated modes. It should be noted that in [8] the limiting mode corresponded to the value of 1243.8 obtained at step 10 of Table 1, although the presented calculation results show the limiting mode at 1250.1993. In the case of $p_4^{\mathrm{Re}}=1250.1994$, the voltage U_3, taking into account fifteen decimal places, is 500.000000000000170, and the unbalance is $454.7474\cdot10^{-15}$.

The results of calculation (Table 2) of mode parameters in a logarithmic scale over a complex multipole for each step p_4^{Re} are given. The parameters S_2, S_3 and S_4 are determined from the active power balances of nodes as follows [8]:

$$\frac{1}{2}\left(\frac{\dot{U}_2^*\dot{U}_1}{\underline{z}_{12}}+\frac{\dot{U}_2\dot{U}_1^*}{\underline{z}_{12}^*}+\frac{\dot{U}_2^*\dot{U}_3}{\underline{z}_{23}}+\frac{\dot{U}_2\dot{U}_3^*}{\underline{z}_{23}^*}+\frac{\dot{U}_2^*\dot{U}_4}{\underline{z}_{24}}+\frac{\dot{U}_2\dot{U}_4^*}{\underline{z}_{24}^*}+\right)=S_2,$$

$$\frac{1}{2}\left(\frac{\dot{U}_3^*\dot{U}_2}{\underline{z}_{23}}+\frac{\dot{U}_3\dot{U}_2^*}{\underline{z}_{23}^*}\right)=S_3=\frac{(U_3)^2}{2}\left(\frac{1}{\underline{z}_{23}}+\frac{1}{\underline{z}_{23}^*}\right)-p_3^{\mathrm{Re}},$$

$$\frac{1}{2}\left(\frac{\dot{U}_4^*\dot{U}_2}{\underline{z}_{24}}+\frac{\dot{U}_4\dot{U}_2^*}{\underline{z}_{24}^*}\right)=S_4=\frac{(U_4)^2}{2}\left(\frac{1}{\underline{z}_{24}}+\frac{1}{\underline{z}_{24}^*}\right)-p_4^{\mathrm{Re}},$$

where

$$S_2=\frac{(U_2)^2}{2}\left(\frac{1}{\underline{z}_{12}}+\frac{1}{\underline{z}_{12}^*}+\frac{1}{\underline{z}_{23}}+\frac{1}{\underline{z}_{23}^*}+\frac{1}{\underline{z}_{24}}+\frac{1}{\underline{z}_{24}^*}\right).$$

Table 1. Four-node power system mode parameters

Step	p_4^{Re}	U_2	Ψ_2	Ψ_3	U_4	Ψ_4
0	500	492,5637	−2,0083	1,6966	481,5826	−9,3567
1	600	489,8857	−3,9833	−0,2924	475,4331	−12,9766
2	700	486,5099	−6,0115	−2,3386	467,9762	−16,7639
3	800	482,2942	−8,1121	−4,4620	458,9055	−20,7819
4	900	477,0080	−10,313	−6,6920	447,7206	−25,1289
5	1000	470,2383	−12,6602	−9,0775	433,5139	−29,9775
6	1100	461,0926	−15,2455	−11,7162	414,2925	−35,6971
7	1200	446,5674	−18,3529	−14,9131	383,2047	−43,5358
8	1225	440,4618	−19,3571	−15,9566	369,7892	−46,5104
9	1237,5	436,0835	−19,9756	−16,6041	359,9997	−48,5514
10	1243,8	432,9471	−20,3673	−17,0168	352,8867	−49,9711
11	1245	432,1875	−20,4557	−17,1103	351,1503	−50,3099
12	1250	426,5797	−21,0298	−17,7230	338,1517	−52,7538
13	1250,1993	425,2433	−21,1460	−17,8485	335,0037	−53,3220

