

Weak form of electric-magnetic duality, electroweak massivation, and color confinement

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Abstract

The notion of electric magnetic duality emerged already two decades ago in the attempts to formulate the Kähler geometry of the "world of classical worlds". Quite recently a considerable step of progress took place in the understanding of this notion. This concept leads to the identification of the physical particles as string like objects defined by magnetic charged wormhole throats connected by magnetic flux tubes. The second end of the string contains particle having electroweak isospin neutralizing that of elementary fermion and the size scale of the string is electro-weak scale would be in question. Hence the screening of electro-weak force takes place via weak confinement. This picture generalizes to magnetic color confinement.

Keywords: Electric-magnetic duality, magnetic monopoles, color confinement, weak confinement, string like objects.

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1 Introduction

The notion of electric-magnetic duality [16] was proposed first by Olive and Montonen and is central in $\mathcal{N} = 4$ supersymmetric gauge theories. It states that magnetic monopoles and ordinary particles are two different phases of theory and that the description in terms of monopoles can be applied at the limit when the running gauge coupling constant becomes very large and perturbation theory fails to converge. The notion of electric-magnetic self-duality is more natural since for CP_2 geometry Kähler form is self-dual and Kähler magnetic monopoles are also Kähler electric monopoles and Kähler coupling strength is by quantum criticality renormalization group invariant rather than running coupling constant. The notion of electric-magnetic (self-)duality emerged already two decades ago in the attempts to formulate the Kähler geometric of world of classical worlds. Quite recently a

considerable step of progress took place in the understanding of this notion [8]. What seems to be essential is that one adopts a weaker form of the self-duality applying at partonic 2-surfaces. What this means will be discussed in the sequel.

Every new idea must be of course taken with a grain of salt but the good sign is that this concept leads to precise predictions. The point is that elementary particles do not generate monopole fields in macroscopic length scales: at least when one considers visible matter. The first question is whether elementary particles could have vanishing magnetic charges: this turns out to be impossible. The next question is how the screening of the magnetic charges could take place and leads to an identification of the physical particles as string like objects identified as pairs magnetic charged wormhole throats connected by magnetic flux tubes.

1. The first implication is a new view about electro-weak massivation reducing it to weak confinement in TGD framework. The second end of the string contains particle having electroweak isospin neutralizing that of elementary fermion and the size scale of the string is electro-weak scale would be in question. Hence the screening of electro-weak force takes place via weak confinement realized in terms of magnetic confinement.
2. This picture generalizes to the case of color confinement. Also quarks correspond to pairs of magnetic monopoles but the charges need not vanish now. Rather, valence quarks would be connected by flux tubes of length of order hadron size such that magnetic charges sum up to zero. For instance, for baryonic valence quarks these charges could be $(2, -1, -1)$ and could be proportional to color hyper charge.
3. The highly non-trivial prediction making more precise the earlier stringy vision is that elementary particles are string like objects in electro-weak scale: this should become manifest at LHC energies.

2 Could a weak form of electric-magnetic duality hold true?

Holography means that the initial data at the partonic 2-surfaces should fix the configuration space metric. A weak form of this condition allows only the partonic 2-surfaces defined by the wormhole throats at which the signature of the induced metric changes. A stronger condition allows all partonic 2-surfaces in the slicing of space-time sheet to partonic 2-surfaces and string world sheets. Number theoretical vision suggests that hyper-quaternionicity *resp.* co-hyperquaternionicity constraint could be enough to fix the initial values of time derivatives of the imbedding space coordinates in the space-time regions with Minkowskian *resp.* Euclidian signature of the induced metric. This is a condition on modified gamma matrices and hyper-quaternionicity states that they span a hyper-quaternionic sub-space.

2.1 Definition of the weak form of electric-magnetic duality

One can also consider alternative conditions possibly equivalent with this condition. The argument goes as follows.

1. The expression of the matrix elements of the metric and Kähler form of WCW in terms of the Kähler fluxes weighted by Hamiltonians of δM_{\pm}^4 at the partonic 2-surface X^2 looks very attractive. These expressions however carry no information about the 4-D tangent space of the partonic 2-surfaces so that the theory would reduce to a genuinely 2-dimensional theory, which cannot hold true. One would like to code to the WCW metric also information about the electric part of the induced Kähler form assignable to the complement of the tangent space of $X^2 \subset X^4$.
2. Electric-magnetic duality of the theory looks a highly attractive symmetry. The trivial manner to get electric magnetic duality at the level of the full theory would be via the identification of the flux Hamiltonians as sums of of the magnetic and electric fluxes. The presence of the induced metric is however troublesome since the presence of the induced metric means that the simple transformation properties of flux Hamiltonians under symplectic transformations -in particular color rotations- are lost.

