On the Higgsless mass generation in the nonlinear generalization of Standard Model

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The results of the experiments, which were set until now for confirmation of Higgs's mechanism, are negative. In connection with the difficulties, which will appear if Higgs's bosons is not discovered, an interest arises in other possible variations of the field theory, which can be accessible for experimental check. Below we will examine the nonlinear quantum field theory (NQFT), which is the generalization of Standard Model and which solves the problem of particle masses without the Higgs mechanism.

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1.0. Introduction. Nonlinear field theories and the Standard Model

1.1."Is the quantum theory linear or is it a nonlinear theory?"

This question, set by W. Heisenberg in 1967 (Heisenberg, 1967), arose in connection with the fact that "practically every problem in theoretical physics is governed by nonlinear mathematical equations, except perhaps quantum theory, and even in quantum theory it is a rather controversial question whether it will finally be a linear or nonlinear theory".

A number of works is devoted to the analysis of this contradiction (Parwani, 2005; Jordan, 2007), but no final solution was found until now.

1.2. Nonlinear electromagnetic basis of modern quantum field theory

The basis of the contemporary theory of elementary particles - Standard Model (SM) - is the nonlinear theory of Yang–Mills. Another special feature of contemporary theory is its close connection with the electromagnetism.

According to modern ideas (Ryder, 1985), the observed substance of the Universe consists of photons, leptons and quarks. Besides electromagnetic interactions, there are strong and weak interactions. All of these interactions are described by the unified theory, which is a substantial generalization of Maxwell's theory. Instead of vectors of the usual electrical and magnetic fields $\vec{\mathrm{E}}$ and $\vec{\mathrm{H}}$, the modern theory contains several similar field vectors $\vec{E}_i$ and $\vec{H}_i$, the waves of which are strictly nonlinear.

The first such generalization of Maxwell's theory was made by C. Yang and R. Mills in 1954. All similar theories are therefore called the Yang-Mills theories. Let us emphasize that the nonlinearity is deeply embedded into the nature of the Yang-Mills fields (Y. Nambu in (Coll. of transl. papers, 1962)): "The generalization of the Maxwell theory is the theory of the Yang-Mills fields or non-Abelian gauge fields. Its equations are nonlinear. In contrast to this, the equations of Maxwell are linear, in other words, Abelian".

Obviously, if we prove that the theory of Dirac's lepton can also be recorded as the nonlinear theory of electromagnetic field, then it is possible to hope for the creation of unified nonlinear electromagnetic field theory.

The first unified nonlinear quantum theory of elementary particles was the theory of W. Heisenberg and his colleagues (Coll. of articles, 1959; Heisenberg, 1966). The universal unified spinor field was accepted as the basis of this theory.

Unfortunately, the mathematical solution of Heisenberg's equation proved to be a difficult problem and theory was not further developed. But one of the ideas of Heisenberg in framework of this theory influenced deeply the development of modern quantum field theory. This was the idea of the spontaneous symmetry breakdown (SSB), which became the basis of Higgs's mechanism of particle mass generation in the Standard Model theory (SM).

1.3. Unified quantum nonlinear Heisenberg's theory of matter, and spontaneous symmetry breakdown

W. Heisenberg's goal was the description of all particles as bound states of a different number of some primary particles.

In order to obtain all necessary particle spins, the primary particles must have spin ½. Therefore, as his initial equation, Heisenberg used Dirac's electron equation of leptons:

$$(i\hat{\alpha}\partial - m)\psi = 0, \qquad (1.1)$$

where γ is the Dirac's matrix, m is the mass of a particle, and ∂ is a four-dimensional gradient.

According to Heisenberg's supposition, the fundamental equation must have the highest possible symmetry. However the mass term in Dirac's equation disrupts the invariance of this equation in relation to a series of transformations (of transformation $\psi \to \gamma_5\psi$, where γ_5 is the fifth matrix of Dirac; of scale transformation $x \to \theta x$ $\psi \to \theta^{-1/2}\psi$, where θ is a certain number, and others). Therefore W. Heisenberg proposed a nonlinear equation without particle mass, which in its simplest version has the following form:

$$\left[i\hat{\alpha}\partial - \lambda\left(\psi^{+}\hat{\gamma}\psi\right)\right]\psi = 0, \qquad (1.2)$$

where $\hat{\gamma}$ is the gamma set of the Dirac matrices and λ according to Heisenberg (Heisenberg, 1966) is an arbitrary constant (which is sometimes called a coupling constant or a constant of self-interaction)..

Since the equation (1.2) has not a term with particle mass, it possess the highest possible symmetry. However it is very well known that the interactions of elementary particles are characterized by different symmetries (isotopic symmetry is lost upon transfer from the strong interaction to the electromagnetic, upon the subsequent transfer to the weak interaction the law of parity conservation ceases to work, etc). It is understandable that it is impossible to create a simple fundamental equation which will automatically have these different symmetries.

