Nonlinear theory of elementary particles: 6.Electrodynamic sense of the quantum forms of Dirac electron theory

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In the present paper it is shown that a fully correspondence between the quantum and the electromagnetic forms of the Dirac electron theory exists, so that each element of the Dirac theory has the known electrodynamics meaning and vice-versa.

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1.0. Introduction. The spinor and bispinor equations of photon

On the basis of the previous chapters we will show here that all the mathematical particularities of the Dirac electron theory have the known electrodynamics meaning.

As is known (Akhiezer and Berestetskiy, 1965), there are many identical mathematical representations of the equation of electron. Since, according to the nonlinear theory of elementary particles (NTEP), an electron has electromagnetic origin, it can be assumed that all these representations have a base in nonlinear electrodynamics. In particular, they must be based on the linear equations of photon (Kyriakos, 2010a) and nonlinear equation of intermediate massive photon (Kyriakos, 2010b). Let us examine the special features of these equations, which must also be reflected in the equations of electron.

As we noted (Kyriakos, 2010a), the quantum equation of photon can be recorded in the form of Maxwell-Lorentz equations, taking into account the quantization of energy according to Planck. In particular, these equations can be written down in the spinor and bispinor form.

The Maxwell-Lorentz equations for a photon, as quantum of electromagnetic wave, in the spinor form looks as follows:

$$\begin{cases} \hat{\varepsilon}\,\Xi + c\hat{\vec{\sigma}}\,\hat{\vec{p}}\,\mathrm{X} = 0, \\ \hat{\varepsilon}\,\mathrm{X} + c\hat{\vec{\sigma}}\,\hat{\vec{p}}\,\Xi = 0, \end{cases} , \qquad (6.1.1)$$

where $\hat{\varepsilon} = i\hbar\frac{\partial}{\partial t}$ and $\hat{\vec{p}} = -i\hbar\vec{\nabla}$ are the operators of energy and momentum; $\hat{\vec{\sigma}}$ are the spin matrices of Pauli: $\hat{\sigma}_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\hat{\sigma}_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, $\hat{\sigma}_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, $\hat{\sigma}_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$; Ξ and X are the wave functions of an photon in the spinor form, described by means of the following matrices:

$$\Xi = \begin{pmatrix} \Xi_1 \\ \Xi_2 \end{pmatrix}, \quad \mathrm{X} = \begin{pmatrix} \mathrm{X}_1 \\ \mathrm{X}_2 \end{pmatrix}, \qquad (6.1.2)$$

Let us examine the photons, which move in the fixed coordinate system X, Y, Z along the y-axis. For such photons, according to electrodynamics, the relationship $\mathrm{E}_y = \mathrm{H}_y = 0$ takes place, which is invariant, relative to the Lorentz transformations. Therefore they are described only by two vectors, perpendicular to *y*-axis.

If we connect the Frenets trihedron $\{\vec{n},\vec{b},\vec{\tau}\}$ to the electrical, magnetic and Poynting vectors $\{\vec{\mathrm{E}},\vec{\mathrm{H}},\vec{\mathrm{S}}\}$ respectively, then the latter will be collinear with the *y*-axis (here $\vec{n}$ is a normal unit vector to the trajectory of the photon, $\vec{b}$ is the unit vector of binormal and $\vec{\tau}$ is the unit vector of tangent to the trajectory of photon (with the rectilinear motion, this vector coincides with the trajectory of the motion of photon, and with the curvilinear it is directed tangentially toward the curve). Additionally the electrical and magnetic vector can be turned relatively to *y*-axis to any angle φ in the limits $0 \le \varphi \le 2\pi$, without changing the physical characteristics of photon. Taking into account this, one should conclude that in the general case there is an infinite number of such photons.

Obviously any linearly polarized photon can be represented as the sum of two photons with mutually perpendicular vectors $\{\mathrm{E}_x,\mathrm{H}_z\}$ and $\{\mathrm{E}_z,\mathrm{H}_x\}$ with a different absolute value. In that case vectors are harmonic functions, this sum is the elliptically-polarized or circularly polarized photon. Since according to the Bose-Einstein theory the monochromatic electromagnetic wave is Bose-condensate of the photons of one frequency, we can present photons as EM wave, and vice versa.

Figure 6.1 depicts a change in the electric field of photon, which moves along the *y*-axis, the polarization plane of which composes the angle $\varphi < \pi/2$ with the plane ZOY. The figure also shows the projections of the motion of electric vector on the plane *ZOY* and *XOY*, which illustrate the photons with its field components $\{\mathrm{E}_x,\mathrm{H}_z\}$ and $\{\mathrm{E}_z,\mathrm{H}_x\}$:

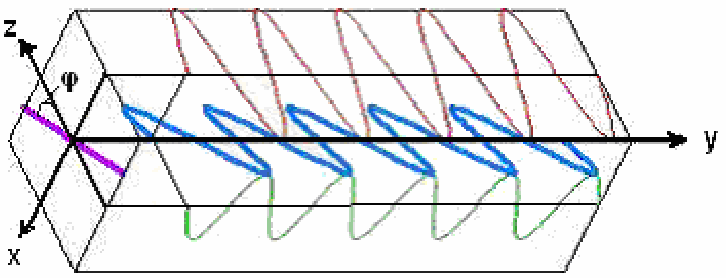

Рис. 6.1.

Further we will have in mind the general case of two separate photons $\{\mathrm{E}_x,\mathrm{H}_z\}$ and $\{\mathrm{E}_z,\mathrm{H}_x\}$, or one circularly polarized photon composed of this pair of EM field vectors (Fig. 6.2):

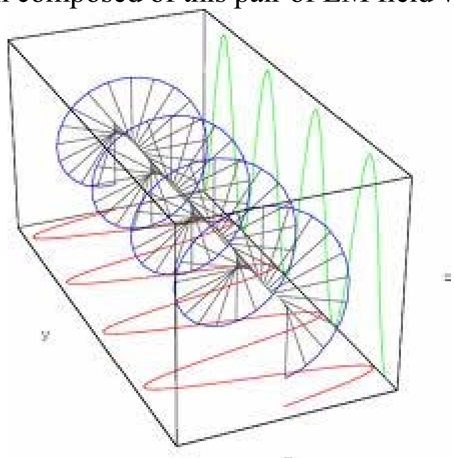

Рис. 6.2.