3. A less trivial formulation of electric-magnetic duality would be as an initial condition which eliminates the induced metric from the electric flux. In the Euclidian version of 4-D YM theory this duality allows to solve field equations exactly in terms of instantons. This approach involves also quaternions. These arguments suggest that the duality in some form might work. The full electric magnetic duality is certainly too strong and implies that space-time surface at the partonic 2-surface corresponds to piece of CP_2 type vacuum extremal and can hold only in the deep interior of the region with Euclidian signature. In the region surrounding wormhole throat at both sides the condition must be replaced with a weaker condition.
4. To formulate a weaker form of the condition let us introduce coordinates (x^0, x^3, x^1, x^2) such (x^1, x^2) define coordinates for the partonic 2-surface and (x^0, x^3) define coordinates labeling partonic 2-surfaces in the slicing of the space-time surface by partonic 2-surfaces and string world sheets making sense in the regions of space-time sheet with Minkowskian signature. The assumption about the slicing allows to preserve general coordinate invariance. The weakest condition is that the generalized Kähler electric fluxes are apart from constant proportional to Kähler magnetic fluxes. This requires the condition

$$J^{03}\sqrt{g_4} = KJ_{12} . \quad (2.1)$$

A more general form of this duality is suggested by the considerations of [7] reducing the hierarchy of Planck constants to basic quantum TGD and also reducing Kähler function for preferred extremals to Chern-Simons terms [?] at the boundaries of CD and at light-like wormhole throats. This form is following

$$J^{n\beta}\sqrt{g_4} = K\epsilon \times \epsilon^{n\beta\gamma\delta} J_{\gamma\delta}\sqrt{g_4} . \quad (2.2)$$

Here the index n refers to a normal coordinate for the space-like 3-surface at either boundary of CD or for light-like wormhole throat. ϵ is a sign factor which is opposite for the two ends of CD . It could be also opposite at the opposite sides of the wormhole throat. Note that the dependence on induced metric disappears at the right hand side and this condition eliminates the potentials singularity due to the reduction of the rank of the induced metric at wormhole throat.

One can consider also a more general variant of the weak self-duality in which appears also the symplectic form of $r_M = \text{constant}$ sphere of light-cone boundary -call it J^1 - defining a magnetic monopole field at ight-cone boundary with tip excluded. This form reads as

$$J^{n\beta}\sqrt{g_4} = K\epsilon \times \epsilon^{n\beta\gamma\delta} (J_{\gamma\delta} + \epsilon J_{\gamma\delta}^1)\sqrt{g_4} . \quad (2.3)$$

Here ϵ is a pure number $\epsilon = 1$ is favored. This condition is very natural if one replaces J with the sum of $J + J_1$ in Kähler action. It would be found that this is forced by very general physical and mathematical constraints.

5. Information about the tangent space of the space-time surface can be coded to the configuration space metric with loosing the nice transformation properties of the magnetic flux Hamiltonians if Kähler electric fluxes or sum of magnetic flux and electric flux satisfying this condition are used and K is symplectic invariant. Using the sum

$$J_e + J_m = (1 + K)J_{12} , \quad (2.4)$$

where J_{12} can denote either CP_2 Kähler form or $J + J_1$, makes it possible to have a non-trivial configuration space metric even for $K = 0$, which could correspond to the ends of a cosmic string like solution carrying only Kähler magnetic fields. This condition suggests that

it can depend only on Kähler magnetic flux and other symplectic invariants. Whether local symplectic coordinate invariants are possible at all is far from obvious, If the slicing itself is symplectic invariant then K could be a non-constant function of X^2 depending on string world sheet coordinates. The light-like radial coordinate of the light-cone boundary indeed defines a symplectically invariant slicing and this slicing could be shifted along the time axis defined by the tips of CD .

2.2 Electric-magnetic duality physically

What could the weak duality condition mean physically? For instance, what constraints are obtained if one assumes that the quantization of electro-weak charges reduces to this condition at classical level?

1. The first thing to notice is that the flux of J over the partonic 2-surface is analogous to magnetic flux

$$Q_m = \frac{e}{\hbar} \oint B dS = n \ .$$

n is non-vanishing only if the surface is homologically non-trivial and gives the homology charge of the partonic 2-surface.

