The Uncertainty Principle and Gribov Copies

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Abstract

In this paper, it is shown that for the hard gluon emitted in 3-jet event, the existence of the Hamiltonian H on a particular gauge orbit is only for the infinitesimal time–period ϕ. During this infinitesimal time–period ϕ, the uncertainty principle implies that there must be certain minimum amount of uncertainty, or quantum fluctuation in the eigenvalue of the Hamiltonian H of the hard gluon emitted in 3-jet event. One can think of these quantum fluctuations as Gribov copies that appear at some time, move along with the real hard gluon and then get annihilated. Like virtual particles, Gribov copies cannot be observed directly with particle detectors, but their indirect effects like anomalous scaling can be observed and measured.

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1. INTRODUCTION:

At present, what is considered as the most important in non-perturbative aspects of QCD is finding co-ordinates on the gauge orbit which entails the elimination (not the inclusion) of the gribov copies. On the other hand, we consider the ordinary Faddeev-Popov methods as fine for the perturbation theory. Against this backdrop, it is shown in this paper that for the hard gluon emitted in 3-jet event, the existence of the Hamiltonian H on a particular gauge orbit is only for the infinitesimal time–period ϕ. During this infinitesimal time–period ϕ, the uncertainty principle implies that there must be certain minimum amount of uncertainty, or quantum fluctuation in the eigenvalue of the Hamiltonian H of the Hard gluon emitted in 3-jet event. One can think of these quantum fluctuations as Gribov copies that appear at some time, move along with the real hard gluon and then get annihilated. Like virtual particles, Gribov copies cannot be observed directly with particle detectors, but their indirect effects like anomalous scaling can be observed and measured.

2. GRIBOV COPIES, 3-JET EVENTS & DELTA PLUS PLUS

In 3-jet event, gluon jets are identified by the particles in the hemisphere opposite to the hemisphere that is containing tagged quark & antiquark jets and is defined by a plane perpendicular to the principal event axis in the 3-jet event [1]. This definition of gluon jet is equivalent to the production of gluon jet from color-singlet point source (i.e., quark-antiquark pair) in perturbative sector. Now, the hard gluon, emitted in the 3-jet event, evidently carries color charge [2, 3] and this color charge of the hard gluon is not in concurrence with the aforesaid production of gluon jet from color-singlet point source in perturbative sector. This non-concurrence has been resolved in published literature [1] by considering the production of gluon jet as creation of gluon-gluon pair for the theoretical description of its internal properties. This creation of gluon-gluon pair is hitherto unobserved [1] and as such, hypothetical in nature. Now, we show why there is no need to look upon this gluon jet emitted from color-singlet point source in perturbative sector as "hypothetical creation of gluon-gluon pair" for the theoretical description of its internal properties [1].

Arthur Jaffe and Edward Witten in their paper [4A] has mentioned that classically, by substituting the abelian group $U(1)$ with a more general compact gauge group $G = SU(3)$, the curvature is changed from $F = dA$ to $F = dA+A\wedge A$, and Maxwell's equations,$0 = dF = d{*F}$, are transformed to the Yang-Mills equations, $0 = d_AF = d_A{*F}$, where* is the Hodge duality operator, A is pure Yang-Mills gauge potential, F is pure Yang-Mills gauge field and d_A denotes the gauge-covariant extension of the exterior derivative. Further, these Yang-Mills equations can be validated by deriving them from the following pure Yang-Mills action L' that is more conveniently expressed as an integral of a pure Yang-Mills Lagrangian L in an appropriate time interval (t_0, t_1) i.e.,

$$L' = (-1/4) \int d^4x \, (F_{\mu,\nu}, F^{\mu,\nu}) \;\; = \int_{t_0}^{t_1} dt \, L \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} \quad (1)$$

where $F_{\mu,\nu} = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x) - ig[A_\mu(x), A_\nu(x)]$ and μ,ν are denoting space-time indices that take value in the range (0, 1, 2, 3) and

$$L = (-1/2) \int_V d^3x \, (\nabla_k(\boldsymbol{A}_\mathbf{k}) A_0 - \acute{\boldsymbol{A}}_\mathbf{k}, \nabla^k(\boldsymbol{A}^\mathbf{k}) A^0 - \acute{\boldsymbol{A}}^\mathbf{k}) - (1/4) \int_V d^3x (F_{kl} (\boldsymbol{A}_\mathbf{k}), F^{kl}(\boldsymbol{A}^\mathbf{k})) \text{\_\_\_\_\_\_\_\_\_} (2)$$

where $F_{kl} = \partial_k A_l(x) - \partial_l A_k(x) - ig[A_k(x), A_l(x)]$; k,l are denoting space indices ranging from 1 to 3; g is arbitrary non-vanishing real parameter; A_0 denotes the time-component of the pure Yang-Mills gauge potential A_μ & the quantity V is closed domain in R^3.

The pure Yang-Mills action L' in equation (1) above remains invariant if A is replaced by gauge transformed field A^U,

$$A \to A^U \equiv U^{-1}AU + U^{-1}\partial U$$

The fields A and A^U lead to same value for the pure Yang-Mills action L' in equation (1) and as such, are called gauge-equivalent fields. As illustrated in Fig. 1 below, the gauge orbit for some gauge field A is nothing but the set of all gauge-equivalent configurations A^U such that the pure Yang-Mills action L' in equation (1) is gauge invariant and all gauge-equivalent configurations A^U on the gauge orbit have got same action L'.

