PARTICLE MASSES WITHOUT HIGGS MECHANISM AND SUPERSYMMETRY

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Abstract

The non-observation of the Higgs boson and supersymmetry in the most recent high energy physics data, suggests to consider the conjectured Planck mass plasma as a potential alternative. In it supersymmetry is replaced by the assumption that the vacuum of space is densely filled in equal numbers with positive and negative Planck mass particles, and the Higgs field by the gravitational field of interacting large positive with likewise large negative mass quasiparticles of the Planck mass plasma, giving these positive-negative mass configurations a small positive gravitational field mass. From this configuration the Dirac equation can be derived, with the fermions of the standard model composed of large positive and negative masses.

The theory of bound states made up from interacting positive with negative masses, where the positive mass is larger than the absolute value of the negative mass, had been studied by Hönl and Papapetrou [1], and in an extension of Hamilton's mechanics by Bopp [2]. These studies were motivated by Schrödinger who showed that the negative energy and hence mass states of the Dirac equation, lead to a luminal "Zitterbewegung" (quivering motion) of a Dirac particle [3].

It was shown by Bopp the presence of negative masses can be accounted in a Lagrange function which also depends on the acceleration. The equations of the motion are there derived from the variational principle:

$$\delta\int L(q_k,\dot{q}_k,\ddot{q}_k)dt=0 \tag{1}$$

or from

$$\delta\int \Lambda(x_a,u_a,\dot{u}_a)ds=0 \tag{2}$$

where $u_a = dx_a/ds,\quad \dot{u}_a = du_a/ds,\quad ds=(1-\beta^2)^{t/2}dt,\quad \beta = \mathrm{v}/c,\quad x_a=(x_1,x_2,x_3,ict),$ and where $L=\Lambda(1-\beta^2)^{1/2}dt$. With the subsidiary condition

$$F=u_a^2=-c^2 \tag{3}$$

One obtains from (2)

$$\frac{d}{ds}\left(\frac{\partial(\Lambda+\lambda F)}{\partial u_a}-\frac{d}{ds}\frac{\partial\Lambda}{\partial u_a}\right)-\frac{\partial\Lambda}{\partial x_a}=0 \tag{4}$$

where λ is a Lagrange multiplier. In the absence of external forces, Λ can only depend on $\dot{u}_a^2$. The simplest assumption is a linear dependence

$$\Lambda=-k_0-(1/2)k_1\dot{u}_a^2 \tag{5}$$

whereby (4) becomes

$$\frac{d}{ds}\left(2\lambda u_a+k_1\ddot{u}_a\right)=0 \tag{6}$$

or

$$2\dot{\lambda}u_a+2\lambda\dot{u}_a+k_1\dddot{u}_a=0 \tag{7}$$

Differentiating the subsidiary condition one has

$$u_a \dot{u}_a = 0,\ \ u_a \ddot{u}_a + \dot{u}_a^2 = 0,\ \ u_a \dddot{u}_a + 3u_a \ddot{u}_a = 0 \tag{8}$$

by which (7) becomes

$$-2\dot{\lambda} - 3k_1 \dot{u}_a \ddot{u}_a = -2\dot{\lambda} - \frac{3}{2} k_1 \frac{d}{ds} \dot{u}_a^2 = 0 \tag{9}$$

It has the integral (summation over ν)

$$2\lambda = k_0 - \frac{3}{2} k_1 \dot{u}_\nu^2 \tag{10}$$

where k_0 appears as a constant of integration. By inserting (10) into (6) the Langrange multiplier is eliminated and one has

$$\frac{d}{ds}\left[(k_0 - \frac{3}{2} k_1 \dot{u}_\nu^2) u_a + k_1 \ddot{u}_a \right] = 0 \tag{11}$$

Writing (11) as follows:

$$\frac{dP_a}{ds} = 0, \quad P_a = (k_0 - \frac{3}{2} k_1 \dot{u}_\nu^2) u_a + k_1 \ddot{u}_a \tag{12}$$

where P_a are the components of the momentum-energy four-vector. For k_1=0 one has p_a=$k_0 u_a$, which by putting k_0=m is the four-momentum of a spinless particle with rest mass m. The mass-dipole moment is therefore given by

$$P_a = k_1 \dot{u}_a \tag{13}$$

as can be seen from the conservation of angular momentum

$$\frac{d}{ds} J_{\alpha\beta} = 0 \tag{14}$$

where

$$J_{\alpha\beta} = [\mathbf{x}, \mathbf{P}]_{\alpha\beta} + [\mathbf{p}, \mathbf{u}]_{\alpha\beta} \tag{15}$$

and where $[\mathbf{x}, \mathbf{P}]_{\alpha\beta} = x_\alpha P_\beta - x_\beta P_\alpha$. For a particle at rest (P_k=0, k=1, 2, 3) one has

$$J_{kl} = [\mathbf{p}, \mathbf{u}]_{kl} = p_k u_l - p_l u_k, \quad k,l = 1,\ 2,\ 3 \tag{16}$$

which is just the spin angular momentum.