2. The expressions of classical electromagnetic and Z^0 fields in terms of Kähler form [15] read as

$$\begin{aligned} \gamma &= \frac{eF_{em}}{\hbar} = 3J - \sin^2(\theta_W)R_{03} \ , \\ Z^0 &= \frac{g_Z F_Z}{\hbar} = 2R_{03} \ . \end{aligned} \quad (2.5)$$

Here R_{03} is one of the components of the curvature tensor in vielbein representation and F_{em} and F_Z correspond to the standard field tensors. From this expression one can deduce

$$J = \frac{e}{3\hbar} F_{em} + \sin^2(\theta_W) \frac{g_Z}{6\hbar} F_Z \ . \quad (2.6)$$

3. The weak duality condition when integrated over X^2 implies

$$\begin{aligned} \frac{e^2}{3\hbar} Q_{em} + \frac{g_Z^2 p}{6} Q_{Z,V} &= K \oint J = Kn \ , \\ Q_{Z,V} &= \frac{I_V^3}{2} - Q_{em} \ , \quad p = \sin^2(\theta_W) \ . \end{aligned} \quad (2.7)$$

Here the vectorial part of the Z^0 charge rather than as full Z^0 charge $Q_Z = I_L^3 + \sin^2(\theta_W)Q_{em}$ appears. The reason is that only the vectorial isospin is same for left and right handed components of fermion which are in general mixed for the massive states.

The coefficients are dimensionless and expressible in terms of the gauge coupling strengths and using $\hbar = r\hbar_0$ one can write

$$\begin{aligned} \alpha_{em} Q_{em} + p \frac{\alpha_Z}{2} Q_{Z,V} &= \frac{3}{4\pi} \times rnK \ , \\ \alpha_{em} &= \frac{e^2}{4\pi\hbar_0} \ , \quad \alpha_Z = \frac{g_Z^2}{4\pi\hbar_0} = \frac{\alpha_{em}}{p(1-p)} \ . \end{aligned} \quad (2.8)$$

4. There is a great temptation to assume that the values of Q_{em} and Q_Z correspond to their quantized values and therefore depend on the quantum state assigned to the partonic 2-surface. The linear coupling of the modified Dirac operator to conserved charges implies correlation between the geometry of space-time sheet and quantum numbers assigned to the partonic 2-surface. The assumption of standard quantized values for Q_{em} and Q_Z would be also seen as the identification of the fine structure constants α_{em} and α_Z . This however requires weak isospin invariance.

2.3 The value of K from classical quantization of Kähler electric charge

The value of K can be deduced by requiring classical quantization of Kähler electric charge.

1. The condition that the flux of $F^{03} = (\hbar/g_K)J^{03}$ defining the counterpart of Kähler electric field equals to the Kähler charge g_K would give the condition $K = g_K^2/\hbar$, where g_K is Kähler coupling constant which should be invariant under coupling constant evolution by quantum criticality. Within experimental uncertainties one has $\alpha_K = g_K^2/4\pi\hbar_0 = \alpha_{em} \simeq 1/137$, where α_{em} is finite structure constant in electron length scale and \hbar_0 is the standard value of Planck constant.
2. The quantization of Planck constants makes the condition highly non-trivial. The most general quantization of r is as rationals but there are good arguments favoring the quantization as integers corresponding to the allowance of only singular coverings of CD and CP_2 . The point is that in this case a given value of Planck constant corresponds to a finite number of pages of the "Big Book". The quantization of the Planck constant implies a further quantization of K and would suggest that K scales as $1/r$ unless the spectrum of values of Q_{em} and Q_Z allowed by the quantization condition scales as r . This is quite possible and the interpretation would be that each of the r sheets of the covering carries (possibly same) elementary charge. Kind of discrete variant of a full Fermi sphere would be in question. The interpretation in terms of anyonic phases [13] supports this interpretation.
3. The identification of J as a counterpart of eB/\hbar means that Kähler action and thus also Kähler function is proportional to $1/\alpha_K$ and therefore to \hbar . This implies that for large values of \hbar Kähler coupling strength $g_K^2/4\pi$ becomes very small and large fluctuations are suppressed in the functional integral. The basic motivation for introducing the hierarchy of Planck constants was indeed that the scaling $\alpha \rightarrow \alpha/r$ allows to achieve the convergence of perturbation theory: Nature itself would solve the problems of the theoretician. This of course does not mean that the physical states would remain as such and the replacement of single particles with anyonic states in order to satisfy the condition for K would realize this concretely.

The weak form of electric-magnetic duality has surprisingly strong implications for basic view about quantum TGD as following considerations show.

3 Magnetic confinement, the short range of weak forces, and color confinement

The weak form of electric-magnetic duality has surprisingly strong implications if one combines it with some very general empirical facts such as the non-existence of magnetic monopole fields in macroscopic length scales.

3.1 How can one avoid macroscopic magnetic monopole fields?

Monopole fields are experimentally absent in length scales above order weak boson length scale and one should have a mechanism neutralizing the monopole charge. How electroweak interactions become short ranged in TGD framework is still a poorly understood problem. What suggests itself is the neutralization of the weak isospin above the intermediate gauge boson Compton length by neutral Higgs bosons. Could the two neutralization mechanisms be combined to single one?

