Quaternions , Spaces , and The Parallel Postulate .

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Abstract In the manuscript is proved that parallel postulate is only in Plane (three points only) and is based on the four Postulates for Constructions, where all properties of Euclidean geometry compactly exist as Extrema on points, lines, planes, circles and spheres. Projective, Hyperbolic and Elliptic geometry is proved to be an Extrema (deviations) in Euclidean geometry where on them Einstein's theory of general relativity is implicated approximately to the properties of physical space . Now is presented that all , Spaces , Anti-Spacs and Sub-Spaces are complex numbers , i.e. quaternions as $\rightarrow \pm d\hat{s} = (x+i.y)^{W}$, and the ± Sub-spaces as $\rightarrow d\hat{s} = (x+i.y)^{1/W}$, all rotated in (PNS) as [$\mathbf{d\hat{s} = x+\nabla i.y = q^{w}}$] and are mutually exclusive and simultaneously Identical , **m^±(ª+ᵭ.∇i) = q^w,** and they all carry intrinally the known conservation laws of the two components . A compact proof of applications to Euclidean spaces is described . The article has been sent for critique and Publication without any sesult , so my decision is this presentation .

Keywords: *Parallel Postulate, Euclid Geometry*

1. Introduction

Euclid’s elements consist of assuming a small set of intuitively appealing axioms and from them, proving many other propositions (theorems). Although many of Euclid’s results have been stated by earlier Greek mathematicians, Euclid was the first to show how these propositions could be fit together into a comprehensive deductive and logical system self consistent. Because nobody until now succeeded to prove the parallel postulate by means of pure geometric logic and under the restrictions imposed to seek the solution , many self consistent non-Euclidean geometries have been discovered based on Definitions, Axioms or Postulates, in order that non of them contradicts any of the other postulates of what actually are or mean .In the manuscript is proved that parallel postulate is only in Plane (three points only) and is based on the four Postulates for Constructions, where all properties of Euclidean geometry compactly exist as Extrema on points, lines, planes, circles and spheres. Projective, Hyperbolic and Elliptic geometry is proved to be an Extrema (deviations) [16] in Euclidean geometry where on them Einstein's theory of general relativity is implicated and calls a segment as line and the disk as plane in physical space. It was shown that the only Space-Energy geometry is Euclidean , on primary and on any vector unit AB, (AB = The Quantization of points and of Energy on AB vector) on the contrary to the general relativity of Space-time which is based on the rays of the non-Euclidean geometries and to the limited velocity of light . Euclidean geometry describes Space-Energy beyond Plank´s length level and also in its deviations which are described as Space-time in Plank´s length level.Quantization is holding only on points and Energy [Space-Energy], where Time is vanished [21], and not on points and Time [Space-time] which is the deviation of Euclidean geometry .

2. Euclid Elements for a Proof of the Parallel Postulate (Axiom)

Axiom or Postulate is a statement checked if it is true and is ascertained with logic (the experiences of nature as objective reality).
Theorem or Proposition is a non-main statement requiring a proof based on earlier determined logical properties.
Definition is an initial notion without any sensible definition given to other notions.
Definitions , Propositions or Postulates created Euclid geometry using the geometrical logic which is that of nature , the logic of the objective reality .
Using the same elements it is posiible to create many other geometries but the true uniting element is the before refereed.

2.1. The First Definitions (D) Of Terms in Geometry

D1: A point is that which has no part (Position)
D2: A line is a breathless length (for straight line, the whole is equal to the parts)
D3: The extremities of lines are points (equation).
D4: A straight line lies equally with respect to the points on itself (identity).
D: A midpoint C divides a segment AB (of a straight line) in two. CA = CB any point C divides all straight lines through this in two.
D: A straight line AB divides all planes through this line in two.
D: A plane ABC divides all spaces through this in two

2.2. Common Notions (Cn)

Cn1: Things which equal the same thing also equal one another.
Cn2: If equals are added to equals, then the wholes are equal.

Cn3: If equals are subtracted from equals, then the remainders are equal.
Cn4: Things which coincide with one another, equal one another.
Cn5: The whole is greater than the part.

2.3. The five postulates (P) for construction

P1. To draw a straight line from any point A to any other B.
P2. To produce a finite straight line AB continuously in a straight line.
P3. To describe a circle with any centre and distance. P1, P2 are unique.
P4. That, all right angles are equal to each other.
P5. That, if a straight line falling on two straight lines make the interior angles on the same side less than two right angles, if produced indefinitely, meet on that side on which are the angles less than the two right angles, or (for three points on a plane)
5a. The same is plane's postulate which states that, from any point M, not on a straight line AB, only one line MM′ can be drawn parallel to AB.
Since a straight line passes through two points only and because point M is the third then the parallel postulate it is valid on a plane (three points only).

3. The Method

AB is a straight line through points A, B and M any other point .When MA+MB > AB then point M is not on AB. (differently if MA+MB = AB then this answers the question of why any line contains at least two points i.e. for any point M on line AB where is holding MA+MB = AB / meaning that lines MA , MB coincide on AB / is thus proved from the other axioms and so D2 is not an axiom).

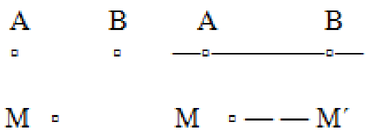


Fig. 0. Any line contains at least two points

To prove that, one and only one line MM′ can be drawn parallel to AB.

To prove the above Axiom is necessary to show:

a. The parallel to AB is the locus of all points at a constant distance **h** from the line AB, and for point M is MA1,
b. The locus of all these points is a straight line.

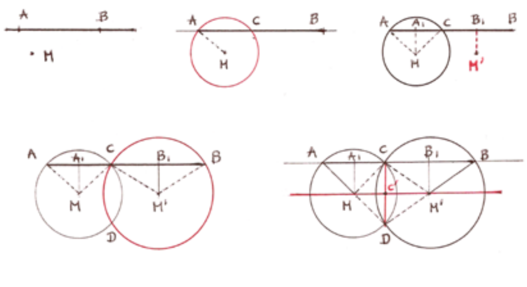


Fig. 1. The Method for proofing one and only one line MM′ can be drawn parallel to AB

Step 1
Draw the circle (M, MA) be joined meeting line AB in C. Since MA = MC, point M is on mid-perpendicular of AC. Let A1 be the midpoint of AC, (it is A1A+A1C = AC because A1 is on the straight line AC. Triangles MAA1, MCA1 are equal because the three sides are equal, therefore angle < MA1A = MA1C (CN1) and since the sum of the two angles < MA1A+MA1C = 180° (CN2, 6D) then angle < MA1A = MA1C = 90 °.(P4) so , MA1 is the minimum fixed distance **h** of point M to AC.