The theory of ferromagnetism, the author of which was Heisenberg, showed him a way to resolve this situation. It was the idea of spontaneous symmetry breaking (SSB): the fundamental equation can have a maximum symmetry, but other symmetries can be introduced by the spontaneous breaking of this symmetry.

One of the most important mechanisms of SSB within the framework of Heisenberg's program was proposed at the beginning of the 1960's by Nambu and Jona-Lazinio (Nambu and Jona-Lasinio, 1961, 1961a). It was taken from the microscopic theory of superconductivity of Bardeen, Cooper and Shriffer (known as the BCSh mechanism).

Mathematically this was like the appearance of a new symmetry - so-called chiral symmetry, which is spontaneously broken. As a result of the breaking of chiral symmetry, in the model of Nambu and Jona-Lasinio mesons appeared, and fermions acquired significant mass.

Heisenberg's equation (1.2) and the equation of superconductivity (nonrelativistic here):

$$\left[i\frac{\partial}{\partial t} + \frac{\nabla^2}{2m} - \lambda\left(\psi^{+}\hat{\gamma}\psi\right)\right]\psi = 0, \qquad (1.3)$$

have similarities. In Heisenberg's theory, in the case of attraction between primary particles, SSB also occurs as the result of formation of Cooper's pairs of primary particles and their Bose condensation.

The generalization of the SSB model in the case of interaction of scalar and vector EM fields was examined by Higgs. In a statical limit, Higgs' model is completely analogous to the theory of Ginsburg-Landau's superconductivity, being its relativistic generalization.

Thus, we come to the conclusion that in order to introduce the required symmetries and particle masses we must take the initial dynamic equations in a mass-free form and use the idea of spontaneous symmetry breakdown (SSB).

Early versions of a unified theory of weak and EM interactions were proposed by Weinberg and Salam. An essential element of this theory was the use of Higgs's model.

1.4. The SSB mechanism and mass generation

The possibility of calculation of the particle masses by means of the SSB is the characteristic property of SM. The mathematical description of this procedure is called Higgs's mechanism. This mechanism is repeatedly described in literature. Therefore we will only consider the conclusions of the theory.

The Higgs field in SM has three important functions:

1) it breaks the gauge symmetries and gives masses to intermediate bosons (W and Z);

2) it breaks the chiral symmetry and gives masses to fermions;

3) it restores the unitarity of the theory.

The last role is very important: if Higgs's boson does not exist, the unitarity of theory in the general case will be broken. In this case it is necessary to exceed the limits of SM. According to present ideas this possibility gives: super-symmetry; the additional measurements of space-time; “great” unificaton of interactions; new internal particle structure of SM (technicolor, little Higgs, etc); superstring, membranes, and the like. But all these versions lie beyond the limitations of the experimental check.

In the Standard Model theory the Higgs's boson mass is not determined. Some estimations, which is based on experimental data, showed that the mass of Higgs's boson must lie approximately in the interval of 96-251 GeV. The results of the experiments, which were set until now for confirmation of Higgs's mechanism, are negative. With a 95% confidence level (ScienceDaily, 2009) the mass of the Higgs boson (within the framework of SM) must be in the limits: m(H) >114 GeV from straight searches on LEP II, and m(H) <160 GeV from the fit of precision measurements on LEP and Tevatron. Also the 1st type of two-doublet Higgs model, in which the different bosons of Higgs are required, was not confirmed.

Other results show that the probability of the Higgs boson detection in a remained, comparatively small, region of energies from 114 to 160 GeV is limited. In connection with the difficulties, which will appear if Higgs's bosons is not discovered, an interest arises in other possible variations of the field theory, which can be accessible for experimental check.

Below we will examine the nonlinear theory of spinor particles, which is possible to solve the problem of particle masses in the framework of SM without the Higgs mechanism. The simplest way to approach the nonlinear theory is the use of the electromagnetic representation of Dirac’s lepton theory.

2.0. Electromagnetic representation of the lepton theory

The possibility of a formal representation of the Schroedinger or Dirac electron equations in a form of linear Maxwell equations was mentioned in several articles and books (Archibald, 1955; Akhiezer and Berestetskii, 1965; Koga, 1975; Campolattoro, 1980; Rodrigues, 2002). However, the sequential and noncontradictory electromagnetic representation of the nonlinear theory of leptons did not exist until now (Kyriakos, 2004b; 2005)