Thus, "*by trading a redundant set of coordinates A^U for locality, the original description of pure Yang-Mills gauge theory is made possible by using only local interactions. However, this local description involving gauge-transformed field A^U comes at a price that there exists a continuous gauge orbit of gauge-equivalent fields to represent each physical situation in continuum Yang-Mills gauge theories. By using gauge-fixing, only gauge inequivalent configurations should be considered in the Feynman path-integral for properly quantizing the pure Yang-Mills theory. In the field configuration space, one essentially describes the gauge* *fixing by a choice of coordinates and thus, one chooses a suitable coordinate system for calculations by gauge fixing. Beyond perturbation theory, the specification of the gauge conditions becomes complicated by gauge algebras of Yang-Mills theory i.e., simple Lie algebras [4] and this complication creates problem in gauge fixing [5].*

Consequent upon the same, the whole field configuration space cannot be covered by a single coordinate system. Hence, the criteria for selecting the representatives of the gauge orbits cannot be characterized by giving a local (and thus single coordinate system) prescription. This problem is called the Gribov-Singer ambiguity with reference to arbitrary covariant gauges but it is present in some form in all gauges for a continuum formulation" [6]. In other words, the Gribov-Singer ambiguity refers to the multiple intersections, of the gauge-fixing surface with the gauge orbit, called as the Gribov copies. This unambiguous quantization of nonabelian gauge theories remains unsolved problem in the non-perturbative regime. The covariant differential gauges are local but plagued by Gribov copies, whereas known Ghost-free axial gauges explicitly violate Lorentz invariance and are non-local. Moreover, beyond perturbation theory the axial gauge always involve a non-local element or reintroduces Gribov copies at the boundary so as not to destroy the Polyakov loop [7].

"Since, two Gribov copies are not infinitesimally close to each other, as otherwise a local distinction would be possible and are therefore separated by non-infinitesimal gauge transformations, so, single representative of any gauge orbit can be specified only by non-local gauge condition in nonperturbative regime and consequently" [6], Gribov-copy-free gauge fixing cannot be expressed as a local function of the pure Yang-Mills potential and a finite number of its derivatives for all spatial points [7]. In other words, the pure Yang-Mills action, with Gribov-copy-free gauge fixing becomes non-local and give rise to non-local quantum field

theory [7]. It is, further, stated in [7] that since, the proof of renormalizability of QCD, the proof of asymptotic freedom, local BRS symmetry and the Schwinger-Dyson equations is based on the pure Yang-Mills action, so, we cannot prove these features of QCD in nonperturbative context with an unreliable basis that is left behind by the nonlocality of the pure Yang-Mills action.

The main conclusion of [7] is that the assumption of having non-perturbative QCD without Gribov copies destroys locality and BRS invariance of the theory; an equally valid point of view is to preserve locality and BRS symmetry as central to the definition of QCD in the non-perturbative regime [8 – 11] and accordingly, this view point implies that Gribov copies are necessarily physically present & gauge orbits are multiply represented in the non-perturbative regime. To understand this implication of the aforesaid view point, we refer to the wave-particle duality concept – the wave-nature and particle-nature of gluon are complementary to each other and both of these natures are never exhibited simultaneously. While in perturbative regime, the particle nature of the gluon is exhibited exclusively in the statement that its Gribov copies, only as free point particles due to the asymptotic freedom, fit in their infinitesimal close separations in perturbative regime, on the other hand, the wave nature of the gluon is exhibited exclusively in the statement that its Gribov copies, as a wave, become physically distinguishable from each other with their increased separations in nonperturbative regime. Let us see how the problem [12 – 18], that the Gribov copies prevent a non-perturbative definition of QCD color charge, can be taken care of by interpreting physically distinguishable wave-nature of Gribov copies as special realization of the hard gluon emitted in 3-jet event.

There are two facets of color in particle physics. First one pertains to the three-valued charge degree of freedom as introduced by Oscar W. Greenberg [19] in 1964 and second one refers to gauge symmetry as introduced by Yoichiro Nambu [20] and by Moo Young Han and by Yoichiro Nambu [21] in 1965. The essential ingredients of QCD are contained in the union of two facets [22]. Although, the notion of color charge had to be introduced as intrinsic property (first facet) of the Lagrangian spin ½ quarks ψ for preserving Dirac-Fermi statistics in the naïve quark model [19] and these Lagrangian quarks ψ of the naive quark model are identified with QCD quarks with an SU(3) color index (second facet), yet, this identification is required to be refined as Lagrangian quarks ψ are not locally gauge invariant on the gauge orbit, $\psi \rightarrow \psi^U \equiv U^{-1}\psi$. But any QCD description of the constituent quarks of any hadron must be locally gauge invariant. "In order to construct such a QCD description, we dress the Lagrangian quarks ψ with dressing function $h^{-1}[\mathrm{A}]$ of the gauge field in such a way that the result is locally gauge-invariant on the gauge orbit, i.e." [12]