Step 2

Let B1 be the midpoint of CB,(it is B1C+B1B = CB because B1 is on the straight line CB) and draw B1M′ = h equal to A1M on the mid-perpendicular from point B1 to CB. Draw the circle (M′, M′B = M′C) intersecting the circle (M, MA = MC) at point D.(P3) Since M′C = M′B, point M′ lies on mid-perpendicular of CB. (CN1)
Since M′C = M′D, point M′ lies on mid-perpendicular of CD. (CN1) Since MC = MD, point M lies on mid-perpendicular of CD. (CN1) Because points M and M′ lie on the same mid-perpendicular (This mid-perpendicular is drawn from point C′ to CD and it is the midpoint of CD) and because only one line MM′ passes through points M , M ′ then line MM′ coincides with this mid-perpendicular (CN4)

Step 3

Draw the perpendicular of CD at point C′. (P3, P1)

a. Because MA1 ⊥ AC and also MC′ ⊥ CD then angle < A1MC′ = A1 CC′. (Cn 2, Cn3, E.I.15) . Because M′B1 ⊥ CB and also M′C′⊥ CD then angle < B1M′C′ = B1CC′. (Cn2, Cn3, E.I.15)
b. The sum of angles A1CC′ + B1CC′ = 180° = A1MC′ + B1M′C′. (6.D), and since Point C′ lies on straight line MM′, therefore the sum of angles in shape A1B1M′M are < MA1B1 + A1B1M ′ + [B1M′ M + M′MA1] = 90° + 90° + 180° = 360 ° (Cn2) , i.e.
c. The sum of angles in a Quadrilateral is 360 and in Rectangle all equal to 90 °. (m)
d. The right-angled triangles MA1B1, M′B1A1 are equal because A1M = B1M′ and A1B1 common, therefore sid e A1 M = B1 M (Cn1) . Triangles A1MM′,B1M′M are equal because have the three sides equal each other, therefore angle < A1MM′ = B1M′M, and since their sum is 180° as before (6 D), so angle < A1MM′ = B1M′M = 90° (Cn2).
e. Since angle < A1MM′ = A1CC′ and also angle < B1M′M = B1CC′ (P4), therefore quadrilaterals A1CC′M , B1CC′M′, A1B1M′M are Rectangles (CN3). From the above three rectangles and because all points (M , M′ and C′) equidistant from AB, this means that C′C is also the minimum equal distance of point C′ to line AB or, h = MA1 = M′ B1 = CD / 2 = C′C (Cn1) Namely, line MM′ is perpendicular to segment CD at point C′ and this line coincides with the mid-perpendicular of CD at this point C′ and points M , M′ ,C′ are on line MM′. Point C′ equidistant ,h, from line AB, as it is for points M ,M′, so the locus of the three points is the straight line MM′, so the two demands are satisfied , (h = C′C = MA1 = M′B1 and also C′C ⊥ AB , MA1 ⊥ AB , M′B1 ⊥ AB). **(o.ε.δ.)**

f. The right-angle triangles A1CM , MCC′ are equal because side MA1= C′C and MC common so angle <A1CM = C′MC , and the Sum of angles C′MC + MCB1 = A1CM + MCB1 =180° as was asked in P5.

3.1. The Succession of Proofs

1. Draw the circle (M , MA) be joined meeting line AB in C and let A1, B 1 be the midpoint of CA , CB.
2. On mid-perpendicular B1M′ find point M′ such that M′B1 = MA1 and draw the circle (M′, M′B = M′C) intersecting the circle (M , MA = MC) at point D.
3. Draw mid-perpendicular of CD at point C′.
4. To show that line MM′ is a straight line passing through point C ′ and it is such that MA1 = M′B1 = C′C = h , i.e. a constant distance **h** from line AB or, also The Sum of angles C′MC + MCB1 = A1CM + MCB1 = 180 °

3.2. Proofed Succession

1. The mid-perpendicular of CD passes through points M , M ′.
2. Angle < A1MC′ = A1 MM′ = A1CC′, Angle < B1M′C′ = B1M′M = B1CC′ <A1MC′ =A1CC′ because their sides are perpendicular among them i.e. MA1┴CA,MC′┴CC′.
   a. In case < A1MM′ +A1CC′=180° and B1M′M +B1CC′ = 180° then angle <A1MM′ = 180° - A1CC′, B1M′M = 180° - B1CC′, and by summation < A1MM′ + B1M′M = 360° - A1CC′- B1CC′ or the Sum of angles < A1MM′ + B1M′M = 360 - (A1CC′ + B1CC′) = 360 -180° = 180°
3. The sum of angles A1 MM′ + B1 M′M = 180° because the equal sum of angles A1CC′ + B1CC′ = 180° , so the sum of angles in quadrilateral MA1B1 M′ is equal to 360°.
4. The right-angled triangles MA1B1, M′B1A1 are equal , so diagonal MB1 = M′A1 and since triangles A1MM′, B1M′M are equal , then angle A1MM′ = B1M′M and since their sum is 180° , therefore angle < A1MM′ = MM′B1 = M′B1A1 = B1A1M = 90 °
5. Since angle A1CC′ = B1CC′ = 90° , then quadrilaterals A1CC′M, B1CC′M′ are rectangles and for the three rectangles MA1CC′, CB1M′C′, MA1B1M′ exists MA1 = M′B1 = C′C
6. The right-angled triangles MCA1 , MCC′ are equal , so angle < A1CM = C′MC and since the sum of angles < A1CM + MCB1 = 180° then also C′MC + MCB1 = 180 ° →

which is the second to show, as this problem has been set at first by Euclid.

All above is a Proof of the Parallel postulate due to the fact that the parallel postulate is dependent of the other four axioms (now is proved as a theorem from the other four).Since AB is common to ∞ Planes and only one Plane is passing through point M (Plane ABM from the three points A, B, M, then the Parallel Postulate is valid for all Spaces which have this common Plane, as Spherical, n-dimensional geometry Spaces . It was proved that it is a necessary logical consequence of the others axioms, agree also with the Properties of physical objects , d + 0 = d and d * 0 = 0 , now is possible to decide through mathematical reasoning , that the geometry of the physical universe is Euclidean. Since the essential difference between Euclidean geometry and the two non-Euclidean geometries , Spherical and hyperbolic geometry, is the nature of parallel line , i.e. the parallel postulate so ,
<<The consistent System of the - non-Euclidean geometry - have to decide the direction of the existing mathematical logic >> . On this , I appreciate any crique .
The above consistency proof is applicable to any line Segment AB on line AB , (segment AB is the first dimentional unit , as AB = 0 → ∞), from any point M not on line AB, [MA + MB > AB for three points only which consist the Plane. For any point M between points A , B is holding MA+MB = AB i.e. from two points M , A or M,B passes the only one line AB . A line is also continuous (P1) with points and discontinuous with segment AB [14] which is the metric defined by non-Euclidean geometries] and it is the answer to the cry about the < crisis in the foundations of Euclid geometry > (F.2)

3.3. A line contains at least two points , is not an axiom because is proved as theorem

Definition D2 states that for any point M on line AB is holding MA+MB = AB which is equal to < segment MA + segment MB is equal to segment AB > , i.e. the two lines MA,MB coincide on line AB and thus this postulate is proved also from the other axioms , thus D2 is not an axiom , which form a system self consistent with its intrinsic real-world meaning . F.0