Introducing spinors in the form:

$$\Xi = \begin{pmatrix} E_x \\ E_z \end{pmatrix}, \quad X = i\begin{pmatrix} H_x \\ H_z \end{pmatrix}, \qquad (6.1.3)$$

and taking into account that $\Xi = \Xi(y),\ X = X(y)$, we will obtain the Maxwell equations of two electromagnetic waves or photons (in the case of the quantization of their energy):

$$\begin{cases} \frac{1}{c}\frac{\partial E_x}{\partial t} - \frac{\partial H_z}{\partial x} = 0, \ (a) \\ \frac{1}{c}\frac{\partial E_z}{\partial t} + \frac{\partial H_x}{\partial z} = 0, \ (b) \\ \frac{1}{c}\frac{\partial H_x}{\partial t} + \frac{\partial E_z}{\partial x} = 0, \ (c) \\ \frac{1}{c}\frac{\partial H_z}{\partial t} - \frac{\partial E_x}{\partial z} = 0, \ (d) \end{cases}, \qquad (6.1.4)$$

It is not difficult to see that two equations (6.1.1) can be recorded in the form of one equation. Actually, introducing a wave function Φ, called bispinor, by means of the following matrix:

$$\Phi = \begin{pmatrix} \Xi \\ X \end{pmatrix} = \begin{pmatrix} \Xi_1 \\ \Xi_2 \\ X_3 \\ X_4 \end{pmatrix} = \begin{pmatrix} \Phi_1 \\ \Phi_2 \\ \Phi_3 \\ \Phi_4 \end{pmatrix} = \begin{pmatrix} E_x \\ E_z \\ iH_x \\ iH_z \end{pmatrix}, \qquad (6.1.5)$$

two spinor equations can be rewritten as one equation:

$$\hat{\varepsilon}\Phi + c\hat{\vec{\alpha}}\,\hat{\vec{p}}\Phi = 0, \qquad (6.1.6)$$

where $\hat{\vec{\alpha}},\ \hat{\beta}$ are the Dirac matrices:

$$\hat{\alpha}_0 = \begin{pmatrix} \hat{\sigma}_0 & 0 \\ 0 & \hat{\sigma}_0 \end{pmatrix}, \ \hat{\vec{\alpha}} = \begin{pmatrix} 0 & \hat{\vec{\sigma}} \\ \hat{\vec{\sigma}} & 0 \end{pmatrix}, \ \hat{\beta} \equiv \hat{\alpha}_4 = \begin{pmatrix} \hat{\sigma}_0 & 0 \\ 0 & -\hat{\sigma}_0 \end{pmatrix}.$$

It is not difficult to prove that using bispinor Φ, we will obtain the same EM equations (6.1.4).

The equations (6.1.4) *a* and *d* correspond to the polarized in the plane *XOY* photon. Equations (6.1.4) *b* and *c* correspond to the polarized in the plane *ZOY* photon. Physically these photons are identical in view of uniformity and isotropism of empty space. But the mathematical record must consider the special features of their propagation depending on the choice of coordinates. The direction of propagation of electromagnetic wave is determined by the Poynting vector and in the Gauss system units takes the form:

$$\vec{S}_P = \frac{c}{4\pi}\left[\vec{E} \times \vec{H}\right], \qquad (6.1.7)$$

For the wave along the *y*-axis we have:

$$\vec{S}_P = -\vec{j}(E_x H_z - E_z H_x), \qquad (6.1.8)$$

where $\vec{j}$ is the unit vector of *y*-axis. As we see, the photon $\{E_x, H_z\}$ moves against the direction of *y*-axis and the equation of field components in this case must be written in the form:

$$\begin{cases} \vec{E} = \vec{E}_o e^{i(\omega t - ky)} \\ \vec{H} = \vec{H}_o e^{i(\omega t - ky)} \end{cases}, \qquad (6.1.9)$$

Photon $\{E_z, H_x\}$ moves along the direction of *y*-axis and the equation of its components must be written in the form:

$$\begin{cases} \vec{\mathrm{E}} = \vec{\mathrm{E}}_o e^{i(\omega t + ky)} \\ \vec{\mathrm{H}} = \vec{\mathrm{H}}_o e^{i(\omega t + ky)} \end{cases}, \qquad (6.1.10)$$

As it follows from the previous chapter (Kyriakos, 2010b), for the formation of electron the photon must, first of all, experience the transformation of rotation in the plane $(\vec{n}, \vec{\tau})$. Obviously, the eventual result of transformation does not depend on the turning of the coordinate system. This means that the transformations, which describe a passage from one Cartesian system to any other, must exist without a change in the result of the rotatory transformation of photon.

By twisting the photon can form a ring either in the plane *XOY* or in the plane *YOZ*. In the process of rotation transformation the ring currents are formed: $\vec{j}_\tau^{\,e} = \dfrac{\omega}{4\pi}\left|\vec{\mathrm{E}}\right|\vec{\tau}$, $\vec{j}_\tau^{\,m} = \dfrac{\omega}{4\pi}\left|\vec{\mathrm{H}}\right|\vec{\tau}$, and "linear" mass-free photon is converted into the massive intermediate boson, which we conditionally call intermediate massive photon. The electric charge of this photon is equal to zero, since it contains alternating current. A question arises of how are fields directed in the intermediate massive photon?

The rotation of photon can be accomplished clockwise and counter-clockwise (looking from the end of the magnetic vector) (Fig. 6.3):

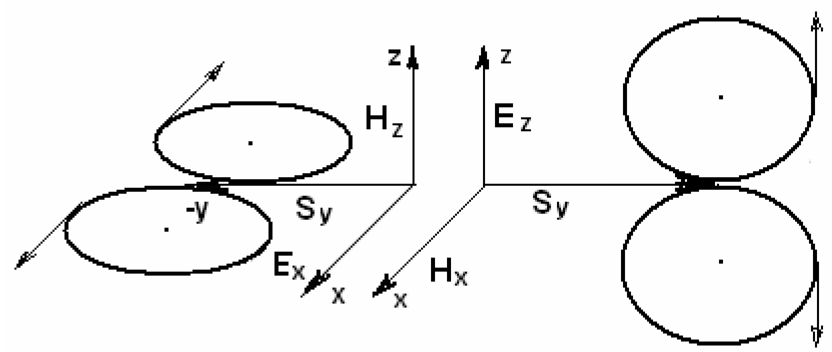

Рис. 6.3

The result will be somehow different if both photons form a circularly polarized photon. We will examine this case in the chapter, dedicated to neutrino.

It follows from above that both the intermediate photons and electrons and positrons must have a set of forms of representations, which do not change the physical sense of equations and particles' characteristics.

2.0. The electrodynamics and quantum forms of Dirac's equation

As we showed in the previous chapter (Kyriakos, 2010c), massive photon can experience spontaneous breaking of symmetry and can be divided into two half-periods – two semi-photons: electron and positron. Accordingly we obtained the equations of these particles. They are known as spinor and bispinor forms.

2.1. The spinor electron equation

For the appearance of the charged particle - electron it is necessary that the rotation of photon occured in the plane, which contains the electric vector and the vector of Poynting. After the breaking of intermediate photon into semi-photons, they can form rings either in the plane *XOY* or in the plane *YOZ*. In this case magnetic currents are not formed. But for the symmetry we will examine the general case of existence of both electric and magnetic currents: $\vec{j}_\tau^{\,e} = \dfrac{\omega}{4\pi}\left|\vec{E}\right|\vec{\tau}$, $\vec{j}_\tau^{\,m} = \dfrac{\omega}{4\pi}\left|\vec{H}\right|\vec{\tau}$, taking into account that for the electron and positron the magnetic currents are equal to zero.