2.1. The bispinor form of Dirac’s lepton equation

In the common form, the Dirac's equation for the free leptons is written in the bispinor form. There are two bispinor Dirac equation forms (Schiff, 1955; Bethe, 1964):

$$\left[\left(\hat{\alpha}_o\hat{\varepsilon} + c\hat{\vec{\alpha}}\ \hat{\vec{p}}\right) + \hat{\beta}\, mc^2\right]\psi = 0\,, \qquad (2.1)$$

$$\psi^+\left[\left(\hat{\alpha}_o\hat{\varepsilon} - c\hat{\vec{\alpha}}\ \hat{\vec{p}}\right) - \hat{\beta}\, mc^2\right] = 0\,, \qquad (2.2)$$

which correspond to two signs of the relativistic expression of the electron energy:

$$\varepsilon = \pm\sqrt{c^2\vec{p}^{\,2} + m^2c^4}\ , \qquad (2.3)$$

Here $\hat{\varepsilon} = i\hbar\dfrac{\partial}{\partial t}$, $\hat{\vec{p}} = -i\hbar\vec{\nabla}$ are the operators of the energy and momentum; ε, $\vec{p}$ are the electron energy and momentum respectively; c is the light velocity; m is the electron mass, and $\hat{\alpha}_o = \hat{1}$, $\hat{\vec{\alpha}}$, $\hat{\alpha}_4 \equiv \hat{\beta}$ are the Dirac matrices:

$$\hat{\alpha}_0 = \begin{pmatrix} \hat{\sigma}_0 & 0 \\ 0 & \hat{\sigma}_0 \end{pmatrix};\ \hat{\vec{\alpha}} = \begin{pmatrix} 0 & \hat{\vec{\sigma}} \\ \hat{\vec{\sigma}} & 0 \end{pmatrix};\ \hat{\beta} \equiv \hat{\alpha}_4 = \begin{pmatrix} \hat{\sigma}_0 & 0 \\ 0 & -\hat{\sigma}_0 \end{pmatrix}, \qquad (2.4)$$

where $\hat{\sigma}_0$, $\hat{\vec{\sigma}}$ are the Pauli matrices.

Note also that for each sign of the equation (2.3) there are two Hermitian-conjugate Dirac's equations.

Let us show that the equations, their elements and also mathematical constructions, which comprise the lasts, have simple electrodynamics sense in the nonlinear theory.

2.2. Electromagnetic form of Dirac's lepton equation

Consider two Hermitian-conjugate equations, corresponding to the minus sign of the expression (2.3):

$$\left[\left(\hat{\alpha}_o\hat{\varepsilon} + c\hat{\vec{\alpha}}\ \hat{\vec{p}}\right) + \hat{\beta}\ mc^2\right]\psi = 0\,, \qquad (2.5')$$

$$\psi^{+}\left[\left(\hat{\alpha}_o\hat{\varepsilon} + c\hat{\vec{\alpha}}\ \hat{\vec{p}}\right) + \hat{\beta}\ mc^2\right] = 0\,, \qquad (2.5'')$$

Let us choose the electromagnetic representation of the wave function, for example, in the form of wave in direction of the y-axis. Then the lepton wave function must contain the following field components:

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix} \equiv \begin{pmatrix} E_x \\ E_z \\ H_x \\ H_z \end{pmatrix},\quad \psi^{+} = \begin{pmatrix} E_x & E_z & -iH_x & -iH_z \end{pmatrix}, \qquad (2.6)$$

Let us emphasize that here the electrical and magnetic fields are not Maxwell's fields, but nonlinear quantized electromagnetic fields, which have the same solutions as the Dirac bispinors (in other words, these fields are not vector, but electromagnetic bispinors).

Using (2.6), from (2.5') and (2.5'') we will obtain EM forms of Dirac's equations for the lepton and antilepton:

$$\begin{cases} \dfrac{1}{c}\dfrac{\partial E_x}{\partial t} - \dfrac{\partial H_z}{\partial y} = -ij_x^e \\ \dfrac{1}{c}\dfrac{\partial H_z}{\partial t} - \dfrac{\partial E_x}{\partial y} = ij_z^m \\ \dfrac{1}{c}\dfrac{\partial E_z}{\partial t} + \dfrac{\partial H_x}{\partial y} = -ij_z^e \\ \dfrac{1}{c}\dfrac{\partial H_x}{\partial t} + \dfrac{\partial E_z}{\partial y} = ij_x^m \end{cases} \quad (2.7'), \qquad \begin{cases} \dfrac{1}{c}\dfrac{\partial E_x}{\partial t} - \dfrac{\partial H_z}{\partial y} = ij_x^e \\ \dfrac{1}{c}\dfrac{\partial H_z}{\partial t} - \dfrac{\partial E_x}{\partial y} = -ij_z^m \\ \dfrac{1}{c}\dfrac{\partial E_z}{\partial t} + \dfrac{\partial H_x}{\partial y} = ij_z^e \\ \dfrac{1}{c}\dfrac{\partial H_x}{\partial t} + \dfrac{\partial E_z}{\partial y} = -ij_x^m \end{cases}\ , \quad (2.7'')$$

where $\vec{j}^{\,e} = \frac{\omega}{4\pi}\vec{E}$ and $\vec{j}^{\,m} = \frac{\omega}{4\pi}\vec{H}$ are own electromagnetic currents of particles, which depend on the wave function, in which $\omega_s = \frac{m_e c^2}{\hbar}$ is the conductivity-like value. As we can see, equations (2.7') and (2.7'') are the Maxwell-like equations with imaginary electric and magnetic currents. It is known that the existence of the magnetic current $\vec{j}^{\,m}$ does not contradict to quantum theory (see Dirac's theory of a magnetic monopole (Dirac, 1931)).