1. In the case of fermions and their superpartners the opposite magnetic monopole would be a wormhole throat. If the magnetically charged wormhole contact is electromagnetically neutral but has vectorial weak isospin neutralizing the weak vectorial isospin of the fermion only the electromagnetic charge of the fermion is visible on longer length scales. The distance of this wormhole throat from the fermionic one should be of the order weak boson Compton length. An interpretation as a bound state of fermion and a wormhole throat state with the quantum numbers of a neutral Higgs boson would therefore make sense. The neutralizing throat would have quantum numbers of $X_{-1/2} = \nu_L \bar{\nu}_R$ or $X_{1/2} = \bar{\nu}_L \nu_R$. $\nu_L \bar{\nu}_R$ would not be neutral Higgs boson (which should correspond to a wormhole contact) but a super-partner of left-handed neutrino obtained by adding a right handed neutrino. This mechanism would apply separately to the fermionic and antifermionic throats of the gauge bosons and corresponding space-time sheets and leave only electromagnetic interaction as a long ranged interaction.
2. One can of course wonder what is the situation for the bosonic wormhole throats feeding gauge fluxes between space-time sheets. It would seem that these wormhole throats must always appear as pairs such that for the second member of the pair monopole charges and I_V^3 cancel each other at both space-time sheets involved so that one obtains at both space-time sheets magnetic dipoles of size of weak boson Compton length. The proposed magnetic character of fundamental particles should become visible at TeV energies so that LHC might have surprises in store!

3.2 Magnetic confinement and color confinement

Magnetic confinement generalizes also to the case of color interactions. One can consider also the situation in which the magnetic charges of quarks (more generally, of color excited leptons and quarks) do not vanish and they form color and magnetic singlets in the hadronic length scale. This would mean that magnetic charges of the state $q_{\pm 1/2} - X_{\mp 1/2}$ representing the physical quark would not vanish and magnetic confinement would accompany also color confinement. This would explain why free quarks are not observed. To how degree then quark confinement corresponds to magnetic confinement is an interesting question.

For quark and antiquark of meson the magnetic charges of quark and antiquark would be opposite and meson would correspond to a Kähler magnetic flux so that a stringy view about meson emerges. For valence quarks of baryon the vanishing of the net magnetic charge takes place provided that the magnetic net charges are $(\pm 2, \mp 1, \mp 1)$. This brings in mind the spectrum of color hyper charges coming as $(\pm 2, \mp 1, \mp 1)/3$ and one can indeed ask whether color hyper-charge correlates with the Kähler magnetic charge. The geometric picture would be three strings connected to single vertex. Amusingly, the idea that color hypercharge could be proportional to color hyper charge popped up during the first year of TGD when I had not yet discovered CP_2 and believed on $M^4 \times S^2$.

p-Adic length scale hypothesis and hierarchy of Planck constants defining a hierarchy of dark variants of particles suggest the existence of scaled up copies of QCD type physics and weak physics. For p-adically scaled up variants the mass scales would be scaled by a power of $\sqrt{2}$ in the most general case. The dark variants of the particle would have the same mass as the original one. In particular, Mersenne primes $M_k = 2^k - 1$ and Gaussian Mersennes $M_{G,k} = (1 + i)^k - 1$ has been proposed to define zoomed copies of these physics. At the level of magnetic confinement this would mean hierarchy of length scales for the magnetic confinement.

One particular proposal is that the Mersenne prime M_{89} should define a scaled up variant of the ordinary hadron physics with mass scaled up roughly by a factor $2^{(107-89)/2} = 512$. The size scale of color confinement for this physics would be same as the weak length scale. It would look more natural that the weak confinement for the quarks of M_{89} physics takes place in some shorter scale and M_{61} is the first Mersenne prime to be considered. The mass scale of M_{61} weak bosons would be by a factor $2^{(89-61)/2} = 2^{14}$ higher and about 1.6×10^4 TeV. M_{89} quarks would have virtually no weak interactions but would possess color interactions with weak confinement length scale reflecting themselves as new kind of jets at collisions above TeV energies.

In the biologically especially important length scale range 10 nm -2500 nm there are as many as four Gaussian Mersennes corresponding to $M_{G,k}$, $k = 151, 157, 163, 167$. This would suggest that the existence of scaled up scales of magnetic-, weak- and color confinement. An especially interesting possibly testable prediction is the existence of magnetic monopole pairs with the size scale in this

range. There are recent claims about experimental evidence for magnetic monopole pairs [18].

