The crisis of ”naked” colour in Quantum Chromodynamics

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Abstract

Quantum Chromodynamics (QCD), as a part of the Standard Model of particle physics, is the most successful theory of the strong interaction at present. However, as shown by the author, it predicts a colour dependence of the electric charge of the quarks. This colour dependence is consistent with all that we know of QCD for three colours and for arbitrary number of colours; except that it brings about a direct conflict with the experimentally well-known coulomb interaction of the electric charges of quarks and leptons. This leads directly to QCD predicting ”naked” colour interaction in electrodynamics at large distances. This is completely unacceptable and is thus a major crisis for QCD. Note that this crisis cannot be swept aside, as the issue of the ”naked” colour is as fundamenatal a prediction of QCD as that of the asypmtotic freedom was.

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The theory of Quantum Chromodynamics (QCD), in the Standard Model, with the group structure $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$, has been very successful for the description of hadrons at high energies [1].

As free colour has not been detected, we confine colour in a baryon and meson as a colour singlet state arising in $3_c \times 3_c \times 3_c = 1 + 8 + 8 + 10$. $\overline{3_c} \times 3_c = 1 + 8$ These hadrons are specified by their baryon number B, electric charge Q, strangeness S etc. Clearly we demand that these quantum numbers be colour-independent too.

In QCD the baryon number of each quark is $\frac{1}{3}$ where the 3 stands for the three colours. Hence for $SU(N_c)$ QCD the baryon number of a quark is $\frac{1}{N_c}$. For a meson the baryon number is $\frac{1}{3} - \frac{1}{3} = 0$ while that of a baryon is $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$. Question: are these baryon numbers colour-independent?

Note that the set of rational numbers Q = $[\ 0,\ \frac{a}{b}; a, b \in Z, b \neq 0\ ]$ where Z is a set of integers, is a group under the binary operation of addition. Here the identity element is 0. Clearly this 0 is a neutral element as to the operation of addition of the set of rational numbers. Hence as baryon number $\frac{1}{3}$ is a rational number, the meson baryon number 0 is a genuine colour-independent quantum number. However for the same reason the baryon number B=1 for the baryons cannot be treated as colour-independent. The colour independence of its additive quantum numbers will depend upon the fact that it belongs to the above set of Q, and that it has its own structure as being a group under addition, with one element, which is its identity, and which should be treated as its neutral element - and which is 0 as above.

Note however that the set of rational numbers Q_0, which is the above set Q minus 0, is a group under the binary operation of multilication. Herein the identity element, as the neutral element of this group, is 1. Hence if this 1 arises out of a multiplicative operation like $N_c \times \frac{1}{N_C}$ then this would be colour-independent under multication of the quantum numbers. (And which the above baryon number of proton in QCD is not!).

We may point out, that it is important to distiguish between the terms "colour-neutral" and "colour-independent". We call the positronium e^+e^- as being electric charge-neutral as to the QED interaction. In the same manner we say that the colour singlet state above is colour-neutral as to the QCD interaction. As to the corresponding quantum numbers, the electric charge of positronium is 0 and hence the state of positronium is charge-independent. Similarly the baryon number of a meson is 0 and hence it is colour-independent. What about the baryon number B=1 for baryons? We know a six-quark system breaks up into two independent systems of three-quarks each. As B=1 arises from three coloured baryon numbers $\frac{1}{N_c}$ of the quarks, so in contrast to the what was expected, B=1 should be viewed as the MAXIMUM coloured baryon number allowed in hadrons in QCD.

To understand how this maximum colour in baryon number of one may manifest itself physically, we look into the structure of the electric charge of quarks and leptons for the SM group $SU(N)_c \otimes SU(2)_L \otimes U(1)_Y$ for arbitrary N_c and which is the group to study QCD for arbitrary number of colour. This has been done by the author in Ref. [2,3]. Given the significance of these results for our purpose here, we give the relevant details of the electric charge [2,3] in an Appendix below, herewith.

