On the existence of magnetic monopoles

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The vortex theory of electromagnetism shows that the concept of magnetic monopoles is inconsistent with the fundamental character of the magnetic field. This is because the magnetic field is a vortex-like field without any sources. Therefore, magnetic monopoles do not exist.

1. Introduction

The existence of the magnetic monopole is apparently compatible with the fully symmetrized Maxwell's equations. It seems that only modification of Maxwell's equations suffices to permit magnetic charges to exist in electrodynamics. However, the existence of a magnetic monopole creates many inconsistencies. This has already been demonstrated in the literature in several different aspects. For example, Zwanziger[1)] and Weinberg[2)] demonstrate that the existence of a magnetic monopole is inconsistent with *S* matrix theory, and Hagen[3)] shows that the inclusion of a magnetic monopole in electrodynamics is inconsistent with relativistic covariance. This inconsistency has been recently demonstrated at a more elementary level by showing that the Hamiltonian for the system of an electric charge interacting with the field of a fixed magnetic monopole does not exist.[4)] This latter demonstration is more interesting when we learn that Dirac has derived his magnetic charge quantization by postulating a Hamiltonian for the same system.[5)] However, his Hamiltonian includes an invalid vector potential instead of a scalar potential for representing the field of the fixed magnetic monopole. By using this invalid Hamiltonian, Dirac predicted that if a magnetic charge is ever found in nature, it must be quantized in units of $\hbar c/2e$, where e is the electron charge value. However, by using an alternative approach, Schwinger obtained the different value of $2\hbar c/e$,[6)] which in hindsight demonstrates the fallacy of the magnetic monopole.

Noticing that there are so many proofs by contradiction, one might ask why we cannot show the impossibility of the magnetic monopole directly based on the theory of electromagnetism. Why do we need to use methods such as *S* matrix theory and the Hamiltonian formulation? Surprisingly, the vortex theory of electromagnetism, which is based on the recently developed rotational theory of relativity[7,8,9)], proves the impossibility of the magnetic monopole in a trivial geometrical manner. This theory shows that the magnetic field is a vorticity-like field without any sources. Through these discoveries, one realizes that the search for a magnetic monopole has been going on for so long, because the underlying geometry of electrodynamics had not been fully appreciated.

The current paper is organized in the following manner. In Section 2, we give a review of the concept of the vorticity and vortex lines in fluid mechanics. Then the fundamental aspects of the rotational theory of relativity are presented in Section 3. Based on this development, a summary of the vortex theory of electromagnetism is given in Section 4. Afterwards, in Section 5, the impossibility of the magnetic monopole is demonstrated based on the vorticity-like character of the magnetic field. In Section 6, it is shown that Dirac has actually used a semi-infinite long thin solenoid (magnet) to obtain his magnetic charge quantization, which has nothing to do with a pure magnetic charge. Finally, Section 7 contains a summary and conclusion.

2. Vorticity in fluid mechanics

From non-relativistic fluid mechanics,[10)] we know that the vorticity vector $\boldsymbol{\zeta}$ is the curl of the velocity vector field $\mathbf{v}$ of the fluid

$$\boldsymbol{\zeta} = \nabla \times \mathbf{v} \tag{1}$$

It is seen that the vorticity $\boldsymbol{\zeta}$ equals twice the angular velocity of the fluid element. Since the divergence of the curl of any vector is identically zero, it follows that

$$\nabla \bullet \boldsymbol{\zeta} = 0 \tag{2}$$

This shows that there can be no sources or sinks of vorticity in the fluid itself; that is, the vorticity vector is source-less. As a result, we notice that the relation (2) is the necessary compatibility condition for the existence of a consistent velocity **v** field for a given vorticity field $\boldsymbol{\zeta}$.

The concept of vortex lines are also useful in fluid mechanics. A vortex line or vorticity line is a line whose tangents are everywhere parallel to the vorticity vector $\boldsymbol{\zeta}$. According to the Helmholtz theorems for vorticity, the compatibility relation (2) shows that a vortex line cannot start or end in the fluid.[10)] Therefore, in general, vortex lines are either closed loops or end at the boundary of the fluid.

