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Quantum Field Theory for hypothetical fifth force

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Abstract
The fifth force is a hypothetical force, which is introduced as a hypothetical additional force, e.g. to describe deviations from gravitational force. Moreover, it is possible, to explain the baryon asymmetry in the universe, an unsolved problem of particle physics, with a hypothetical fifth force. This research shows, how the concept of a fifth force and its quantization can be used as a model for baryon asymmetry.

Introduction
In physics, there are four fundamental forces known. The weakest force of them is the gravity, which acts with an inverse-square-root law and has only an attractive direction [1]. Gravity can be described by Einstein's theory of General Relativity, but in Quantum mechanics, it is difficult, to derive a quantization of Gravity. A stronger fundamental force is electromagnetism, which can act either attractive or repulsive; the exchanged particle in quantum field theory is the Photon. Furthermore, there exists the weak interaction, which acts in nuclear decay or nuclear fusion (exchange bosons are for example the Z-boson) and the strong interaction, where gluons exerting a force between the quarks, the elementary particles of hadrons. The deviation of gravity from standard gravitational laws was examined due to gravity experiments in the Greenland ice cap, see [2] and [3]. Another experiments for measuring gravitational deviations were done in [4]. Such deviations in the Standard gravity law (e.g. Newton's law of gravity) could be explained by a hypothetical fifth force. From quantum field theory, it is known that the dirac equation [5] for Spin-1/2-particles has the form

$$(-i\gamma^\mu D_\mu + m)\psi = 0 \tag{1}$$

with the particle mass m, the covariant derivative D_μ, the Dirac matrices γ^μ and the spinor ψ. The Dirac matrices obeyeing the relationship

$$\{\gamma^\mu, \gamma^\nu\}_+ = 2g^{\mu\nu} \tag{2}$$

with the metric tensor $g^{\mu\nu}$. If the gravity is acting on a Spin-1/2-particle, the Dirac matrices must be coupled on a general-relativistic vierbein e^μ_a, which is the relevant contribution for spacetime curvature, such as $\gamma^\mu \mapsto e^\mu_a \gamma^a$ and the covariant derivative must have an additional spin connection term. For the description of the fifth force, there can be introduced a deviation of the gravity laws due to the change of Dirac matrices, i.e. γ^μ is dependent on spacetime. A concept to make an additional matrix-valued quantum field γ^μ possible is a topological map from the category of scalar-valued functions (i.e. the category of mathematical fields) s to the category to matrix-valued functions m. The dependence of Dirac matrices on spacetime would imply that the scalar product for the quantum-mechanical probability density $\psi^\dagger\gamma^0\psi$ (in Standard model there holds the relationship $\gamma^0 = diag(1, 1, -1, -1)$) can have deviations in the diagonal components. Hence, by the fact that the distribution of matter and antimatter is changed, the baryon asymmetry in the universe can be explained.

There are given other models for explaining baryon asymmetry, for example in [6]. For derivation of a quantum field theory with matrices, the morphism connection fuctor

$$F : Mor(s_1, s_2) \mapsto Mor(m_1, m_2) \tag{3}$$

of the two categories must be known. By introducing the characteristic invariances for these morphisms, the functor can be derived. This invariances leading to gauge invariance of the fifth force field, so that a quantum field theory can be concepted for the fifth fundamental force, that is similar to quantum electrodynamics. The coupling constant of this field theory must be very low, because the effect of fifth fundamental force interaction is very weak.

Physical theory

For simplification, the theory of fifth force is treated with absence of gravity, i.e. $g^{\mu\nu} = \eta^{\mu\nu}$ with the Minkowski metric $\eta^{\mu\nu}$. The quantum field for the fifth force (also called Deviaton field) $\gamma^\mu \in m$ is the object of matrix category $C^{4\times4}$ with the field of complex numbers C. Because all Dirac matrices have the properties

$$tr(\gamma^\mu) = 0, \tag{4}$$

this category must have morphisms of traceless matrix transformations $Mor^\times(\beta_1, \beta_2)$ with $tr(\beta_1) = tr(\beta_2) = 0$. May be $q \in Mor^\times(\beta_1, \beta_2)$ and $\mathcal{M}_1, \mathcal{M}_2$ two topological spaces with an invariant matrix trace. Then, the mapping

$$q : \mathcal{M}_1 \to \mathcal{M}_2 \tag{5}$$

is a homeomorphism. The mapping q is the kernel of the gauge invariant space $\mathcal{G}$, which is induced by the mapping

$$\pi : \mathcal{M}_1 \otimes \mathcal{M}_2 \to \mathcal{G}, \tag{6}$$

where the two correlating topological structures, which are interlinked with q-morphism, are covered with the mapping π to the gauge space. From the relationship (2) for two fifth force fields, the tensor product can be made equivalent with equivalence relation (with generalization of Kronecker's delta $\delta(X - Y) = \delta_{XY}$):

$$E := \{\{\mathcal{M}_1, \mathcal{M}_2\}_+ = 2\delta(\mathcal{M}_1 - \mathcal{M}_2)\}. \tag{7}$$