Table 2. Mode parameters on a logarithmic scale

step	p_4^{Re}	$\ln \mathrm{MAX}(2)$	$\ln \lvert S_2\rvert$	$\ln \mathrm{MAX}(3)$	$\ln \lvert S_3\rvert$	$\ln \mathrm{MAX}(4)$	$\ln \lvert S_4\rvert$
0	500	8.7079	7.2533	8.7079	5.6038	8.2651	6.8197
1	600	8.7025	7.2424	8.7025	5.6038	8.2468	6.9129
2	700	8.6955	7.2286	8.6955	5.6038	8.2240	6.9963
3	800	8.6868	7.2112	8.6868	5.6038	8.1958	7.0711
4	900	8.6758	7.1891	8.6758	5.6038	8.1601	7.1383
5	1000	8.6615	7.1606	8.6615	5.6038	8.1135	7.1981
6	1100	8.6419	7.1213	8.6419	5.6038	8.0485	7.2497
7	1200	8.6099	7.0573	8.6099	5.6038	7.9385	7.2884
8	1225	8.5961	7.0297	8.5961	5.6038	7.8891	7.2931
9	1237.5	8.5861	7.0097	8.5861	5.6038	7.8523	7.2929
10	1243.8	8.5789	6.9953	8.5789	5.6038	7.8251	7.2910
11	1245	8.5771	6.9918	8.5771	5.6038	7.8184	7.2903
12	1250	8.5641	6.9657	8.5641	5.6038	7.7677	7.2827
13	1250.1993	8.5609	6.9594	8.5609	5.6038	7.7552	7.2803

Parameters $\ln \mathrm{MAX}(2)$, $\ln \mathrm{MAX}(3)$ and $\ln \mathrm{MAX}(4)$ at each step p_4^{Re} corresponded to the same branches and were determined by the formulas:

$$\ln \mathrm{MAX}(2) = \ln \mathrm{MAX}(3) = \left(\frac{U_2 U_3}{|z_{23}|}\right),$$

$$\ln \mathrm{MAX}(4) = \left(\frac{U_2 U_4}{|z_{24}|}\right).$$

It must be noted that for all nodes v of the power system (Table 2) the condition is satisfied

$$\ln \mathrm{MAX}(v) > \ln |S_v|,$$

and accordingly, given in [8]

$$|S_U| \leqslant \max_k \left(\frac{U_v U_k}{|z_{vk}|}\right). \tag{11}$$

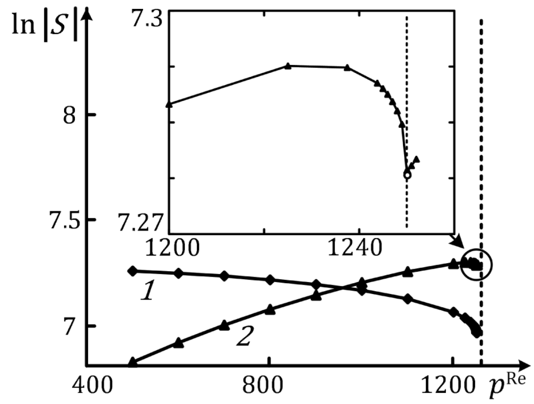

Fig. 5. Parameter dependencies $\ln|S|$ from p^{Re}: *1* – $\ln|S_2|$; *2* – $\ln|S_4|$.

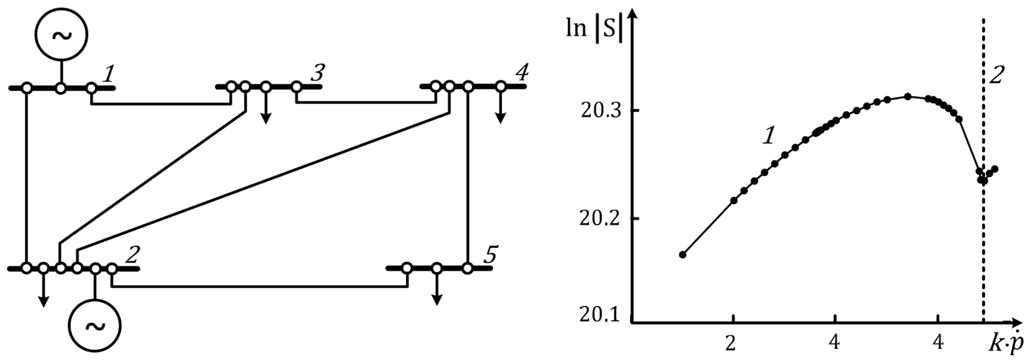

Fig. 6. Scheme *a* IEEE 5 buses and dependencies *b* *1* – $\ln|S_5|$ from $k\sum\dot{p}$; *2* – $\dot{p}_{\max}$.

