

# Quark Masses in the Planck Model

Bernard Riley<sup>1</sup>

The mass of each up-type quark, like the mass of the electron, is simply related to the Planck Mass. Since the quarks of each generation are symmetrically arranged about prominent mass levels in the Planck Model, the mass of each down-type quark may be calculated. The quark masses of the model are 2.18 MeV (up), 4.89 MeV (down), 96.1 MeV (strange), 1.27 GeV (charm), 4.28 GeV (bottom) and 174 GeV (top). The model explains why in generations 2 and 3 the up-type quark is heavier than its down-type partner.

## 1 Introduction

The Bohr radius,  $a_0 = 0.529$  nm, has been shown [1] to be simply related to the Planck Length,  $l_{Planck} = 1.616199(97) \times 10^{-35}$  m [2]:

$$a_0 = \left(\frac{\pi}{2}\right)^{125} l_{Planck} (= 0.529 \text{ nm}) \quad (1)$$

For the electron mass,  $m_{electron} = 0.511$  MeV, one may immediately write:

$$m_{electron} = \alpha^{-1} \left(\frac{\pi}{2}\right)^{-125} m_{Planck} (= 0.511 \text{ MeV}) \quad (2)$$

where  $\alpha$  is the fine structure constant and  $m_{Planck}$  is the Planck Mass,  $1.220932(73) \times 10^{19}$  GeV [2]. Several other scales of atomic and particle physics have been found to be related to Planck scale through multiplication by powers of  $(\pi/2)^{25}$  and  $\alpha$  [1]. For example, the GUT scale of  $2.1 \times 10^{16}$  GeV is given by:

$$m_{GUT} = \alpha^{-1} \left(\frac{\pi}{2}\right)^{-25} m_{Planck} (= 2.09 \times 10^{16} \text{ GeV}) \quad (3)$$

For the Hartree energy,  $E_h = 27.2$  eV, one may write, on the basis of (1):

---

<sup>1</sup> bernardriley@hotmail.co.uk

$$E_h = \alpha \left(\frac{\pi}{2}\right)^{-125} E_{Planck} (= 27.2 \text{ eV}) \quad (4)$$

Since the up and top quark masses have been found to be simply related in scale to the Hartree energy [1], equations were sought within the Planck Model for the masses of the other quarks.

## 2 The Quark Masses

The masses of the up-type quarks are given by:

$$m_{up} = \alpha \left(\frac{\pi}{2}\right)^{-100} m_{Planck} (= 2.177 \text{ MeV}) \quad (5)$$

$$m_{charm} = \alpha^2 \left(\frac{\pi}{2}\right)^{-75} m_{Planck} (= 1.271 \text{ GeV}) \quad (6)$$

$$m_{top} = \alpha \left(\frac{\pi}{2}\right)^{-75} m_{Planck} (= 174.1 \text{ GeV}) \quad (7)$$

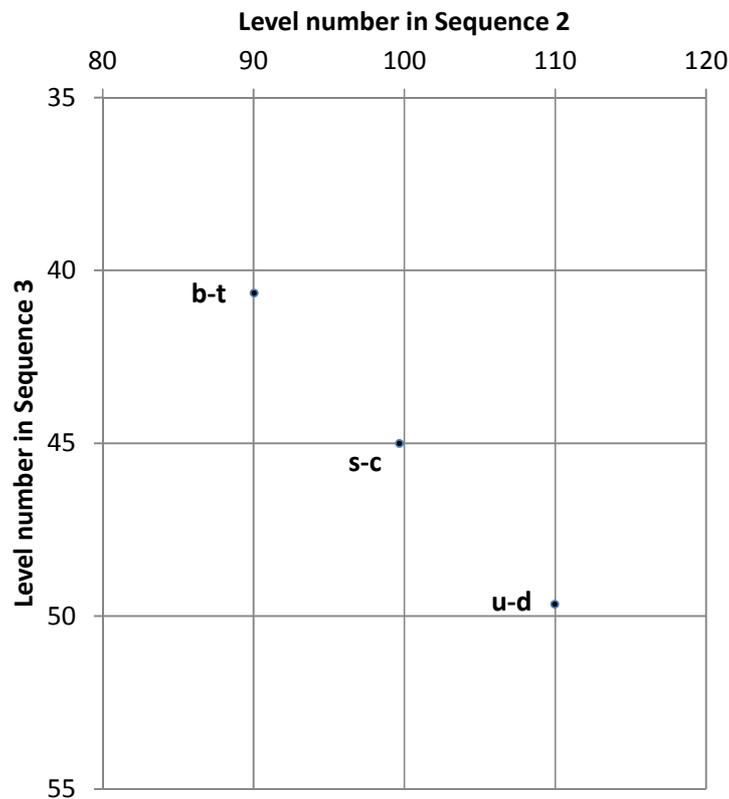
The most recent Particle Data Group evaluations of quark mass are shown in Table 1.