It is easy to verify that the correct choice of wave functions can be obtained by cyclic transposition of indices, or by a canonical transformation of matrices and wave functions

Analysis shows (Kyriakos, 2005) that particles' own currents in the general case are alternating currents. In this case, the particle charge as integral of the current is equal to zero. Such particles are neutral and can be identified with neutral leptons with contrary spirality – neutrino and antineutrino.

In a more special case (Kyriakos, 2004b) there are particles with direct electric currents and zero magnetic currents. In this case according to the equations (2.7') and (2.7") there are two identical particles with a different sign of currents. Obviously, leptons of such type with positive and negative electric charges can be identified with the electron and positron.

It should be noted that there are no particles with direct magnetic currents, possibly because magnetic currents do not ensure the stability of particles.

2.3. Electrodynamical sense of bilinear forms of Dirac's lepton equation

It is well known that there are 16 Dirac matrices of 4 x 4 dimensions. We will exploit the same set of matrices, which Dirac used, and name it as α -set.

The values $O = \psi^{+}\hat{\alpha}\,\psi$, where $\hat{\alpha}$ is any of the Dirac's matrices, are called bilinear forms of Dirac's theory.

It can be shown that the tensor dimensions of a bilinear form follows from the tensor dimensions of nonlinear electrodynamics forms. Let us enumerate the Dirac's matrices as follows (Akhiezer and Berestetskii, 1965; Bethe, 1964; Schiff, 1955):

Here we have:

1) scalar $\hat{\alpha}_4 \equiv \hat{\beta}$,

2) 4-vector $\hat{\alpha}_\mu = \{\hat{\alpha}_0, \hat{\vec{\alpha}}\} \equiv \{\hat{\alpha}_0, \hat{\alpha}_1, \hat{\alpha}_2, \hat{\alpha}_3, \hat{\alpha}_4\}$,

3) pseudoscalar $\hat{\alpha}_5 = \hat{\alpha}_1 \cdot \hat{\alpha}_2 \cdot \hat{\alpha}_3 \cdot \hat{\alpha}_4$,

4) 4-pseudovector $\hat{\alpha}_\mu^A = \hat{\alpha}_5 \cdot \hat{\alpha}_\mu$.

Let us calculate (Kyriakos, 2004a) the electrodynamics values corresponding to the above matrices using ψ according to (2.6).

1) $\psi^{+}\hat{\alpha}_4\,\psi = \left(E_x^{\,2} + E_z^{\,2}\right) - \left(H_x^{\,2} + H_z^{\,2}\right) = \vec{E}^2 - \vec{H}^2 = 8\pi\, I_1$, where I_1 is the *first scalar* (invariant) of electrodynamics;

2) $\psi^{+}\hat{\alpha}_o\psi = \vec{E}^2 + \vec{H}^2 = 8\pi u$, where u is the *energy density* of the electromagnetic field;

$\psi^{+}\hat{\alpha}_y\psi = -\frac{8\pi}{c}\vec{S}_{Py} = -8\pi\, c\vec{g}_y$, where $\vec{g}_y$ is a *momentum density* of an electromagnetic field; the value $\left\{\frac{1}{c}u, \vec{g}\right\}$ is a 4-vector of the energy-momentum of EM field.

3) $\psi^{+}\hat{\alpha}_5\,\psi = 2\left(E_x H_x + E_z H_z\right) = 2\left(\vec{E}\cdot\vec{H}\right)$ is a *pseudoscalar* of electromagnetic field, and $\left(\vec{E}\cdot\vec{H}\right)^2 = I_2$ is the second scalar (invariant) of electromagnetic field theory.

It is not difficult to show that the statistical interpretation of wave function within the framework of SM is equivalent to electromagnetic representation of wave function in NQFT, if we take into account the normalization of energy and momentum densities of particle field relatively to the particle mass. Actually, the probability continuity equation can be obtained from the Dirac's equation (Schiff, 1955; Bethe, 1964):

$$\frac{\partial\ P_{pr}(\vec{r},t)}{\partial\ t} + div\ \vec{S}_{pr}(\vec{r},t) = 0, \qquad (2.8)$$

Here, $P_{pr}(\vec{r},t) = \psi^{+}\hat{\alpha}_0\ \psi$ is the probability density, and $\vec{S}_{pr}(\vec{r},t) = -c\psi^{+}\hat{\vec{\alpha}}\ \psi$ is the probability flux density. Using the above results, we can obtain: $P_{pr}(\vec{r},t) = 8\pi\ u$ and $\vec{S}_{pr} = c^2\vec{g} = 8\pi\ \vec{S}$. Then, an electromagnetic form of equation (2.8) can be presented in the following form:

$$\frac{\partial\ u}{\partial\ t} + div\ \vec{S} = 0, \qquad (2.9)$$

which is the form of *law of energy-momentum conservation* of electromagnetic wave.