At the same time, in [8] in the range from $p_4^{\text{Re}} = 1200$ to $p_{4\max}^{\text{Re}}$ (1243.8) condition (11) is violated, that makes it possible to identify a subregion within the region of existence of the regime, the exit of which precedes the limit regime.

It should be noted that compliance with condition (11) in the existence region of the regime is consistent with the qualitative results obtained for the circuit in Fig. 1. In addition, we can segregate the node *4*, in which the dependence $\ln|S_4|$ has a maximum (Fig. 5 and Table 2, cell with fill), similar to that shown in Fig. 3b (see box). To check the conditions (11) and the possibility of using the $\ln|S|$ dependencies when identifying the region preceding the limiting mode, the calculations were carried out on the IEEE 5-bus test circuit (Fig. 6a) with standard initial data presented in Table 3. The parameter values in the table are given in relative units at basic power values of 100 MBA and voltage of 100 kV. The circuit takes into account the capacitive conductivity B of the network lines and a PV-type generator is connected to the *2* bus.

As the power system mode became heavier, the load of all nodes $\sum\dot{p} = \dot{p}_3+\dot{p}_4+\dot{p}_5$ was increased simultaneously, by multiplying by coefficient k. As a result of the calculations, it was established that for all nodes (buses) of the power system, conditions (11) are met, and for node *5* the maximum of dependence $\ln|S_5|$ from $k\sum\dot{p}$, preceding the limiting mode $\dot{p}_{\max}$ (see Fig. 6b) was observed. It should be noted that for arbitrary values of node loads and line resistances conditions (11) are met and there is the indicated maximums.

Studies out of the scalability of the proposed method and criterion on circuits with a large number of buses (standard IEEE 30-bus circuit), in which only PQ nodes of loads and generators were specified, as well as in the presence of PV generators were carried. For all the cases considered, only some (several) nodes connected to generators exhibit growth and maxima in the dependencies $\ln|S|$,

Table 3. IEEE 5-Bus Power System Test Circuit Source Data

Information on network nodes					Information on network branches				
no.	Node type	V, o.e..	p^{Re}, o.e.	p^{Im}, o.e.	Branches		Resistance and conductivity		
							R, o.e.	X, o.e.	B, o.e.
1	Slack bus	1.20	–	–	1	2	0.02	0.06	0.06
					1	3	0.08	0.24	0.05
2	Generator (PV)	1.20	0.40	–	2	3	0.06	0.18	0.04
					2	4	0.06	0.18	0.04
3	Load	–	0.45	0.15	2	5	0.04	0.12	0.03
4	Load	–	0.60	0.10	3	4	0.01	0.03	0.02
5	Load	–	0.40	0.05	4	5	0.08	0.24	0.05

which can be used to determine the approach to the region boundary of the permissible power system modes. For other nodes and branches of the power system, these dependencies decrease with increasing load, as shown in Fig. 5, curve *1*.

Thus, studies show that as the regime becomes heavier, the maximum of dependence $\ln|S|$ for power system nodes can be used to determine the boundary beyond which precedes the regime reaching the boundary of its domain of existence.

5. CONCLUSION

1. It is possible to identify the lack of balance in the power system, and, accordingly, whether the mode under consideration belongs to the region of an unstable state, by monitoring the value of the specified voltage of the PV-type generating unit (bus) during calculations.

2. In the region where the mode exists, conditions (11) are satisfied for each node (bus) of the power system; they cannot be used to identify the subregion, the exit beyond which precedes the limit mode.