Interestingly, the vorticity defined here may be called circular vorticity, because of the circular character of rotational motion of the fluid elements. This means that we can also define hyperbolic vorticity in a mathematical context. It turns out that the concept of circular and hyperbolic rotations and vorticities are fundamental in understanding the theory of relativity and geometry of electromagnetism. This geometrical vortex theory of electromagnetism proves the impossibility of magnetic monopole directly, as we demonstrate in the following sections.

3. Rotational theory of relativity

The rotational theory of relativity is based on postulating the fundamental relation between space-time and particles.[7,8,9)] This development is basically an extension of Poincaré's theory of relativity, which establishes that there is only one theory of relativity in physics. This extended theory of relativity postulates that every particle specifies its own space-time body frame, in which the particle has its attached four-vector velocity with magnitude c in the time direction. This has several important consequences, such as:

1. The relative motion of the particles is in fact the result of the relative four-dimensional rotation of their corresponding space-time body frames in a universal entity, which could be called the ether. This four-dimensional rotation is a combination of a circular and a hyperbolic rotation. The hyperbolic part of this rotation is actually what is known as accelerating motion. This is the reason why the relative motion follows the rules of non-Euclidean geometry.

2. The orthogonal transformations similar to Lorentz transformations are not restricted to relative uniform motion. The relative motion of accelerating particles is also represented by varying orthogonal transformations. However, the general Lorentz transformation must be written for the attached four-vector velocities, not positions. This is the completion of Poincare's relativity for accelerating systems. For the special case of constant Lorentz transformation, we can integrate and obtain the traditional Lorentz transformation for four-vector positions in inertial systems.

3. The fundamental interaction is a four-dimensional vorticity-like field, which is a combination of a circular and a hyperbolic vorticity. This is the non-Euclidean geometrical theory of interaction. As a result, we notice that the Maxwellian theory of electromagnetism is a vortex theory and is a model for any other fundamental interaction. This geometrical theory enables us to resolve some existing ambiguities, such as the speculation about the magnetic monopole.

4. The Lorentz force acting on a point charged particle has amazing geometrical and mechanical interpretations. From the geometrical viewpoint, this force is simply the rotational effect of the body frame under the vortex field. From the mechanical viewpoint, the Lorentz force is analogous to the lift force in aerodynamics.

4. Vortex theory of electromagnetism

Consider an inertial reference frame $x_1x_2x_3x_4$ such that $x_1x_2x_3$ represents space, and x_4 is the axis measuring time with imaginary values, such that $x_4 = ict$. Relative to this inertial system, the electric charges create an electric field $\mathbf{E}$ and magnetic field $\mathbf{B}$. The governing Maxwell's equations in SI units are[11,12)]

$$\nabla \bullet \mathbf{B} = 0 \qquad \text{(Gauss's law for magnetism)} \tag{3}$$

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0 \qquad \text{(Faraday's law)} \tag{4}$$

$$\nabla \bullet \mathbf{E} = \rho / \varepsilon_0 \qquad \text{(Gauss's law)} \tag{5}$$

$$\nabla \times \mathbf{B} - \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} = \mu_0 \mathbf{J} \qquad \text{(Ampere's law)} \tag{6}$$

Here ρ is the electric charge density and $\mathbf{J}$ is the electric current density vector, which satisfy the continuity equation

$$\frac{\partial \rho}{\partial t} + \nabla \bullet \mathbf{J} = 0 \tag{7}$$

By defining the vector potential $\mathbf{A}$ and the scalar potential ϕ, the electric and magnetic fields $\mathbf{E}$ and $\mathbf{B}$ can be represented as

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} - \nabla \phi \tag{8}$$

$$\mathbf{B} = \nabla \times \mathbf{A} \tag{9}$$

One can see that these representations are compatible with the homogeneous Maxwell's equations (3) and (4), which are trivially satisfied.