3.3 Magnetic confinement and stringy picture in TGD sense

The connection between magnetic confinement and weak confinement is rather natural if one recalls that electric-magnetic duality in super-symmetric quantum field theories means that the descriptions in terms of particles and monopoles are in some sense dual descriptions. Fermions would be replaced by string like objects defined by the magnetic flux tubes and bosons as pairs of wormhole contacts would correspond to pairs of the flux tubes. Therefore the sharp distinction between gravitons and physical particles would disappear.

The reason why gravitons are necessarily stringy objects formed by a pair of wormhole contacts is that one cannot construct spin two objects using only single fermion states at wormhole throats. Of course, also superpartners of these states with higher spin obtained by adding fermions and antifermions at the wormhole throat but these do not give rise to graviton like states [10]. The upper and lower wormhole throat pairs would be quantum superpositions of fermion antifermion pairs with sum over all fermions. The reason is that otherwise one cannot realize graviton emission in terms of joining of the ends of light-like 3-surfaces together. Also now magnetic monopole charges are necessary but now there is no need to assign the entities X_{\pm} with gravitons.

Graviton string is characterized by some p-adic length scale and one can argue that below this length scale the charges of the fermions become visible. Merenne hypothesis suggests that some Mersenne prime is in question. One proposal is that gravitonic size scale is given by electronic Mersenne prime M_{127} . It is however difficult to test whether graviton has a structure visible below this length scale.

What happens to the generalized Feynman diagrams is an interesting question. It is not at all clear how closely they relate to ordinary Feynman diagrams. All depends on what one is ready to assume about what happens in the vertices. One could of course hope that zero energy ontology could allow some very simple description allowing perhaps to get rid of the problematic aspects of Feynman diagrams.

1. Consider first the recent view about generalized Feynman diagrams which relies zero energy ontology. A highly attractive assumption is that the particles appearing at wormhole throats are on mass shell particles. For incoming and outgoing elementary bosons and their superpartners they would be positive it resp. negative energy states with parallel on mass shell momenta. For virtual bosons they the wormhole throats would have opposite sign of energy and the sum of on mass shell states would give virtual net momenta. This would make possible twistorial description of virtual particles allowing only massless particles (in 4-D sense usually and in 8-D sense in TGD framework). The notion of virtual fermion makes sense only if one assumes in the interaction region a topological condensation creating another wormhole throat having no fermionic quantum numbers
2. The addition of the particles X^{\pm} replaces generalized Feynman diagrams with the analogs of stringy diagrams with lines replaced by pairs of lines corresponding to fermion and $X_{\pm 1/2}$. The members of these pairs would correspond to 3-D light-like surfaces glued together at the vertices of generalized Feynman diagrams. The analog of 3-vertex would not be splitting of the string to form shorter strings but the replication of the entire string to form two strings with same length or fusion of two strings to single string along all their points rather than along ends to form a longer string. It is not clear whether the duality symmetry of stringy diagrams can hold true for the TGD variants of stringy diagrams.
3. How should one describe the bound state formed by the fermion and X^{\pm} ? Should one describe the state as superposition of non-parallel on mass shell states so that the composite state would be automatically massive? The description as superposition of on mass shell states does not conform with the idea that bound state formation requires binding energy. In TGD framework the notion of negentropic entanglement has been suggested to make possible the analogs of bound states consisting of on mass shell states so that the binding energy is zero [14]. If this kind of states are in question the description of virtual states in terms of on mass shell states is not lost. Of course, one cannot exclude the possibility that there is infinite number of this kind of states serving as analogs for the excitations of string like object.

4. What happens to the states formed by fermions and $X_{\pm 1/2}$ in the internal lines of the Feynman diagram? Twistorial philosophy suggests that only the higher on mass shell excitations are possible. If this picture is correct, the situation would not change in an essential manner from the earlier one.

The highly non-trivial prediction of the magnetic confinement is that elementary particles should have stringy character in electro-weak length scales and could be coming to become manifest at LHC energies. This adds one further item to the list of non-trivial predictions of TGD about physics at LHC energies [12].

3.4 Also S^2 monopole charges are necessary

The generalization of the the weak form of self-duality to $J^{n\beta} = \epsilon^{n\beta\gamma\delta} K(J_{\gamma\delta} + \epsilon J_{\gamma\delta}^1)$ was already mentioned and the following argument suggests that this generalization is unavoidable if one wants quantization of electromagnetic charge classically.

