Target Type Tracking with a new Probabilistic Belief Transformation

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***Abstract*—In this paper we analyze the performances of a new probabilistic belief transformation, denoted DSmP, for the sequential estimation of target ID from classifier outputs in the Target Type Tracking problem (TTT). We complicate here a bit the TTT problem by considering three types of targets (Interceptor, Fighter and Cargo) and show through Monte-Carlo simulations the advantages of DSmP over the classical pignistic transformation which is classically used for decision-making under uncertainty when dealing with belief assignments. Based on our previous works for the justification of rules of combination for TTT problem, only the Proportional Conflict Redistribution rule and the hybrid fusion rules are considered in this work for their ability to deal consistently with high conflicting sources of evidence with three different belief assignment modelings.**

Keywords: Information Fusion, DSmT, Subjective probability, Probabilistic Information Content, Pignistic probability, DSmP.

I. INTRODUCTION

In order to improve the performances of Generalized Data Association (GDA) in tracking algorithms [13] (Chap. 12), we investigate here the possibility of using uncertain classifier attribrute decisions coupled with a sequential fusion mechanism based either on a Proportional Conflict Redistribution (PCR [13]) fusion rule or on a hybrid (DSmH [11]) rule. The novelty of this paper lies (aside the fusion mechanism itself) in the way the decision-making is carried out for tracking the types of target under observation. In this work we analyze and show the difference of performances obtained for the decision-making support by the classical betting/pignistic probability (BetP) introduced in nineties by Smets [15], and the new probabilistic transformation, denoted DSmP, developed by Dezert and Smarandache in [5]. We will show that BetP and DSmP yield to same performances when the optimistic PCR fusion rule is used, but that DSmP outperforms slightly BetP if the more prudent/cautious DSmH fusion rule is preferred by the fusion system designer. This paper extends and improves our previous works on the Target Type Tracking problem (TTT) published in [3] and [13].

In section II and III, we briefly introduce DSmT (Dezert-Smarandache Theory) and its two main rules of combination: the PCR rule no. 5 and the DSm hybrid rule[1]. In section IV, we recall the classical pignistic transformation of a belief mass into a subjective probability measure and we also present our new probabilistic transformation DSmP which provides in general a better Probabilistic Information Content (PIC) than with BetP. In section V, we present the general mechanism for solving the TTT problem and simulations results and comparisons presented and discussed in section VI. The section VII concludes this work.

II. A SHORT INTRODUCTION OF DSMT

In Dempster-Shafer Theory (DST) framework [9], one considers a frame of discernment $\Theta = \{\theta_1, \dots, \theta_n\}$ as a finite set of n exclusive and exhaustive elements (*i.e.* Shafer's model denoted $\mathcal{M}^0(\Theta)$). The *power set* of Θ is the set of all subsets of Θ. The order of a power set of a set of order/cardinality $|\Theta| = n$ is 2^n. The power set of Θ is denoted 2^Θ. For example, if $\Theta = \{\theta_1, \theta_2\}$, then $2^\Theta = \{\emptyset, \theta_1, \theta_2, \theta_1 \cup \theta_2\}$. In Dezert-Smarandache Theory (DSmT) framework [11], [13], one considers $\Theta = \{\theta_1, \dots, \theta_n\}$ be a finite set of n exhaustive elements only (*i.e.* free DSm-model denoted $\mathcal{M}^f(\Theta)$). Eventually some integrity constraints can be introduced in this free model depending on the nature of problem we have to cope with. The *hyper-power set* of Θ (*i.e.* the free Dedekind's lattice) denoted D^Θ [11] is defined as:

1) $\emptyset, \theta_1, \dots, \theta_n \in D^\Theta$.
2) If $A, B \in D^\Theta$, then $A \cap B$ and $A \cup B$ belong to D^Θ.
3) No other elements belong to D^Θ, except those obtained by using rules 1 or 2.

If $|\Theta| = n$, then $|D^\Theta| \leq 2^{2^n}$. Since for any finite set Θ, $|D^\Theta| \geq |2^\Theta|$, we call D^Θ the *hyper-power set* of Θ. For example, if $\Theta = \{\theta_1, \theta_2\}$, then $D^\Theta = \{\emptyset, \theta_1 \cap \theta_2, \theta_1, \theta_2, \theta_1 \cup \theta_2\}$. The *free DSm model* $\mathcal{M}^f(\Theta)$ corresponding to D^Θ allows to work with vague concepts which exhibit a continuous and relative intrinsic nature. Such concepts cannot be precisely refined in an

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[1]DSmH is a natural extension of Dubois and Prade fusion rule [6] for dealing with dynamical frames of discernments.

absolute interpretation because of the unreachable universal truth. It is clear that Shafer's model $\mathcal{M}^0(\Theta)$ which assumes that all elements of Θ are truly exclusive is a more constrained model than the free-DSm model $\mathcal{M}^f(\Theta)$ and the power set 2^Θ can be obtained from hyper-power set D^Θ by introducing in $\mathcal{M}^f(\Theta)$ all exclusivity constraints between elements of Θ. Between the free-DSm model $\mathcal{M}^f(\Theta)$ and Shafer's model $\mathcal{M}^0(\Theta)$, there exists a wide class of fusion problems represented in term of the DSm hybrid models denoted $\mathcal{M}(\Theta)$ where Θ involves both fuzzy continuous hypothesis and discrete hypothesis. The main differences between DST and DSmT frameworks are (i) the model on which one works with, and (ii) the choice of the combination rule and conditioning rules [11], [13]. In the sequel, we use the generic notation G^Θ for denoting either D^Θ (when working in DSmT with free DSm model) or 2^Θ (when working in DST with Shafer's model).