$$\boldsymbol{\psi} = h^{-1}[\mathrm{A}]\psi \qquad (3)$$

"In-fact, the equation (3) was the original idea of Dirac [23]. The dressed matter field $\boldsymbol{\psi}$ is locally gauge invariant on the gauge orbit provided that dressing function $h^{-1}[\mathrm{A}]$ transforms as" [12]

$$h^{-1}[\mathrm{A}] \rightarrow h^{-1}[\mathrm{A}^U] = h^{-1}[\mathrm{A}]U \qquad (4)$$

where the gauge field transforms as $A \rightarrow A^U \equiv U^{-1}AU + U^{-1}\partial U$. "This specification of the dressing function in equation (4) is enough for ensuring gauge-invariance" [12]. "Gauge-fixing choices may be used to construct the dressing function $h^{-1}[\mathrm{A}]$. Suppose $\chi(A) = 0$ be the gauge fixing condition. For any given field configuration A, there exists a transformation $h[\mathrm{A}]$ which takes the given field A to the point on the gauge orbit where the following gauge fixing condition is satisfied" [12].

$$\chi(A^{h[A]}) = 0 \qquad (5)$$

"The same must be true for any gauge transformed field A^U,

$$\chi\left(A^{U\,h[A^U]}\right) = 0. \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (6)" [12]}$$

As discussed above, any local gauge fixing condition $\chi(A) = 0$, in general, must contain multiple Gribov copies at the points where any gauge orbit intersects the chosen gauge slice $\chi(A) = 0$ and further, due to increased separations in non-perturbative regime, any Gribov copy as a wave is physically distinguishable from other Gribov copies of a hard gluon emitted in 3-jet event. This physical distingushability means that the gauge condition in equation (6) above is satisfied uniquely and separately for each Gribov copy of the hard gluon emitted in 3-jet event. Thus, while comparing the equations (5) and (6) above, the aforesaid uniqueness implies that $A^{h[A]} = A^{U\,h[A^U]}$ [12] so that

$$h[A] = Uh[A^U] \implies h^{-1}[A^U] = h^{-1}[A]U. \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (7) [12]}$$

This equation (7) is in-fact exactly the equation (4) above that is required for the dressing function h^{-1}[A]. Actually, the dressing function in the above equation (7) can be viewed as the transformation which takes any gauge field A to a particular Gribov copy on a particular gauge slice. In other words, the dressing of the Lagrangian quark ψ in equation (3) above involves **simultaneous dressing with multiple individual dressing functions h^{-1}[A] corresponding to each and every Gribov copy of the hard gluon emitted in 3-jet event** in such a way that the combined dressing function does not remain unchanged at the place of occurrence of the Gribov copies on the gauge orbit and thereby, makes the dressed matter field ψ locally gauge invariant on the gauge orbit.

Let us now consider a special case of a baryon called Δ^{++} (delta plus plus) that comprises three identical Lagrangian up quarks in same quantum state neglecting the color degree of freedom. This same quantum state can be easily inferred from the fact that these three 'up' quarks are having zero orbital angular momentum with respect to each other and are having their individual intrinsic spin angular momentum aligned in the same direction. Since, Δ^{++} is in-fact a resonance having very short life-time of the order of 10^{-24} seconds and hadronization takes place in nonperturbative regime, so, all identical Lagrangian 'up' quarks of Δ^{++} are expected to remain in nonperturbative regime during the very short life-time of Δ^{++}. As such, for nonperturbative QCD description of constituent quarks of Δ^{++}, we dress these three identical Lagrangian up quarks with glue in such a way that the result is gauge-invariant on the same gauge-orbit as constituent quarks of Δ^{++} are in same quantum state. Since, Lagrangian quarks are not gauge invariant on the gauge orbit, so, for preserving Dirac-Fermi statistics in the naïve quark model, these three identical Lagrangian up quarks need to orient themselves at three distinct points on the same gauge orbit in such a way that the result is three separate red, blue and green colored Lagrangian up quarks in Δ^{++}.

Now, a single gluon, as a sole representative of the gauge orbit, occupies a unique point on the gauge orbit and as such, cannot be used alone for the simultaneous nonperturbative dressing of these three identical up Lagrangian quarks positioned at three distinct points on the same gauge orbit in Δ^{++}. Thus, the aforesaid simultaneous dressing, with multiple individual dressing functions $h^{-1}[A]$ corresponding to each and every Gribov copy of the hard gluon emitted in 3-jet event, becomes an inescapable necessity for QCD description of constituent quarks of Δ^{++} and accordingly, we use the Gribov copies as special realization of a gluon for simultaneous nonperturbative dressing of three identical up Lagrangian quarks in Δ^{++}.

Also, the nonperturbative QCD description, that three separate red, blue and green colored QCD up quarks are positioned at three distinct points on the same gauge orbit in Δ^{++}, endows each of the Gribov copies with unique non-identical color charge such that the combined color charge of all the Gribov copies of a gluon is color-singlet one. Consequent upon this color-singlet nature of Gribov copies, there is no need to look upon the gluon jet in 3-jet event as "hypothetical creation of gluon-gluon pair" for the theoretical description of its internal properties [1].