Introducing φ and χ as the wave functions of an electron in the spinor form by means of the following matrices:

$$\varphi = \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} = \begin{pmatrix} E_x \\ E_z \end{pmatrix}, \quad \chi = \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} = i \begin{pmatrix} H_x \\ H_z \end{pmatrix}, \qquad (6.2.1)$$

we obtain Dirac's equation in the spinor form:

$$\begin{cases} \hat{\varepsilon}\varphi + c\hat{\vec{\sigma}}\,\hat{\vec{p}}\chi + mc^2\varphi = 0, \\ \hat{\varepsilon}\chi + c\hat{\vec{\sigma}}\,\hat{\vec{p}}\varphi - mc^2\chi = 0, \end{cases}, \qquad (6.2.2)$$

Taking into account that $\varphi = \varphi(y)$, $\chi = \chi(y)$, we will obtain Maxwell's equations in the complex form with electrical and magnetic currents:

$$\begin{cases} \frac{1}{c}\frac{\partial E_x}{\partial t} - \frac{\partial H_z}{\partial x} + i\frac{\omega}{c}E_x = 0, \\ \frac{1}{c}\frac{\partial E_z}{\partial t} + \frac{\partial H_x}{\partial z} + i\frac{\omega}{c}E_z = 0, \\ \frac{1}{c}\frac{\partial H_x}{\partial t} + \frac{\partial E_z}{\partial x} - i\frac{\omega}{c}H_x = 0, \\ \frac{1}{c}\frac{\partial H_z}{\partial t} - \frac{\partial E_x}{\partial z} - i\frac{\omega}{c}H_z = 0, \end{cases}, \qquad (6.2.3)$$

where $\omega/c = mc/\hbar$. In the quantum form this system is more conveniently written in the form of one bispinor equation of Dirac.

2.2. The bispinor electron equation (Dirac's electron equation)

Dirac's equation more frequently is written in a bispinor form. Introducing a wave function by means of the following matrix:

$$\psi = \begin{pmatrix} \varphi \\ \chi \end{pmatrix} = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix} = \begin{pmatrix} E_x \\ E_z \\ iH_x \\ iH_z \end{pmatrix}, \qquad (6.2.4)$$

two spinor equations of electron can be rewritten as one Dirac's electron equation:

$$\hat{\varepsilon}\psi + c\hat{\vec{\alpha}}\,\hat{\vec{p}} + \hat{\beta}mc^2\psi = 0, \qquad (6.2.5)$$

2.3. Quantum forms of Dirac's electron and positron equations

There are two bispinor Dirac equations (Akhiezer and Berestetskii, 1965; Bethe, 1964; Schiff, 1955; Fermi, 1960):

$$\left[\left(\hat{\alpha}_o\hat{\varepsilon} + c\hat{\vec{\alpha}}\,\hat{\vec{p}}\right) + \hat{\beta}mc^2\right]\psi = 0, \qquad (6.2.6)$$

$$\left[\left(\hat{\alpha}_o\hat{\varepsilon} - c\hat{\vec{\alpha}}\,\hat{\vec{p}}\right) - \hat{\beta}mc^2\right]\psi = 0, \qquad (6.2.7)$$

which correspond to two signs of relativistic expression of an electron energy:

$$\varepsilon = \pm\sqrt{c^2\vec{p}^2 + m^2c^4}, \qquad (6.2.8)$$

Moreover, for each sign of expression (6.2.8), there are two Hermitian-conjugate Dirac equations. Thus, there are two Hermitian-conjugate equations corresponding to a minus sign of the expression (6.2.8):

$$\left[\left(\hat{\alpha}_o\hat{\varepsilon} + c\hat{\vec{\alpha}}\,\hat{\vec{p}}\right) + \hat{\beta}mc^2\right]\psi = 0, \qquad (6.2.9')$$

$$\psi^{+}\left[\left(\hat{\alpha}_{o}\hat{\varepsilon}+c\hat{\vec{\alpha}}\,\hat{\vec{p}}\right)+\hat{\beta}mc^{2}\right]=0\,, \qquad (6.2.9'')$$

and two equations that correspond to plus signs of (6.2.8):

$$\left[\left(\hat{\alpha}_{o}\hat{\varepsilon}-c\hat{\vec{\alpha}}\,\hat{\vec{p}}\right)-\hat{\beta}mc^{2}\right]\psi=0\,, \qquad (6.2.10')$$

$$\psi^{+}\left[\left(\hat{\alpha}_{o}\hat{\varepsilon}-c\hat{\vec{\alpha}}\,\hat{\vec{p}}\right)\;-\hat{\beta}mc^{2}\right]=0\,, \qquad (6.2.10'')$$

We will further use the wave function of a circular-polarized EM wave that is moving as in the previous chapters along the y - axis:

$$\psi=\begin{pmatrix}E_x\\E_z\\iH_x\\iH_z\end{pmatrix},\;\psi^{+}=\left(E_x\quad E_z\quad -iH_x\quad -iH_z\right), \qquad (6.2.11)$$

2.4. EM forms of Dirac's electron equation

Let us consider first two Hermitian-conjugate equations, corresponding to a minus sign of expression (6.2.8):

Using (6.2.11), we obtain from (6.2.9') and (6.2.9''):

$$\begin{cases}\frac{1}{c}\frac{\partial E_x}{\partial t}-\frac{\partial H_z}{\partial y}=-\vec{j}^{\,e}_x\\ \frac{1}{c}\frac{\partial H_z}{\partial t}-\frac{\partial E_x}{\partial y}=\vec{j}^{\,m}_z\\ \frac{1}{c}\frac{\partial E_z}{\partial t}+\frac{\partial H_x}{\partial y}=-\vec{j}^{\,e}_z\\ \frac{1}{c}\frac{\partial H_x}{\partial t}+\frac{\partial E_z}{\partial y}=\vec{j}^{\,m}_x\end{cases},\ (6.2.12'),\quad \begin{cases}\frac{1}{c}\frac{\partial E_x}{\partial t}-\frac{\partial H_z}{\partial y}=\vec{j}^{\,e}_x\\ \frac{1}{c}\frac{\partial H_z}{\partial t}-\frac{\partial E_x}{\partial y}=-\vec{j}^{\,m}_z\\ \frac{1}{c}\frac{\partial E_z}{\partial t}+\frac{\partial H_x}{\partial y}=\vec{j}^{\,e}_z\\ \frac{1}{c}\frac{\partial H_x}{\partial t}+\frac{\partial E_z}{\partial y}=-\vec{j}^{\,m}_x\end{cases},\ (6.2.12'')$$

where

$$\vec{j}^{\,e}=i\frac{\omega}{4\pi}\vec{E}=i\frac{1}{4\pi}\frac{c}{r_C}\vec{E}\,, \qquad (6.2.13')$$

$$\vec{j}^{\,m}=i\frac{\omega}{4\pi}\vec{H}=i\frac{1}{4\pi}\frac{c}{r_C}\vec{H}\,, \qquad (6.2.13'')$$

are complex currents.