From any finite discrete frame Θ, we define a belief assignment as a mapping $m(.) : G^\Theta \to [0,1]$ associated to a given body of evidence $\mathcal{B}$ which satisfies

$$m(\emptyset) = 0 \qquad \text{and} \qquad \sum_{A \in G^\Theta} m(A) = 1 \tag{1}$$

$m(A)$ is the *generalized* basic belief assignment/mass (bba) of A. The belief and plausibility functions are defined as:

$$\text{Bel}(A) \triangleq \sum_{\substack{B \subseteq A \\ B \in G^\Theta}} m(B) \text{ and } \text{Pl}(A) \triangleq \sum_{\substack{B \cap A \neq \emptyset \\ B \in G^\Theta}} m(B) \tag{2}$$

These definitions are compatible with the Bel and Pl definitions given in DST when $\mathcal{M}^0(\Theta)$ holds. When the free DSm model $\mathcal{M}^f(\Theta)$ holds, the pure conjunctive consensus, called DSm classic rule (DSmC), is performed on $G^\Theta = D^\Theta$. DSmC of two independent[2] sources associated with gbba $m_1(.)$ and $m_2(.)$ is thus given $\forall C \in D^\Theta$ by [11]:

$$m_{DSmC}(C) = \sum_{\substack{A,B \in D^\Theta \\ A \cap B = C}} m_1(A) m_2(B) \tag{3}$$

D^Θ being closed under $\cup$ and $\cap$ operators, DSmC guarantees that $m(.)$ is a proper bba.

III. DSmH and PCR5 combination rules

A. DSmH combination rule

When $\mathcal{M}^f(\Theta)$ does not hold (some integrity constraints exist), one deals with a proper DSm hybrid model $\mathcal{M}(\Theta) \neq \mathcal{M}^f(\Theta)$. DSm hybrid rule (DSmH) for $k \geq 2$ independent sources is thus defined for all $A \in D^\Theta$ as [11]:

$$m_{DSmH}(A) \triangleq \phi(A) \cdot \Big[S_1(A) + S_2(A) + S_3(A)\Big] \tag{4}$$

where $\phi(A)$ is the *characteristic non-emptiness function* of a set A, i.e. $\phi(A) = 1$ if $A \notin \boldsymbol{\emptyset}$ and $\phi(A) = 0$ otherwise, where $\boldsymbol{\emptyset} \triangleq \{\boldsymbol{\emptyset}_\mathcal{M}, \emptyset\}$. $\boldsymbol{\emptyset}_\mathcal{M}$ is the set of all elements of D^Θ which have been forced to be empty through the constraints of the model $\mathcal{M}$ and $\emptyset$ is the classical/universal empty set. $S_1(A) \equiv m_{\mathcal{M}^f(\theta)}(A)$, $S_2(A)$, $S_3(A)$ are defined by

$$S_1(A) \triangleq \sum_{\substack{X_1, X_2, \ldots, X_k \in D^\Theta \\ (X_1 \cap X_2 \cap \ldots \cap X_k) = A}} \prod_{i=1}^{k} m_i(X_i) \tag{5}$$

$$S_2(A) \triangleq \sum_{\substack{X_1, X_2, \ldots, X_k \in \boldsymbol{\emptyset} \\ [\mathcal{U} = A] \vee [(\mathcal{U} \in \boldsymbol{\emptyset}) \wedge (A = I_t)]}} \prod_{i=1}^{k} m_i(X_i) \tag{6}$$

$$S_3(A) \triangleq \sum_{\substack{X_1, X_2, \ldots, X_k \in D^\Theta \\ u(X_1 \cap X_2 \cap \ldots \cap X_k) = A \\ (X_1 \cap X_2 \cap \ldots \cap X_k) \in \boldsymbol{\emptyset}}} \prod_{i=1}^{k} m_i(X_i) \tag{7}$$

where each element is in the disjunctive normal form (i. e. disjunctions of conjunctions); $\mathcal{U} \triangleq u(X_1) \cup \ldots \cup u(X_k)$ where $u(X)$ is the union of all θ_i that compose X, $I_t \triangleq \theta_1 \cup \ldots \cup \theta_n$ is the total ignorance. $S_1(A)$ is nothing but the DSmC rule for k independent sources based on $\mathcal{M}^f(\Theta)$; $S_2(A)$ is the mass of all relatively and absolutely empty sets which is transferred to the total or relative ignorances associated with non existential constraints (if any, like in some dynamic problems); $S_3(A)$ transfers the sum of relatively empty sets directly onto the canonical disjunctive form of non-empty sets. DSmH generalizes DSmC and allows to work on Shafer's model. It is definitely not equivalent to Dempster's rule since these rules are different. DSmH works for any models (free DSm model, Shafer's model or any hybrid models) when manipulating *precise* bba and is actually an extension of Dubois and Prade's rule for working with static or dynamic frames as well [11].