"*This hitherto unobserved "hypothetical creation of gluon-gluon pair" for the theoretical description of the internal properties of the gluon jet in 3-jet event has been considered in [24 – 28]. Due to color octet neutralization of the gluon field by another gluon, this hitherto "hypothetical creation of gluon-gluon pair" has the signature of a neutral leading system separated from the rest of low energy particles by a large rapidity gap devoid of any hadrons. To the contrary, the color triplet fragmentation, that involves creation of quark-antiquark pairs, in the string stretched between emitted quark & antiquark in 2-jet events results in the production of charged leading systems in excellent agreement with string Monte Carlo Models Like JETSET [29]. On the similar lines, in 3-jet event, JETSET (ARIANDE) model stretches two strings – one between gluon and quark and other between gluon and antiquark.*

As such, JETSET (and ARIANDE) model predicts charged leading systems when the aforesaid two strings in 3-jet event fragments by creation of quark-antiquark pairs in the color triplet fragmentation. But the preliminary experimental data obtained in [30 – 33] revealed excess of neutral leading systems with large rapidity gap in gluon jets than the one predicted by JETSET (and ARIANDE) model. This excess of neutral leading systems in gluon jets indicates the color octet neutralization of the gluon field by another gluon after hitherto unobserved "hypothetical creation of gluon-gluon pair" in gluon jet. Also, this excess of neutral leading system with large rapidity gap in gluon jet can also be produced by another altogether different mechanism called color reconnection [34].

But the experimentally observed data in [32, 35] clearly discards color reconnection mechanism as implemented in Monte Carlo simulations for reproducing the observed excess of neutral leading systems in gluon jet [36]. Thus, as proposed by Minkowski and Ochs [24, 25], the color octet neutralization of the gluon field by another gluon after hitherto unobserved "hypothetical creation of gluon-gluon pair" in gluon jet is only left as viable option to account for the observed excess of neutral leading systems in gluon jet. In [24, 25], it has been proposed to enhance the possible contribution of the color octet neutralization process in accounting the observed excess of neutral leading systems in gluon jet by selecting those 3-jet events that have large rapidity gap in gluon jet.

This enhancement with larger rapidity gap in gluon jet is due to the absence of multiple gluon emissions and related color neutralization processes of small rapidity range [24]. In [36], the experimental observation of this enhancement with larger rapidity gap in gluon jet has been reported. In the limit rapidity gap becomes very large, it is expected to have cent percent neutral leading system in gluon jet. Here we consider such a configuration: Consider a 3-jet event in which a hard gluon after production travels without any gluon radiation for a while and as a result, forms a jet with a very large rapidity gap devoid of any hadrons. But the probability of finding such events decreases exponentially with rapidity gap in the light of Sukadov form factor [37]. In this case, the octet field of hard gluon, which is not distorted by multiple gluon emissions and related color neutralization processes of small rapidity range, builds up to such an extent that its neutralization with equally strong octet field of another gluon will become clearly visible if it exists. But such a gluon-gluon pair production is hitherto unobserved and as such, is hypothetical in nature [1]. This forces us to focus our attention on the aforesaid color-singlet nature of Gribov copies for providing physical explanation to the *production of hard gluon in 3-jet event with very large rapidity gap and with exponentially decreasing probability according to the Sukadov form factor.*

Firstly, we characterize the phase space distribution of the particles emitted in the form of jets during high energy collisions by using the term rapidity y *that is based on the maximum rapidity distance between adjacent particles in a jet and is accordingly defined in [35] as*

$$y = \frac{1}{2}\ln\left(\frac{E+p_{\|}}{E-p_{\|}}\right) \qquad (8)$$

Where E *is the energy of the particle and* $p_{\|}$ *is the component of its 3-momentum along the jet-axis. Due to the aforesaid color-singlet nature of Gribov copies in gluon jet, the whole energy* E *of the hard gluon emitted in 3-jet event becomes equal to* $p_{\|}$ *as all its Gribov copies, while representing the same physics to the outside world, moves parallel to the same jet-axis. This equality of* E *and* $p_{\|}$ *gives rise to* ***infinite rapidity*** y *in above equation (8) for the hard gluon emitted in 3-jet event.*

Also, if the Gribov copies are treated as physical entities, then the color octet neutralization of these Gribov copies would ultimately result into cent percent neutral leading particle systems at low invariant masses and with infinite rapidity gap in the gluon jet. This prediction of infinite rapidity and cent percent neutral leading particle systems at low invariant masses in gluon jet is in-fact the limiting case of the trend in the experimental observation [36, 38] that the excess of neutral leading particle systems, with typically lower invariant masses, increases with increasing rapidity gap size and with gluon purity in gluon jet. It is pertinent to mention here that no excess in the low mass regions is exhibited by the corresponding mass spectra of leading systems in quark jets. Actually, due to Sudakov form factor, there is *considerable loss of statistics with the requirement of having a rapidity gap as large as possible."* [36]

3. CANONICAL QUANTISATION:

In preceding section 2, the central assumption recurring throughout the text is that the Gribov copies should be treated as physical entities and that they can be interpreted as special realization of the hard gluon emitted in 3-jet event. But this central assumption appears to contradict the published work [39, 40] about the problem of Gribov copies in non-abelian gauge field theories. In order to remove this contradiction, we now turn to the derivation of canonical theory **in perturbative sector** for the hard gluon (and its Gribov copies) emitted in 3-jet event under the following boundary conditions.