The vortex theory of electromagnetism shows that the negative of the four vector potential $\mathsf{A} = (\mathbf{A}, i\frac{1}{c}\phi)$ is a four-dimensional velocity-like field, which is called the electromagnetic four-vector velocity field.[7,8,9)] The four-dimensional curl of this four-vector velocity is the electromagnetic strength four-tensor vorticity-like F defined as

$$F_{\mu\nu}(x) = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{10}$$

We can easily verify that the definition (10) is equivalent to (8) and (9). The four-tensor vorticity-like $\mathbf{F}$ can be written in terms of electric and magnetic fields $\mathbf{E}$ and $\mathbf{B}$ as

$$\mathbf{F} = \begin{bmatrix} 0 & B_3 & -B_2 & -iE_1/c \\ -B_3 & 0 & B_1 & -iE_2/c \\ B_2 & -B_1 & 0 & -iE_3/c \\ iE_1/c & iE_2/c & iE_3/c & 0 \end{bmatrix} \tag{11}$$

This representation shows that the electromagnetic four-tensor vorticity field $\mathbf{F}$ is a combination of hyperbolic electromagnetic vorticity $\frac{1}{c}\mathbf{E}$ and circular electromagnetic vorticity $-\mathbf{B}$. As a result, the homogeneous Maxwell's equations (3) and (4) are the necessary compatibility equations for the circular and hyperbolic electromagnetic vorticities. Moreover, we can also see that the inhomogeneous Maxwell's equations (5) and (6) govern the motion of these vorticities.

Interestingly, the developed theory shows that the four-vector current density $\mathbf{J} = (\mathbf{J}, i\rho c)$ represents the four-vector mean curvature vector $\mathbf{K} = (\mathbf{K}, K_4)$ of the electromagnetic field, where the space and time components are

$$\mathbf{K} = -\frac{1}{6}\mu_0 \mathbf{J} \tag{12}$$

$$K_4 = -i\frac{1}{6c}\frac{1}{\varepsilon_0}\rho \tag{13}$$

Based on the developed rotational theory of relativity,[7,8,9)] the electric charge q of a particle has the property of a kinematical coupling, which maps the four-dimensional electromagnetic vorticity $\mathbf{F}$ at the position of the particle to the angular velocity $\mathbf{\Omega}$ of its body frame, where

$$\mathbf{\Omega}(\mathbf{x}) = \frac{q}{m}\mathbf{F}(\mathbf{x}) \tag{14}$$

This is the geometrical character of interaction, which shows that the electromagnetic Lorentz force is simply the rotational effect of the body frame under the vortex field.

Interestingly, the geometrical theory of electromagnetism decides the fate of the magnetic monopole in a fundamental way as follows.

5. Impossibility of magnetic monopole

As we mentioned, the magnetic field $\mathbf{B}$ is the space electromagnetic vorticity induced to the ether relative to the inertial reference frame, which is the curl of the electromagnetic velocity vector field $\mathbf{A}$

$$\mathbf{B} = \nabla \times \mathbf{A} \tag{9}$$

that is repeated here for convenience. As a result, we may recall

$$\nabla \bullet \mathbf{B} = 0 \tag{3}$$

which shows that the magnetic vorticity field $\mathbf{B}$ is source-less. This is analogous to the vorticity field $\boldsymbol{\zeta}$ in the rotational fluid flow. The condition (3) is the necessary kinematical condition for the existence of the vector potential $\mathbf{A}$ for a given magnetic field $\mathbf{B}$. Therefore, it is seen that the existence of magnetic monopoles would violate this kinematical compatibility equation as follows.

Let us assume that there is a point magnetic monopole of strength q_m at the origin. Therefore, in SI units

$$\nabla \bullet \mathbf{B} = \mu_0 q_m \delta^{(3)}(\mathbf{x}) \tag{15}$$

and the static magnetic field is then given by

$$\mathbf{B} = \frac{\mu_0}{4\pi}\frac{q_m}{r^2}\hat{\mathbf{r}} \tag{16}$$

However, the relation (15) contradicts the kinematical compatibility (3). Therefore, the magnetic field of a magnetic monopole cannot be represented by a vector potential $\mathbf{A}$. Based on the Helmholtz decomposition theorem, this field can only be represented by a scalar potential[12,13,14)]

$$\phi_m(\mathbf{x}) = \frac{\mu_0}{4\pi}\frac{q_m}{r} \tag{17}$$

where the magnetic field $\mathbf{B}$ is given by

$$\mathbf{B} = -\nabla\phi_m \tag{18}$$

Nonetheless, this is absurd because the electromagnetic vorticity vector field $\mathbf{B}$ has to be always represented by the curl of the electromagnetic velocity vector $\mathbf{A}$. Therefore, magnetic monopoles cannot exist within the classical theory. Perhaps it should be noted that no one has ever found a magnetic monopole in nature.