4. The Types of Geometry

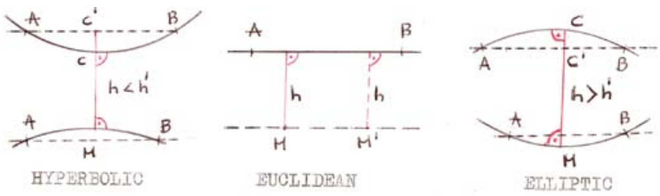


Fig. 2. (Hyperbolic Euclidean Elliptic)
The structure of Euclidean geometry

Any single point A constitutes a Unit without any Position and dimension (non-dimensional = Empty Space) simultaneously zero, finite and infinite. The unit meter of Point is equal to 0.
Any single point B , not coinciding with A , constitutes another one Unit which has also dimension zero. Only one Straight line (i.e. the Whole is equal to the Parts) passes through points A and B ,which consists another un-dimensional Unit,since is consisted of infinite points with dimension zero. A line Segment AB between points A and B (either points A and B are near zero or are extended to the infinite), consists the first Unit with one dimensional, the length AB, beginning from Unit A and a regression ending in Unit B . AB = 0→ ∞ , is the one-dimensional Space. The unit meter of AB is m = 2.(AB/2) = AB because only one middle point exists on AB and since also is composed of infinite points which are filling line, then nature of line is that of Point (the all is one for Lines and Points).

Adding a third point C , outside the straight line AB , (CA+CB >AB) , then is constituted a new Unit (the Plane) without position, since is consisted of infinite points, without any position. Shape ABC enclosed between parts AB, AC, BC is of two dimensional, the enclosed area ABC, and since is composed of infinite Straight lines which are filling Plane , then, nature of Plane is that of Line and that of Points (the all is one for Planes , Lines and Points) . Following the harmony of unit meter AB=AC=BC, then Area ABC = 0 → ∞, is the two-dimensional Space with unit meter equal to m = $2.(\pi.AB/\sqrt{2})^2 = \pi.AB^2$, i.e. one square equal to the area of the unit circle.
Four points A,B,C,D (....) not coinciding, consist a new Unit (the Space or Space Layer) without position also, which is extended between the four Planes and all included, forming Volume ABCD and since is composed of infinite Planes which are filling Space, then, nature of Space is that of Plane and that of Points (the all is one for Spaces, Planes, Lines and Points). Following the same harmony of the first Unit, shape ABCD is the Regular Tetrahedron with volume ABCD = 0 → ∞, and it is the three-dimensional Space.
The dimension of Volume is 4 - 1 = 3. The unit measure of volume is the side X of cube X^3 twice the volume of another random cube of side a = AB such as $X^3 = 2.a^3$ and $X = \sqrt[3]{2}.a$, i.e Geometry measures Volumes with side X related to the problem of doubling of the cube. In case that point D is on a lower Space Layer, then all Properties of Space, or Space Layer are transferred to the lower corresponding Unit, i.e. from volume to Plane from Plane to the Straight line and then to the Point. This Concentrated (Compact) Logic of geometry[CLG] exists for all Space – Layers and is very useful in many geometrical and physical problems. (exists, Quality = Quantity, since all the new Units are produced from the same, the first one , dimensional Unit AB).
N points represent the N-1 dimentional Space or the N-1 Space Layer ,DL, and has analogous properties and measures.Following the same harmony for unit AB, (AB = 0 → ∞) then shape ABC...M (i.e. the ∞ spaces AB = 1, 2,..nth) is the Regular Solid in Sphere ABC...M = 0 → ∞. This N Space Layer is limiting to ∞ as N → ∞. Proceeding inversely with roots of any unit AB = 0→ ∞ (i.e. the Sub-Spaces are the roots of AB, $\sqrt[2]{AB}$, $\sqrt[3]{AB}$,.. $\sqrt[n]{AB}$ then it is $\sqrt[n]{AB} = 1$ as n → ∞), and since all roots of unit AB are the vertices of the Regular Solids in Spheres then this n Space Layer is limiting to 0 as n → ∞ The dimensionality of the physical universe is unbounded (∞) but simultaneously equal to (1) as the two types of Spaces and Sub-Spaces show.
Because the unit-meters of the N-1 dimensional Space Layers coincide with the vertices of the nth-roots of the first dimensional unit segment AB as AB = ∞ → 0 ,which is point, (the vertices of the n-sided Regular Solids) , therefore the two Spaces are coinciding (the Space Layers and the Sub-Space Layers are in superposition on the same monads).[F.5]

That is to say, Any point on the Nth Space or Space-Layer, of any unit AB = 0→ ∞, jointly exists partly or whole, with all Subspaces of higher than N Spaces, N = (N+1) -1 = (N+2) -2 = (N+N)-N...= (N+∞) - ∞, where (N+1),…(N+∞) are the higher than N Spaces, and with all Spaces of lower than N Subspaces, N = (N-1) +1 = (N-2)+2 = (N-N)+N = (N -∞) +∞, where (N-1), (N-2), (N-N), (N-∞) are the lower than N Spaces.
The boundaries of N points, corresponding to the Space, have their unit meter of the Space and is a Tensor of N dimension (i.e. the unit meters of the N roots of unity AB), simultaneously, because belonging to the Sub-Space of the Unit Segments > N, have also the unit meter of all spaces. [F.5]

1. The Space Layers : (or the Regular Solids) with sides equal to AB ⇒ 0∞ The Increasing Plane Spaces with the same Unit. (F.3)

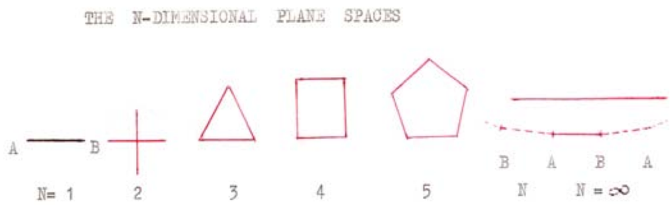


Fig. 3. The n-dimensional plane Spaces

2. The Sub-Space Layers : (or the Regular Solids on AB) as Roots of AB = 0 ∞ The Decreasing Plane Spaces with the same Unit. (F.4)

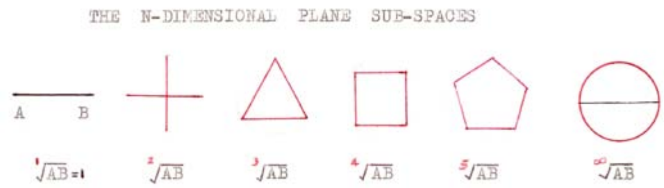


Fig. 4. The n-dimensional plane sub-Spaces

3. The superposition of Plane Space Layers and Sub-Space Layers : (F.5) The simultaneously co-existence of Spaces and Sub-Spaces of any Unit AB = 0→ ∞ , i.e. Euclidean, Elliptic, Spherical, Parabolic, Hyperbolic, Geodesics, metric and non-metric geometries have Unit AB as common.