3. It is possible to identify the boundary within the existence region of the regime, the exit beyond which precedes the limiting regime, by determining the maximum of the values of $\ln|S|$ node (bus) of the power system. This will make it possible, at a low cost of computational resources, to determine the nodes that are critical in terms of weighting and to obtain additional information to enter the regime into a more stable region.

FUNDING

The work was carried out with the financial support of the Priority 2030 program (grant no. 122060300035-2).

APPENDIX

Expressions (4) and (5) are obtained from (3) as follows:

$$U^2 + \left(p^{\mathrm{Re}} - jp^{\mathrm{Im}}\right)(R + jX) = EUe^{-j\Psi_U},$$
$$U^2 + p^{\mathrm{Re}}R + p^{\mathrm{Im}}X + j\left(p^{\mathrm{Re}}X + p^{\mathrm{Im}}R\right) = EUe^{-j\Psi_U}. \tag{A.1}$$

The balance of modules of expression (A.1) is reduced to a quadratic equation for the unknown $\hat{U} = U^2$:

$$a\hat{U}^2 + b\hat{U} + c = 0, \tag{A.2}$$

where

$$a = 1, \quad b = 2\left(p^{\mathrm{Re}}R + p^{\mathrm{Im}}X\right) - E^2, \quad c = \left(R^2 + X^2\right)\left[\left(p^{\mathrm{Re}}\right)^2 + \left(p^{\mathrm{Im}}\right)^2\right].$$

The solution to (A.2) is expression (4). The angle Ψ_U in expression (5) is determined by substituting the found expression (4) for U into equation (A.1).

Expression (6) of the article is obtained from the load bus power equation:

$$\dot{p} = p^{\mathrm{Re}} + jp^{\mathrm{Im}} = \dot{U}\left(\frac{\dot{E} - \dot{U}}{R + jX}\right)^*. \tag{A.3}$$

From (A.3) we obtain

$$\frac{p^{\mathrm{Im}}}{p^{\mathrm{Re}}} = \tan\phi = \frac{EX\cos\Psi_U - UX + ER\sin\Psi_U}{ER\cos\Psi_U - UR - EX\sin\Psi_U}. \tag{A.4}$$

Let's express the voltage modulus U of the load bus from (A.4)

$$U = E\left(\cos\Psi_U + \sin\Psi_U\left(\frac{R + X\tan\phi}{X - R\tan\phi}\right)\right)$$

and put it into the expression for active power obtained from (A.3):

$$p^{\mathrm{Re}} = \frac{U}{(R^2 + X^2)}\left[R\left(E\cos\Psi_U - U\right) - EX\sin\Psi_U\right]. \tag{A.5}$$

Taking the derivative of (A.5) with respect to the angle Ψ_U and equating it to zero, we obtain the expression

$$\frac{d\,p^{\mathrm{Re}}}{d\,\Psi_U} = \frac{\sin\left(2\Psi_U\right)\left(R + X\tan\phi\right) - \cos\left(2\Psi_U\right)\left(R\tan\phi - X\right)}{\left(X - R\tan\phi\right)^2} = 0 \tag{A.6}$$

from which we determine

$$\tan\left(2\Psi_U\right) = \frac{R\tan\phi - X}{R + X\tan\phi}. \tag{A.7}$$

The resulting expression (A.7) is equivalent to equation (6).

REFERENCES

1. Venikov, V.A., Stroev, V.A., Idelchik, V.I., and Tarasov, V.I., Estimation of Electrical Power System Steady State Stability in Load Flow Calculations, *IEEE Trans. Power App. Syst.* 1975, vol. 94, no. 3, pp. 1034–1041.

2. Dobson, A. and Lu, L., New Methods for Computing a Closest Saddle Node Bifurcation and Worst Case Load Power Margin for Voltage Collapse, *IEEE Trans. Power Syst.* 1993, vol. 8, no. 3, pp. 905–911.