The author has derived the expression of the electric charge in this model as below [2,3]. Surprisingly it has colour quantum number sitting inside the electric charge, which though is a property of QED:

$$Q(u) = \frac{1}{2}(1 + \frac{1}{N_c})$$

$$Q(d) = \frac{1}{2}(-1 + \frac{1}{N_c}) \tag{1}$$

where $\frac{1}{N_c}$ = B, the baryon number. And hence the charge of proton is (for $N_c = 3$ below):

$$Q(p) = \frac{1}{2}(-1 + \frac{1}{N_c}) + \frac{1}{2}(1 + \frac{1}{N_c}) + \frac{1}{2}(1 + \frac{1}{N_c})$$

$$= \frac{1}{2} + \frac{1}{2}(\frac{1}{N_c} + \frac{1}{N_c} + \frac{1}{N_c})$$

$$= \frac{1}{2} + \frac{1}{2}B \tag{2}$$

where B=1 as above is maximum colour-dependent and hence surprisingly the electric charge of the proton is colour-dependent too. (For the pion for example for π^+, $Q = (\frac{1}{2} + \frac{1}{2N_c}) - (\frac{-1}{2}) + \frac{1}{2N_c}) = 1$ is colour-independent as the colour cancels out).

Now as in QED $U(1)_{em}$ is long ranged, the coulomb interaction between two isolated protons at a distance r is proportional to the product of the charges

$$Q_1 Q_1 = (\frac{1}{2} + \frac{1}{2}B)(\frac{1}{2} + \frac{1}{2}B)$$

$$= \frac{1}{4} + \frac{1}{2}B + \frac{1}{4}B^2 \tag{3}$$

This is coloured and so at a distance there is explicit colour dependence in coulomb interaction for the protons in QCD. Thus there is "naked" colour here (we borrow the term from cosmology where there may be a "naked" singularity). This is most amazing and exteremely puzzling! Hence surprisingly, an electron will see a proton as having a charge of only 1/2. This is an explicit example of the effect of "naked" colour discussed above

Note that the colour-neutrality of the singlet state in QCD is not guaranteeing its colour-indepedence of the quark charges as to its behaviour in the QED interactions. This is, because it is the relevant quantum numbers and their colour dependence, which runs across to QED in the process of the Spontaneous Symmetry Breaking of $SU(N_c) \otimes SU(2)_L \otimes U(1)_Y \rightarrow SU(N_c) \otimes U(1)_{em}$. It was the anomaly cancellation [2] which surprisingly brought colour dependence into the electric charge (a property of QED) in the first place. It is the same anomaly which again is raising its head to bring about these puzzling "naked" colour effects through the electric charge quantum numbers which is cutting across the boudaries of QCD to QED.

Let us put the issue more directly and simply as follows. Imagine an incoming beam of electrons interacting with u- and d- quarks inside the baryons in deep inelastic scattering. As the colour dependent electric charge of the quarks in eqn.(1) is a sum of two terms, the first one is colour independent and the second one colour dependent, and as the the electric charge of electron is immune to the colour degree of freedom; it will see the u- and the d- quarks as having charges 1/2 and -1/2 respectively. This is disastrously against experiment which sees u- and d- quarks as having charges 2/3 and -1/3 respectively. This QCD prediction of the electric charge of the quarks is dead against experiments!

One possibility is that one may cast doubts on the validity and/or relevance of the above colour dependent electric charge of quarks in QCD and hope that it may be that somehow the electric charge in the SM be indepedent of colour - that is it be always 2/3 and -1/3 for u- and d- quarks respectively, for any arbitrary N_c. This was precisly what was done by Witten and collaborators [4,5] in a study of QCD for arbitary number of colours. They reasoned that it was possible to ignore electrodynamics of the quark charges as this was much weaker than the colour charge. In such a limit the baryons have finite size but the mass going as N_C. However we find that if we take Witten fixed charges 2/3 and -1/3, then it is disastrous for the whole concept of the study of QCD for large number of colours [2]. With these fixed charges for the quarks, the Coulomb self energy of this composite proton would go as ${N_c}^2$ and this will add to the proton mass and mess up the whole QCD analysis. Though Witten had actually ignored this fact, the above problem was clearly demonstrated in ref [2] by the author. Hence we can not take fixed charges 2/3 and -1/3 for arbitrary number of colours. It was also shown that if we take the correct colour dependent charges of quarks as given above, then the proton charges for any arbitrary N_c comes out to be always unity (and neutron always neutral). In Ref. [2] it was extensively discussed as to why the colour dependent charges of quarks shown here, are the correct charges to take for a study of QCD for arbitrary number of colours, and not the rigid and colour-independent quark charges of 2/3 and -1/3 as done by Witten et. al. [4,5].