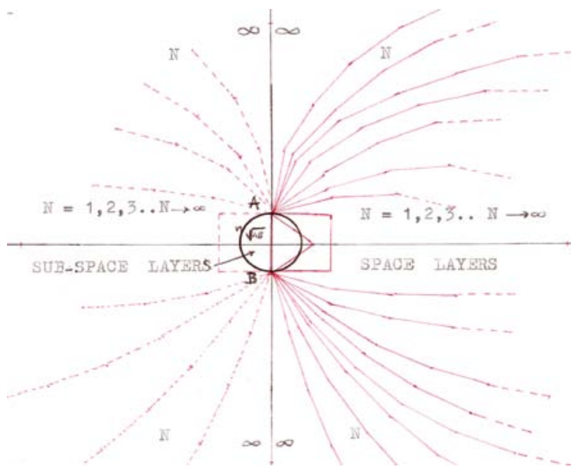


Fig. 5. The Interconnection of Homogeneous and Heterogeneous Spaces, and Subspaces of the Universe. [**dŝ = x+∇i.y**]

4. A linear shape is the shape with N points on a Plane bounded with straight lines. A circle is the shape on a Plane with all points equally distance from a fix point O. A curved line is the shape on a Plane with points not equally distance from a fix point O. Curved shapes are those on a Plane bounded with curved lines. Rotating all the above axial-centrifugally (machine AB$\perp$ AC) is obtained Flat Space, Conics, Sphere, Curved Space, multi Curvature Spaces, Curved Hyperspace etc. The fact that curvature changes from point to point, is not a property of one Space only but that of the common area of more than two Spaces, namely the result of the Position of Points. Euclidean manifold (Point, sectors, lines, Planes, all Spaces etc) and the one dimentional Unit AB is prooved to be the same thing (according to Euclid έν το πάν). [F.5]

Since Riemannian metric and curvature is on the great circles of a Sphere which consist a Plane, say AMA΄, while the Parallel Postulate is consistent with three points only, therefore the great circles are not lines (this is because is holding MA + MA΄ > AA΄) and the curvature of Space is that of the circle in this Plane , i.e. that of the circle (O, OA), which are more than three points . Because Parallel Axiom is for three points only, which consist a Plane, then the curvature of < empty space > is equal to 0, (has not metric or intrinsic curvature).

The physical laws are correlated with the geometry of Spaces and can be seen, using CLG, in Plane Space as it is shown in figures F3 - F5. A marvelous Presentation of the method can be seen on Dr Geo-Machine Macro-constructions.

Perhaps, Inertia is the Property of a certain Space Layer, which is the conserved work as a field, and the Interaction of Spaces happening at the Commons (Horizon of Space, Anti-Space) or those have been called Concentrated Logic = Spin , and so create the motion. [21]

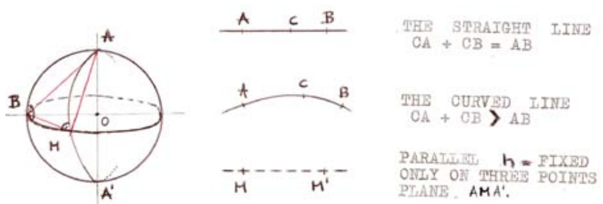


Fig. 6. Hyperbolic geometry and straight lines

The parallel axiom (the postulate) on any Segment AB in empty Space is experimentally verifiable, and in this way it is dependent of the other Axioms and is logically consistent, and since this is true then is accepted and so the Parallel Postulate as has been shown is a Parallel Axiom, so all Nature (the Universe as objective reality) is working according to the Principles (the patterns), the Properties and the dialectic logic of the Euclidean geometry. [17]

Hyperbolic and Projective geometry transfers the Parallel Axiom to problem of a point M and a Plane AB-C instead of problem of three points only, i.e it is a Plane , which such it is (F.6) .
Vast (the empty space) is simultaneously ∞ and 0 for every unit AB, as this is for numbers. Uniformity (P4) , Homogenous Plane) of Empty Space creates, all the one dimensional units , the Laws of conservation for Total Impulse , and the moment of Inertia in Mechanics, independently of the Position of Space and regardless the state of motion of other sources. (Isotropic Spaces) Uniformity (Homogenous) of Empty Time creates, the Laws of conservation of the Total Energy regardless of the state of motion (Time is not existing here , since Timing is always the same as zero) and Time Intervals are not existing. [17]

In Special Relativity events from the origin are determined by a velocity and a given unit of time, and the position of an observer is related with that velocity after the temporal unit .

Since all Spaces and Subspaces co-exist, then Past, Present and Future simultaneously exist on different Space Layer. Odd and Even Spaces have common and opposite Properties,(the regular Odd and Even regural Polygons on any dimensional Unit) so for Gravity belonging to different Layers as that of particles, is also valid in atom Layers . Euclidean geometry with straight lines is extended beyond Standard Model (AB < 10^{-} 33m) from that of general relativity where Spaces may be simultaneously Flat or Curved or multi-Curved, and according to the Concentrated, (Compact) Logic of the Space , are below Plank's length Level , so the changing curvature from point to point is possible in the different magnitudes of particles. In Planck length level and Standard Model , upper speed is that of light , while beyond Planck length as it is now , a new type of light is needed to see what is happening.

5. Respective Figures

5.1. Rational Figured Numbers or Figures

This document is related to the definition of “ Heron ” that gnomon is as that which , when added to anything , a number or figure, makes the whole similar to that to which it is added. In general the successive gnomonic numbers for any polygonal number, say, of n sides have n-2 for their common difference. The odd numbers successively added were called gnomons. See Archimedes (Heiberg 1881, page 142,ε΄.) The Euclidean dialectic logic of an axiom is that which is true in itself.

τών δέ άνίσων, όμοίων δέ τά κέντρα τών βαρέων όμοίως έσσείται κείμενα. όμοίως δέ λέγομες σαμεία κείσθαι ποτί τά όμοία σχήματα. άφ΄ ών επί τάς ίσας γωνίας άγόμεναι εύθείαι ποιέοντι γωνίας ίσας ποτί τάς όμολόγους πλευράς. (Επιπέδων ίσορροπιών ή κέντρα βαρών επιπέδων α΄).