Summarizing the results (5),(6), and (7), there exists the following short exact sequence:

$$0 \longrightarrow \mathcal{M}_1 \otimes \mathcal{M}_2/E \xrightarrow{\pi} \mathcal{M}_1 \xrightarrow{q} \mathcal{M}_2 \longrightarrow 0 \tag{8}$$

For $\mathcal{M}_1 \otimes \mathcal{M}_2 = E$, the homeomorphism property (5) holds. The short exact sequence (8) is defines the morphisms of the category m. The connection between

$Mor^{\times}(m_1, m_2)$ and $Mor(s_1, s_2)$ (with toplological spaces for scalars $\mathcal{S}_1, \mathcal{S}_2$) is given by the following commutative diagram:

$$\begin{array}{ccc} \mathcal{M}_1 & \xrightarrow{q} & \mathcal{M}_2 \\ \downarrow{\scriptstyle c_1} & & \downarrow{\scriptstyle c_2} \\ \mathcal{S}_1 & \xrightarrow{q'} & \mathcal{S}_2 \end{array} \quad . \tag{9}$$

The mappings c_1, c_2 are the forget mappings, i.e. the property of the trace (4) is forgotten. Then, by the relation (9) (i.e. $q' \circ c_1 = c_2 \circ q$) and with the convention that the relation $c_i \subset B$ hols for the forgetting group B it follows that q' is given by the transformation:

$$q' = c_2 \circ q \circ c_1^{-1}. \tag{10}$$

Since q is a homeomorphism in the category $Mor^{\times}(m_1, m_2)$, from (10) must follow $q' \subset Mor(m_1, m_2)$. By the functor definition (3), the functor F is the inclusion of scalar morphisms to matrix morphisms with generating of trace condition (4). The short sequence (8) can be expanded to a longer exact sequence, so that the connection between matrix category and scalar category is given. Consider the n-th Milnor K-Theory over a mathematical field k with its abelization $k^{\#}$[7]:

$$K_n(k) = T^n k^{\#} / (a \otimes (1 - a)). \tag{11}$$

The T^n denotes the n-th tensor product algebra and a are the elements, which generates the ideal $(a \otimes (1 - a))$. May be $St(A)$ the Steinberg group of a ring A with the group homomorphism $\phi : St(A) \to GL(A)$. Then the following short and exact sequence holds:

$$0 \longrightarrow K_2(A) \longrightarrow St(A) \longrightarrow GL(A) \longrightarrow K_1(A) \longrightarrow 0 \tag{12}$$

The comparison of (8) with (12) shows that there can be found isomorphical mappings from K-theories and topological spaces in the sequence (8). For the topological equivalence class $\mathcal{M}_1 \otimes \mathcal{M}_2 / E$ can be constructed an isomorphism:

$$i : \mathcal{M}_1 \otimes \mathcal{M}_2 / E \to K_2. \tag{13}$$

Analogous, the topological spaces $\mathcal{M}_1, \mathcal{M}_2$ can be mapped isomorphic to the groups $St(A), GL(A)$. These isomorphisms are possible, if the groups $St(A), GL(A)$ are not generated from any mapping π or q from the corresponding fiber spaces. The expansion of the exact sequence (8) is the following diagram with exact rows and columns:

$$\begin{array}{ccccccccccc} 0 & \longrightarrow & \mathcal{M}_1 \otimes \mathcal{M}_2 / E & \longrightarrow & \mathcal{M}_1 & \longrightarrow & \mathcal{M}_2 & \longrightarrow & \mathcal{X} & \longrightarrow & 0 \\ & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \\ 0 & \longrightarrow & K_2(A) & \longrightarrow & St(A) & \longrightarrow & GL(A) & \longrightarrow & K_1(A) & \longrightarrow & 0 \end{array} \tag{14}$$

From (14), the new topological space $\mathcal{X}$ is isomorphic to $K_1(A)$ for $\mathcal{M}_2 = 0$. Moreover, if $\mathcal{X} \cong K_1(A)$, if:

$$\mathcal{M}_1 \otimes \mathcal{M}_2/E \cong \mathcal{M}_1. \tag{15}$$

By settimg $\mathcal{M}_2 = 0$ in (15), the isomorphy (15) is automatically satisfied, if the quantization condition E holds. In Milnor's K-theory, it holds the relation $K_1 = GL(A)^{\#}$ with abelization operator $\#$, so that $\mathcal{X}$ is the set of all commutative matrices with nonvanishing determinant. Hence, the topological space $\mathcal{X}$ can be represented by forget mappings (forgetting of the noncommutativity property of matrices) acting on $\mathcal{M}_1$. Formally, this kind of forget mapping f is defined as:

$$f : \mathcal{M}_1 \to GL^{\#}. \tag{16}$$

For two given matrices α, β, there holds the relation $tr(\alpha\beta) = tr(\beta\alpha)$. May be $\alpha, \beta \in GL^{\#}$ and $tr(\alpha) = tr(\beta) = 0$. Then $tr(\alpha\beta - \beta\alpha) = tr(0) = 0$ and the trace vanishing condition can be forgotten only by this abelization condition. Hence, the mapping f is isomorphic to c_1 in commutative diagram (9). Since $\mathcal{M}_1 \subset K_2(A)$, the interlinking forget mapping f has the form:

$$f : K_2(A) \to K_1(A). \tag{17}$$

Comparing (17) with (9) by us ing $f \cong c_1$, it yields the commutative diagram

(18)

with the isomorphic mappings r_m (for matrices) and r_s (for scalars). Hence, the diagram (18) implies that the matrix-valued field γ^μ is from the set of abelian matrices or $K_1(A)$, because the matrix-valued field is treated similar to a scalar-valued field. In quantum electrodynamics (QED), the gauge transformation of a scalar-valued field (electromagnetic potentials) A_μ is given by

$$A_\mu \mapsto A_\mu + \frac{\partial}{\partial x^\mu}\Lambda \tag{19}$$

for arbitrary gauge function Λ [8]. The gauge invariance (19) is satisfied, if the electromagnetic field strength tensor has the form:

$$F_{\mu\nu} = \frac{\partial}{\partial x^\mu}A_\nu - \frac{\partial}{\partial x^\nu}A_\mu. \tag{20}$$

Then, the Lagrangian (density) of the elctromagnetic field can be written as:

$$\mathcal{L} = \lambda F_{\mu\nu}F^{\mu\nu} \tag{21}$$

with the coupling constant of QED λ. In analogy to (19), (20) and (21), the field for fifth force can be quantized. May be Ξ an arbritary matrix in $K_1(A)$. Then, the gauge transformation of $K_1(A)$ is given by:

$$\gamma^\mu \mapsto \gamma^\mu + \frac{\partial}{\partial x_\mu}\Xi. \tag{22}$$

The matrix-valued field γ^μ is a matrix in $K_2(A)$. By introducing the fifth force field strength tensor

$$E_{\mu\nu} = \frac{\partial}{\partial x_\mu}\gamma^\nu - \frac{\partial}{\partial x_\nu}\gamma^\mu, \tag{23}$$

this form of field strength tensor is independent on the gauge transformation (22). In equation (21), the tensor elements $F^{\mu\nu}$ and $F_{\mu\nu}$ are commutative. The commutativity of the fifth force field strength tensor elements must be commutative, too. With the introduction of the wedge product mapping

$$w(K_2(A), K_2(A)) : K_2(A) \times K_2(A) \to K_1(A); w(x,y) := x \vee y, \tag{24}$$

the relation $E_{\mu\nu} \vee E^{\mu\nu} = E^{\mu\nu} \vee E_{\mu\nu}$ is satisfied. The standard product $E_{\mu\nu}E^{\mu\nu}$ can be expressed as a wedge product via:

$$E_{\mu\nu}E^{\mu\nu} = E_{\mu\nu} \vee E^{\mu\nu} + [E_{\mu\nu}, E^{\mu\nu}]_{-,K_1}. \tag{25}$$

The commutator $[x,y]_{-,K_1}$ vanishes, if $xy - yx = 0$. Otherwise, there can be set $[x,y]_{-,K_1} = xy - yx$. May be $\mathcal{L}_D$ the Lagrangian of the Dirac field, then the Lagrange function

$$L = \int d^4x(\mathcal{L}_D + gE_{\mu\nu} \vee E^{\mu\nu}) \tag{26}$$

with the fifth force coupling constant g, which has a very low value, must be minimized. For canonical quantization, the relation

$$\{\gamma^\mu(X), \gamma^\nu(Y)\}_+ = 2g^{\mu\nu}\delta(X - Y) \tag{27}$$

for two arbitrary points in spacetime X, Y must hold. Becuase equation (26) has the same form as the QED Lagrangian, the Lagrange function is Lorentz invariant. By (26), the source of the fifth force fields is coming from the contribution of Dirac's equation.

Conclusions

The Lagrange function (26) with its quantization condition (27) shows that it is possible to develop a quantum field theory for an additional fifth force. With the concept of a deviation in Dirac matrices, there can be constructed a model for measured deviations in gravity. Furthermore, the baryon asymmetry can be explained as a deviation in the matrix γ^0, which occured at the high energy values of the Big Bang. These high energy values imply that the sources of fifth force field (the contributions in Dirac's equation) become larger and hence, the deviations in γ^0 are significant. Moreover, from the concept of non-constant Dirac matrices, there could be derived alternative models for gravitation for further theoretical researches.

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