The boundary conditions for the hard gluon (and its Gribov copies) emitted from color-singlet point source in 3-jet event can be stated as "the pure Yang-Mills gauge potential A_μ pertaining exclusively to the hard gluon (and its Gribov copies) would be coming into existence at the time t_0 of its emission in 3-jet event in the region of the space somewhere inside a finite closed hemispherical volume V (as referred to in Equation (2) above) but would be zero at the surface of the hemispherical volume V".

As a first step towards canonical quantization, we convert the classical Lagrangian L of Equation (2) into a Hamiltonian one. For this conversion, we follow the standard procedure by defining the canonical momenta $\pi_a^{\ \mu}$ as under:

$$\pi_a^{\ \mu} = \partial L'' / \partial \acute{A}^a_{\ \mu} = - F_a^{\ \mu 0} \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (9)}$$

where $\acute{A}^a_{\ \mu}$ are the generalized velocities and L'' denotes the Lagrangian density such that $L = \int_V d^3x\, L''$

It can easily observed that the canonical momenta $\pi_a^{\ 0}$ corresponding to the time index $\mu = 0$ in the above Equation (9) vanishes due to antisymmetry of $F^{\mu\nu}$ and accordingly, the Gauss law ($\nabla_k(A)\ \pi_a^{\ k} = 0$) is absent.

For implementing the Gauss law at classical Lagrangian level as a constraint [41], first of all we solve the following non-abelian Gauss law,

$$\nabla_k(A)\nabla^k(A)A^0 - \nabla_k(A)\ \acute{\boldsymbol{A}}^{\mathbf{k}} = 0 \qquad (10)$$

where the space indices k = 1,2,3 and $\nabla_k(A)$ is 'covariant gradient',

By treating the above Equation (10) as a system of linear, elliptic partial differential equations, the (matrix valued) potential component A_0, for given value of the space components $\boldsymbol{A}_{\mathbf{k}}$ & their time derivatives $\partial_0\boldsymbol{A}_{\mathbf{k}}$, has been determined in [41] by assuming that the unique solution A_0 as a functional of $\boldsymbol{A}_{\mathbf{k}}$ & their time derivatives $\partial_0\boldsymbol{A}_{\mathbf{k}}$ does exist i.e.,

$$A_0 = A_0\{\ \boldsymbol{A}_{\mathbf{k}}, \partial_0\boldsymbol{A}_{\mathbf{k}}\} \qquad (11)$$

Now, for implementing the Gauss law at classical Lagrangian level as a constraint [41] we substitute the above Equation (11) into the classical Lagrangian Equation (2) to get new Lagrangian L_0, i.e.,

$$L_0 = (-1/2) \int_V d^3x\ (\nabla_k(\boldsymbol{A}_{\mathbf{k}})\ A_0\{\boldsymbol{A}_{\mathbf{k}}, \partial_0\boldsymbol{A}_{\mathbf{k}}\} - \acute{\boldsymbol{A}}_{\mathbf{k}}, \nabla^k(\boldsymbol{A}^{\mathbf{k}})\ A^0\{\boldsymbol{A}^{\mathbf{k}}, \partial^0\boldsymbol{A}^{\mathbf{k}}\} - \acute{\boldsymbol{A}}^{\mathbf{k}})$$

$$- (1/4) \int_V d^3x(G_{kl}\,(\boldsymbol{A}_{\mathbf{k}}), G^{kl}(\boldsymbol{A}^{\mathbf{k}})) \qquad (12)$$

where $G_{kl} = \partial_k A_l(x) - \partial_l A_k(x) - ig[A_k(x), A_l(x)]$ and k,l are denoting space indices ranging from 1 to 3 & the quantity V is a finite hemispherical closed volume encompassing only the hard gluon (and its Gribov copies) at the time t_0 of its emission in 3-jet event. This new Lagrangian L_0 must reproduce the Lagrange equations of motion for k = 1,2,3 when Hamilton's action principle is invoked.

Now, we proceed to the Hamiltonian construction, for which the substitution of L_0 of the Equation (12) into the Equation (9) above leads to

$$\pi^k = \partial L_0''/\partial \acute{A}^k = (\nabla^k(A)\, A^0\{\boldsymbol{A}^{\mathbf{k}}, \partial^0 \boldsymbol{A}^{\mathbf{k}}\} - \acute{\boldsymbol{A}}^{\mathbf{k}}) \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (13)}$$

where L_0'' denotes the Lagrangian density such that $L_0 = \int_V d^3x\, L_0''$

From the above expression for canonical momentum π^k, it is impossible to find generalized velocity $\acute{A}^k$ in terms of π^k and $\boldsymbol{A}^{\mathbf{k}}$ because here A^0 is a functional of $\boldsymbol{A}^{\mathbf{k}}$ and their time derivatives $\acute{\boldsymbol{A}}^{\mathbf{k}} = \partial^0 \boldsymbol{A}^{\mathbf{k}}$. We impose the generalized Coulomb gauge fixing condition at Lagrangian level on the non-abelian Gauss law of Equation (10) for converting the aforesaid impossibility into possibility i.e.,

$$\nabla_k(A)\, \acute{\boldsymbol{A}}^{\mathbf{k}} = 0 \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (14)}$$

With the substitution of Equation (14) in equation (10), we get

$$(\nabla_k(A)\, \pi_a^{\ k}) = \nabla_k(A)\nabla^k(A)A^0 = 0 \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (15)}$$

In view of Equations (14) & (15) above, we use the transversal part of the momentum $\pi_a^{\ k}$ (i.e., $\pi_a^{\ k\perp}$) and transversal component $A_k^{\ a\perp}$. Further, the equation (11) is accordingly modified as $A_0 = A_0\{\boldsymbol{A}_{\mathbf{k}}^{\perp}\}$ in view of Equation (15).