This logic exists in nature (objective logic) and is reflected to our minds as dialectic logic of mind. Shortly for ancient Greeks was, (μηδέν εν τη νοήσει ειμή πρότερον εν τοί αισθήσοι) i.e. there is nothing in our mind unless it passes through.our senses . Since the first dimentional Unit is any Segment AB , it is obvious that all Rational Segments are

multiples of AB potentially the first polygonal number of any form , and the first is 2AB = AB +AB, which shows that multiplication and Summation is the same action with the same common base, the Segment AB .
To Prove : F.7

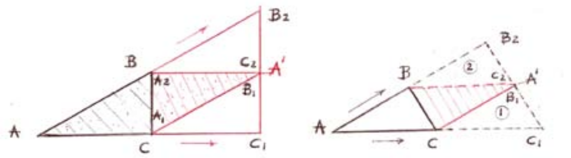


Fig. 7. AB2C1 is a triangle with sides twice the initial ABC

The triangle with sides AC1,AB2,C1B2 twice the length of initial segments AC,AB,CB preserves the same angles < A = BAC, B = ABC, C = ACB of the triangle. Proof :

a. Remove triangle ABC on line AC such that point A coincides with point C (A1). Triangles CB1C1, ABC are equal , so CA′ = AB, C1A′ = CB
b. Remove triangle ABC on line AB such that point A coincides with point B (A2). Triangles BB2C2, ABC are equal , so BC2 = AC , B2C2 = BC
c. The two circles (C,CB1 = AB) and (B,BC2 = AC) determine by their intersection point A′, so triangles CBA′, CBA are equal ,and also equal to the triangles CC1B1 , BB2C2, and this proposition states that sides CB1 = CA′, BC2 = BA′. Point A′ must simultaneously lie on circles (C1, C1 B1), (B2, B2C2) , which is not possible unless point A′ coincides with points B1 and C2.
d. This logic exists in Mechanics as follows : The linear motion of a Figure or a Solid is equivalent to the linear motion of the gravity centre because all points of them are linearly displaced, so
1st Removal ---- BB1 = AC, CB1 = AB, BC = BC
2nd Removal ---- CC2 = AB, BC2 = AC, BC = BC
1st +2nd Removal ---- CB1 = AB, BC2 = AC, BC = BC which is the same .

Since all degrees of freedom of the System should not be satisfied therefore points B1, C2, A′ coincide.
e. Since circles (C1,C1B1 = C1A′= CB), (B2, B2C2 = B2A′= CB) pass through one point A′, then C1A′B2 is a straight line, this because C1A′+A′B2 = C1B2, and A′ is the midpoint of segment B2C1.
f. By reasoning similar to what has just been given, it follows that the area of a triangle with sides twice the initials, is four times the area of the triangle.
g. Since the sum of angles < C1A′C + CA′B + BA′B2 = 180▫ (6 D) and equal to the sum of angles CBA + CAB + ACB then the Sum of angles of any triangle ABC is 180▫, which is not depended on the Parallel theorem or else-where.
This proof is a self consistent logical system .

Verification :

Let be the sides a=5, b=4, c =3 of a given triangle and from the known formulas of area S = (a + b + c) / 2 = 6 , Area = √ 6.1.2.3 = 6 For a=10, b=8, c=6 then S = 24/2 = 12 and Area =√ 12.2.4.6 .= 24 = 4 x 6 (four times as it is)

5.2. A given point P and any circle (O, OA)

To Prove : F.8

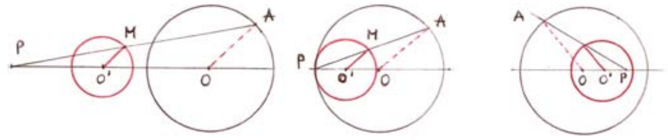


Fig. 8. Point P and any circle of radius OA

1.. Point P is outside the circle.
2.. Point P on circle.
3.. Point P in circle.
To Prove :
The locus of midpoints M of segments PA, is a circle with center O′ at the middle of PO and radius O′M = OA / 2 where , P is any point on a Plane , A is any point on circle (O, OA) and M is mid point of segment PA .
Proof :

Let O′ and M be the midpoints of PO, PA. According to the previous given for Gnomon, the sides of triangle POA are twice the size of PO′M, or PO = 2.PO′ and PA = 2.PM therefore as before , OA = 2.O′M, or O′M = OA/2.

Assuming M found , and Since O′ is a fixed point, and O′M is constant, then (O′, O′M = OA/2) is a circle. For point P on the circle : The locus of the midpoint M of chord PA is the circle (O′,O′M = PO / 2) and it follows that triangles OMP, OMA are equal which means that angle < OMP = OMA = 90 ▫, i.e. the right angle < PMO = 90▫ and exists on diameter PO (on arc PO), and since the sum of the other two angles < MPO + MOP exist on the same arc PO = PM + MO, it follows that the sum of angles in a rectangle triangle is 90 + 90 = 180 ▫

5.3. The two angles problem

Any two angles α =AOB, β = A′O′B′ with perpendicular sides are equal. (F.9)

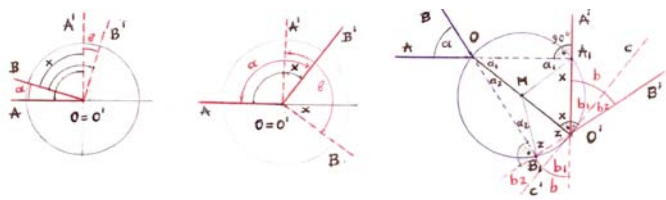


Fig. 9. Equal angles have perpendicular sides

O = O ′	O = O ′	O # O′
angle α ≤ 90 ▫	angle α > 90 ▫	any angle
Rotation of α=β	Rotating of α=β	Displacing of α = β

Let AOB = a be any given angle and angle A′O′B′ = b such that AO ⊥ O′A′, OB ⊥ O′B′.
To proof that angle b is equal to a.

Proof :
CENTRE O′ = O, α ≤ 90▫
Angle < AOA′ = 90▫ = AOB + BOA′ = α + x (1)
Angle < BOB ′ = 90= BOA′ + A′OB′ = x + β (2) , subtracting (1), (2) → angle β = α

CENTRE O′ = O, 90▫ < α < 180▫
Angle < AOA′ = 90▫ = AOB′ - B′OA′ = α - x (1)
Angle < BOB ′ = 90 - A′OB - A′OB′ = β - x (2), subtracting (1), (2) → angle β = α
CENTRE O′ # O.
Draw circle (M, MO = MO′) with OO′ as diameter intersecting OA,O′B′ produced to points A1,B1.
Since the only perpendicular from point O to O′A′ and from point O′ to OB is on circle (M, MO)
then, points A1, B1 are on the circle and angles O′A1O, O′B1O are equal to 90▫.

The vertically opposite angles a = a1 + a2, b = b1 + b2 where O′C ⊥ OO′.

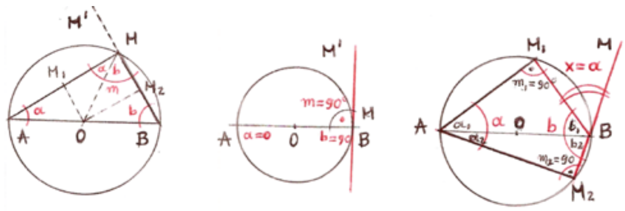
Since MO = MA1 then angle < MOA1 = MA1O = a1.
Since MA1 = MO ′ then angle < MA1O′ = MO′A1 = x
Since MO′ = MB1 then angle < MO′B1 = MB1O′ = z

Angle MO′C = 90▫ = x + b1 = z + b2.
Angle O′A1O = 90▫ = x + a1 = x + b1 → a1 = b1
Angle O′B1O = 90▫ = z + a2 = z + b2. → a2 = b2

By summation a1 + a2 = b1 + b2 or b = a (ο.ε.δ)

i.e. any two angles a,b, having their sides perpendicular among them are equal .
From upper proof is easy to derive the Parallel axiom, and more easy from the Sum of angles on a right-angled triangle .