Thus, the equations (6.2.9') and (6.2.9'') are Maxwell's equations with complex currents, where *Hermitian-conjugate equations* (6.2.12) *and* (6.2.13*) differ by current directions.*

Let us consider now equations that correspond to a plus signs of (6.2.8). An electromagnetic form of equation (6.2.10') is:

$$\begin{cases}\frac{1}{c}\frac{\partial E_x}{\partial t}+\frac{\partial H_z}{\partial y}=-\vec{j}^{\,e}_x\\ \frac{1}{c}\frac{\partial H_z}{\partial t}+\frac{\partial E_x}{\partial y}=\vec{j}^{\,m}_z\\ \frac{1}{c}\frac{\partial E_z}{\partial t}-\frac{\partial H_x}{\partial y}=-\vec{j}^{\,e}_z\\ \frac{1}{c}\frac{\partial H_x}{\partial t}-\frac{\partial E_z}{\partial y}=\vec{j}^{\,m}_x\end{cases}, \qquad (6.2.14)$$

Obviously, an electromagnetic form of equation (6.2.10'') will have the opposite signs of currents with regard to (6.2.14). Comparing (6.2.14) and (6.2.12), we can consider *equation* (6.2.14) *as the Maxwell's equation of the retarded wave in relation to Maxwell's equation of advanced wave* (6.2.12).

So, if we do not want to use the retarded wave, we can transform the wave function of the retarded wave to the following form:

$$\psi_{ret} = \begin{pmatrix} E_x \\ -E_z \\ iH_x \\ -iH_z \end{pmatrix}, \qquad (6.2.15)$$

Then, contrary to the system (6.2.14), we get the system (6.2.12). The transformation from the function ψ_{ret} to the function ψ_{adv} is called a charge conjugation operation. Thus, we can say that the electron and positron wave functions can be considered as retarded and advanced waves.

Note that the above result relates to the theory of advanced waves of Wheeler and Feynman (Wheeler and Feynman, 1945; Wheeler, 1957). (See also the Dirac's work on time-symmetric classical electrodynamics (Dirac, 1938) and Konopinski's book on the same topic (Konopinski, 1980).

3.0. Electrodynamics' meaning of bilinear forms

This is well known that there are 16 Dirac matrices of 4x4 dimensions. We exploit the same set of matrices used by Dirac, and name it as α -set.

The values $O = \psi^+ \hat{\alpha}\ \psi$, where $\hat{\alpha}$ is any of the Dirac's matrices, are called bilinear forms of Dirac's electron theory.

It can be shown that the tensor dimension of a bilinear form follows from the tensor's nonlinear electrodynamics forms. Let us enumerate the Dirac's matrices as follows (Akhiezer and Berestetskii, 1965; Bethe, 1964; Schiff, 1955):

1) $\hat{\alpha}_4 \equiv \hat{\beta}$, (6.3.1)

2) $\hat{\alpha}_\mu = \left\{\hat{\alpha}_0, \hat{\vec{\alpha}}\right\} \equiv \left\{\hat{\alpha}_0, \hat{\alpha}_1, \hat{\alpha}_2, \hat{\alpha}_3, \hat{\alpha}_4\right\}$, (6.3.2)

3) $\hat{\alpha}_5 = \hat{\alpha}_1 \cdot \hat{\alpha}_2 \cdot \hat{\alpha}_3 \cdot \hat{\alpha}_4$, (6.3.3)

4) $\hat{\alpha}_\mu^A = \hat{\alpha}_5 \cdot \hat{\alpha}_\mu$, (6.3.4)

5) $\hat{\alpha}_{\mu\nu} = -\hat{\alpha}_{\nu\mu} = \begin{cases} i\hat{\alpha}_\nu \hat{\beta}\ \hat{\alpha}_\mu, & \mu \neq \nu \\ 0, & \mu = \nu \end{cases}$, (6.3.5)

Here we have: 1) scalar, 2) 4-vector, 3) pseudoscalar, 4) 4-pseudovector, 5) antisymmetrical tensor of a second rank.

Let us calculate electrodynamics values corresponding to the above matrices using ψ according to (6.2.11).

1) $\psi^+ \hat{\alpha}_4 \psi = \left(E_x^{\ 2} + E_z^{\ 2}\right) - \left(H_x^{\ 2} + H_z^{\ 2}\right) = \vec{E}^2 - \vec{H}^2 = 8\pi I_1$, where I_1 is the *first scalar* (invariant) of Maxwell theory; it is also Lagrangian of an electromagnetic field in vacuum;

2) $\psi^+ \hat{\alpha}_o \psi = \vec{E}^2 + \vec{H}^2 = 8\pi u$, where u is the *energy density* of the electromagnetic field;

$\psi^+ \hat{\alpha}_y \psi = -\frac{8\pi}{c}\vec{S}_{Py} = -8\pi\, c\vec{g}_y$, where $\vec{g}_y$ is a *momentum density* of an electromagnetic wave's field moving along the *Y*-axis. As it is well known, the value $\left\{\frac{1}{c}u, \vec{g}\right\}$ is a 4-vector of the energy-momentum.

3) $\psi^+ \hat{\alpha}_5 \psi = 2(E_x H_x + E_z H_z) = 2(\vec{E}\cdot\vec{H})$ is a *pseudoscalar* of electromagnetic field, and $(\vec{E}\cdot\vec{H})^2 = I_2$ is the second scalar (invariant) of electromagnetic field theory. We will show subsequently that this bilinear form is related to spirality of particles.

4) $\psi^+ \hat{\alpha}_5 \hat{\alpha}_0 \psi = 2(E_x H_x + E_z H_z) = 2(\vec{E}\cdot\vec{H})$

$\psi^+ \hat{\alpha}_5 \hat{\alpha}_1 \psi = -2i(E_x E_z - H_x H_z)$,

$\psi^+ \hat{\alpha}_5 \hat{\alpha}_2 \psi = 0$,

$\psi^+ \hat{\alpha}_5 \hat{\alpha}_3 \psi = -i(E_x^2 - E_z^2 - H_x^2 + H_z^2)$.

In quantum mechanics, three-dimensional components of the spin's tensor are expressed through these matrices (Sokolov and Ivanenko, 1952).

5) Tensor $\psi^+ \hat{\alpha}_{\mu\nu} \psi$ can be presented in a compact form as follows:

$$(\alpha_{\mu\nu}) = \begin{pmatrix} 0 & E_x^2 - E_z^2 + H_x^2 - H_z^2 & 0 & -2(E_x H_z + E_z H_x) \\ -(E_x^2 - E_z^2 - H_x^2 + H_z^2) & 0 & 2(E_x E_z - H_x H_z) & 0 \\ 0 & -2(E_x E_z - H_x H_z) & 0 & -2(E_x H_x - E_z H_z) \\ 2(E_x H_z + E_z H_x) & 0 & 2(E_x H_x - E_z H_z) & 0 \end{pmatrix}$$

In quantum mechanics, tensor $\hat{\alpha}_{\mu\nu}$ (Levich, 1969) describes magnetic and electric moments of an electron. In our theory, the bilinear form of this tensor describes also the Lorentz's force acting to an electron.