As such, one can now straightforwardly express the generalized velocity $\acute{A}^{k\perp}$ in terms of generalized co-ordinate and momenta variables by using $A_0 = A_0\{\boldsymbol{A}_{\mathbf{k}}^{\perp}\}$ in the Equation (13) i.e., $\acute{A}^{k\perp} = (\nabla^k(A)\, A^0\{\boldsymbol{A}^{\mathbf{k}\perp}\} - \pi^{k\perp})$. Therefore, the use of $\acute{A}^{k\perp} = (\nabla^k(A)\, A^0\{\boldsymbol{A}^{\mathbf{k}\perp}\} - \pi^{k\perp})$ and substitution of $L = L_0$ from the Equation (12) & substitution of Gauss law constraint $(\nabla_k(A)\, \pi_a^{\ k}) = 0$ from Equation (15) in the mathematical construction $[H = \int_V (\pi_a^{\ k}, \acute{A}_k^{\ a})\, d^3x - L]$ of the Hamiltonian H through Legendre transformation yields

$$H = (-1/2) \int_V d^3x\, J^{-1}[\boldsymbol{A}_{\mathbf{k}}^{\perp}]\{(\nabla_k(\boldsymbol{A}_{\mathbf{k}})\, A_0\{\boldsymbol{A}_{\mathbf{k}}^{\perp}\} - \acute{\boldsymbol{A}}_{\mathbf{k}}^{\perp}\}, J[\boldsymbol{A}_{\mathbf{k}}^{\perp}]\{\nabla^k(\boldsymbol{A}^{\mathbf{k}})\, A^0\{\boldsymbol{A}^{\mathbf{k}\perp}\} - \acute{\boldsymbol{A}}^{\mathbf{k}\perp})\} + (1/4) \int_V d^3x (F_{kl}(\boldsymbol{A}_{\mathbf{k}}^{\perp}), F^{kl}(\boldsymbol{A}^{\mathbf{k}\perp})) \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (16)}$$

Where $J[\boldsymbol{A}_{\mathbf{k}}^{\perp}]$ is the Faddeev-Popov determinant, interpreted as the Jacobian of the transformation.

However, the above derivation of the Hamiltonian H in the Equation (16) is plagued by a **technical complication** [42] i.e., when the above generalized coulomb gauge fixing condition of the Equation (14) is in force for straightforwardly expressing the generalized velocity in terms of generalized co-ordinate and momenta variables, one cannot define canonical momentas by Equation (9) above as these generalized velocities are no longer independent quantities. In order to solve this technical complication, we focus our attention, in the next few paragraphs, on the nature of the generalized coulomb gauge fixing condition of the Equation (14).

Let us assume that the hard gluon, at the time t_0 of its emission in the 3-jet event, satisfies, by default, the above Hamiltonian H in the Equation (16). Since, the Gauss law constraint of the Equation (15) is not present naturally on its own and has been satisfied identically (in principle) by construction in the Equation (10) for the sole purpose of the derivation of Hamiltonian H of the Equation (10) from the Lagrangian L_0, so, **momentarily after t_0 i.e., at t_0^+ the time development of the Hamiltonian H of the Equation (16) lands the hard gluon on the adjacent gauge orbit along which the latter has natural instinct, with passage of time, to undergo time-independent gauge transformation in the absence of Equation (15)**. Because the gauge transformation is usually understood as the relationship between two A_μ fields defined on the same space-time point and hence, the identification of this time-independent gauge transformation on the adjacent gauge orbit with time development is further probed below.

In the Yang – Mills theory, gauge transformation ω is not global one. In figure 1, the general gauge potential A_k, valid at some time instant t_0^+, transforms as a connection in the adjoint representation under local gauge transformation ω to A_{lk}, say valid at some later time instant t_1 ,[42] i.e.,

$$A_{lk} = \omega A_k \omega^{-1} - (i/g)(\partial_k \omega)\omega^{-1} \qquad (17)$$

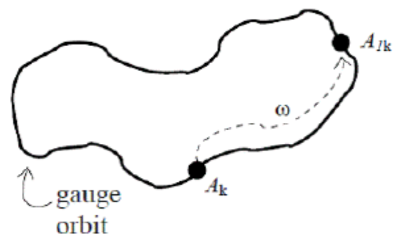

Figure 1 Illustration of the gauge orbit [7]

Differentiating above Equation (17) with respect to time t and then, applying 'covariant gradient' $\nabla_k(A)$ on both sides [41], we get

$$\nabla_k(A)\partial_0 A_{lk} = \omega[\nabla_k(A)\, \partial_0 A^{\,k} - \nabla_k(A)\nabla^k(A)\, X_0\,)]\omega^{-1} \quad \text{where } X_0 = (i/g)(\omega^{-1})(\partial_0\omega)$$