3.0. Nonlinear lepton equation and its Lagrangian

Further we will derive the general type of the nonlinear equation of electron and construct its Lagrangian.

3.1. Self-action and the nonlinear equation of leptons

The stability of a semi-photon is only possible because of the self-action of the semi-photon. This self-action forms the particle itself, and the particle's internal parameters must ensure this self-action. The basic parameters which determine the behavior of a particle are the energy and momentum of the particle's fields. This shows how self-action can be introduced into the equation.

Since Dirac's equation (2.5) does not have other parameters, the internal parameters of electron must be connected with the free term: $\hat{\beta}\ m_e c^2$. Linearizing the conservation law of energy-momentum $\varepsilon^2 - c^2\vec{p}^2 - m_e c^2 = 0$ according to Dirac's method, namely $\varepsilon_{\pm} = \pm\sqrt{c^2\vec{p}^2 + m_e^2 c^4} = \pm\left(c\hat{\vec{\alpha}}\vec{p} + \hat{\beta} m_e c^2\right)$, we obtain the linear equivalent of this relationship: the linear expression of the energy-momentum conservation law (in present case for the internal (in) field

$$\hat{\beta}\ m_e c^2 = -\varepsilon_{in} - c\hat{\vec{\alpha}}\ \vec{p}_{in} = -e\varphi_{in} - e\hat{\vec{\alpha}}\ \vec{A}_{in}, \qquad (A)$$

(note that here $\varepsilon_{in} = e\varphi_{in}$ and $p_{in} = ec\vec{A}_{in}$ are not operators, but the energy and momentum of field; φ_{in} and $\vec{A}_{in}$ are the scalar and vector potentials correspondingly). Substituting (A) into Dirac's equation, we obtain the following equation:

$$\left[\hat{\alpha}_0(\hat{\varepsilon} - \varepsilon_{in}) + c\hat{\vec{\alpha}}\cdot\left(\hat{\vec{p}} - \vec{p}_{in}\right)\right]\psi = 0, \qquad (3.1)$$

Here, the inner energy ε_{in} and momentum p_{in} can be expressed using the inner energy density u and the inner momentum density $\vec{g}$ (or Poynting vector $\vec{S}$) of an EM wave:

$$\varepsilon_{in} = \frac{1}{8\pi}\iiint\limits_{x,y,z}\left(\vec{E}^2 + \vec{H}^2\right)dxdydz = \int_0^{\tau} u\,d\tau, \qquad (3.2)$$

$$\vec{p}_{in} = \iiint\limits_{x,y,z}\left[\vec{E}\times\vec{H}\right]dxdydz = \int_0^{\tau}\vec{g}\,d\tau = \frac{1}{c^2}\int_0^{\tau}\vec{s}\,d\tau, \qquad (3.3)$$

assuming that the upper limit of integration for the space is variable ($0 \le x, y, z < \infty$) or conditionally ($0 \le \tau < \infty$), where $d\tau = dxdydz$.

Taking into account the EM form of ψ - function (see chapters 3 -4), we obtain the quantum forms of u and $\vec{S}$ as follows:

$$u = \frac{1}{8\pi}\left(\vec{E}^2 + \vec{H}^2\right) = \frac{1}{8\pi}\psi^+\hat{\alpha}_0\psi, \qquad (3.4)$$

$$\vec{S} = \frac{c}{4\pi}\left[\vec{E}\times\vec{H}\right] = c^2\vec{g} = -\frac{c}{8\pi}\psi^+\hat{\vec{\alpha}}\psi, \qquad (3.5)$$

Substituting expressions (3.2) and (3.3) into the electron equation (3.1), and taking into account (3.4) and (3.5), we will obtain the *nonlinear integro-differential equation in both electromagnetic and quantum forms.*

We assume that equation (3.1) *is the basic nonlinear equation of the electron, which describes both the electron's motion and structure.*

Actually, taking into account the relationship (A), the equation (3.1) is reduced to the usual Dirac's equation (3.1), which describes *motion* of an electron.

For the description of the electron field *structure* apparently it is necessary to solve the nonlinear equation. The difficulty of solving such equations is already noted by Heisenberg (Heisenberg, 1967). The solution is usually anticipated by the analysis of the properties of the equation symmetry and by the possibility of its conversion into the system of linear equations. A large number of papers is devoted to this (Zakharov and Takhtadzhyan, 1979; Fushchich and Shtelen, 1983; Fushchich and Zhdanov, 1987. ; Fushchich and Zhdanov, 1988), etc.

In order to study the properties of symmetry, let us find the approximate quantum form of the equation (3.1). Then the nonlinear equation of Heisenberg occurs unexpectedly, which properties of symmetry are well studied.