B. PCR5 combination rule

Instead of distributing equally the total conflicting mass onto elements of 2^Θ as within Dempster's rule through the normalization step, or transferring the partial conflicts onto partial uncertainties as within DSmH rule, the idea behind the Proportional Conflict Redistribution rules [12], [13] is to transfer conflicting masses (total or partial) proportionally to non-empty sets involved in the model according to all integrity constraints. The general principle of PCR rules is then to :

1) calculate the conjunctive rule of the belief masses of sources ;
2) calculate the total or partial conflicting masses ;
3) redistribute the conflicting mass (total or partial) proportionally on non-empty sets involved in the model according to all integrity constraints.

The way the conflicting mass is redistributed yields actually to several versions of PCR rules. These PCR fusion rules work for any degree of conflict in $[0,1]$, for any DSm models (Shafer's model, free DSm model or any hybrid DSm model) and both in DST and DSmT frameworks for static or dynamical fusion problems. We just now present only the

[2] While independence is a difficult concept to define in all theories managing epistemic uncertainty, we consider that two sources of evidence are independent (i.e. distinct and noninteracting) if each leaves one totally ignorant about the particular value the other will take.

most sophisticated proportional conflict redistribution rule no. 5 (PCR5) since this rule is what we feel the most efficient PCR fusion rule proposed so far for sequential fusion problem like in this TTT problem. The PCR5 combination rule for only two sources is defined by: $m_{PCR5}(\emptyset) = 0$ and $\forall X \in G^\Theta \setminus \{\emptyset\}$

$$m_{PCR5}(X) = m_{12}(X) + \sum_{\substack{Y \in G^\Theta \setminus \{X\} \\ X \cap Y = \emptyset}} [\frac{m_1(X)^2 m_2(Y)}{m_1(X) + m_2(Y)} + \frac{m_2(X)^2 m_1(Y)}{m_2(X) + m_1(Y)}] \quad (8)$$

where each element X, and Y, is in the disjunctive normal form. $m_{12}(X)$ corresponds to the conjunctive consensus on X between the two sources. All denominators are different from zero. If a denominator is zero, that fraction is discarded. No matter how big or small is the conflicting mass, PCR5 mathematically does a better redistribution of the conflicting mass than Dempster's rule and other rules since PCR5 goes backwards on the tracks of the conjunctive rule and redistributes the partial conflicting masses only to the sets involved in the conflict and proportionally to their masses put in the conflict, considering the conjunctive normal form of the partial conflict. PCR5 is quasi-associative and preserves the neutral impact of the vacuous belief assignment.

C. How to choose between PCR5 and DSmH

It is important to note that we don't claim that PCR5 is better than DSmH, neither the opposite, since they apply differently. All depends actually on the point of view the fusion system designer and the risk he/she is ready to accept. If the fusion system designer is pessimistic (not confident) about the singletons of the frame, then it is safer to use DSmH and transfer the partial conflicting mass to the corresponding partial ignorance. But if he/she is optimistic (confident) about the singletons of the frame, then he/she can apply PCR5 to transfer the conflicting mass back to the singletons involved in that conflict for more specificity. In short summary, the main differences between DST and DSmT are (1) the model on which one works with, and (2) the choice of the combination rule and its possibility to deal with qualitative beliefs as well [13].

IV. PROBABILISTIC BELIEF TRANSFORMATIONS

In order to take a decision from a basic belief assignment $m(.)$, a common adopted approach consists in approximating the bba $m(.)$ by a subjective probability measure $P(.)$ through a given probabilistic transformation and then choose the element of the frame which has the highest probability. Several transformations have been proposed in the literature mainly by Smets in nineties [15], later by Sudano [17]–[20] and last year by Cuzzolin [1], [2]. In a companion paper of this one, we proposed a new probabilistic transformation, denoted $DSmP(.)$, which outperforms all previous transformations in term of maximum of Probabilistic Information Content (PIC) [17], [18], [20]. In this paper, we focuse our presentation and comparison only on Smets' pignistic transformation, denoted $BetP(.)$ and on our new $DSmP(.)$ since $BetP(.)$ is well known and generally adopted by the community of researchers and engineers working with belief functions. A detailed comparison of all main probabilistic transformations of bba can be found in [5].

A. Classical and generalized pignistic probabilities

The basic idea of Smets' pignistic transformation [15], denoted $BetP(.)$ consists in transferring the positive mass of belief of each non specific element (also called partial or total ignorance) onto the singletons involved in that element split by the cardinality of the proposition. In Dempster-Shafer framework [9] (when working with normalized basic belief assignments (bba's) and with $m(\emptyset) = 0$), $BetP(.)$ is defined by $BetP(\emptyset) = 0$ and $\forall X \in 2^\Theta \setminus \{\emptyset\}$ by:

$$BetP(X) = \sum_{\substack{Y \in 2^\Theta \\ X \subseteq Y}} \frac{1}{|Y|} m(Y) = m(X) + \sum_{\substack{Y \in 2^\Theta \\ X \subset Y}} \frac{1}{|Y|} m(Y) \quad (9)$$

where 2^Θ is the power set of the finite and discrete frame Θ with Shafer's model, i.e. all elements of Θ are assumed truly exclusive. This transformation has been generalized in DSmT for any model of the frame (free DSm model, hybrid DSm model and Shafer's model as well) [11]. It is defined by $BetP(\emptyset) = 0$ and $\forall X \in G^\Theta \setminus \{\emptyset\}$ by