1. The original form of weak self-duality gives a quantization of the electromagnetic charge in the case of single monopole throat. If one however considers electric flux over a 2-surface enclosing monopole pairs only, the Kähler charge vanishes if weak confinement takes place in the proposed manner. Electromagnetic charge is however of form $Q_{em} = Q_K + I_{3V}$ for leptons and $Q_{em} = Q_K/3 + I_{3V}$ for quarks. This requires that Q_K is non-vanishing also in length scales longer than electro-weak scale but this cannot be the case for the original formula.
2. The solution of the problem would be that electro-weak massivation involves a 2-D topological condensation of the wormhole throats representing particles to a 2-surface which is homologically non-trivial in δM_{\pm}^4 . In this case one would Q_K would reduce to $Q_K = \epsilon Q_{1,m}$. I have suggested that this kind of condensation happens even in astrophysical scales and leads to anyonization [13]. The requirement that the electromagnetic charge is conserved in the process implies that the homology charge of the "partonic" - or rather anyonic- 2-surface is proportional to the electromagnetic charge. Quantization requires that ϵ is integer valued and the most natural value is $\epsilon = \pm 1$. If $J + J_1$ replaces Kähler form in Kähler action one has $\epsilon = 1$.
3. The mechanism involves both C breaking and parity breaking since the orientation of the anyonic surface correlates with the sign of the electromagnetic charge. This macroscopic parity breaking seems to have no obvious relation to the electro-weak parity breaking. Matter-antimatter asymmetry could be interpreted as S^2 monopole condensation favoring same orientation for all individual 2-surfaces winding around the tip of CD s. Matter-antimatter symmetric states would have vanishing or very small S^2 homology charge and this might make them unstable. The effects of Coulomb repulsion would be minimized because the charges are at different sheets of the covering.
4. A two surface possessing n units of S^2 magnetic charge can be obtained from sphere by a deformation for which the circles parallel to equator wind n times around z-axis before closing. In δM_{\pm}^4 homological nontriviality for $n > 1$ is not possible without self intersections but in $\delta M_{\pm}^4 \times CP_2$ situation is different. In the simplest situation CP_2 coordinates are n -valued functions of the angle coordinate ϕ of S^2 : this would give a singularity at poles but a small deformation removes the singularities.

3.5 Should $J + J_1$ appear in Kähler action?

The presence of J_1 in self-duality condition is required by the above consistency argument as well as the argument about the reduction to almost topological QFT to be described in the next subsection. This raises the question whether one should replace J with $J + J_1$ in the Kähler action. This would not affect basic non-vacuum extremals but would modify the vacuum degeneracy of the Kähler action. Canonically imbedded M^4 would become a monopole configuration with an infinite magnetic energy and Kähler action due to the monopole singularity at the line connecting tips of the CD . Action and energy can be made small by drilling a small hole around origin. This is however not consistent with the weak form of electro-weak duality. Amusingly, the modified Dirac equation reduces to ordinary massless Dirac equation in M^4 .

This extremal can be transformed to a vacuum extremal by assuming that the solution is also a CP_2 magnetic monopole with opposite contribution to the magnetic charge so that $J + J_1 = 0$ holds true. This is achieved if one can regard space-time surface as a map $M^4 \rightarrow CP_2$ reducing to a map $(\Theta, \Phi) = (\theta, \pm\phi)$ with the sign chosen properly projecting the homologically non-trivial $r_M = \text{constant}$ spheres of CD to the homologically non-trivial geodesic sphere of CP_2 . Symplectic transformations of $S^2 \times CP_2$ produce new vacuum extremals of this kind. Using Darboux coordinates in which one has $J = \sum_{k=1,2} P_k dQ^k$ and assuming that (P_1, Q_1) corresponds to the CP_2 image of S^2 , one can take either P_2 or Q^2 to be an arbitrary function of (t, r_M) to obtain even more general vacuum extremals with 3-D CP_2 projection. Also P_1 or Q_1 can be assumed to be an arbitrary function of (t, r_M) . Therefore the spectrum of vacuum extremals, which is very relevant for the TGD based description of gravitation in long length scales because it allows to satisfy Einstein's equations as an additional condition, is much richer than for the original option. Robertson-Walker cosmologies must be slightly deformed meaning a slight breaking of the cosmological principle. For the simplest option the dependence of CP_2 coordinates on lightcone proper time a is $P_2 = f(a)$ or $Q^2 = f(a)$, f an arbitrary function. The induced metric of X^4 deviates extremely little from Robertson-Walker form for the simplest solutions. From the point of classical gravitation this option is obviously more promising than the original one.

The objection is that J_1 is a radial monopole field and this breaks Lorentz invariance to $SO(3)$. Lorentz invariance is broken to $SO(3)$ for a given CD also by the presence of the preferred time direction defined by the time-like line connecting the tips of the CD becoming carrying the monopole charge but is compensated since Lorentz boosts of CD s are possible. Could one consider similar compensation also now? Certainly the extremely small breaking of Lorentz invariance and the vanishing of the monopole charge for the vacuum extremals is all that is needed at the space-time level. No new gauge fields would be introduced since only the Kähler field part of photon and Z^0 boson would receive an additional contribution.