3.1.1. The derivation of the nonlinear equation of Heisenberg

Let us find the approximate quantum form of the equation (3.1).

Taking into account that the solution of Dirac's equation for a free electron is the plane wave

$$\psi = \psi_0 \exp\left[i(\omega t - ky)\right], \qquad (3.6)$$

we can approximately write (3.2) and (3.3) as follows:

$$\varepsilon_p = u\Delta\tau = \frac{\Delta\tau}{8\pi}\psi^+\hat{\alpha}_0\psi = \frac{\Delta\tau}{8\pi}\left(\vec{E}^2 + \vec{H}^2\right), \qquad (3.7)$$

$$\vec{p}_p = \vec{g}\,\Delta\tau = -\frac{\Delta\tau}{8\pi c}\psi^+\hat{\vec{\alpha}}\psi = \frac{\Delta\tau}{4\pi c}\left[\vec{E}\times\vec{H}\right], \qquad (3.8)$$

where $\Delta\tau$ is the volume that contains the main part of the semi-photon's energy. If we assume that the fields of the particle apply to infinity, then apparently the cutting of integral will lead to the violation of the unitarity of theory. This must be taken into account in the use of this (approximate) equation for the description of particles.

Using (3.7) and (3.8) we can find the approximate form of the equation (3.1) as follows:

$$\frac{\partial\psi}{\partial t} - c\hat{\vec{\alpha}}\vec{\nabla}\psi + i\frac{\Delta\tau}{8\pi c}\left(\psi^+\hat{\alpha}_0\psi - \hat{\vec{\alpha}}\psi^+\hat{\vec{\alpha}}\psi\right)\psi = 0, \qquad (3.9)$$

If instead of using the α-set of Dirac's matrices we use the γ-set matrices, from the equation (3.9) we obtain the equation of Heisenberg in a form, which is known from the theory (Heisenberg, 1966; Paper translation collection, 1959):

$$\gamma_\mu\frac{\partial\psi}{\partial x_\mu} + \frac{1}{2}i\lambda\left[\gamma_\mu\psi\left(\bar{\psi}\gamma_\mu\psi\right) + \gamma_\mu\gamma_5\psi\left(\bar{\psi}\gamma_\mu\gamma_5\psi\right)\right] = 0, \qquad (3.10)$$

where $\lambda = \dfrac{\Delta\tau}{4\pi c}$ is a positive constant.

The nonlinear equation (3.10) was postulated by Heisenberg. Unlike, the equation (3.9) was obtained in a logical and correct way, and the constant λ automatically appears in this equation as a self-action constant.

In order to understand the connection of this theory with the contemporary results, let us find the Lagrangian, which corresponds to equation (3.9-3.10).

3.2. The Lagrangian of the nonlinear lepton theory

The linear type Lagrangian is presented in quantum form in Dirac's electron theory as follows (Schiff, 1955):

$$L_D = \psi^+\left(\hat{\varepsilon} + c\hat{\vec{\alpha}}\ \hat{\vec{p}} + \hat{\beta} m_e c^2\right)\psi, \qquad (3.11)$$

It is not difficult to find its electromagnetic form:

$$L_D = \frac{\partial\ u}{\partial\ t} + div\ \vec{S} - i\frac{\omega}{8\pi}\left(\vec{E}^2 - \vec{H}^2\right), \qquad (3.12)$$

(Note that in the case of a variation procedure we must distinguish the complex conjugate field vectors $\vec{E}^*, \vec{H}^*$ and $\vec{E}, \vec{H}$).

The Lagrangian of nonlinear theory can be obtained from the Lagrangian (3.11) using the same method that we used to find the nonlinear equation. Substituting relationship (A) into this equation, we obtain:

$$L_N = \psi^+\left(\hat{\varepsilon}\ - c\hat{\vec{\alpha}}\cdot\hat{\vec{p}}\right)\psi + \psi^+\left(\varepsilon_{in} - c\hat{\vec{\alpha}}\cdot\vec{p}_{in}\right)\psi, \qquad (3.13)$$

We will assume that (3.13) *represents the general form of the Lagrangian of nonlinear electron theory.*

In order to confirm this, let us compare (3.13) with the known results from classical and quantum physics. For this purpose let us find electromagnetic and quantum approximations of this Lagrangian.