4.0. On statistical interpretation of the wave function

Everything, which was said in the chapter (Kyriakos, 2010a) about the normalized and non-normalized representation of the wave function of photon (see paragraph "3.0. Normalized and non-normalized representation of the wave function of the photon"), is valid for the wave function of electron. In other words, the wave function of electron in the non-normalized form is the projection of the strength vector of the nonlinear electromagnetic field at a certain time and position. Accordingly the square of the non-normalized function of electron is the energy density of electron. In the normalized form this square can be considered as the density of the probability to find the electron at a certain time and position. For confirmation of this assertion we will additionally examine the probability continuity equation of electron theory.

As it is well known, the probability continuity equation can be obtained from the Dirac's equation (Akhiezer and Berestetskii, 1965; Bethe, 1964; Schiff, 1955; Fermi, 1960):

$$\frac{\partial P_{pr}(\vec{r},t)}{\partial t} + div\, \vec{S}_{pr}(\vec{r},t) = 0, \qquad (6.4.1)$$

Here, $P_{pr}(\vec{r},t) = \psi^+ \hat{\alpha}_0 \psi$ is the probability density, and $\vec{S}_{pr}(\vec{r},t) = -c\psi^+ \hat{\vec{\alpha}} \psi$ is the probability flux density. Using the above results, for non-normalized wave function we can obtain: $P_{pr}(\vec{r},t) = 8\pi\, u$ and $\vec{S}_{pr} = c^2 \vec{g} = 8\pi\, \vec{S}$. Then, an electromagnetic form of equation (6.3.15) can be presented in the following form:

$$\frac{\partial u}{\partial t}+div\,\vec{S}=0\,, \qquad (6.4.2)$$

which is the form of *law of energy conservation* of electromagnetic field.

5.0. Electrodynamical meaning of matrices' choice

According to Fermi (Fermi, 1960) "it can prove that all the physical consequences of Dirac's equation do not depend on the special choice of Dirac's matrices… In particular it is possible to interchange the roles of the four matrices by unitary transformation. So, their differences are only apparent".

The matrix sequence $(\hat{\alpha}_1,\hat{\alpha}_2,\hat{\alpha}_3)$ agrees with an electromagnetic wave that has $-y$-direction. The question arises: how to describe the waves that have x and z - directions?

Introducing the axes' indexes that indicate the direction of an electromagnetic wave, we can write three groups of matrices each of which corresponds to one, and only one, wave direction:

$$(\hat{\alpha}_{1x},\hat{\alpha}_{2y},\alpha_{3z}),\ (\hat{\alpha}_{2x},\hat{\alpha}_{3y},\hat{\alpha}_{1z},),\ (\hat{\alpha}_{2z},\hat{\alpha}_{1y},\hat{\alpha}_{3x})\,.$$

Let us choose now the wave function forms which give correct Maxwell equations for x and z - directions. Taking into account (6.2.11) as an initial form of the $-y$ - direction, we will get other forms from it by means of indexes' transposition around the circle (see. Fig. 6.4).

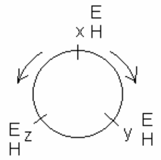

Fig. 6.4.

Since in this case the Poynting vector has the minus sign, we can assume that the transposition must be counterclockwise. Let us examine this supposition checking the Poynting vector's values:

The sets $(\hat{\alpha}_{1x},\hat{\alpha}_{2y},\alpha_{3z})$, $(\hat{\alpha}_{2x},\hat{\alpha}_{3y},\hat{\alpha}_{1z},)$, $(\hat{\alpha}_{2z},\hat{\alpha}_{1y},\hat{\alpha}_{3x})$ correspond to wave functions

$$\psi(y)=\begin{pmatrix}E_x\\E_z\\iH_x\\iH_z\end{pmatrix},\ \psi(x)=\begin{pmatrix}E_z\\E_y\\iH_z\\iH_y\end{pmatrix},\ \psi(z)=\begin{pmatrix}E_y\\E_x\\iH_y\\iH_x\end{pmatrix}$$

and to non-zero Poynting vectors

$\psi^{+}\hat{\alpha}_{2y}\psi=-2\left[\vec{E}\times\vec{H}\right]_y$, $\psi^{+}\hat{\alpha}_{2x}\psi=-2\left[\vec{E}\times\vec{H}\right]_x$, $\psi^{+}\hat{\alpha}_{2z}\psi=-2\left[\vec{E}\times\vec{H}\right]_z$ respectively. As we can see, we obtained a correct result.

We can assume now that the wave functions will describe electromagnetic waves by a clockwise indexes' transposition. These wave functions move in a positive direction along the different co-ordinate axes. Let us prove this:

The sets $(\hat{\alpha}_{1x},\hat{\alpha}_{2y},\alpha_{3z})$, $(\hat{\alpha}_{2x},\hat{\alpha}_{3y},\hat{\alpha}_{1z})$, $(\hat{\alpha}_{2z},\hat{\alpha}_{1y},\hat{\alpha}_{3x})$ correspond to wave functions

$$\psi(y)=\begin{pmatrix}E_z\\E_x\\iH_z\\iH_x\end{pmatrix},\ \psi(x)=\begin{pmatrix}E_y\\E_z\\iH_y\\iH_z\end{pmatrix},\ \psi(z)=\begin{pmatrix}E_x\\E_y\\iH_x\\iH_y\end{pmatrix}$$

and to non-zero Poynting vectors

$\psi^+\hat{\alpha}_{2_y}\psi = 2\left[\vec{E}\times\vec{H}\right]_y$, $\psi^+\hat{\alpha}_{2x}\psi = 2\left[\vec{E}\times\vec{H}\right]_x$, $\psi^+\hat{\alpha}_{2z}\psi = 2\left[\vec{E}\times\vec{H}\right]_z$ respectively. As we can see, once again, we get correct results.