4 Could Quantum TGD reduce to almost topological QFT?

There seems to be a profound connection with the earlier unrealistic proposal that TGD reduces to almost topological quantum theory in the sense that the counterpart of Chern-Simons action assigned with the wormhole throats somehow dictates the dynamics. This proposal can be formulated also for the modified Dirac action action. I gave up this proposal but the following argument shows that Kähler action with weak form of electric-magnetic duality effectively reduces to Chern-Simons action plus Coulomb term.

1. Kähler action density can be written as a 4-dimensional integral of the Coulomb term $j_K^\alpha A_\alpha$ plus and integral of the boundary term $J^{n\beta} A_\beta \sqrt{g_4}$ over the wormhole throats and of the quantity $J^{0\beta} A_\beta \sqrt{g_4}$ over the ends of the 3-surface.
2. If the self-duality conditions generalize to $J^{n\beta} = 4\pi\alpha_K \epsilon^{n\beta\gamma\delta} J_{\gamma\delta}$ at throats and to $J^{0\beta} = 4\pi\alpha_K \epsilon^{0\beta\gamma\delta} J_{\gamma\delta}$ at the ends, the Kähler function reduces to the counterpart of Chern-Simons action evaluated at the ends and throats. It would have same value for each branch and the replacement $\hbar_0 \rightarrow r\hbar_0$ would effectively describe this. Boundary conditions would however give $1/r$ factor so that \hbar would disappear from the Kähler function! The original attempt to realize quantum TGD as an almost topological QFT was in terms of Chern-Simons action but was given up. It is somewhat surprising that Kähler action gives Chern-Simons action in the vacuum sector defined as sector for which Kähler current is light-like or vanishes.

Holography encourages to ask whether also the Coulomb interaction terms could vanish. This kind of dimensional reduction would mean an enormous simplification since TGD would reduce to an almost topological QFT. The attribute "almost" would come from the fact that one has non-vanishing classical Noether charges defined by Kähler action and non-trivial quantum dynamics in M^4 degrees of freedom. One could also assign to space-time surfaces conserved four-momenta which is not possible in topological QFTs. For this reason the conditions guaranteeing the vanishing of Coulomb interaction term deserve a detailed analysis.

1. For the known extremals j_K^α either vanishes or is light-like ("massless extremals" for which weak self-duality condition does not make sense [11]) so that the Coulombic term vanishes identically in

the gauge used. The addition of a gradient to A induces terms located at the ends and wormhole throats of the space-time surface but this term must be cancelled by the other boundary terms by gauge invariance of Kähler action. This implies that the M^4 part of WCW metric vanishes in this case. Therefore massless extremals as such are not physically realistic: wormhole throats representing particles are needed.

2. One can ask whether the contribution of Coulomb term could vanish quite generally without leading to trivial WCW metric in M^4 degrees of freedom if one modifies the weak form of self-duality. Besides CP_2 Kähler form there is the Kähler form assignable to the light-cone boundary reducing to that for $r_M = \text{constant}$ sphere - call it J^1 . Also the Hamilton-Jacobi coordinates for M^4 define a family of slicings of M^4 by string world sheets and 2-surfaces and one can assign to the 2-surface a natural Kähler form playing a key role in the construction of extremals of Kähler action [11]. Recall that the generalization of the the weak form of self-duality is $J^{n\beta} = \epsilon^{n\beta\gamma\delta} K(J_{\gamma\delta} + \epsilon J_{\gamma\delta}^1)$. This form implies that the boundary term gives a non-trivial contribution to the M^4 part of the WCW metric. Kähler charge is not affected unless the partonic 2-surface contains the tip of CD in its interior. In this case the value of Kähler charge is shifted by a topological contribution.
3. The Coulombic interaction term is not invariant under gauge transformations. The good news is that this might allow to find a gauge in which the Coulomb term vanishes. The vanishing condition fixing the gauge transformation ϕ is

$$j_K^\alpha \partial_\alpha \phi = -j^\alpha A_\alpha \quad . \quad (4.1)$$

This differential equation can be reduced to an ordinary differential equation along the flow lines j_K by using $dx^\alpha/dt = j_K^\alpha$. Global solution is obtained only if one can combine the flow parameter t with three other coordinates- say those at the either end of CD to form space-time coordinates. The condition is that the parameter defining the coordinate differential is proportional to the covariant form of Kähler current: $dt = \phi j_K$. This condition in turn implies $d^2t = d(\phi j_K) = d\phi \wedge j_K + \phi dj_K = 0$ implying $j_K \wedge dj_K = 0$ or more concretely,

$$\epsilon^{\alpha\beta\gamma\delta} j_\beta^K \partial_\gamma j_\delta^K = 0 \quad . \quad (4.2)$$

j_K is a four-dimensional counterpart of Beltrami field [17] and could be called generalized Beltrami field.