Thus the problem of the "naked" colour for the electromagnetic effects of proton is real and cannot be dispensed with. For quantum chromodynamics, the most successful model of the strong interaction, this is a crisis of the most serious magnitude. We would like to point out that as shown here, the issue of "naked" colour in QCD is a fundamental prediction of QCD - actuallly as fundamental as that of the asymptotic freedom in it! One, the asymptotic freedom, proved to be a blessing for the theory of QCD; and the other one, that of the "naked" colour in it, is proving to be a "curse". And how to get around it? - that is the big question now!

Appendix

To demostrate charge quantization as an intrinsic property of the SM, the complete machinery which makes the SM as what it fully is, is required. As demanded by the SM, one takes the repetitive structure for each generation of the fermions. Let us start by looking at the first generation of quarks and leptons (u, d, e,ν) and assign them to $SU(N_c) \otimes SU(2)_L \otimes U(1)_Y$ representation as follows [2,3].

$$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L , (N_c, 2, Y_q)$$

$$u_R; (N_c, 1, Y_u)$$

$$d_R; (N_c, 1, Y_d)$$

$$l_L = \begin{pmatrix} \nu \\ e \end{pmatrix} ; (1, 2, Y_l)$$

$$e_R; (1, 1, Y_e) \tag{4}$$

$N_c = 3$ corresponds to the Standard Model case. To keep things as general as possible this brings in five unknown hypercharges.

Let us now define the electric charge in the most general way in terms of the diagonal generators of $SU(2)_L \otimes U(1)_Y$ as

$$Q' = a' I_3 + b' Y \tag{5}$$

We can always scale the electric charge once as $Q = \frac{Q'}{a'}$ and hence ($b = \frac{b'}{a'}$)

$$Q = I_3 + bY \tag{6}$$

In the SM $SU(N_c) \otimes SU(2)_L \otimes U(1)_Y$ is spontaniously broken through the Higgs mechanism to the group $SU(N_c) \otimes U(1)_{em}$. In this model the Higgs is assumed to be doublet ϕ with arbitrary hypercharge Y_ϕ. The isospin $I_3 = -\frac{1}{2}$ component of the Higgs develops a nonzero vacuum expectation value $< \phi >_o$. Since we want the $U(1)_{em}$ generator Q to be unbroken we require $Q < \phi >_o = 0$. This right away fixes b in (3) and we get

$$Q = I_3 + (\frac{1}{2Y_\phi})Y \tag{7}$$

Next requiring that in electrodynamics, the left-handed and the right-handed electric charges of a particle be equal and also by demanding that the triangular anomaly cancels (to ensure renormaligability of the theory) (see [2,3] for details); one obtaines all the unknown hypercharge in terms of the unknown Higgs hypercharge Y_ϕ. Ultimately Y_ϕ is cancelled out and one obtains the correct charge quantization as follows.

$$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L, Y_q = \frac{Y_\phi}{N_c},$$

$$Q(u) = \frac{1}{2}(1 + \frac{1}{N_c}), Q(d) = \frac{1}{2}(-1 + \frac{1}{N_c})$$

$$u_R, Y_u = Y_\phi(1 + \frac{1}{N_c}), Q(u_R) = \frac{1}{2}(1 + \frac{1}{N_c})$$

$$d_R, Y_d = Y_\phi(-1 + \frac{1}{N_c}), Q(d_R) = \frac{1}{2}(-1 + \frac{1}{N_c})$$

$$l_L = \begin{pmatrix} \nu \\ e \end{pmatrix}, Y_l = -Y_\phi, Q(\nu) = 0, Q(e) = -1$$

$$e_R, Y_e = -2Y_\phi, Q(e_R) = -1 \tag{8}$$

A repetitive structure gives charges for the other generation of fermions also [2,3].

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