$$BetP(X) = \sum_{Y \in G^\Theta} \frac{\mathcal{C}_\mathcal{M}(X \cap Y)}{\mathcal{C}_\mathcal{M}(Y)} m(Y) \quad (10)$$

where G^Θ corresponds to the hyper-power set including all the integrity constraints of the model (if any); $\mathcal{C}_\mathcal{M}(Y)$ denotes the DSm cardinal of Y, i.e. the number of parts of Y in the Venn diagram of the model $\mathcal{M}$ of the frame Θ under consideration [11] (Chap. 7). The formula (10) reduces to (9) when G^Θ reduces to classical power set 2^Θ when one adopts Shafer's model.

B. A new generalized pignistic transformation

In the companion paper [5], we have developed a new generalized quantitative pignistic transformation denoted $DSmP(.)$ to avoid confusion with the previous $BetP(.)$ transformation. $DSmP(.)$ has also been extended in [5] to deal with qualitative belief assignments but it is out of the scope of this paper and this will not be presented here. The new $DSmP(.)$ transformation is straight, different from Smets', Sudano's and Cuzzolin's transformations. The two last ones are more refined than Smets' approach but less interesting and efficient in our opinions than $DSmP(.)$ as proved in [5]. The basic idea of our $DSmP(.)$ transformation consists in a new way of proportionalizations of the mass of each partial ignorance such as $A_1 \cup A_2$ or $A_1 \cup (A_2 \cap A_3)$ or $(A_1 \cap A_2) \cup (A_3 \cap A_4)$, etc and the mass of the total ignorance $A_1 \cup A_2 \cup \ldots \cup A_n$, to the elements involved in the ignorances. The main innovation in this new transformation is to take into account both the values of the belief masses and the cardinality of elements in the redistribution process, contrariwise to previous transformations proposed in the literature so far. We first recall what is the Probabilistic Information Content (PIC) of any given discrete probability measure $P(.)$ and then we briefly present the $DSmP(.)$ formula. In the next section, after presenting the Target Type Tracking problem, we will show how $DSmP(.)$ performs with respect to $BetP(.)$ from on Monte Carlo simulations based on classifier decisions in a three-targets-type scenario.

1) The Probabilistic Information Content (PIC): PIC is a criteria introduced by John Sudano [18] for depicting the strength of a critical decision by a specific probability distribution. PIC is an essential measure in any threshold-driven automated decision system. A PIC value of one indicates the total knowledge (i.e. minimal entropy) or information to make a correct decision (one hypothesis has a probability value of one and the rest of zero). A PIC value of zero indicates that the knowledge or information to make a correct decision does not exist (all the hypothesis have an equal probability value), i.e. one has the maximal entropy. Mathematically, the PIC of a probability measure $P\{.\}$ associated with a probabilistic source over a discrete frame $\Theta = \{\theta_1, \ldots, \theta_n\}$ is defined by:

$$PIC(P) = 1 + \frac{1}{H_{\max}} \cdot \sum_{i=1}^{n} P\{\theta_i\} \log_2(P\{\theta_i\}) \qquad (11)$$

The PIC is nothing but the dual of the normalized Shannon entropy and thus is actually unit less. $PIC(P)$ takes its values in $[0, 1]$. $PIC(P)$ is maximum, i.e. $PIC_{\max} = 1$ with any *deterministic* probability and it is minimum, i.e. $PIC_{\min} = 0$, with the uniform probability over the frame Θ. The simple relationships between Shannon's entropy $H(P)$ and $PIC(P)$ are $PIC(P) = 1 - \frac{H(P)}{H_{\max}}$ and $H(P) = H_{\max} \cdot (1 - PIC(P))$ where $H_{\max}$ is the maximum value achievable by Shannon's entropy, i.e. $H_{\max} = -\sum_{i=1}^{n} \frac{1}{n} \log_2(\frac{1}{n}) = \log_2(n)$.

2) The DSmP formula: Let's consider a discrete frame Θ with a given model (free DSm model, hybrid DSm model or Shafer's model), the DSmP transformation is defined by $DSmP_\epsilon(\emptyset) = 0$ and $\forall X \in G^\Theta \setminus \{\emptyset\}$ by

$$DSmP_\epsilon(X) = \sum_{Y \in G^\Theta} \frac{\displaystyle\sum_{\substack{Z \subseteq X \cap Y \\ \mathcal{C}(Z)=1}} m(Z) + \epsilon \cdot \mathcal{C}(X \cap Y)}{\displaystyle\sum_{\substack{Z \subseteq Y \\ \mathcal{C}(Z)=1}} m(Z) + \epsilon \cdot \mathcal{C}(Y)} m(Y) \qquad (12)$$

where $\epsilon \geq 0$ is a tuning parameter and G^Θ corresponds to the hyper-power set including eventually all the integrity constraints (if any) of the model $\mathcal{M}$; $\mathcal{C}(X \cap Y)$ and $\mathcal{C}(Y)$ denote the DSm cardinals[3] of the sets $X \cap Y$ and Y respectively.