The integrability conditions follow also from the construction of the extremals of Kähler action [11]. The conjecture was that for the extremals the 4-dimensional Lorentz force vanishes (no dissipation): this requires $j_K \wedge J = 0$. One manner to guarantee this is the topologization of the Kähler current meaning that it is proportional to the instanton current: $j_K = \phi j_I$, where $j_I = *(J \wedge A)$ is the instanton current, which is not conserved for 4-D CP_2 projection. The conservation of j_K implies the condition $j_I^\alpha \partial_\alpha \phi = \partial_\alpha j^\alpha \phi$ and from this ϕ can be integrated if the integrability condition $j_I \wedge dj_I = 0$ holds true implying the same condition for j_K . By introducing at least 3 or CP_2 coordinates as space-time coordinates, one finds that the contravariant form of j_I is purely topological so that the integrability condition fixes the dependence on M^4 coordinates and this selection is coded into the scalar function ϕ . These functions define families of conserved currents $j_K^\alpha \phi$ and $j_I^\alpha \phi$ and could be also interpreted as conserved currents associated with the critical deformations of the space-time surface.

4. There are gauge transformations respecting the vanishing of the Coulomb term. The vanishing condition for the Coulomb term is gauge invariant only under the gauge transformations $A \rightarrow A + \nabla\phi$ for which the scalar function the integral $\int j_K^\alpha \partial_\alpha \phi$ reduces to a total divergence a giving an integral over various 3-surfaces at the ends of CD and at throats vanishes. This is satisfied if the allowed gauge transformations define conserved currents

$$D_\alpha(j^\alpha \phi) = 0 \quad . \quad (4.3)$$

As a consequence Coulomb term reduces to a difference of the conserved charges $Q_\phi^e = \int j^0 \phi \sqrt{g_4} d^3x$ at the ends of the CD vanishing identically. The change of the imons type term is trivial if the total weighted Kähler magnetic flux $Q_\phi^m = \sum \int J \phi dA$ over wormhole throats is conserved. The existence of an infinite number of conserved weighted magnetic fluxes is in accordance with the electric-magnetic duality. How these fluxes relate to the flux Hamiltonians central for WCW geometry is not quite clear.

5. The gauge transformations respecting the reduction to almost topological QFT should have some special physical meaning. The measurement interaction term in the modified Dirac interaction corresponds to a critical deformation of the space-time sheet and is realized as an addition of a gauge part to the Kähler gauge potential of CP_2 . It would be natural to identify this gauge transformation giving rise to a conserved charge so that the conserved charges would provide a representation for the charges associated with the infinitesimal critical deformations not affecting Kähler action. The gauge transformed Kähler potential couples to the modified Dirac equation and its effect could be visible in the value of Kähler function and therefore also in the properties of the preferred extremal. The effect on WCW metric would however vanish since K would transform only by an addition of a real part of a holomorphic function. Kähler function is identified as a Dirac determinant for Chern-Simons Dirac action and the spectrum of this operator should not be invariant under these gauge transformations if this picture is correct. This is achieved if the gauge transformation is carried only in the Dirac action corresponding to the Chern-Simons term: this assumption is motivated by the breaking of time reversal invariance induced by quantum measurements. The modification of Kähler action can be guessed to correspond just to the Chern-Simons contribution from the instanton term.
6. A reasonable looking guess for the explicit realization of the quantum classical correspondence between quantum numbers and space-time geometry is that the deformation of the preferred extremal due to the addition of the measurement interaction term is induced by a $U(1)$ gauge transformation induced by a transformation of $\delta CD \times CP_2$ generating the gauge transformation represented by ϕ . This interpretation makes sense if the fluxes defined by Q_ϕ^m and corresponding Hamiltonians affect only zero modes rather than quantum fluctuating degrees of freedom.

To sum up, one could understand the basic properties of WCW metric in this framework. Effective 2-dimensionality would result from the existence of an infinite number of conserved charges in two different time directions (genuine conservation laws plus gauge fixing). The infinite-dimensional symmetric space for given values of zero modes corresponds to the Cartesian product of the WCWs associated with the partonic 2-surfaces at both ends of CD and the generalized Chern-Simons term decomposes into a sum of terms from the ends giving single particle Kähler functions and to the terms from light-like wormhole throats giving interaction term between positive and negative energy parts of the state. Hence Kähler function could be calculated without any knowledge about the interior of the space-time sheets and TGD would reduce to almost topological QFT as speculated earlier. Needless to say this would have immense boost to the program of constructing WCW Kähler geometry.

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