Now, we will prove that the above choice of matrices and wave functions gives correct electromagnetic equation forms. If we use, for instance, equation (6.2.10') and transpose the indexes clockwise, then we correspondingly obtain for the positive direction of electromagnetic wave the following results for x, y, z-directions:

$$\begin{cases} \frac{1}{c}\frac{\partial E_y}{\partial t} + \frac{\partial H_z}{\partial x} = -j_y^e \\ \frac{1}{c}\frac{\partial H_z}{\partial t} + \frac{\partial E_y}{\partial x} = j_z^m \\ \frac{1}{c}\frac{\partial E_z}{\partial t} - \frac{\partial H_y}{\partial x} = -j_z^e \\ \frac{1}{c}\frac{\partial H_y}{\partial t} - \frac{\partial E_z}{\partial x} = j_y^m \end{cases}, \quad \begin{cases} \frac{1}{c}\frac{\partial E_z}{\partial t} + \frac{\partial H_x}{\partial x} = -j_z^e \\ \frac{1}{c}\frac{\partial H_x}{\partial t} + \frac{\partial E_z}{\partial x} = j_x^m \\ \frac{1}{c}\frac{\partial E_x}{\partial t} - \frac{\partial H_z}{\partial x} = -j_x^e \\ \frac{1}{c}\frac{\partial H_z}{\partial t} - \frac{\partial E_x}{\partial x} = j_z^m \end{cases}, \quad \begin{cases} \frac{1}{c}\frac{\partial E_x}{\partial t} + \frac{\partial H_y}{\partial x} = -j_x^e \\ \frac{1}{c}\frac{\partial H_y}{\partial t} + \frac{\partial E_x}{\partial x} = j_y^m \\ \frac{1}{c}\frac{\partial E_y}{\partial t} - \frac{\partial H_x}{\partial x} = -j_y^e \\ \frac{1}{c}\frac{\partial H_x}{\partial t} - \frac{\partial E_y}{\partial x} = j_x^m \end{cases}, \qquad (6.5.1)$$

Therefore, we have obtained three equation groups each of which contains four equations, *as this is necessary for description of all electromagnetic wave's directions*. In the same way, analogous results can be obtained for all other forms of Dirac equation.

Obviously, it is also possible, using canonical transformations, to choose the Dirac matrices in such a way that electromagnetic wave will have any direction. Let us show this.

4.1. EM meaning of canonical transformations of Dirac's matrices and bispinors

The choice of the α-set matrices is not unique (Akhiezer and Berestetskii, 1965; Schiff, 1955; Fock, 1932). As it is well known, there is a free transformation of a kind of $\alpha = S\ \alpha' S^+$ (where S is a unitary matrix called a canonical transformation operator), to which the wave transformation of functions ψ' corresponds: $\psi = S\ \psi'$. This does not change the results of the theory.

If we choose matrices α' as

$$\hat{\alpha}'_1 = \begin{pmatrix} \hat{\sigma}_x & 0 \\ 0 & \hat{\sigma}_x \end{pmatrix},\ \hat{\alpha}'_2 = \begin{pmatrix} \hat{\sigma}_y & 0 \\ 0 & -\hat{\sigma}_y \end{pmatrix},\ \hat{\alpha}'_3 = \begin{pmatrix} \hat{\sigma}_z & 0 \\ 0 & \hat{\sigma}_z \end{pmatrix},\ \hat{\alpha}'_4 = \begin{pmatrix} 0 & -i\hat{\sigma}_y \\ i\hat{\sigma}_y & 0 \end{pmatrix}, \qquad (6.5.2)$$

then the functions ψ will be associated with functions ψ' according to the relationships:

$$\psi_1 = \frac{\psi'_1 - \psi'_4}{\sqrt{2}}, \psi_2 = \frac{\psi'_2 + \psi'_3}{\sqrt{2}}, \psi_3 = \frac{\psi'_1 + \psi'_4}{\sqrt{2}}, \psi_4 = \frac{\psi'_2 - \psi'_3}{\sqrt{2}}, \qquad (6.5.3)$$

A unitary matrix S, which corresponds to this transformation, is equal to:

$$S = \frac{1}{\sqrt{2}}\begin{Bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 \end{Bmatrix}, \qquad (6.5.4)$$

It is not difficult to verify, that by means of this transformation we will also obtain equations of Maxwell theory. In fact, it is easy to get the following using (6.2.11) and (6.5.3):

$$\frac{\psi'_1 - \psi'_4}{\sqrt{2}} = E_x, \frac{\psi'_2 + \psi'_3}{\sqrt{2}} = E_z, \frac{\psi'_1 + \psi'_4}{\sqrt{2}} = iH_x, \frac{\psi'_2 - \psi'_3}{\sqrt{2}} = iH_z, \qquad (6.5.5)$$

whence:

$$\psi' = \frac{\sqrt{2}}{2}\begin{pmatrix} E_x + iH_x \\ E_z + iH_z \\ E_z - iH_z \\ -E_x + iH_x \end{pmatrix}, \qquad (6.5.6)$$

Substituting these functions into the Dirac's equation, we will obtain correct Maxwell equations for electromagnetic waves in double quantity. It is possible to assume that functions ψ' correspond to an electromagnetic wave moving at angle of 45 degrees to both coordinate axes.

Thus, it follows from the above result that *every choice of Dirac matrices defines only the direction of an initial electromagnetic wave*. Obviously, *this is a physical reason why "the physical consequences of the Dirac's equation do not depend on the special choice of Dirac's matrices*" (Fermi,1960).

6.0. An electromagnetic form of the electron theory's Lagrangian

The Lagrangian of the Dirac theory can have the following form (Schiff, 1955):

$$L = \psi^+\left(\hat{\varepsilon} + c\hat{\vec{\alpha}}\,\hat{\vec{p}} + \hat{\beta}\, mc^2\right)\psi, \qquad (6.6.1)$$

If an electromagnetic wave is moving along the $(-y)$-axis, then equation (6.6.1) can be rewritten as follows:

$$L = \frac{1}{c}\psi^+\frac{\partial\psi}{\partial t} - \psi^+\hat{\alpha}_y\frac{\partial\psi}{\partial y} - i\frac{mc}{\hbar}\psi^+\hat{\beta}\psi, \qquad (6.6.2)$$

Transforming each term in (6.6.2) to electrodynamics form, we obtain an electromagnetic form of Lagrangian of Dirac's theory:

$$L = \frac{\partial u}{\partial t} + div\,\vec{S} - i\frac{\omega}{4\pi}\left(\vec{E}^2 - \vec{H}^2\right), \qquad (6.6.3)$$

(Note that in a case of variation procedure we must distinguish the complex conjugate field vectors $\vec{E}^*, \vec{H}^*$ and $\vec{E}, \vec{H}$). Using complex electrical and "magnetic" currents (6.2.13') and (6.2.13''), we have:

$$L = \frac{\partial u}{\partial t} + div\,\vec{S} - \left(\vec{j}^{\,e}\vec{E} - \vec{j}^{\,m}\vec{H}\right), \qquad (6.6.4)$$

It is interesting that since $L = 0$ due to (6.2.11), we can take the following equation:

$$\frac{\partial u}{\partial t} + div\,\vec{S} - \left(\vec{j}^{\,e}\vec{E} - \vec{j}^{\,m}\vec{H}\right) = 0, \qquad (6.6.5)$$

which has a form of the law of conservation of energy-momentum for Maxwell equation with current.

7.0. An expression of Lorentz force in EM representation

According to our theory, the force that is perpendicular to the trajectory of motion of EM fields must appear to provide stability of EM particles. However, the tangential force (by our choice – along the y-axis) must absent in this case, since it would provoke a tangential acceleration of electric charge.