Consequently, we should realize that the magnetic field $\mathbf{B}$ is only generated by moving electric charges. Furthermore, it should be noted that there is no claim about the existence of the point source for the analogous fluid vorticity $\boldsymbol{\zeta}$.

It has been long speculated that magnetic monopoles might not exist, because there is no complete symmetry between $\mathbf{B}$ and $\mathbf{E}$. This is due to the fact that $\mathbf{B}$ is a pseudo or axial vector, but $\mathbf{E}$ is a real or polar vector. What we have here is the confirmation of this correct speculation that there is no duality between $\mathbf{E}$ and $\mathbf{B}$ in electrodynamics. The magnetic field $\mathbf{B}$ has the character of a circular vorticity field and is divergence free. Therefore, the magnetic field lines cannot intersect each other.

Interestingly, we have realized that the electric field $\mathbf{E}$ has the character of a hyperbolic vorticity with electric charges as its sources in Gauss' law (5).

As mentioned, the electric charge q of a particle has the property of a kinematical coupling, which maps the four-dimensional electromagnetic vorticity $\mathbf{F}$ at the position of the particle to the angular velocity $\boldsymbol{\Omega}$ of its body frame, as defined in (14). We have shown that the electric charge is the only possible coupling and there is no need for any other coupling.

6. Dirac magnetic monopole

It seems that the concept of magnetic monopole is the result of the apparent similarity of magnetic field of a thin solenoid (magnet) and electric field of an electric dipole. Consider a very thin solenoid with length L and uniform magnetic moment per unit length M , which is placed along the z-axis as shown in Figure 1.

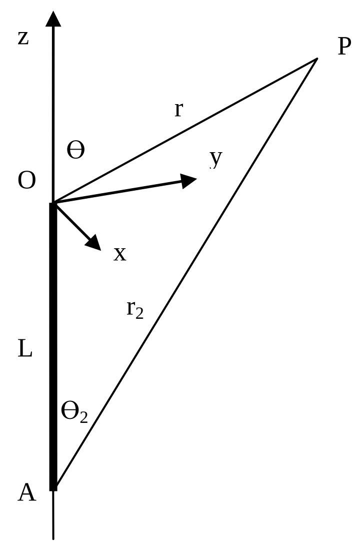

Figure 1

The magnetic field is given by[13,14)]

$$\mathbf{B}(\mathbf{x}) = \frac{\mu_0}{4\pi}\frac{M}{r^2}\hat{\mathbf{r}} - \frac{\mu_0}{4\pi}\frac{M}{r_2^2}\hat{\mathbf{r}}_2 \qquad \mathbf{x} \notin OA \tag{19}$$

This may appear as if there are two point magnetic poles with charges q_m, and $-q_m$ at points O and A, where

$$q_m = M \tag{20}$$

This is only an interesting mathematical result governing the physical phenomenon. The apparent similarity to an electric dipole should not be misleading. We should realize that the magnetic field $\mathbf{B(x)}$ on the axis of the solenoid is not defined by (19). This field can only be represented by the vector potential

$$\mathbf{A}(\mathbf{x}) = \frac{\mu_0}{4\pi}\frac{q_m}{r\sin\theta}(\cos\theta_2 - \cos\theta)\hat{\boldsymbol{\varphi}} \qquad \mathbf{x} \notin OA \tag{21}$$

where

$$\mathbf{B}(\mathbf{x}) = \nabla \times \mathbf{A}(\mathbf{x}) \tag{22}$$

We have emphasized that the vector potential **A** and the magnetic field **B(x)** are not defined on the axis of the solenoid in (21).