The energy of a pole-dipole particle at rest, and for which u=icγ, is determined by the fourth component

$$\mathbf{P}_4 = imc = i(k_0 - \frac{3}{2}k_1\dot{u}_v^2)c\gamma \tag{17}$$

For the transition to quantum mechanics one needs the equation of motion in canonical form. There we separate the space and time derivative, whereby $L = -\Lambda ds/dt = L(\mathbf{r},\dot{\mathbf{r}},\ddot{\mathbf{r}})$. Setting c=1 we have

$$\left.\begin{aligned} L &= -(k_0 + \frac{1}{2}k_1\dot{u}_a^2)(1-\mathrm{v}^2)^{1/2} \\ \dot{u}_a^2 &= \frac{1}{\left[(1-\mathrm{v}^2)^{1/2}\right]^4}\left[\dot{\mathbf{v}}^2 + \left(\frac{\mathbf{v}\bullet\dot{\mathbf{v}}}{(1-\mathrm{v}^2)^{1/2}}\right)^2\right] \end{aligned}\right\} \tag{18}$$

From

$$\mathbf{P} = \frac{\partial L}{\partial \mathbf{v}} = \frac{d}{dt}\frac{\partial L}{\partial \mathbf{v}}\mathbf{s} = \frac{\partial L}{\partial \dot{\mathbf{v}}} \tag{19}$$

one has to compute the Hamilton function

$$H = \mathbf{v}\cdot\mathbf{P} + \dot{\mathbf{v}}\cdot\mathbf{s} - L \tag{20}$$

From $\mathbf{s} = \partial L/\partial\dot{\mathbf{v}}$ one obtains

$$\begin{aligned} \mathbf{s} &= \frac{1}{\left[\sqrt{(1-\mathrm{v}^2)}\right]^3}\left[\dot{\mathbf{v}}^2 + \left(\frac{(\mathbf{v}\bullet\dot{\mathbf{v}})\,\mathbf{v}}{(1-\mathrm{v}^2)}\right)\right] \\ \dot{\mathbf{v}} &= \frac{\left[\sqrt{(1-\mathrm{v}^2)}\right]^3}{k_1}\left[\mathbf{s} - (\mathbf{v}\cdot\mathbf{s})\mathbf{v}\right] \end{aligned} \tag{21}$$

by which together with (18) $\dot{\mathbf{v}}\bullet\mathbf{s}$ can be expressed in terms of $\mathbf{v}$ and $\mathbf{s}$. In these variables the angular momentum conservation law (14) assumes the form

$$\mathbf{r}\times\mathbf{P} + \mathbf{v}\times\mathbf{s} = \mathbf{const} \tag{22}$$

with the vector $\mathbf{s}$ equal the mass dipole moment. For the Hamilton function (20) one then finds

$$H = \mathbf{v}\cdot\mathbf{P} + k_0(1-\mathrm{v}^2)^{1/2} - (1/2k_1)(1-\mathrm{v}^2)^{3/2}\left[\mathbf{s}^2 - (\mathbf{s}\cdot\mathbf{v})^2\right] \tag{23}$$

Putting

$$\mathbf{P} = \frac{\hbar}{i}\frac{\partial}{\partial \mathbf{r}}$$
$$\mathbf{v} = \mathbf{a} \tag{24}$$
$$(1-\mathrm{v}^2)^{1/2} = \alpha^4$$

where $\alpha = \{\mathbf{a}, \alpha_4\}$ are the Dirac matrices, one finally obtains the Dirac equation

$$\frac{\hbar}{i}\frac{\partial \psi}{\partial t} + H\psi = 0 \tag{25}$$

where

$$H = \alpha_1 P_1 + \alpha_2 P_2 + \alpha_3 P_3 + \alpha_4 m$$
$$\alpha_\beta \alpha_\nu + \alpha_\nu \alpha_\beta = 2\delta_{\beta\nu} \tag{26}$$

with the mass given by

$$m = k_0 - (1/2k_1)(1-\mathrm{v}^2)^{3/2}\left[\mathbf{s}^2 - (\mathbf{s}\cdot\mathbf{v})^2\right] \tag{27}$$

This result can be directly applied to the Planck mass plasma where positive and negative mass quasiparticles form gravitational bound Dirac particle fermions [4].

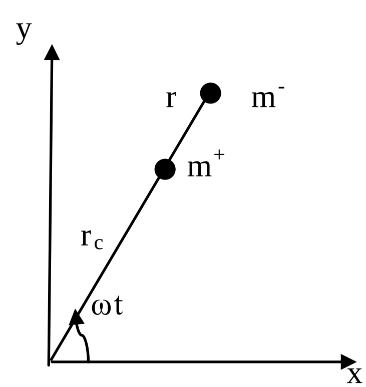

Fig. 1: Pole-dipole particle configuration.

