A non-empirical alternative to the Koide formula

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A non-empirical alternative to the Koide formula is shown to approximate the charged lepton mass ratios.

In the 1980s the empirical Koide formula [1] [2] established that

$$\frac{\left(\sqrt{a}+\sqrt{b}+\sqrt{c}\right)^2}{a+b+c} \approx \frac{3}{2} \tag{1}$$

when a, b, and c are the experimental electron, muon, and tau masses (see [3] for an historical overview).

Interestingly, if

$$a = x^0 \tag{2a}$$
$$b = 3x^3 \tag{2b}$$
$$c = 3x^5 \tag{2c}$$

then the function

$$f(x) = \frac{\left(\sqrt{x^0}+\sqrt{3x^3}+\sqrt{3x^5}\right)^2}{x^0+3x^3+3x^5} \tag{3}$$

gives

$$f(4.1) \approx 1.5001087 \tag{4a}$$
$$f(5.1) \approx 1.4002429 \tag{4b}$$
$$f(6.1) \approx 1.3335168 \tag{4c}$$
$$f(7.1) \approx 1.2858222 \tag{4d}$$
$$f(8.1) \approx 1.2500473 \tag{4e}$$
$$f(9.1) \approx 1.2222258 \tag{4f}$$
$$f(10.1) \approx 1.1999733 \tag{4g}$$
$$f(11.1) \approx 1.1817709 \tag{4h}$$
$$f(12.1) \approx 1.1666058 \quad , \tag{4i}$$

a set of regularities suggesting that the simple approximation

$$f(x) \approx 1 + \frac{2}{x - 1/10} \tag{5a}$$

holds at least for

$$4.1 \le x \le 12.1 \quad . \tag{5b}$$

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(Note that within the interval denoted by Eq. (5b), inspection suggests that using 1/10 in Eq. (5a) is always more accurate than using either 1/9 or 1/11.)

Although the use of x^0, $3x^3$, and $3x^5$ in Eq. (3) is empirically inspired by [4] (and to a lesser degree by [5] and [6]), the simplicity with which Eq. (3) can be approximated makes it mathematically interesting even when viewed in isolation. Hence, Eq. (3) and its approximation are non-empirical.

There is, however, the issue of whether Eq. (3) and its approximation are mathematically unique. It is conceivable that by altering both Eq. (3) and Eq. (5a) one can produce an equally remarkable fit with equal generality. This could destroy any claim that Eq. (3) and its approximation have to uniqueness.

Be that as it may, Eq. (3) shows that the proportion

$$m_e : m_\mu : m_\tau \approx 1 : 3 \times 4.1^3 : 3 \times 4.1^5 \tag{6}$$

occurs naturally in conjunction with $f(4.1) \approx 1.5001087$. In [4] the above muon-electron mass ratio is calculated to be accurate to roughly 1 part in 40,000, and the above tau-electron mass ratio to roughly 1 part in 2,000.

So what is one to make of all this?

Equation (3) is a logical rendering of Eq. (1) in that it allows the simple approximation Eq. (5a) to hold. And the effectiveness of the expression $x - 1/10$ in Eq. (5a) justifies the use of values for x of 4.1, 5.1, etc. in Eq. (3), as it is just these values that cause Eq. (5a) to produce simple values such as 3/2, 7/5, etc.

It follows that all of the ratios on the right side of Eq. (6) derive from

$$\frac{\left(\sqrt{a}+\sqrt{b}+\sqrt{c}\right)^2}{a+b+c}$$

without any need for extraneous assumptions, the use of $x = 4.1$ in Eq. (3) suggesting itself for the reasons given above. That these ratios derive from the above simple expression through pure mathematics is a remarkable outcome given that experiment produces roughly these same ratios in connection with the electron, muon, and tau masses.

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