

**The economical expression of the muon-, neutron-, and  
proton-electron mass ratios**

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**Abstract**

It is demonstrated that the proton-, neutron-, and muon-electron mass ratios may be expressed precisely and economically with the aid of two constants that derive from twin approximations of the fine structure constant.

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## I. Introduction

It will be shown that the proton-, neutron-, and muon-electron mass ratios may be expressed precisely and economically with the aid of two constants that derive from twin approximations of the fine structure constant. This article extends earlier work [1,2,3,4,5].

## II. Two fine structure constant approximations

The value of the fine structure constant inverse depends inversely on the square of the electron's charge  $e$

$$\frac{1}{\alpha} = 4\pi\epsilon_0\hbar c/e^2 \approx 137.036 \quad . \quad (1)$$

As this usage of electron charge is arbitrary, one could instead employ the square of the d-quark's charge of  $e/3$  to produce a "d-quark fine structure constant inverse" that is ninefold larger than usual.

With this in mind let

$$k_2 = 100 \quad k_3 = 1000$$

$$m_2 = 1/9 \quad m_3 = 3$$

so that

$$\frac{k_3 - k_2^{-1}}{m_3} + \frac{\left(10 - \frac{1}{300k_2}\right)^2}{m_2} = 9 \times 137.036\,000\,001\,111 \dots \quad (2a)$$

and

$$\frac{k_2 - k_3^{-1}}{m_2} + \frac{\left(10 - \frac{1}{300k_3}\right)^3}{m_3} = 9 \times 137.036\,000\,000\,012 \dots \quad (2b)$$

Each of these equations reproduces nine times the value of the experimental fine structure constant inverse, where the value 137.036 differs from the 2006 CODATA value of 137.035 999 679 (94) by about 2.3 parts per billion [6]. See [5] for more on Eqs. (2a) and (2b).

*Note that Eqs. (2a) and (2b) are identical after the  $2 \leftrightarrow 3$  exchange of subscripts and powers.*

Moreover, with the aid of two definitions that exploit the first term of Eq. (2a)

$$j_2 = \frac{k_3 - k_2^{-1}}{m_3} = 333.33 \quad , \quad (3a)$$

and the second term of Eq. (2b)

$$j_3 = \frac{\left(10 - \frac{1}{300k_3}\right)^3}{m_3} = 333.333\,000\,000\,111 \dots \quad , \quad (3b)$$

the muon-, neutron-, and proton-electron mass ratios can be expressed economically.

### III. The proton-, neutron-, and muon-electron mass ratios

To see how, let

$$Q = 6k_2k_3 \quad , \quad (4a)$$

$$Q' = 1.01Q \quad , \quad (4b)$$

$$Q'' = \frac{Q'}{0.99} \quad , \quad (4c)$$

and

$$L = 4.1^3 \quad , \quad (4d)$$

$$L' = \frac{L}{0.99} \quad , \quad (4e)$$

$$L'' = \frac{L'}{0.99} \quad , \quad (4f)$$

where the values 0.99 and 1.01 are merely  $1 - 1/k_2$  and  $1 + 1/k_2$ , respectively.

This allows the proton-, neutron-, and muon-electron mass ratios to be reproduced as

$$\frac{m_p}{m_e} = \frac{Q'' - L''}{\frac{k_3}{3}} = 1\,836.152\,675\,23 \dots \quad (5a)$$

$$\frac{m_n}{m_e} = k_3 \frac{k_2 L + Q'}{j_3 k_3 - 1} = 1\,838.683\,654\,7 \dots \quad (5b)$$

$$\frac{m_\mu}{m_e} = k_2 \frac{k_3 L - 1}{j_2 k_2 - 1} = 206.768\,270\,7 \dots \quad (5c)$$

The above calculated mass ratios differ from their 2006 CODATA values of 1 836.152 672 47 (80), 1 838.683 660 5 (11), and 206.768 282 3 (52) by 1.5, 3.1, and 56 parts per billion, respectively, and are each close to their limits of experimental error [6].

*Note that, with the exception of their constants  $Q'$  and  $-1$ , Eqs. (5b) and (5c) are identical after the  $2 \leftrightarrow 3$  exchange of subscripts.*

#### IV. The tau-electron mass ratio

Interestingly, the constant 4.1 used in Eq. (4d) already appeared in connection with a study of the electron, muon, and tau masses. These masses were shown to reflect the proportion

$$m_e : m_\mu : m_\tau = 1 : 3 \times 4.1^3 : 3 \times 4.1^5 , \quad (6)$$

which fits the experimental muon-electron mass ratio to about 1 part in 40 000, and the less precisely measured tau-electron mass ratio to about 1 part in 2000 [4].

## V. Analysis and conclusion

It is unlikely that the above equations fit the proton-, neutron-, and muon-electron mass ratios merely by chance, given their precision, simplicity, and given that several of their key constants can be derived independently from other experimental data. To be specific:

- The constant 4.1 of Eq. (4d) also helps reproduce the tau-electron mass ratio in Eq. (6).
- The constants  $j_2$  and  $j_3$  help reproduce the fine structure constant in Eqs. (2a) and (2b).

Moreover, Eq. (5a) is especially compact, while Eqs. (5b) and (5c) are uncannily alike. It is logical, therefore, to suspect that there is an underlying physical basis for the precision and economy of the above mass equations.

## References

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