The parameter ϵ allows to reach the maximum value the Probabilistic Information Content (PIC) of the approximation of $m(.)$ into a subjective probability measure. The smaller ϵ is, the better/bigger PIC value is. In some particular degenerate cases however, the $DSmP_{\epsilon=0}(.)$ values cannot be derived, but the $DSmP_{\epsilon>0}(.)$ values can however always be derived by choosing ϵ as a very small positive number, say $\epsilon = 1/1000$ by example in order to be as close as we want to the maximum of the PIC (see examples in [5]). It is interesting to note also that when $\epsilon = 1$ and when the masses of all elements Z having $\mathcal{C}(Z) = 1$ are zero, the DSmP formula (12) reduces to the formula (10), i.e. $DSmP_{\epsilon=1}(.) = BetP(.)$. The passage from a free DSm model to a Shafer's model involves the passage from a structure to another one, and the cardinals change as well in DSmP formula.

3) Advantages of DSmP: It has been shown in [5] that among all main probabilistic belief transformations proposed so far, only $DSmP(.)$ transformations yields to highest PIC value and its main advantage is that it works for all models (free, hybrid and Shafer's) - while other transformations work for Shafer's model only. In order to apply other transformations we had to first refine the frame Θ (on the cases when it is possible!) in order to work with Shafer's model, and then apply their formulas. In the case when it is possible to build a ultimate refined frame, then one can apply the other subjective probabilities on the refined frame. $DSmP_\epsilon(.)$ works on the refined frame as well and gives the same result as it does on the non-refined frame. Thus $DSmP_{\epsilon>0}$ transformation works on any models and so is very general and appealing. $DSmP_\epsilon(.)$ can be seen as a combination of Sudano's $PrBel(.)$ transformation [19] and Smet's $BetP(.)$. The advantages and limitations of Smets [15], Sudano [17]–[20] and Cuzzolin [1], [2] transformations have been discussed in details in [5].

V. The Target Type Tracking Problem

The Target Type Tracking Problem can be simply stated as follows [3], [4]:

- Let $k = 1, 2, ..., k_{max}$ be the time index and consider M possible target types $T_i \in \Theta = \{\theta_1, \ldots, \theta_M\}$ in the environment; for example in air target surveillance systems Θ could be $\Theta = \{Interceptor, Fighter, Cargo\}$ and $T_1 \triangleq Interceptor$, $T_2 \triangleq Fighter$, $T_3 \triangleq Cargo$, in ground target surveillance systems Θ could be $\Theta = \{Tank, Truck, Car, Bus\}$ [8], etc.
- at each instant k, a target of true type $T(k) \in \Theta$ (not necessarily the same target) is observed by an attribute-sensor (we assume a perfect target detection probability here).
- the attribute measurement of the sensor (say noisy Radar Cross Section for example) is then processed through a classifier which provides a decision $T_d(k)$ on the type of the observed target at each instant k.
- The sensor is in general not totally reliable and is characterized by a $M \times M$ confusion matrix

$$\mathbf{C} = [c_{ij} = P(T_d = T_j | TrueTargetType = T_i)]$$

[3]We have omitted the index of the model $\mathcal{M}$ for notation convenience.

We had proposed and analyzed in [3], [4] a method for solving the Target Type Tracking Problem which was based on Shafer's model for the frame of Target Types Θ and the sequential/temporal combination of basic belief assignments (measurements) with prior belief mass available at previous step/scan to update at each current step the belief in each target type. So we gave a solution to estimate $T(k)$ from the sequence of declarations done by the unreliable classifier up to time k, i.e. we built an estimator $\hat{T}(k) = f(T_d(1), T_d(2), \ldots, T_d(k))$ of $T(k)$. The decision about the target type was then taken from Smet's $BetP(.)$ transformations of the updated belief assignment/mass. We had shown the efficiency of PCR5 fusion rule with respect to its main alternatives to track efficiently the true target type of the target under observation at each scan. Our Target Type Tracker consisted in the sequential combination of the current basic belief assignment (drawn from classifier decision, i.e. our *measurements*) with the prior bba estimated up to current time from all past classifier declarations and can be sketched by the following steps:

- a) Initialization step (i.e. $k = 0$). Select the target type frame $\Theta = \{\theta_1, \ldots, \theta_M\}$ and set the prior bba $m^-(.)$ as vacuous belief assignment, i.e $m^-(\theta_1 \cup \ldots \cup \theta_M) = 1$ since one has no information about the first target type that will be observed.
- b) Generation of current *observation*: We investigate in this paper three possible modelings for building $m_{obs}(.)$ from the current decision $T_d(k)$ and the confusion matrix. Let's assume that $T_d(k) = T_j$, $j \in \{1, 2, \ldots, M\}$ and let's denote by S_j the sum of the j-th column of the confusion matrix $\mathbf{C}$, i.e. $S_j = \sum_{i=1,M} c_{ij}$.
  - Modeling #1 (probabilistic bba modeling) : For $i = 1, \ldots, M$, one takes $m_{obs}(\theta_i) = c_{ij}/S_j$.
  - Modeling #2: We commit a belief only to θ_j and to the 2D partial ignorances which include θ_j, i.e. one takes $m_{obs}(\theta_i \cup \theta_j) = c_{ij}/S_j$.
  - Modeling #3: We commit a belief only to θ_j and the full ignorance, i.e. one takes $m_{obs}(\theta_j) = c_{jj}/S_j$ and $m_{obs}(\theta_1 \cup \ldots \cup \theta_M) = 1 - c_{jj}/S_j$.
- c) Combination of current bba $m_{obs}(.)$ with prior bba $m^-(.)$ to get the estimation of the current bba $m(.)$. Symbolically we will write the generic fusion operator as $\oplus$, so that $m(.) = [m_{obs} \oplus m^-](.) = [m^- \oplus m_{obs}](.)$. The combination $\oplus$ is done according either with DSmH fusion rule (i.e. $m(.) = m_{DSmH}(.)$) or with PCR5 rule (i.e. $m(.) = m_{PCR5}(.)$) to show what happens in simulation if one adopts a pessimistic or an optimistic point of view of the fusion process.
- d) Estimation of True Target Type is obtained from $m(.)$ by taking the singleton of Θ, i.e. a Target Type, having the maximum of $BetP(.)$ or the maximum of $DSmP(.)$.
- e) set $m^-(.) = m(.)$; do $k = k + 1$ and go back to b).