Thus, when the condition of equation (14) is imposed on the final potential A_{lk} by equating right hand side of the above equation to zero [42], we get

$$\nabla_k(A)\nabla^k(A)\, X_0 = \nabla_k(A)\partial_0 A^{\,k} \qquad (18)$$

If we consider any time slice at some in-between time instant t_i such that ($t_0^+ < t_i < t_1$), then the above elliptic linear partial differential equation (18), in-general, pertains to some in-between value of A_{ki} at the time instant t_i along the dotted gauge transformation path on the gauge orbit in above Figure 1. In other words, the Lie-algebra valued quantity $X_0 = (i/g)(\omega^{-1})(\partial_0\omega)$ of the elliptic linear partial differential Equation (18) is exclusively defined at some in-between time instant t_i such that ($t_0^+ < t_i < t_1$) and remains non-zero along the dotted gauge transformation path on the gauge orbit in Figure.1 during the time–period $\phi = (t_1 - t_0^+)$ only. Accordingly, this gauge transform ω is uniquely determined in [42] at some fixed spatial point x by the following exponential time-integral. [42]

$$\omega(x, t_1) = [T \exp. (ig) \int_{t_0^+}^{t_1} dt\, X_0(t)]\ \omega(x, t_0^+) \text{ where T indicates time-ordering.}$$

Since, the gauge transformation $A_k \rightarrow A_{lk}$, as illustrated in Figure.1 above, is time-independent one on the gauge orbit and the time-independent gauge transformations, on the gauge orbit, are generated **in infinitesimal form** by the generator ($\nabla_k(A)\ \pi_a^{\ k}$) on the left hand side of the Gauss Law Equation (15), so, the time-independent gauge transformation $A_k \rightarrow A_{lk}$, as illustrated in Figure.1 above, is also **infinitesimal** one and is, however, shown in Figure.1 in exaggerated form for the sake of clarity. This **infinitesimal** nature the time-independent gauge transformation $A_k \rightarrow A_{lk}$ implies that the integration limits t_1 and t_0^+ of above time-integral correspond to infinitesimal time–period $\phi = (t_1 - t_0^+)$. It is this infinitesimal nature of the time–period ϕ that leads to transient existence of X_0 in Equation (18) and hence, transient existence of $\nabla_k(A)\partial_0 A_k = 0$ as hypersurface in configuration space. Further, this transitory nature of $\nabla_k(A)\partial_0 A_k = 0$ of Equation (8) solves the aforesaid **technical complication,** that plagues the derivation of Hamiltonian H in the Equation (16) in the following manner.

In the Einstein causality, the transient existence of covariant derivative null vector (i.e., $\nabla_k(A)\partial_0 A_k = 0$) at any spatial point x in Minkowski space V leads to local gauge-fixing procedure that allows non-emergence of the Faddeev – Popov ghosts; imposition of the equal time canonical commutation relations $[A_i^{a\perp}(x), \pi_j^{b\perp}(y)] = \delta_{ij}\ \delta^{ab}\ \delta^3(x - y)$ & the generalized velocities $\partial_0 A_k$ to be treated as independent quantities for all intent & purpose during the transient existence of $\nabla_k(A)\partial_0 A_{lk} = 0$ at any spatial point x in Minkowski space V. At time t_1, the

Gauss law constraint of the Equation (15) is satisfied again identically (in principle) by construction to obtain the Hamiltonian Equation (16) once more and accordingly, the Hamiltonian system is again ready to undergo time development to land at next adjacent gauge orbit momentarily after t_1 i.e., at t_1^+. This cycle goes on repeating itself until the hard gluon fragmentation in non-perturbative regime.

Thus, we see that the existence of the Hamiltonian Equation (16) on a particular gauge orbit is only for the infinitesimal time–period $\phi = (t_1 - t_0^+)$. During this infinitesimal time–period ϕ, **the uncertainty principle** implies that there must be certain minimum amount of uncertainty, or quantum fluctuation in the eigenvalue of the Hamiltonian H in Equation (16) above. One can think of these quantum fluctuations as Gribov copies that appear at some time, move along with the real hard gluon and then get annihilated. Like virtual particles, Gribov copies cannot be observed directly with particle detectors, but their indirect effects like anomalous scaling can be observed and measured. In next section 4, we have discussed about the anomalous scaling as indirect effect of the Gribov copies. Thus, the uncertainty principle helps in removing the contradiction concerning Gribov copies as mentioned above in the beginning of this section 3.

4. DISCUSSION:

Until now, we have focused on the canonical approach to quantizing the pure Yang – Mills theory in perturbative sector for the case of hard gluon emitted in 3-jet event. In-fact, there are many approaches for quantizing the pure Yang – Mills theory. But, the most natural approach in this regard is that of the functional integral. Since, the canonical approach and the functional integral approach are equivalent to each other, so, the various concepts developed in the derivation of canonical theory in perturbative sector in section 3 of this paper can be extended to functional integral approach in non-perturbative sector also. For instance, the important equivalence principle, stating that a physical configuration for the pure Yang – Mills theory is not a given field A_μ, but rather a class of gauge equivalent fields, is taken into account in canonical approach by the identification of the time-independent gauge transformation in Figure1 on the adjacent gauge orbit with time development of the Hamiltonian system. Likewise, this equivalence principle is also taken into account in the functional integral approach in non-perturbative sector by integrating over classes of equivalent fields as discussed below.