Quark	Mass
up	$2.3^{+0.7}_{-0.5} \text{ MeV}$
down	$4.8 \pm_{-0.3}^{+0.5} \text{ MeV}$
strange	$95 \pm 5 \text{ MeV}$
charm	$1.275 \pm 0.025 \text{ GeV}$
bottom	$4.18 \pm 0.03 \text{ GeV}$
top	$173.07 \pm 0.52 \pm 0.72 \text{ GeV}$

**Table 1:** Quark mass evaluations of the Particle Data Group [3]

In the Planck Model, particles occupy mass levels that descend in geometric sequence from the Planck Mass [4, 5, 6]. There are three sequences: Sequence 1 of common ratio  $1/\pi$ ,

Sequence 2 of common ratio  $2/\pi$  and Sequence 3 of common ratio  $1/e$ . A level of number  $n$  within a sequence of common ratio  $r$  is of mass  $m_P r^n$ . Mass levels are of integer and fractional number. Singlet states lie upon mass levels, while isospin doublets are arranged symmetrically about levels. The quarks of each generation are arranged symmetrically about levels of integer number within Sequences 2 and 3; the integers,  $n_2$  and  $n_3$ , are multiples of 5. The level numbers  $n_2$  and  $n_3$  calculated for the geometric mean of each generation's quark masses are shown in Figure 1. The markers are constrained to lie on a straight line since  $n_2$  and  $n_3$  are in constant ratio.



**Figure 1:** Quark doublets on the mass levels of Sequences 2 and 3. Sequences 2 and 3 descend in geometric sequence from the Planck Mass with common ratios  $2/\pi$  and  $1/e$ , respectively. The doublets are represented by the geometric mean of the two central values of Particle Data Group quark mass evaluation [3]). Marker size is an indication of uncertainty.

From Figure 1, it is clear that the up and down quarks are arranged symmetrically about Level 110 in Sequence 2. The mass of the down quark is then given by the square of the Level 110 mass divided by the up quark mass given by (5):

$$m_{down} = \alpha^{-1} \left(\frac{\pi}{2}\right)^{-120} m_{Planck} (= 4.888 \text{ MeV}) \quad (8)$$

The strange and charm quarks are arranged symmetrically about Level 45 in Sequence 3. The mass of the strange quark is then given by the square of the Level 45 mass divided by the charm quark mass given by (6):

$$m_{strange} = \alpha^{-2} e^{-90} \left(\frac{\pi}{2}\right)^{75} m_{Planck} (= 96.13 \text{ MeV}) \quad (9)$$

The bottom and top quarks are arranged symmetrically about Level 90 in Sequence 2. The mass of the bottom quark is then given by the square of the Level 90 mass divided by the top quark mass given by (7):

$$m_{bottom} = \alpha^{-1} \left(\frac{\pi}{2}\right)^{-105} m_{Planck} (= 4.275 \text{ GeV}) \quad (10)$$

The quark masses of the Planck Model are shown in Table 2.

Quark	Mass $\div m_{Planck}$	Mass
up	$\alpha \left(\frac{\pi}{2}\right)^{-100}$	2.177 MeV
down	$\alpha^{-1} \left(\frac{\pi}{2}\right)^{-120}$	4.888 MeV
strange	$\alpha^{-2} e^{-90} \left(\frac{\pi}{2}\right)^{75}$	96.13 MeV
charm	$\alpha^2 \left(\frac{\pi}{2}\right)^{-75}$	1.271 GeV
bottom	$\alpha^{-1} \left(\frac{\pi}{2}\right)^{-105}$	4.275 GeV
top	$\alpha \left(\frac{\pi}{2}\right)^{-75}$	174.1 GeV

**Table 2:** The quark masses of the Planck Model

### 3 Discussion

The Planck Model was based on the premise that the individual quarks should occupy mass levels within a geometric sequence that descends from the Planck Mass [4]. To identify the sequence, the occupation by hadrons of mass sublevels<sup>2</sup> was sought, and found for singlet states. Isospin doublets were found to be arranged symmetrically about mass sublevels. The quarks, as weak isospin doublets, have now been found to be arranged similarly to the hadronic isospin doublets, but about mass levels whose level-numbers are integers and multiples of 5. Individual quarks do not occupy mass levels of integer level-number.

The masses of the up-type quarks are related to Planck scale through multiplication by powers of  $(\pi/2)^{25}$  and  $\alpha$ , as are various essential scales of atomic and particle physics. Since the quarks of each generation take up a symmetrical arrangement about a mass level that falls within a network, as shown in Figure 1, whether the up-type quark or the down-type quark of a generation is the heavier seems to be accidental.

Mass levels descend from the Planck Mass and may derive from the locations of boundaries within an extra-dimensional geometry by way of exponential factors of distance from the Planck brane. Singlet states live on boundaries. Doublets, formed by the breaking of symmetries, are arranged symmetrically about boundaries; the masses of the partners are arranged symmetrically about mass levels.

The factor  $(\pi/2)^{25}$  may derive from the length,  $\pi/2$ , of an  $S^1/(Z_2 \times Z_2')$  orbifold with Planck-scale compactification radii [1, 6]. The factor  $\alpha$  may also derive from an extra-dimensional length [7].

### 4 References

1. Riley BF, viXra:1305.0061
2. CODATA 2010
3. J Beringer et al. (Particle Data Group), Phys. Rev. **D86**, 010001 (2012) and 2013 partial update for the 2014 edition
4. Riley BF, arXiv:physics/0306098
5. Riley BF, arXiv:physics/0509076
6. Riley BF, arXiv:0809.0111
7. Riley BF, viXra:1301.0087

---

<sup>2</sup> Levels of fractional level-number