Using (3.7) and (3.8), we can represent (3.11) in the following quantum approximation:

$$L_N = i\hbar\left[\frac{\partial}{\partial t}\left[\frac{1}{2}\left(\psi^+\psi\right)\right] - c\,div\left(\psi^+\hat{\vec{\alpha}}\psi\right)\right] + \frac{\Delta\tau}{8\pi}\left[\left(\psi^+\psi\right)^2 - \left(\psi^+\hat{\vec{\alpha}}\psi\right)^2\right], \qquad (3.14)$$

In order to obtain an EM form of (3.14), we initially substitute the normalized ψ-function using the expression $L'_N = \frac{1}{8\pi\ mc^2}L_N$. Then, using (3.4) and (3.5), we obtain the following electromagnetic approximation:

$$L'_N = i\frac{\hbar}{2m_e}\left(\frac{1}{c^2}\frac{\partial\ u}{\partial\ t} + div\ \vec{g}\right) + \frac{\Delta\tau}{m_e c^2}\left(u^2 - c^2\vec{g}^2\right), \qquad (3.15)$$

We can transform the second term using a known identity in electrodynamics:

$$(8\pi)^2\left(U^2 - c^2\vec{g}^2\right) = \left(\vec{E}^2 + \vec{H}^2\right)^2 - 4\left(\vec{E}\times\vec{H}\right)^2 = \left(\vec{E}^2 - \vec{H}^2\right)^2 + 4\left(\vec{E}\cdot\vec{H}\right)^2, \qquad (3.16)$$

Taking into account that $L_D = 0$, and using (3.12) and (3.16), we can represent (3.15) in the following form:

$$L'_N = \frac{1}{8\pi}\left(\vec{E}^2 - \vec{H}^2\right) + \frac{\Delta\tau}{(8\pi)^2 mc^2}\left[\left(\vec{E}^2 - \vec{H}^2\right)^2 + 4\ \left(\vec{E}\cdot\vec{H}\right)^2\right], \qquad (3.17)$$

As we can see, the approximation of the Lagrangian of the nonlinear equation for the transformed quantum of an EM wave contains only invariants of Maxwell's theory. It is similar to the known Lagrangian of photon-photon interaction (Akhiezer and Berestetskii, 1965).

Now, let us analyze the quantum form of the Lagrangian density (3.17). The equation (3.14) can be written in the form:

$$L_Q = \psi^+ \hat{\alpha}_\mu \partial_\mu \psi \ + \frac{\Delta\tau}{8\pi}\left[\left(\psi^+ \hat{\alpha}_0 \psi\right)^2 - \left(\psi^+ \hat{\vec{\alpha}}\ \psi\right)^2\right], \qquad (3.18)$$

We can see that in quantum form, the electrodynamics correlation (3.16) takes the form of the known Fierz identity (Cheng and Li, 1984; 2000):

$$\left(\psi^+ \hat{\alpha}_0 \psi\right)^2 - \left(\psi^+ \hat{\vec{\alpha}}\ \psi\right)^2 = \left(\psi^+ \hat{\alpha}_4 \psi\right)^2 + \left(\psi^+ \hat{\alpha}_5 \psi\right)^2, \qquad (3.19)$$

Using (3.19), we obtain from (3.18):

$$L_Q = \psi^+ \hat{\alpha}_\mu \partial_\mu \psi \ + \frac{\Delta\tau}{8\pi}\left[\left(\psi^+ \hat{\alpha}_4 \psi\right)^2 - \left(\psi^+ \hat{\alpha}_5 \psi\right)^2\right], \qquad (3.20)$$

If instead of using the α-set of Dirac's matrices we use the γ-set matrices, *the Lagrangian* (3.20) *coincides with the Lagrangian of Nambu – Jona-Lazinio* (Nambu and Jona-Lazinio, 1961; 1961a).

Let us note some special features of the results, obtained in the nonlinear quantum field theory (NQFT) in comparison with the results that were obtained in the contemporary theory.

Since the Lagrangian of Nambu – Jona-Lazinio is a Lagrangian of weak interaction of the type (V - A), the nonlinear theory NQFT covers not only electromagnetic, but also weak interactions. To this corresponds the fact that in the general case Dirac's equation describes massive neutrino with a conserved inner helicity (Kyriakos, 2005).

The NQFT show that Lagrangian of Nambu – Jona-Lazinio is approximate. Therefore its use can cause different violations of the type of violations of unitarity. This is connected to the fact that the probability distribution density must behave under the Lorenz transformation as time component of the four-dimensional vector, whose divergence is equal to zero. But the Lagrangian of Nambu - Jona-Lazinio contains the strengths of electromagnetic field. As is known, from the strengths of electromagnetic field it is not possible to compose the bilinear combination, which forms the four-dimensional vector, whose divergence would be equal zero. However, this value can be constructed, relying on the integral values - energy and momentum, which compose a completely determined 4-vector.

It is understandable that in order to avoid these difficulties there is no need to use some additional models; it is sufficient to use the precise Lagrangian (3.13).

As we noted, Heisenberg's equation has a high degree of symmetry because of the absence of mass, but a special mathematical mechanism SSB is required for the primary particles of equation (3.10) to become massive.