5.4. Any two angles having their sides perpendicular among them are equal or Supplementary . F.10



F.10

Fig. 10. Equal or supplementary angles have perpendicular sides

AB = Diameter M → B, AB ⊥ BM′ AM1⊥BM1, AM2⊥BM2 = AM2⊥BM

Let angle < M1AM 2 = a and angle < M1BM 2 = b, which have side AM1 ⊥ BM1 and side AM 2 ⊥ BM 2 ⊥ BM

Show :

1. Angle < M1AM2 = M1BM = a
2. Angle < M1AM2 + M1BM2 = a + b = 180 ▫
3. The Sum of angles in Quadrilateral AM1BM2 is 360 ▫
4. The Sum of angles in Any triangle AM1M2 is 180▫.

Proof :

1. In figure 10.3, since AM1 ⊥ BM1 and AM2 ⊥ BM2 or the same AM2 ⊥ BM, then according to prior proof, AB is the diameter of the circle passing through points M1, M2, and exists a1 +b1 = m1 = 90▫, a2 +b2 = m2 = 90▫ and by summation (a1 +b1) + (a2 + b2) = 180▫ or (a1 + a2) + (b1+ b2) = a + b = 180▫, and since also x + b = 180 therefore angle < x = a
2. Since the Sum of angles M1BM2 + M1BM = 180▫ then a + b = 180 ▫
3. The sum of angles in quadrilateral AM1BM2 is a + b + 90 +90 = 180 + 180 = 360 ▫
4. Since any diameter AB in Quadrilateral divides this in two triangles, it is very easy to show that diamesus M1M2 form triangles AM1M2, BM1M2 equal to 180 each .

so,

1. Any angle between the diameter AB of a circle is right angle (90▫).
2. Two angles with vertices the points A, B of a diameter AB, have perpendicular sides
3. and are equal or supplementary.
4. Equal angles exist on equal arcs, and central angles are twice the inscribed angles.
5. The Sum of angles of any triangle is equal to two right angles.

i.e two Opposite angles having their sides perpendicular between them, are Equal or Supplementary between them. This property has been used in proofs of Parallel Postulate and is also a key to many others .[20]
Many theorems in classical geometry are easily proved by this simple logic.
Conclutions , and how useful is this invention is left to the reader.

5.5.1. A Point M on a circle of any diameter AB = 0 → ∞ , F.11

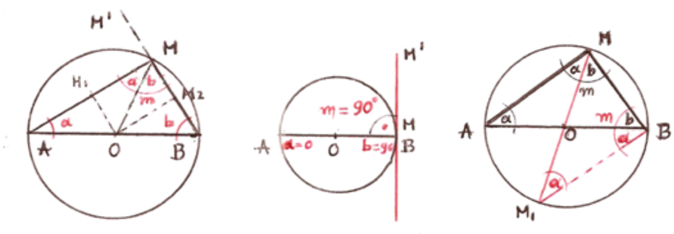


Fig. 11. A Point M on a circle of any diameter AB

AB = Diameter M → B, AB ⊥ BM′
Δ [AMB = MBM1]

Let M be any point on circle (O,OM = OA = OB) ,and M 1,M 2 the middle points of MA, MB and in second figure MM′ ⊥ BA at point B (angle AMM′ = 90 ▫).
In third figure MM1 is a diameter of the circle .

Show :

1. Angle < AMB = MAB + MBA = a + b = m
2. Triangles MBM1, MBA are always equal and angle < MBM1 = AMB = 90 ▫
3. The Sum of angles on triangle MAB are < AMB + MAB + MBA = 180▫.

Proof :

1. Since OA = OM and M1A = M1M and OM1 common, then triangles OM1A, OM1M are equal and angle < OAM = OMA = BAM = a →(a)

Since OM = OB and M2B = M2M and OM2 common, then triangles OM2B, OM2M are equal and angle < OBM = OMB = ABM = b → (b) By summation (a), (b) BAM + ABM = (OMA + OMB) = AMB = a + b = m..(c) i.e. When a Point M lies on the circle of diameter AB, then the sum of the two angles at points A, B is constantly equal to the other angle at M. Concentrated logic of geometry exists at point B, because as on segment AB of a straight line AB, which is the one dimensional Space, springs the law of Equality, the equation AB = OA + OB i.e. The whole is equal to the parts, so the same is valid for angles of all points on the circumference of the circle (O,OM), [as Plane ABM and all angles there exist in the two dimensional Space], and it is m = a + b. In figure (11), when point M approaches to B, the Side BM΄ of angle < ABM tends to the perpendicular on BA and when point M coincides with point B, then angle < ABM = 90▫ and < OAM = BAM = 0, therefore angle < AMB = 90▫ and equation (c) becomes : BAM + ABM = AMB → 0 + 90▫ = AMB → AMB = 90▫, (i.e. AM ⊥ BM) and the sum of angles is (BAM + ABM) + AMB = 90▫ + 90 ▫ = 180▫, or BAM + ABM + AMB = 180▫

2. Triangles MBA, MBM1 are equal because they have diameter MM1 = AB, MB common and angle < OBM = OMB = b (from isosceles triangle OMB). Since Triangles MBA, MBM1 are equal therefore angle < MM1B = MAB = a, and from the isosceles triangle OM1B, angle < OBM 1 = OM1B = a The angle at point B is equal to MBM 1 = MBA + ABM 1 = b + a = m = AMB . Rotating diameter MM1 through centre O so that points M, M1 coincides with B, A then angle < MBM1 = MBA + ABM1 = BBA + ABA = 90▫ + 0 = 90▫ and equal to AMB i.e. The required connection for angle MBM1 = AMB = m = a + b = 90 ▫.(ο.ε.δ)
3. 3. Since the Sum of angles a + b = 90 ▫, and also m = 90 ▫ then a + b + m = 90+90 = 180 ▫. It is needed to show that angle m is always constant and equal to 90▫ for all points on the circle. Since angle at point B is always equal to MBM 1 = MBO + OBM 1 = b + a = m = AMB, by Rotating triangle MBM1 so that points M,B coincide then MBM1 = BBA +ABA = 90+0 = m Since angle < AMB = a + b = m and is always equal to angle < MBM1, of the rotating unaltered triangle MBM1, and since at point B angle < MBM1 of the rotating triangle MBM1 is 90 ▫, then is always valid, angle < AMB = MBM1 = 90 ▫ (ο.ε.δ),

2a. To show , the Sum of angles a + b = constant = 90 ▫ = m. F.11
M is any point on the circle and MM1 is the diameter. Triangles MBA,MBM1 are equal and by rotating diameter MM1 through centre O, the triangles remain equal.
Proof :

a. Triangles MBA, MBM1 are equal because they have MM1 = AB, MB common and angle < OBM = OMB = b (from isosceles triangle OMB) so MA = BM1.
b. Since Triangles MBA, MBM1 are equal therefore angle < MM1B = MAB = a, and from isosceles triangle OM1B, angle < ABM1 = OBM 1 = OM1B = a
c. The angle at point B is always equal to MBM1 = MBO + OBM 1 = b + a = m = AMB Rotating triangle MBM1 so that points M,B coincide then MBM1 = ABB +ABA = 90+0 = m .Since angle AMB = a + b = m and is equal to angle <MBM1, of the rotating unaltered triangle MBM1 and which at point B has angle m = 90 ▫, then is valid angle <AMB = MBM1 = 90 ▫ i.e. the required connection for angle AMB = m = a + b = 90 ▫. (ο.ε.δ), 22 / 4 /2010.