In this paper, we follow the same Target Type Tracking approach as in [3], [4] but we complicate a bit the Target Type Tracking scenario and we want to see how the new proposed $DSmP(.)$ transformation performs with respect to $BetP(.)$ with the different bba modelings for observations. For doing this we examine the PCR5 and DSmH fusion rules for the sequential update of belief mass of target types. The two fusion rules correspond actually to the confidence the fusion system designer has in the singletons of the frame. If the fusion system designer is not confident in the singletons, then he/she would prefer to use DSmH, otherwise he/she would prefer to use PCR5.

VI. Simulations results

In order to analyze, evaluate and to compare fairly the performances of both probabilistic belief transformations (BetP(.) and the new DSmP(.) one), for the sequential (temporal) estimation of target ID in the considered here Target Type Tracking problem, we did a set of Monte-Carlo simulations, based on an assumed scenario for a 3D Target Type frame, i.e. $\Theta = \{(I)nterceptor, (F)ighter, (C)argo\}$ for a given classifier, corresponding to the following confusion matrix:

$$\mathbf{C} = \begin{bmatrix} 0.7 & 0.2 & 0.1 \\ 0.15 & 0.7 & 0.15 \\ 0.1 & 0.2 & 0.7 \end{bmatrix}$$

The confusion matrix is asymmetric, reflecting the degree of mutual discrimination between the considered target types. In our scenario we consider that there are three closely-spaced targets: one interceptor, one fighter and one cargo. Due to circumstances, attribute measurements received are predominately from one or another, and all three targets generate actually one single (unresolved kinematics) track. In the real world, the tracking system should in this case maintain three separate tracks: one for interceptor, one for fighter and one for cargo, and then, based on the classification, allocate the measurement to the proper track. But in difficult scenario like this one, there is no way in advance to know the true number of targets because they are unresolved and that's why only a single track is maintained. Of course, the single track can further be split into three separate tracks as soon as three different targets are declared based on the attribute tracking. This is not the purpose of our work however since we only want to examine how works the new probabilistic belief transformation and to compare its performance with the well known BetP transformation for Target Type Tracking.

To simulate such scenario, a true Target Type sequence over 200 scans was generated according to figure 1. The target type sequence (Groundtruth) is characterized with variable switches' time steps, starting with the observation of a Cargo Type (called Type 3) during the first 20 scans. Then the observation of the Target Type switches four times: onto Fighter Type (called Type 2) during time duration of 25 scans; again onto Cargo Type during the next 25 scans; onto Interceptor Type (called Type 1) during the next 15 scans and finally, again to Cargo Type during the last 115 scans. As a simple analogy, tracking the target type changes committed to the same (hidden unresolved) track can be interpreted as tracking color changes of a chameleon moving in a tree on its leaves and on its trunk.

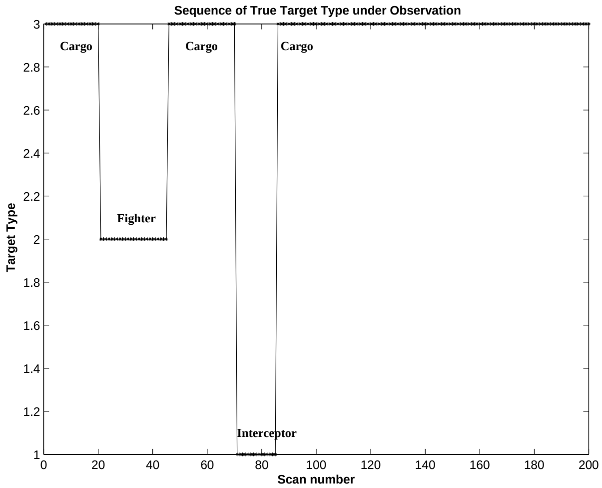

Figure 1. Sequence of True Target Type (Groundtruth)

Our simulation consists in 500 Monte-Carlo runs and we compute, show and analyze in the sequel the averaged performances of the two probabilistic belief transformations, applied over the results of sequential fusion, performed via PCR5 and DSmH combinational rules. At each time step k the decision $T_d(k)$ is randomly generated according to the corresponding row of the confusion matrix of the classifier given the true Target Type (known in simulations). Then the algorithm presented in the previous section is applied.