As it is known from (Jackson, 1999), an expression of Lorentz's force in a vector form is described by the expression: $\vec{F} = e\vec{E} + \frac{e}{c}\vec{\upsilon}\times\vec{H}$, where $\vec{\upsilon}$ is the charge velocity. Introducing the charge density $\rho = de/d\tau$, it is possible to rewrite this expression in a form:

$\vec{F} = \int\limits_{(\tau)} \left(\rho\vec{E} + \frac{1}{c}\vec{j}\times\vec{H} \right) d\tau$, where τ is the volume occupied by charge. The expression in brackets is called a Lorentz force's density $\vec{f} = \frac{d\vec{F}}{d\tau} = \rho\vec{E} + \frac{1}{c}\vec{j}\times\vec{H}$, which acts to any part of the charge (electron) itself. Since $\vec{j} = \rho\vec{\upsilon}$ (where, in case of NEPT, $|\vec{\upsilon}| = c$ inside the electron), then we can rewrite this expression as $\vec{f} = \frac{1}{c} j\vec{E} + \frac{1}{c}\vec{j}\times\vec{H}$.

If a photon undergoes the rotation transformation around the *OZ* axis, we obtain:

$$^{OZ} f_x = \frac{1}{c} j(E_x + H_z)$$

If a photon undergoes the rotation transformation around the *OX* axis, we obtain:

$$^{OX} f_z = \frac{1}{c} j(E_z - H_x)$$

(the upper left index shows the spinning axis *OZ* or *OX*).

The Lorentz's force density in classical electrodynamics can be expressed through the symmetrical energy-momentum tensor of electromagnetic field τ_μ^ν (Tonnelat, 1959; Ivanenko and Sokolov, 1949):

$$f_\mu = -\frac{1}{4\pi}\frac{\partial \tau^\nu{}_\mu}{\partial x^\nu} \equiv -\frac{1}{4\pi}\partial_\nu \tau_\mu{}^\nu, \qquad (6.7.1)$$

where τ_μ^ν is determined by the following expressions:

$$\tau_{ij} = -\left(\mathrm{E}_i\mathrm{E}_j + \mathrm{H}_i\mathrm{H}_j\right) + \frac{1}{2}\delta_{ij}\left(\vec{\mathrm{E}}^2 + \vec{\mathrm{H}}^2\right),$$

$$\tau_{i4} = 4\pi\, S = \left[\vec{\mathrm{E}}\times\vec{\mathrm{H}}\right]_i,$$

$$\tau_{44} = 4\pi\, u = \frac{1}{2}\left(\vec{\mathrm{E}}^2 + \vec{\mathrm{H}}^2\right),$$

Here, indices $\mu,\nu = 1,2,3,4$, $i, j = 1,2,3$; $\delta_{ij} = 0$, when $i = j$ and $\delta_{ij} = 1$ for $i \neq j$. Moreover, a 4-vector of the space-time has the form $x_\mu = \{x_i, x_4\} = \{\vec{r}, x_4\} = \{x, y, z, ict\}$.

Using (6.7.1), the force components can be written as:

$$f_x = f_z = 0, \;\; f_y \equiv -\left(\frac{\partial\vec{g}}{\partial t} + grad\, U\right), \qquad (6.7.2)$$

$$f_4 = -\left(\frac{1}{c}\frac{\partial u}{\partial t} + c\, div\,\vec{g}\right), \qquad (6.7.3)$$

Here, first three components describe the Lorentz force density vector, and the fourth component corresponds to law of energy conservation.

As we can see, if we use a symmetrical energy-momentum tensor, then we do not obtain the needed components of the force. Actually, in our case, the components f_x and f_y must be equal to zero, but the f_y component not.

It appears that the right result can be obtained if we use antisimmetrical tensor $\alpha_{\mu\nu}$ (6.3.5). Then, we have:

$$f_\mu = -\frac{1}{4\pi}\frac{\partial \alpha_\mu^\nu}{\partial x^\nu} \equiv -\frac{1}{4\pi}\partial_\nu \alpha_\mu^\nu, \qquad (6.7.4)$$

Or, using the tensor components:

$$\begin{cases} f_x = -\left(\dfrac{\partial\ \alpha_{12}}{\partial\ x_2} + \dfrac{\partial\ \alpha_{14}}{\partial\ x_4}\right) \\ f_y = 0 \\ f_x = -\left(\dfrac{\partial\ \alpha_{32}}{\partial\ x_2} + \dfrac{\partial\ \alpha_{34}}{\partial\ x_4}\right) \\ f_0 = 0 \end{cases}, \qquad (6.7.5)$$

Using (6.2.11) and (6.3.5), we obtain components of the Lorenz's force:

$$2\pi\ f_x = E_x\left(\frac{1}{c}\frac{\partial H_z}{\partial t} - \frac{\partial E_x}{\partial y}\right) + H_z\left(\frac{1}{c}\frac{\partial E_x}{\partial t} - \frac{\partial H_z}{\partial y}\right) +$$
$$+ H_x\left(\frac{\partial E_z}{\partial t} + \frac{\partial H_x}{\partial y}\right) + E_z\left(\frac{1}{c}\frac{\partial H_x}{\partial t} + \frac{\partial E_z}{\partial y}\right)$$
$$f_y = 0, \qquad (6.7.6)$$
$$2\pi\ f_x = E_x\left(\frac{1}{c}\frac{\partial H_x}{\partial t} - \frac{\partial E_z}{\partial y}\right) - H_z\left(\frac{1}{c}\frac{\partial E_z}{\partial t} - \frac{\partial H_x}{\partial y}\right) +$$
$$+ H_x\left(\frac{\partial E_x}{\partial t} + \frac{\partial H_z}{\partial y}\right) - E_z\left(\frac{1}{c}\frac{\partial H_z}{\partial t} + \frac{\partial E_x}{\partial y}\right)$$
$$f_4 = 0,$$

(Note that we obtained here the duble number of brackets since bispinor (6.2.11) contains two plane polarized waves of the same direction, which turn around the different axes).

For a "linear" photon, all expressions in brackets in (6.7.6) are equal to zero according to Maxwell equation. It means that no forces appear in the linear EM wave quantum. When photon rotates around any axis which are perpendicular to y -axis, we will get additional current terms

$$\vec{j} = i\omega\vec{E} \qquad (6.7.7)$$

where the imaginary unit indicates that the tangential current is perpendicular to electric vector of wave.

For the transformed photon (E_x, H_z), the force components are:

$${}^{OZ}f_x = i\frac{1}{4\pi}\frac{\omega}{c}E_x\left(E_x + H_z\right) = \frac{1}{c}j_\tau \cdot \left(E_x + H_z\right), \qquad (6.7.8)$$

For the transformed photon (E_z, H_x), we have:

$${}^{OX}f_z = -i\frac{1}{4\pi}\frac{\omega}{c}E_z\left(E_z - H_x\right) = -\frac{1}{c}j_\tau \cdot \left(E_z - H_x\right), \qquad (6.7.9)$$

$$f_y = 0, \qquad (6.7.10)$$

$$f_4 = 0, \qquad (6.7.11)$$

As we can see, the results (6.7.8) – (6.7.11) correspond to our representations of dynamics of a semi-photon.