In our case the nonlinear integro-differential equation (3.1) does not contain mass, and "the mechanism", through which the mass is introduced into the quantum field equations, is the relationship (A) (Kyriakos, 2009):

$$-\varepsilon_{in} - c\hat{\vec{\alpha}}\ \vec{p}_{in} = -e\varphi_{in} - e\hat{\vec{\alpha}}\ \vec{A}_{in} = \hat{\beta}\ m_e c^2\,, \qquad (A').$$

This relationship, recorded here in the reverse order, clearly reflects the process of symmetry breaking, since we substitute the term of high degree of symmetry with a term of low degree of symmetry. Moreover, it is possible to show that the relationship (A') reflects the result of the rotation transformation of the internal symmetry of particle, which is mathematically equivalent to the gauge transformation result (Kyriakos, 2009).

The special feature of this mechanism is that it does not require the introduction of additional particles and at the same time it does not lead to the necessity to exceed the limits of SM.

Heisenberg poses a problem to obtain all the remaining particles in the form of bound states of a different number of primary particles on the basis of some primary spinor particles.

If we consider the spinor particles as the primary building elements of matter, then it is really possible, using spinor equations, to obtain the equations of all other particles (Kyriakos, 2009).

Now, on the basis of the obtained results it is possible to propose the solution of the difficulties of the theory, which can arise in connection with the possible absence of Higgs's boson in nature.

4.0. The symmetry breaking and mass generation without Higgs's mechanism

In the introduction we analyzed the difficulties, which arose in Heisenberg's attempt to obtain a linear equation for the massive particle from the mass-free nonlinear equation. In order to overcome these difficulties Heisenberg proposed to use a SSB of the ground state.

To overcome the same difficulties within the framework of SM the introduction of SSB with additional particles - Goldstone bosons and Higgs's bosons – was also nessesery.

The essence of the Nambu – Jona-Lazinio and Higgs mechanisms of mass generation is described in the majority of books about the quantum field theory. We will only note nodal points of this description, which are necessary for the comparison.

As it is known (Okun, 1988; Ryder, 1985) the SSB according to Higgs lies in the fact that from the mass-free vector field (similar to photon field), which has two spin states, and the mass-free doublet of scalar field ϕ (two particles φ^{+}, φ^{0} and two antiparticles $\varphi^{-}, \overline{\varphi}^{0}$), appers the massive vector particle with three projections of isospin. In this case from four scalar fields as a result of SSB three particles "are eaten" and one particle acquires mass. These massive bosons are called Higgs's bosons.

It is remarkable that SSB appears by the nonlinear interaction of the field ϕ with itself; the self-interaction energy can be written down in the form of the potential $V(\phi) = \lambda^2 \left(|\phi|^2 - \eta^2 \right)^2$, where $|\phi|^2 = \overline{\varphi}^{+}\varphi^{+} + \overline{\varphi}^{0}\varphi^{0}$ is isoscalar, λ is the dimensionless parameter, η is the parameter, which has the dimensionality of mass.

Within the framework of NQFT (Kyriakos, 2009) the process of the particles mass generation is connected with the generation of particles themselves, so they acquire all their parameters and characteristics in the process of this generation. Nevertheless, analogy with the process of particles' mass generation according to Higgs's mechanism is obvious. Let us compare these mechanisms step by step.

1) Similarly to Higgs's mechanism (HM) the initial stage of the introduction of mass in NQFT is the mass-free field of photon.

2) In the second stage in NQFT occurs the process of the symmetry breaking of initial field. But in contrast to HM here, there is no need of introducing any additional particles. Symmetry breaking occurs as a result of rotation transformation of the photon fields. Due to this process the photon acquires mass and becomes the "massive photon", i.e. the intermediate boson. In this case, similarly with HM, mass appears as a result of self-interaction of photon fields (conditionally speaking, photon becomes an additional particle by itself).

It is important to emphasize that the result of the rotation transformation is equivalent to the result of gauge transformation . Therefore the process of acquisition of the particles' masses in NQFT always remains an invariance relatively to gauge transformations.

3) Similar to Higgs's mechanism, in NQFT the intermediate boson ("massive photon") is necessary for the generation of other massive particles - leptons and hadrons. But in contrast to SM in NQFT elementary particles (excluding photons) are never mass-free. This corresponds to the experimental observations: in nature no mass-free elementary particles were ever observed, except for photons. With the spontaneous symmetry breaking of massive intermediate photon all particles are born massive. In particular, under certain conditions the particles appear in form of pairs of particle-antiparticle in reaction of the intermediate boson disintegration.

Let us also note that the mathematical description of all these stages has similarity both in SM and NQFT. This allows us to assume that HM is the abstraction of mathematical description of the simple mechanisms, which occur in nature and are adequately described by NQFT. Let us emphasize that NQFT, as the generalization of Standard Model, contradicts in nothing with the results of the latter.

Conclusion

The negative result of the experiments, set until now for the confirmation of Higgs's mechanism, implies that in nature a different version of the generation of masses takes place. Complete agreement of NQFT with SM and with the existing experimental results makes the version, proposed by NQFT, the basic candidate to the role of the theory, which is adequate to the reality.

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