3. Ayuev, B.I., Davydov, V.V., and Erokhin, P.M., Optimization Model of Limit Modes of Electrical Systems, *Elektrichestvo*, 2010, no. 11, pp. 2–12.

4. Voropai, N.I., Golub, I.I., Efimov, D.N., et al., Spectral and Modal Methods for Studying Stability and Control of Electric Power Systems, *Autom. Remote Control*, 2020, vol. 81, no. 10, pp. 1751–1774.

5. Wang, Y., Lopez, J.A., and Sznaier, M., Convex Optimization Approaches to Information Structured Decentralized Control, *IEEE Trans. Autom. Control*, 2018, vol. 63, no. 10, pp. 3393–3403.

6. Matveev, A.S., MacHado, J.E., Ortega, R., et al., Tool for Analysis of Existence of Equilibria and Voltage Stability in Power Systems with Constant Power Loads, *IEEE Trans. Autom. Control*, 2020, vol. 65, no. 11, pp. 4726–4740.

7. Ghiocel, S.G. and Chow J.H., A Power Flow Method Using a New Bus Type for Computing Steady-State Voltage Stability Margins, *IEEE Trans. Power Syst.*, 2014, vol. 29, no. 2, pp. 958–965.

8. Kirshtein, B.K. and Litvinov, G.L., Analyzing Stable Regimes of Electrical Power Systems and Tropical Geometry of Power Balance Equations Over Complex Multifields, *Autom. Remote Control*, 2014, vol. 75, no. 10, pp. 1802–1813.

9. Su, H.Y. and Liu, C.W., Estimating the Voltage Stability Margin Using PMU Measurements, *IEEE Trans. Power Syst.*, 2016, vol. 31, no. 4, pp. 3221–3229.

10. Ayuev, B.I., Davydov, V.V., and Erokhin, P.M., Fast and Reliable Method of Searching Power System Marginal States, *IEEE Trans. Power Syst.*, 2016, vol. 31, no. 6, pp. 4525–4533. https://doi.org/10.1109/TPWRS.2016.2538299

11. Sharov, Ju.V., About Development of Analysis Methods Static Stability of Electric Power Systems, *Elektrichestvo*, 2017, no. 1, pp. 12–18.

12. Sharov, Ju.V., Application Modal Approach for Solving the Problem of Ensuring Power System Static Stability, *Izvestiya RAN. Energetika*, 2017, no 2, pp. 13–29.

13. Rao, S., Tylavsky, D., and Feng, Y., Estimating the Saddle-Node Bifurcation Point of Static Power Systems Using the Holomorphic Embedding Method, *Int. J. Electr. Power Energ. Syst.*, 2017, vol. 84, pp. 1–12.

14. Liu, C., Wang, B., Hu, F., Sun, K., and Bak, C.L., Online Voltage Stability Assessment for Load Areas Based On the Holomorphic Embedding Method, *IEEE Trans. Power Syst.*, 2018, vol. 33, no. 4, pp. 3720–3734.

15. Qiu, Y., Wu, H., Song, Y., and Wang, J., Global Approximation of Static Voltage Stability Region Boundaries Considering Generator Reactive Power Limits, *IEEE Trans. Power Syst.*, 2018, vol. 33, no. 5, pp. 5682–5691.

16. Wang, L. and Chiang, H.D., Group-Based Line Switching for Enhancing Contingency-Constrained Static Voltage Stability, *IEEE Trans. Power Syst.*, 2020, vol. 35, no. 2, pp. 1489–1498.

17. Ali, M., Gryazina, E., Khamisov, O., and Sayfutdinov, T., Online Assessment of Voltage Stability Using Newton-Corrector Algorithm, *IET Generat., Transmiss. Distribut.*, 2020, vol. 14, no. 19, pp. 4207–4216.