Interestingly, we notice that for the case of a semi-infinite solenoid (magnet), where $L \to \infty$ ($\theta_2 \to 0$), we have

$$\mathbf{A}(\mathbf{x}) = \frac{\mu_0}{4\pi}\frac{q_m(1-\cos\theta)}{r\sin\theta}\hat{\boldsymbol{\varphi}} \qquad \theta \neq \pi \tag{23}$$

and

$$\mathbf{B(x)} = \frac{\mu_0}{4\pi}\frac{q_m}{r^2}\hat{\mathbf{r}} \qquad \theta \neq \pi \tag{24}$$

As before $\mathbf{A}(\mathbf{x})$ and $\mathbf{B}(\mathbf{x})$ are not defined along the negative z -axis. This is because the magnet or solenoid is laid on this axis, which represents the distribution of the source current. Strangely enough, vector fields have been used by Dirac to represent a monopole field.[5)] However, a real monopole should generate an isotropic spherical field

$$\mathbf{B(x)} = \frac{\mu_0}{4\pi}\frac{q_m}{r^2}\hat{\mathbf{r}} \tag{25}$$

which is not equivalent to (24). As mentioned before, $\mathbf{B}$ can only be represented by the scalar potential

$$\phi_m(\mathbf{x}) = \frac{\mu_0}{4\pi}\frac{q_m}{r} \tag{17}$$

repeated here for clarity.

Attempting to use the results for a semi-infinite solenoid in (23) and (24) to represent the field of a magnetic monopole is obviously not valid. These results are based on the current generating magnetostatics,[13,14)] which has nothing to do with a pure monopole. Dirac's derivation[5)] is based on the single-valuedness of the wave function of an interacting electric charge around an artificial semi-infinite solenoid, called a 'Dirac string' attached to the magnetic monopole. Interestingly, we have shown that the Hamiltonian for the system of an electric point charge interacting with a fixed magnetic monopole does not exist.[4)] The line of singularity has a physical meaning for the solenoid (magnet), but it has been artificially created for the monopole by using the irrelevant vector potential (23). It should also be noticed that the Dirac derivation is essentially based on the same ideas related to the gauge transformations in quantum mechanics to explain the Aharonov-Bohm effect, which involves an infinite solenoid (magnet).[15)]

7. Conclusion

The vortex theory of electromagnetism gives a clear geometrical explanation of electrodynamics, which enables us to resolve decisively some existing ambiguities, such as the speculation about the magnetic monopole. The circular vortical character of the magnetic field shows that a magnetic monopole cannot exist. Therefore, electric charges are the only source of the electromagnetic field. It is naïve to assume that a simplistic modification of Maxwell's equations suffice to allow the existence of the magnetic charges in electrodynamics.

References

1) D. Zwanziger, Phys. Rev. B **137**, 647 (1965).

2) S. Weinberg, Phys. Rev. B **138**, 988 (1965).

3) C.R. Hagen, Phys. Rev., B **140**, 804 (1965).

4) A. R. Hadjesfandiari, Inconsistency of magnetic monopole, http://vixra.org/abs/1210.0132 .

5) P. A. M. Dirac, Proc. R. Soc. London **A133**, 60 (1931).

6) J. Schwinger, Science **165**, 757 (1969).

7) A. R. Hadjesfandiari, New perspectives in electrodynamics and geometry of space-time, http://imechanica.org/files/Geom_Electrodynamics_0.pdf, (2010).

8) A. R. Hadjesfandiari, New perspectives on the classical theory of motion, interaction and geometry of space-time, http://www.gsjournal.net/old/ntham/hadjes.pdf , (2010), http://vixra.org/abs/1011.0058, (2010).

9) A. R. Hadjesfandiari, Vortex theory of electromagnetism and its non-Euclidean character, http://imechanica.org/files/Electro_Vortex_IMECHANICA.pdf. (2012)

10) G. K. Batchelor, *An Introduction to Fluid Dynamics* (Cambridge Mathematical Library series, Cambridge University Press, 1967).

11) J. D. Jackson, *Classical Electrodynamics* (John Wiley, New York, 1999).

12) D. F. Griffiths, *Introduction to Electrodynamics*, 2nd ed. (Prentice Hall, New Jersey, 1989).

13) A. R. Hadjesfandiari, Character of the magnetic monopole, International Journal of Materials and Product Technology, **34**, 37-53 (2009).

14) A. R. Hadjesfandiari, Field of the magnetic monopole. http://arxiv.org/ftp/physics/papers/0701/0701232.pdf. (2007).

15) Y. Aharonov, and D. Bohm, Phys. Rev. 115, 485 (1959).