2b. When point M moves on the circle, Euclidean logic is as follows :
Accepting angle ABM΄ = b at point B , automatically point M is on the straight line BM΄
and the equation at point B is for (a = 0, b = 90▫, m= 90▫) → 0 + 90▫ = m and also
equal to, 0 + b – b + 90▫ = m or the same → b + (90▫ - b) = m …… (B)
In order that point M be on the circle of diameter AB, is necessary → m = b + a...(M)
where , a , is an angle such that straight line AM (the direction AM) cuts BM΄, and is
b + (90▫ - b) = m = b + a or → 90▫ - b = a and → a + b = 90▫ = constant,
i.e. the demand that the two angles , a , b , satisfy equation (M) is that their sum must be constant and equal to 90 ▫. (ο.ε.δ)

3. In figure 3,according to prior proof , triangles MBA,MBM1 are equal . Triangles AM1B,AMB
are equal because AB is common, MA = BM1 and angle < MAB = ABM1, so AM1 = MB.
Triangles ABM1, ABM are equal because AB is common MB = AM1 and AM = BM1
therefore angle < BAM1 = ABM = b and so, angle MAM1 = a + b = MBM1
Since angle AMB = AM1B = 90 ▫ then AM ⊥ BM and AM1 ⊥ BM1.
Triangles OAM1, OBM are equal because side OA = OB, OM = OM1 and angle
< MOB = AOM1, therefore segment M1A = MB.
Rotating diameter MM1 through O to a new position Mx, M1x any new segment is
MxB = M1xA and the angle < MxBM1x = MxBA + ABM1x and segment BMx = AM1x.
Simultaneously rotating triangle MxBM1x through B such that BMx ⊥ AB then angle
< MxBM1x = BBA + ABA = 90▫ + 0 = 90▫,
i.e. in any position Mx of point M angle < AMxB = MxBM1x = 90▫

i.e. two Equal or Supplementary between them opposite angles , have their sides perpendicular between them. (the opposite to that proved) .

Following the proofs , then any angle between the diameter of a circle is right angle (90 ▫), central angles are twice the inscribed angles , angles in the same segments are equal to one another and then applying this logic on the circumscribed circle of any triangle ABM , then is proofed that the Sum of angles of any triangle is equal to two right angles or < BAM + ABM + AMB = 180▫

5.5.2. A Point M on a circle of any diameter

AB = 0 →∞ (F.12)

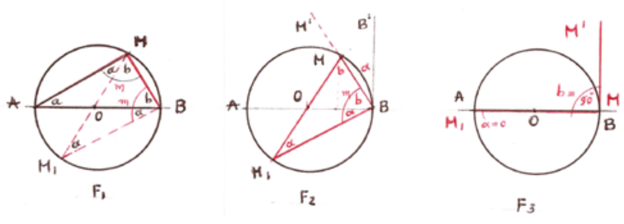


Fig. 12. a point M on a circle of any diameter AB

BB′ ⊥ BA , MM ' ⊥ BA
To show that angle < AMB = m = 90 ▫
BB′ ⊥ BA (angle ABB ′ = 90 ▫), MM " ⊥ AB

F.12.1 : It has been proved that triangles AMB, MBM1 are equal and angle <AMB = MBM1 = m for all positions of M on the circle. Since triangles OMB, OAM1 are equal then chord BM = AM1 and arc BM = AM1.

F.12.2 : The rotation of diameter MM1 through centre O is equivalent to the new position Mx of point M and simultaneously is the rotation of angle < M′MxBM1 = M′BM1 through point B, and this because arc BM = AM1, BMx = AM1x, i.e. when point M moves with BMM′ to a new position Mx on the circle, diameter MOM1 = MxOM1x is rotated through O, the points M,M1 are sliding on sides BMM′, BM1 because point M1 to the new position M1x is such that AM1x = BMx and angle < M′BM1 is then rotated through B. (analytically as below)

F.12.3 : When diameter MM1 is rotated through O, point M lies on arc MB = AM1 and angle < M′BM1 is not altered (this again because MB = AM1) and when point M is at B, point M1 is at point A, because again arc BM = AM1 = 0, and angle a = BM1M = 0, or angle < M1BM = M1BM′ = ABM′ = 90▫ = m = a + b

Conclusion 1.
Since angle < AMB is always equal to MBM1 = M′BM1 and angle M′BM1 = 90 ▫ therefore angle < AMB = a + b = m = 90 ▫
Conclusion 2.
Since angle < ABB′ = 90 ▫ = ABM + MBB′ = b + a, therefore angle MBB′ = a, i.e. the two angles < BAM, MBB′ which have AM ⊥ BM and also AB ⊥ BB′ are equal between them.

Conclusion 3.
Any angle < MBB′ on chord BM and tangent BB′ of the circle (O, OA = OB), where is holding (BB′⊥ BA), is equal to the inscribed one, on chord BM.

Conclusion 4.
Drawing the perpendicular MM " on AB, then angle BMM " = MAB = MBB ′, because they have their sides perpendicular between them, i.e. since the two lines BB′, MM " are parallel and are cut by the transversal MB then the alternate interior angles MBB′, BMM " are equal.

Conclusion 5.
In Mechanics, the motion of point M is equivalent to, a curved one on the circle, two Rotations through points O, B, and one rectilinear in the orthogonal system M′BM1 = MBM1.

5.5.3. A Point M on a circle of any diameter

AB = 0 →∞

Show that angle MBM1 is unaltered when plane MBM1 is rotated through B to a new position MxBM1x . F.13

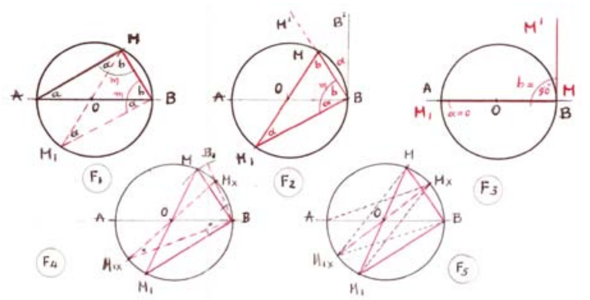


Fig. 13. a point M on a circle of any diameter AB

Proof :
Let Plane (MBM1), (F13) be rotated through B, to a new position B1BM1x such that :
1. Line BM → BB1 intersects circle (O,OB) at point Mx and the circle (B,BM = BB1), at point B1.
2. Line BM1 → BM1x extended intersects circle (O,OB) at the new point M1x.
3. Angle < M1BM1x = MBB1 = MBMx, is angle of rotation.
Proof :
Since angle < M1BM1x = MBMx, therefore angle < M1BM is unaltered by rotation →
i.e. Angle < M1BM = M1xBMx and diameter MM1 is sliding uniformly on their sides.
Data + Remarks.