Following Hönl and Papapetrou [1], we analyze the simple classical mechanical two body pole-dipole model shown in Figure 1. It consists of a positive mass m^+ and a negative mass m^-. In a two body problem with both masses positive and with an attractive force in between, the two bodies can execute a circular motion around their center of mass. In case one of the masses is negative, but with both together having a positive mass pole $m_0=m^+-|m^-|$, the circular motion persists, except that the center of mass is no more in between the masses, even though it is still located on the line connecting m^+ and m^-. As a consequence, the pole-dipole particle executes a rotational motion which causes the spin. This motion has the same property as the “Zitterbewegung” derived by Schrödinger.

If $|m^+| > |m^-|$, the distance of m^- from the center of mass is larger than for m^+, and we assume that m^+ is at a distance r_c, with m^- at a distance r_c+r. Furthermore, if $m_0 << m^+ \sim |m^-|$, one has $r << r_c$. Defining $\gamma_+ = (1 - \mathrm{v}_+^2 / c^2)^{-1/2}$, with $\mathrm{v}_+ = r_c \omega$ where ω is the angular velocity around the center of mass, and $\mathrm{v}_- = (1 - \mathrm{v}_-^2 / c^2)^{-1/2}$. With $\mathrm{v}_- = (r_c + r)\omega$, momentum conservation leads to

$$m^+ \gamma_+ r_c = | m^- | \gamma_- (r_c + r) \tag{28}$$

For $r << r_c$ and henceforth putting $\gamma_+ = \gamma$ one can expand:

$$\gamma_- = \gamma(1 + \frac{r_c r \omega^2 \gamma^2}{c^2} + ...) \tag{29}$$

For the mass dipole moment one has

$$p = m^+ r \simeq | m^- | r = \frac{m^+ \gamma - | m^- | \gamma_-}{\gamma_-} r_c \tag{30}$$

With the help of (29) and for $\gamma >> 1$ one finds

$$r_c \simeq p \gamma^2 / m_0 \tag{31}$$

and for the energy

$$E / c^2 = m = m^+ \gamma - | m^- | \gamma_- \simeq p\gamma / r_c \tag{32}$$

and finally, for the angular momentum (putting $\omega r_c \sim c$):

$$J = \left[m^+ \gamma r_c^2 - | m^- | \gamma_- (r_c + r)^2 \right] \omega \simeq -p\gamma c \simeq -mcr_0 \tag{33}$$

The correct spin angular momentum is obtained from the Dirac equation for $r_c \simeq \hbar / 2mc$. From (31) and (32) one has

$$m = m_0 / \gamma \tag{34}$$

In a co-rotating of the pole-dipole particle the gravitational interaction energy is positive, the gravitational interaction energy is positive and for $m^+ - | m^- | << | m^\pm |$, given by

$$E = m_0 c^2 = -\frac{Gm^+ m^-}{r} \simeq \frac{G|m^\pm|^2}{r} \tag{35}$$

and according to (34) the mass in a system at rest

$$mc^2 = \frac{G|m^\pm|^2}{\gamma r} \tag{36}$$

With $p \simeq |m^\pm| r$ given by (30) and $r_c = \hbar / 2mc$, one obtains from (32)

$$2\gamma |m^\pm| r_c = \hbar \tag{37}$$

which can be used to eliminate r from (36), with the result that

$$m = 2G|m^\pm|^3 / \hbar c = 2|m^\pm|^3 / m_p^2 \tag{38}$$

where $m_p = \sqrt{\hbar c / G}$ is the Planck mass.

This is the gravitational field mass of a positive mass interacting with a likewise negative mass. It replaces the mass a zero rest mass fermion acquires in the standard model by the Higgs mechanism.

Equation (38) can also be written as follows:

$$\frac{m}{m_p} = 2\left(\frac{|m^\pm|}{m_p}\right)^3 \tag{39}$$

In the Planck mass plasma [4] the largest gravitationally unbound quasiparticle is a roton with a mass $|m^\pm| \simeq 0.1 m_p$. There, $m/m_p \simeq 10^{-3} m_p$, about equal the mass of the GUT unification scale.

Rewriting (39) as follows

$$\frac{|m^\pm|}{m_p} = 2^{-1/3}\left(\frac{m}{m_p}\right)^{1/3} \tag{40}$$

one obtains the values for the masses $|m^\pm|$of which a given fermionic elementary particle is made up. For baryons one has m~100GeV, hence with $m_p \sim 10^{19} GeV$ that $m/m_p \approx 10^{-19}$, whereby $|m^\pm| \sim 5 \times 10^{12} GeV$.

More generally, one can see from (39) that the assumption of negative masses can bridge the huge gap in between the Planck mass and the typical masses of elementary particles.

References

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