The dimensional transmutation analysis, as published in Section 3 of [43], can be directly understood as the functional integral approach for the hard gluon emitted in 3-jet event by considering the generating functional W(B) being dependent upon initial field configuration at emission time t_0 and final field configuration at time t_2 ($>t_1$) just before hard gluon fragmentation in non-perturbative regime. Accordingly, the equation (4) of [43] is an integral over a certain set of gauge-inequivalent (i.e., gauge-fixed) configurations such that for each gauge-inequivalent (i.e., gauge-fixed) configuration, the evaluation of the Faddeev-Popov determinant $\det((\nabla_\mu + ga_\mu)\nabla_\mu)$ in the vicinity of the gauge-fixing surface is carried out under infinitesimal gauge transformation along the gauge orbit, as depicted in Figure 1. This is nothing but extension of the concept of cycle repetition of canonical approach in perturbative sector, as mentioned at the end of section 3 of this paper, to the functional integral approach in non-perturbative sector.

Thus, the infinitesimal gauge transformation along the gauge orbit, as depicted in Figure 1, establishes direct equivalence between canonical approach, as outlined in Section 3 of this paper and functional integral approach of Section 3 of [43]. In Equation (16) of [43], the Faddeev-Popov determinant $\det((\nabla_\mu + ga_\mu)\nabla_\mu)$ is then written in terms of the functional integral over grassman algebra for expressing the generating functional W(B) as a series in the powers of the coupling constant. The zero order in coupling constant term in Equation (16) of [43] diverge and the same is trivially regularized in [43]. After regularization, the numerical coefficient of the classical action term in Equation (21) of [43] is focused upon.

Since, during the infinitesimal time–period ϕ, the uncertainty principle implies about the transient existence of the Gribov copies as quantum fluctuations in the eigenvalue of Hamiltonian H in Equation (16), so, the aforesaid formal manipulations involving the Faddeev-Popov determinant are multiply counted for each gauge-inequivalent configuration in the integral of Equation (4) of [43]. As a result of this multiple counting, the dimensionless numerical coefficient g of the classical action in perturbative sector (where Gribov copies cannot be aware of each other due to asymptotic freedom) becomes dimensionful in Equation (21) of [43] for non-perturbative sector (where Gribov copies are accounted for in aforesaid multiple counting). This dimensional transmutation can be intuitively understood in the light of the fact that the aforesaid multiple counting physically means spatial and temporal coherence of the waves associated with the hard gluon and its Gribov copies i.e., overlapping or merger in space and time of the wave-packets of hard gluon & its Gribov copies, that are representing the same quantum state i.e., the gauge orbit.

In other words, as one moves from perturbative regime to nonperturbative regime, the separation distance between the Gribov copies increases with the lowering of energy and at certain energy scale Λ_{QCD} in non-perturbative regime, this separation distance between the Gribov copies becomes ideal for the occurrence of spatial and temporal coherence of the waves associated with the hard gluon & its Gribov copies. This occurrence of spatial and temporal coherence of the waves associated with the hard gluon & its Gribov copies at the said certain energy scale Λ_{QCD} is not laser like gain because the number of the hard gluon and its Gribov copies in the said coherence remains conserved with further lowering of energy. As such, a prerequisite for Bose-Einstein Condensation (BEC) is fulfilled to form **Gribov BEC glueball** at the said certain energy scale Λ_{QCD} as Gribov copies are always representing the same quantum state i.e., the gauge orbit and gluons have been experimentally confirmed in [44, 45] to possess spin one. Further, the Bose–Einstein condensation of photons at room temperature has already been reported in [46].

Thus, due to the aforesaid Bose-Einstein Condensation (BEC) of hard gluon & its Gribov copies, the quantity g of the classical action in perturbative sector losts its meaning as dimensionless coupling constant or color charge at the said certain energy scale Λ_{QCD} and instead, becomes dimensionful quantity having dimension of energy for the aforesaid **Gribov BEC glueball**. In-fact, the aforesaid Bose-Einstein Condensation (BEC) is nothing but the stationary wave-pattern of the waves, associated with the hard gluon & its Gribov copies and formed in some inertial reference frame S that is uniformly moving with respect to the laboratory reference frame and as such, there exists a mass-gap in the pure Yang-Mills theory in the laboratory reference frame due to the formation of the aforesaid stationary wave-pattern in moving reference frame S. Accordingly, one defines the mass scale Λ_{QCD} of QCD to be the energy at which the color charge or the coupling constant equals some dimensionlful value, say 1. Then, via this phenomenon of dimensional transmutation 1) one can calculate all the observables of QCD in terms of dynamically generated mass scale and there remains no adjustable parameter in QCD and 2) one can introduce a physical scale Λ_{QCD} at which color confinement occurs as the **Gribov BEC glueball** of hard gluon & its Gribov copies is color-singlet one as already stated in the section 2 of this paper.

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