A. Results based on DSmH fusion

The lack of confidence about the singletons of the frame justifies the application of DSmH combination rule and we test the three modelings for measurement's basic belief assignment as proposed in step b) of TTT algorithm described in the section V.

Figures 2 and 3 show the performances of DSmP and BetP probabilistic belief transformations obtained by our Target Type Tracker based on DSmH fusion rule for the three measurement's bba modelings. We have set the tuning parameter $\epsilon = 0.0001$ when applying $DSmP(.)$ transformation.

From these figures, one clearly sees the advantage of DSmP transformation with respect to BetP transformation since the level of probabilty of the true target type under observation is clearly bigger with DSmP than with BetP. DSmP shows a faster reaction to the target type changes than BetP, shortening that way the time for correct decision-making in comparison to BetP. It is also interesting to note that modelings 2 and 3 provide significantly higher PIC than with modeling 1. This is because modelings 2 and 3 are less strict than modeling 1 and thus offer a better ability to readapt after Target ID switches.

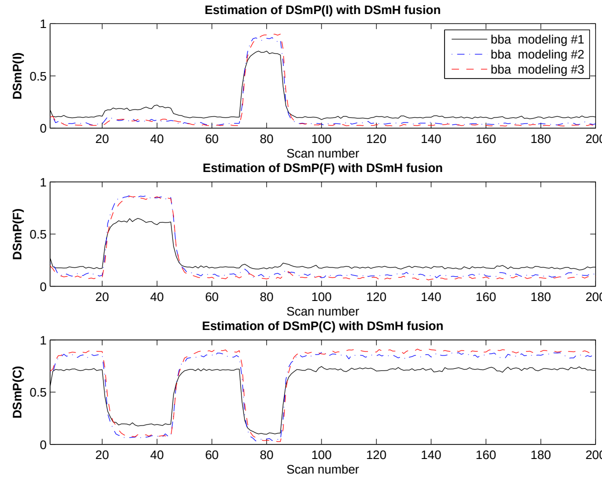

Figure 2. $DSmP(.)$ results after using DSmH rule of combination

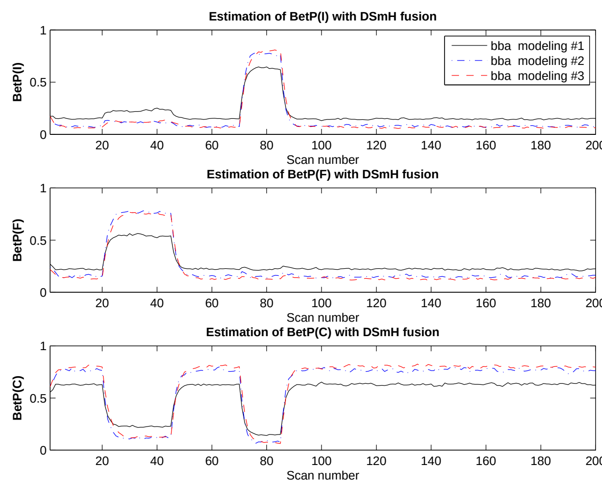

Figure 3. $BetP(.)$ results after using DSmH rule of combination

B. Results based on PCR5 fusion

The possible confidence of the fusion system designer about the singletons of the frame justifies the application of PCR5 combination rule of our TTT algorithm. Figures 4 and 5, show the performances of DSmP and BetP probabilistic belief transformations obtained by our Target TypeTracker based on PCR5 fusion rule, according to Interceptor, Fighter and Cargo types respectively and for the three measurements' bba modelings considered in this work. Here again $\epsilon = 0.0001$ when applying $DSmP(.)$ transformation.

It had been proven in [3], [4] and it can be seen again here, considering the 3D TTT problem, that the TTT based on PCR5 fusion rule tracks properly the quick changes of target type, with a very short latency delay in order to produce the correct target type decision. Since PCR5 reacts faster to the target target changes, accelerating that way to reach the correct decision. Then the mass of ignorance quickly decreases, because of the strict redistribution of conflicting mass (total or partial) proportionally on non-empty sets involved in the considered

