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Smarandache semiquasi near-rings[1]

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Abstract G. Pilz [1] has defined near-rings and semi-near-rings. In this paper we introduce the concepts of quasi-near ring and semiquasi-near ring. We have also defined Smarandache semiquasi-near-ring. Some examples are constructed. We have posed some open problems.

Keywords Near-ring, semi-near-ring, quasi-near-ring, semiquasi-near-ring, Smarandache semiquasi-near-ring.

§1. Introduction

In the paper [2] W.B. Kandasamy has introduced a new concept of Smarandache semi-near ring. These are associative rings. We have defined a new concepts of quasi-near ring and Smarandache semiquasi-near-ring. These are non associative rings.

Definition 1.1. An algebraic structure $(Q, +, \cdot)$ is called a quasi-near-ring (or a right quasi-near-ring) if it satisfies the following three conditions:

1.$(Q, +)$ is a group (not necessarily abelian).

2.$(Q, \cdot)$ is a quasigroup.

3.$(n_1 + n_2) \cdot n_3 = n_1 \cdot n_3 + n_2 \cdot n_3$ for all $n_1, n_2, n_3 \in Q$ (right distributive law).

Example 1.1. Let $Q = \{1, 2, 3, 4\}$ and the two binary operations are defined on Q by the following tables;

+	1	2	3	4
1	1	2	3	4
2	2	3	4	1
3	3	4	1	2
4	4	1	2	3

.	1	2	3	4
1	4	2	1	3
2	1	3	4	2
3	3	1	2	4
4	2	4	3	1

Definition 1.2. An algebraic system $(S, +, \cdot)$ is called a semiquasi-near-ring (or right semiquasi-near-ring) if it satisfies the following three conditions:

1. $(S, +)$ is a quasigroup (not necessarily abelian).

2. $(S, \cdot)$ is a quasigroup.

3. $(n_1 + n_2) \cdot n_3 = n_1 \cdot n_3 + n_2 \cdot n_3$ for all $n_1, n_2, n_3 \in S$ (right distributive law).

Example 1.2. Consider the algebraic system $(S, +, \cdot)$ where $S = \{1, 2, 3, 4\}$ defined by the following tables;

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+	1	2	3	4
1	1	3	4	2
2	4	2	1	3
3	2	4	3	1
4	3	1	2	4

.	1	2	3	4
1	4	2	1	3
2	1	3	4	2
3	3	1	2	4
4	2	4	3	1

Example 1.3. We know that integers $\mathbf{Z}$ with subtraction (-) forms a quasigroup. $(\mathbf{Z}, \cdot)$ is a quasigroup and subtraction of integers distributes over multiplication. Thus $(\mathbf{Z}, -, \cdot)$ is a semiquasi-near-ring.

Definition 1.3 We know that integers $\mathbf{Z}$ with subtraction (-) forms a quasigroup. $(\mathbf{Z}, \cdot)$ is a quasigroup and subtraction of integers distributes over multiplication. Thus $(\mathbf{Z}, -, \cdot)$ is a semiquasi-near-ring.

Example 1.4. Consider the semiquasi-near-ring $(S, +, \cdot)$ defined by the following tables;

+	1	2	3	4
1	1	3	4	2
2	4	2	1	3
3	2	4	3	1
4	3	1	2	4

.	1	2	3	4
1	4	3	1	2
2	1	2	4	3
3	3	4	2	1
4	2	1	3	4

one can easily verify that addition distributes over multiplication from right as well as S contains $N = \{4\}$ properly which is a quasi-near-ring.
Thus $(S, +, \cdot)$ is a Smarandache semiquasi-near-ring.

Example 1.5. Let R be the set of reals. We know that $(R, +)$ is a group and hence a quasigroup. Also, R w.r.t. division is a quasigroup, that is $(R, \div)$ is a quasigroup. More over, addition distributes over division from right. Thus $(R, +, \div)$ is a semiquasi-near-ring.

Let Q be the set of non-zero rationals. Then $(Q, +)$ is a group. Also, $(Q, \div)$ is a quasigroup. Addition distributes over division. Hence $(Q, +, \div)$ is a quasi-near-ring.
We know that $R \supset Q$. Therefore, $(R, +, \div)$ is a Smarandache semiquasi-near-ring. We now show by an example that there do exist semiquasi-near -rings which are not Smarandache semiquasi-near-rings.
Consider example 1.2 where we can not have a quasi-near-ring contained in S.
We give below the example of a smallest Smarandache semiquasi-near-ring which is not a near-ring.

Example 1.6. Consider the semiquasi-near ring $(S = \{1, 2, 3\}, +, \cdot)$ defined by the following tables;

+	1	2	3
1	1	3	2
2	3	2	1
3	2	1	3

.	1	2	3
1	3	2	1
2	1	3	2
3	2	1	3

One can easily verify that $(S = \{1, 2, 3\}, +, \cdot)$ is a semiquasi-near-ring. Moreover, $N = \{3\} \subset S$ and $(N, +, \cdot)$ is a quasi-near ring. Therefore $(S = \{1, 2, 3\}, +, \cdot)$ is a Smarandache semiquasi-near-ring.

We now show by an example that there do exist semiquasi-near -rings which are not Smarandache semiquasi-near-rings.

Consider example 1.2 where we can not have a quasi-near-ring contained in S.

We give below the example of a smallest Smarandache semiquasi-near-ring which is not a near-ring.

Definition 1.4. N is said to be an Anti-Smarandache semiquasi-near-ring if N is a quasi-near-ring and has a proper subset A such that A is a semiquasi-near-ring under the same operations as of N.

Example 1.7. In example 1.5 $(R, +, \div)$ is also a quasi-near-ring which contains a semiquasi-near-ring $(Q, +, \div)$.

Thus we can say that $(R, +, \div)$ is an Anti-Smarandache semiquasi-near-ring.

We propose the following:

Problem 1. Do there exist a finite Smarandache semiquasi-near-ring such that the order of the quasi-near-ring contained in it is greater than 1 ?

Problem 2. How to construct finite Anti-Smarandache semiquasi-near-rings ?

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