Emergence of the Electroweak Scale from Fractal Spacetime

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Abstract

As of today, neither one of the mass generation mechanisms of the Standard Model (SM) can convincingly explain the source of the electroweak scale. In this brief report we argue that the onset of fractal spacetime near the electroweak interaction provides a natural motivation for the emergence of this scale.

Key words: Electroweak scale, Fractal spacetime, Cantor set, Vacuum expectation value, Higgs boson, Dimensional Regularization.

1. Introduction

There are two primary mass-generation mechanisms in the SM: the *Higgs mechanism* of electroweak (EW) symmetry breaking, accounting for the spectrum of massive gauge bosons and fermions, and *dimensional transmutation*, partially responsible for the mass of baryonic matter [1-2]. While technical aspects of both mechanisms are well under control, neither one is able to uncover the origin of the EW scale *or* of the Higgs boson mass [3, 16]. Building on previous contributions, currently outside the mainstream research [4-12], this brief report hints that the EW scale follows from the principles of dimensional regularization and the onset of fractal spacetime near or above the EW interaction. We base our approach on the idea that, near or above this scale, spacetime dimension turns into a *continuous variable*. This transition is driven by a parameter describing arbitrarily small deviations from the four-dimensionality of ordinary spacetime ($d_0 = 4$), i.e.

$$\varepsilon = 4 - d_0\,,\ \varepsilon << 1 \tag{1}$$

The conceptual benefits of this viewpoint are discussed in [4-7]. In particular, spacetime with low-level fractality ($\varepsilon << 1$) favors the formation of a weakly coupled *Higgs-like condensate* of gauge bosons, as in

$$\Phi_C = \tfrac{1}{4}[(W^+ + W^- + Z^0 + \gamma + g) + (W^+ + W^- + Z^0 + \gamma + g)] \tag{2}$$

Here, $W^{\pm}, Z^0$ denote the triplet of massive $SU(2)$ bosons and g, γ stand for gluon and photon, respectively. Relation (2) implies that the scalar condensate Φ_C acquires a mass in close agreement with the mass of the SM Higgs boson ($m_H = 125.6\,GeV$).

Our short report has four sections. Next section covers a brief introduction to the geometry of fractal sets. The derivation of the EW scale is developed in the third section and results are summarized in the last section. Needless to say, our findings are strictly preliminary and in need for independent validation.

2. A primer of fractals and multifractals

To make the paper self-contained, we highlight here few basic concepts and terminology pertaining to fractals and multi-fractals. Fractals are geometrical objects with non-integer dimensions that display self-similarity on all scales of observation [14-15]. The concept of *dimension* plays a key role in the geometry of fractal sets. It is customary to characterize fractals by an ensemble of three dimensions, namely:

1) The Euclidean dimension "$d = 1, 2, 3...$" represents the dimension of the space where the object resides and is always an integer.

2) The topological dimension "$d_T \leq d$" describes the dimensionality of continuous primitive objects such as points, curves, surfaces or volumes ($d_T = 0, 1, 2, 3$ in ordinary four-dimensional spacetime).

3) The definition of the fractal (or *Hausdorff*) dimension is as follows: Cover the fractal object by $d-$dimensional balls of radius "Δ" and let "$N(\Delta)$" be the minimum number of balls needed for this operation. The fractal dimension "D" satisfies the inequality $d_T \leq D \leq d$ and is given by

$$\lim_{\Delta \to 0} N(\Delta) = \Delta^{-D} \tag{3}$$

leading to

$$D = \lim_{\Delta \to 0} [\frac{\log N(\Delta)}{\log \Delta^{-1}}] \tag{4}$$

Many of the self-similar structures of fractal geometry are built recursively, a typical example being the *Cantor set*. To construct a Cantor set in one dimension ($d = 1$), take a line segment called the *generator*, split it into thirds and remove the middle third. Iterate this process arbitrarily many times. One is left with a countable set of isolated points having a non-integer fractal dimension D, with $d_T = 0 \leq D \leq d = 1$. A simple Cantor set generated from segments of equal length is defined by a single scaling factor $r = 1/3 < 1$. By contrast, more general fractals (such as *multifractals*) can be created using generator segments of different scaling factors $r_i < 1$, $i = 1, 2, ..., N$ satisfying the closure relation

$$\boxed{\sum_{i=1}^{N} r_i^D = 1} \tag{5}$$

Many strange attractors of nonlinear dynamical systems represent multifractals and are typically characterized by a continuous spectrum of Hausdorff dimensions [15].

3. Derivation of the EW scale

As pointed out in the Introduction, the emergence of fractal geometry in QFT arises from the approach to criticality near the EW scale. The most reliable description of criticality is through the tools of the Renormalization Group program (RG), in general, and dimensional regularization, in particular [14-15]. A key parameter of the RG is the dimensionless ratio (μ/Λ_{UV}), in which μ is the sliding scale and $\Lambda_{UV} >> \mu$ the high-energy cutoff of the underlying theory. With reference to a field theory embedded in four dimensions $(d_0 = 4)$, the connection between the parameter $\varepsilon = 4 - d_0$ and Λ_{UV} is given by [4, 6]

$$\varepsilon \sim \frac{1}{\log(\frac{\Lambda_{UV}^2}{\mu^2})} \tag{6}$$

The large numerical disparity between μ and Λ_{UV} enables one to redefine ε' as in

$$\varepsilon' \sim (\frac{\mu}{\Lambda_{UV}})^2 \quad (7)$$

Furthermore, to highlight the contribution of the electroweak scale (M_{EW}), we choose to recast (7) in the following form

$$\varepsilon'' \sim \varepsilon' \cdot (\frac{\Lambda_{UV}}{M_{EW}})^2 \sim (\frac{\mu}{M_{EW}})^2 \quad (8)$$

under the assumption that $\mu < M_{EW}$. Relation (8) brings up a natural analogy between ε'' and the scaling factor r previously introduced in connection with the construction of the one-dimensional Cantor set. In light of this analogy we opt for $\varepsilon'' = r$, take D to lie close to its upper limit ($D \approx 1$) and assign the spectrum of RG scales (μ) to the particle content of the SM. By (5), these steps lead to

$$\boxed{(\frac{m_H}{M_{EW}})^2 + (\frac{m_W}{M_{EW}})^2 + (\frac{m_Z}{M_{EW}})^2 + \sum_f (\frac{m_f}{M_{EW}})^2 = 1} \quad (9)$$

where the sum in the left-hand side of (9) extends over all SM fermions (leptons and quarks). From (2) and (9) one obtains

$$\boxed{M_{EW} \sim \mathrm{v} = 246.2\ GeV} \quad (10)$$

in good agreement with the vacuum expectation value of the SM Higgs boson (v) [3, 13]. In closing, we mention that the numerical relation (10) was first brought up in [17], with no attempt of formulating a theoretical interpretation.

4. Summary

Our brief report suggests that the EW scale is a direct outcome of fractal spacetime equipped with arbitrarily small deviations from four-dimensions. By analogy with one-dimensional Cantor sets, massive SM fields can be interpreted as *primary generators* involved in the construction of such sets.

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