8.0. EM and QM representation of interaction Lagrangian and Hamiltonian of nonlinear theory

The Hamiltonian and Lagrangian of NEPT, considered as a nonlinear theory, must contain all possible invariants of nonlinear electromagnetic field theory. Thus, we can assume that the Lagrangian must be some function of field invariants:

$$L = f_L(I_1, I_2)\,, \qquad (6.8.1)$$

where $I_1 = \left(\vec{E}^2 - \vec{H}^2\right), I_2 = \left(\vec{E}\cdot\vec{H}\right)$.

Hamiltonian is fully defined by the Lagrangian. Thus, if function (6.8.1) is known, then it is easy to calculate the Hamiltonian using formulas (1.13), which will be now functions of various powers of electromagnetic field vectors:

$$\mathrm{H} = f_{\mathrm{H}}(\vec{E}, \vec{H})\,, \qquad (6.8.2)$$

Apparently, the functions f_L and f_{H} must have its special form for each problem. This form is unknown before the problem's solution. As it is known, an approximate form of function f_{H} can be found on the basis of Schroedinger's or Dirac's wave equations using the so-called perturbation method. Here, we assume that there is an expansion of function f_{H} in the Taylor–MacLaurent power series with unknown expansion coefficient. Then, the problem is reduced to calculation of these coefficients. The solution is searched for each term of the expansion separately, starting from the first. Usually, this is a problem for a free particle, whose solution is already known. Then, using an equation with two first terms, we find the coefficient of the second term. Using further an equation for the first three terms, we find the coefficient for the third term of expansion, etc. In many cases, it is possible to obtain the solution by this method with any desirable accuracy.

In case of function of two variables $\xi = f(x, y)$, the Taylor – MacLaurent power series in the vicinity of the point (x_0, y_0) is:

$$f(x,y) = f(x_0,y_0) + \sum_{k=1}^{n}\frac{1}{k!}\left((x-x_0)\frac{\partial}{\partial x} + (y-y_0)\frac{\partial}{\partial y}\right)^k f(x_0,y_0) + O\left(\rho^n\right), \qquad (6.8.3)$$

where $\rho = \sqrt{(x-x_0)^2 + (y-y_0)^2}$,

$$\left((x-x_0)\frac{\partial}{\partial x} + (y-y_0)\frac{\partial}{\partial y}\right) f(x_0,y_0) \equiv (x-x_0)\frac{\partial f(x_0,y_0)}{\partial x} + (y-y_0)\frac{\partial f(x_0,y_0)}{\partial y}\,, \qquad (6.8.4)$$

$$\begin{aligned}\left((x-x_0)\frac{\partial}{\partial x} + (y-y_0)\frac{\partial}{\partial y}\right)^2 f(x_0,y_0) &\equiv (x-x_0)^2\frac{\partial f^2(x_0,y_0)}{\partial x^2} + \\ &+ 2(x-x_0)(y-y_0)\frac{\partial^2 f(x_0,y_0)}{\partial x \partial y} + (y-y_0)^2\frac{\partial f^2(x_0,y_0)}{\partial y^2}\end{aligned}\,, \qquad (6.8.5$$

etc. (In case when $x_0 = 0,\ y_0 = 0$, we obtain the MacLaurent series).

Obviously, for the most types of functions $f_L(I_1, I_2)$, the expansion contains approximately the same set of terms that differ only by constant coefficients, any of which can be equal to zero (as an example, see expansions of quantum electrodynamics Lagrangian for a particle in the presence of physical vacuum (Akhiezer and Berestetskii,. 1965; Schwinger, 1951; Weisskopf, 1936). In general, the expansion will look like:

$$L_M = \frac{1}{8\pi}\left(\vec{E}^2 - \vec{B}^2\right) + L'\,, \qquad (6.8.6)$$

where

$$L' = \alpha\left(\vec{E}^2 - \vec{B}^2\right)^2 + \beta\left(\vec{E}\cdot\vec{B}\right)^2 + \gamma\left(\vec{E}^2 - \vec{B}^2\right)\left(\vec{E}\cdot\vec{B}\right) + \\ + \xi\left(\vec{E}^2 - \vec{B}^2\right)^3 + \varsigma\left(\vec{E}^2 - \vec{B}^2\right)\left(\vec{E}\cdot\vec{B}\right)^2 + \ldots \quad , \qquad (6.8.7)$$

is the part which is responsible for the nonlinear interaction (here, $\alpha, \beta, \gamma, \xi, \varsigma, \ldots$ are constants).

The corresponding Hamiltonian will be defined as follows:

$$H = \sum_i E_i \frac{\partial L}{\partial E_i} - L = \frac{1}{8\pi}\left(\vec{E}^2 + \vec{B}^2\right) + \overline{H}', \qquad (6.8.8)$$

where the Hamiltonian part responsible for the nonlinear interaction is:

$$\hat{H}' = \alpha\left(\vec{E}^2 - \vec{B}^2\right)\left(3\vec{E}^2 - \vec{B}^2\right) + \beta\left(\vec{E}\cdot\vec{B}\right)^2 + \\ + \xi\left(\vec{E}^2 - \vec{B}^2\right)\left(5\vec{E}^2 + \vec{B}^2\right) + \varsigma\left(3\vec{E}^2 - \vec{B}^2\right)\left(\vec{E}\cdot\vec{B}\right)^2 + \ldots \quad , (6.8.9)$$

It is not difficult to obtain a quantum representation of the Hamiltonian (6.8.9) of nonlinear theory. Replacing vectors of electromagnetic wave field by quantum wave function, we will obtain the series of the following type:

$$\hat{\mathrm{H}} = \left(\psi^+\hat{\alpha}_0\psi\right) + \sum c_{1i}\left(\psi^+\hat{\alpha}_i\psi\right)\left(\psi^+\hat{\alpha}_j\psi\right) + \\ + \sum c_{2i}\left(\psi^+\hat{\alpha}_i\psi\right)\left(\psi^+\hat{\alpha}_j\psi\right)\left(\psi^+\hat{\alpha}_k\psi\right) + \ldots \quad , \qquad (6.8.10)$$

where $\hat{\alpha}_i, \hat{\alpha}_j, \hat{\alpha}_k$ are Dirac's matrixes, c_i are coefficients of expansion.

As we can see, the terms of Lagrangian and Hamiltonian series contain the same elements, such as $\left(\vec{E}^2 + \vec{B}^2\right)$, $\left(\vec{E}\cdot\vec{B}\right)^2$, $\left(\vec{E}^2 - \vec{B}^2\right)$, and some others. It is possible to assume that each element of series has some particular physical meaning. In this case, it is possible to see analogy with the expansion of fields of electromagnetic moments (2.23), and also with the decomposition of S-matrix on the elements (Akhiezer and Berestetskii,, 1965), each of which corresponds to particularities of interaction of separate particles.

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