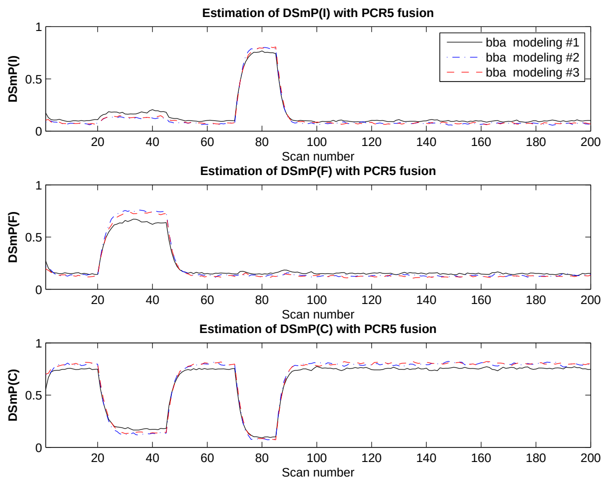

Figure 4. $DSmP(.)$ results after using PCR5 rule of combination

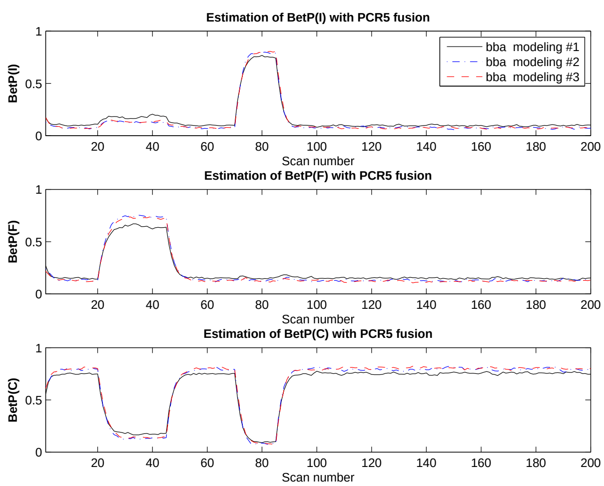

Figure 5. $BetP(.)$ results after using PCR5 rule of combination

model. In parallel the mass to be transferred to singletons decreases very fast. Because of this, the behavior of both probabilistic belief transformations (DSmP and BetP) converge very quickly. When the mass, assigned to the ignorance becomes zero, DSmP and BetP coincide. Here again we see the advantage of using bba modeling 2 and 3 with respect to bba modeling 1, even now the difference between performances is less important than when using DSmH fusion rule.

C. Results based on Dempster-Shafer rule

We provide here on figures 6 and 7 the results obtained when applying Dempster-Shafer rule of combination for this scenario with same inputs and bba modelings 1, 2 or 3. We clearly see that this rule under same conditions cannot track correctly the types of targets under observation whichever probabilistic transformation $DSmP(.)$ or $BetP(.)$ is chosen for decision-making.

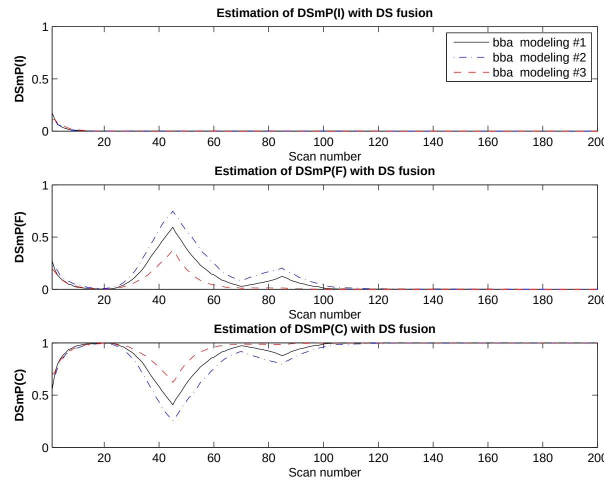

Figure 6. $DSmP(.)$ results after using DS rule of combination

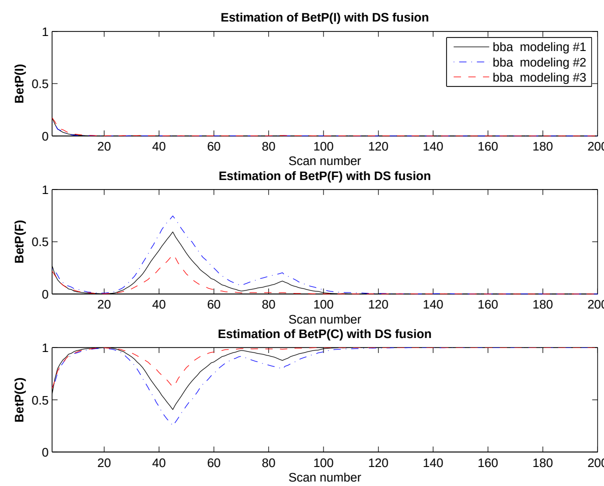

Figure 7. $BetP(.)$ results after using DS rule of combination

VII. Conclusions

This paper concerned the application of a new probabilistic belief transformation, denoted DSmP, for solving the Target Type Tracking problem (TTT). We have considered three types of targets (Interceptor, Fighter and Cargo) in our scenario and have shown how the types of each target can be efficiently estimated from the sequential outputs/decisions of a classifier and its confusion matrix when using different belief mass modelings with DSmT fusion rules couples with DSmP. The advantages of DSmP over the classical pignistic transformation have been shown through Monte-Carlo simulations. Based on our previous works for the justification of rules of combination for the TTT problem, only the Proportional Conflict Redistribution rule no. 5 and the DSm hybrid fusion rules were considered in this work for their ability to deal consistently with high conflicting sources of evidence in an optimistic or a pessimistic/cautious way.

From our analysis one can clearly conclude on the advantage of the new DSmP transformation with respect to BetP whenever the cautious DSmH fusion rule is used. When PCR5 fusion rule is preferred, DSmP and BetP provide very quickly almost the same performancesbecause PCR5 reduces efficiently and quickly the masses committed to ignorances (partial or total) and in such case, DSmP and BetP mathematically coincide. We can claim that DSmP provides a stable and faster reacting behavior than BetP and reduces the delay for correct decision-making in comparison with BetP. Our simulation results show also the advantage of using uncertain bba modelings of type 2 and/or 3 over the probabilistic bba modeling 1 in term of higher level of probability of correct ID estimation.

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