18. Bulatov, Yu.N., Kryukov, A.V., Suslov K.V., et al., Timely Determination of Static Stability Margins in Power Supply Systems Equipped with Distributed Generation Installations, *Vestnik Irkut. Gos. Tekh. Univ.*, 2021, vol. 25, no. 1(156), pp. 31–43. https://doi.org/10.21285/1814-3520-2021-1-31-43

19. Bulatov, Y., Kryukov, A., Suslov, K., et al., A Stochastic Model for Determining Static Stability Margins in Electric Power Systems, *Computation*, 2022, vol. 10, no. 5. https://doi.org/10.3390/computation10050067

20. Weng, Y., Yu, S., Dvijotham, K., and Nguyen, H.D., Fixed-Point Theorem-Based Voltage Stability Margin Estimation Techniques for Distribution Systems with Renewables, *IEEE Transact. Industr. Inform.*, 2022, vol. 18, no. 6, pp. 3766–3776. https://doi.org/10.1109/TII.2021.3112097

21. Zhang, W., Wang, T., and Chiang, H.D., A Novel FFHE-Inspired Method for Large Power System Static Stability Computation, *IEEE Trans. Power Syst.*, 2022, vol. 37, no. 1, pp. 726–737. https://doi.org/10.1109/TPWRS.2021.3093236

22. Ali, M., Gryazina, E., Dymarsky, A., and Vorobev, P., Calculating Voltage Feasibility Boundaries for Power System Security Assessment, *Int. J. Electr. Power Energ. Syst.*, 2023, vol. 146, 108739. https://doi.org/10.1016/j.ijepes.2022.108915

23. Ali, M., Ali, M.H., Gryazina, E., and Terzija, V., Calculating Multiple Loadability Points in the Power Flow Solution Space, *Int. J. Electr. Power Energ. Syst.*, 2023, vol. 148, 108915. https://doi.org/10.1016/j.ijepes.2022.108739

24. Machado, J.E., Grino, R., Barabanov, N., et al., On Existence of Equilibria of Multi-Port Linear AC Networks with Constant-Power Loads, *IEEE Transact. Circuits and Systems. Part 1: Regular Papers*, 2017, vol. 64, no. 10, pp. 2772–2782.https://doi.org/10.1109/TCSI.2017.2697906

25. Danilov, M.I. and Romanenko, I.G., Determination of Power Flows and Temperature of Electrical Network Wires of a Power System Steady State, *Power Technol. Engineer.*, 2023, vol. 56, no. 5, pp. 739–750. https://doi.org/10.1007/s10749-023-01583-z

26. Karimi, M., Shahriari, A., Aghamohammadi, M.R., et al., Application of Newton-Based Load Flow Methods for Determining Steady-State Condition of Well and Ill-Conditioned Power Systems: A Review, *Int. J. Electr. Power Energ. Syst.*, 2019, vol. 113, pp. 298–309.

27. Zorin, I.A. and Gryazina, E.N., An Overview of Semidefinite Relaxations for Optimal Power Flow Problem, *Autom. Remote Control*, 2019, vol. 80, no. 5, pp. 813–833. https://doi.org/10.1134/S0005231019050027

28. Danilov, M.I. and Romanenko, I.G., Identification of Unauthorized Electric-Power Consumption in the Phases of Distribution Networks with Automated Metering Systems, *Power Technol. Engineer.*, 2022, vol. 56, no. 3, pp. 414–422. https://doi.org/10.1007/s10749-023-01530-y

29. Danilov, M.I. and Romanenko, I.G., Operational Identification of Resistances of Wires of 380 V Distribution Networks by Automated Accounting Systems, *Energetika. Izv. Vuzov i energ. ob"edinenii SNG*, 2023, vol. 66, no. 2, pp. 124–140. https://doi.org/10.21122/1029-7448-2023-66-2-124-140

30. Bonchuk, I.A., Shaposhnikov, A.P., Sozinov, M.A., and Erokhin, P.M., Optimization of the Operating Modes of Power Plants in Isolated Electrical Power Systems, *Power Technol. Engineer.*, 2021, vol. 55, no. 3, pp. 445–453. https://doi.org/10.1007/s10749-021-01380-6

This paper was recommended for publication by M.V. Khlebnikov, a member of the Editorial Board