1. Diameter MM1 is sliding in angle M1BM which means that points M1, M lie on the circle (O,OB) and on lines BM1, BM respectively, and also sliding to the other sides BM1x, BMx of the equal angle < M1xBMx. Any line segment M1xMw= MM1 is also diameter of the circle.
2. Only point Mx is simultaneously on circle (O,OB) and on line BB1.
3. The circle with point M1x as centre and radius M1xMw = MM1 intersect circle (O, OB) at only one unique point Mw.
4. Since angle < M1xBB1= M1BM and since segment M1xMw = MM1 then chord M1xMw must be also on sides of angles M1xBB1, M1BM, i.e. Point Mw must be on line BB1.
5. Ascertain 2 and 4 contradict because this property belongs to point Mx, unless this unique point Mw coincides with Mx and chord MxM1x is diameter of circle (O,OB).

Point Mx is simultaneously on circle (O, OB) , on angle < M1xBB1 = M1xBMx and is sliding on line BB1. We know also that the unique point Mw has the same properties as point Mx , i.e. point Mw must be also on circle (O, OB) and on line BB1, and the diameter M1xMw is sliding also on sides of the equal angles M1xBB1, M1xBMx , M1BM.
Since point Mw is always a unique point on circle (O,OB) and also sliding on sides of angle M1BM = M1xBMx and since point Mx is common to circle (O,OB) and to line BB1 = BMx, therefore, points Mw, Mx coincide and chord MxM1x is diameter on the circle (O,OB) , i.e. The Rotation of diameter MM1 through O , to a new position MxM1x, is equivalent to the Rotation of Plane (MBM1) through B and exists angle < MBM1 = MxBM1x, so angle < MBM1 = MxBM1x = AMB = 90 ▫ = m = a + b … ο.ε.δ

Since angle < MBMx = M1BM1x is the angle of rotation, and since also arc MMx = M1M1x (this because triangles OMMx, OM1M1x are equal) then : Equal inscribed angles exist on equal arcs.

6. General Remarks

6.1. Axiom not satisfied by Hyperbolic or other geometry.

It has been proved that quadrilateral MA1CC΄ is Rectangle (F1.d) and from equality of triangles MA1C, MC΄C then angle < C΄MC = MCA1. Since the sum of angles < MCA1 + MCB = 180 ▫ , also, the sum of angles < C΄MC + MCB = 180 ▫ which answers to Postulate P5 , as this has been set (F1.e). Hyperbolic geometry, Lobachevski, non-Euclidean geometry, in Wikipedia the free encyclopedia, states that there are TWO or more lines parallel to a given line AB through a given point M not on AB. If this is true, for second angle C2MC, exists also the sum of angles < C2MC + MCB = 180▫ , which is Identity (C2MC = C΄MC), i.e. all (the called parallels) lines coincide with the only one parallel line MM΄, and so again the right is to Euclid geometry.

Definitions, Axioms or Postulates create a geometry, but in order this geometry to be right must follow the logic of Nature , the objective reality, which is the meter of all logics, and has been found to be the first dimensional Unit AB = 0→ ∞ (F.2.2) i.e. the reflected Model of the Universe. Lobachevski's and Riemann's Postulate may seem to be good attempts to prove Euclid's Fifth Postulate by contradiction, and recently by "compromising the opposites "in the Smarandache geometries . Non of them contradicts any of the other Postulates of what actually are or mean. From any point M on a straight line AB, springs the logic of the equation (the whole AB is equal to the parts MA, MB as well as from two points passes only one line –theorem-), which is rightly followed (intrinsically) in Euclidean geometry only, in contradiction to the others which are based on a confused and muddled false notion (the great circle or segment is line, disk as planes and others) , so all non-Euclidean geometries basically contradict to the second definition (D2) and to the first Euclidean Postulate (P 1).

6.2. Hyperbolic geometry satisfies the same 4 axioms as Euclidean geometry, and the error if any in Euclidean derivation of the 5th axiom .

An analytical trial is done to answer this question.
Postulate 1 : States that "Let it have been postulated to draw a straight-line from any point to any point". As this can be done by placing the Ruler on any point A to any point B, then this is not in doubt by any geometry. The world "line" in Euclid geometry is straight line (the whole is equal to the parts, where lines on parts coincide) and axioms require that line to be as this is (Black color is Black and White color is White). In ancient Greek < Ευθεία γραμμή έστιν, ήτις εξ΄ ίσου τοίς εαυτής σημείοις κείται. >
Postulate 2 : states that, " And to produce a finite straight-line " Marking points A, B which are a line segment AB, and by using a Ruler then can produce AB in both sides continuously, not in doubt by any geometry.
Postulate 3 : states that, " And to draw a circle with any centre and radius " Placing the sting of a Compass at any point A (centre) and the edge of pencil at position B and (as in definition 15 for the circle) Radiating all equal straight lines AB, is then obtained the figure of the circle (the circumference and the inside), not in doubt by any geometry.
Postulate 4 : states that, " And all right angles are equal to one another " In definition D8 is referred as Plane Angle, to be the inclination of two lines in a plane meeting one another, and are not laid down straight-on with respect to one another, i.e. the angle at one part of a straight line. In definition D9 is stated " And when the lines containing the angle are straight then the angle is called rectilinear " and this because straight lines divide the plane, and as plane by definition is 360▫ then the angles on a straight line are equal to 180▫ In definition D10 is stated that a perpendicular straight line stood upon (another) straight-line makes adjacent angles (which are) equal to one another, each of the equal angles is a right-angle and this because as the two adjacent angles are equal and since their sum is 180 ▫ , then the two right-angles are 90 ▫ each and since this happen to any two perpendicular straight-lines, then all right angles are equal to one another, not in doubt by any geometry.
Postulate 5 : This postulate is referred to the Sum of the two internal angles on the same side of a straight- line falling across two (other) straight lines, being produced to infinity, and be equal to 180 Because this postulate, beside all attempts to prove it , was standing for centuries, mathematicians created new geometries to step aside this obstruction. In my proposed article the followings have been geometrically proved :
From any point M to any line AB (the three points consist a Plane) is constructed by using the prior four Postulates, a system of three rectangles MA1CC΄, C΄CB1M΄, MA1B1M΄ which solve the problem . (2.d)
The Sum of angles C΄MC and MCB is < C΄MC + MCB = 180 ▫ , which satisfies initial postulate P5 of Euclid geometry, and as this is now proved from the other four postulates , then it is an axiom.
The extended Structure of Euclidean-geometry to all Spaces (Spaces and Sub-spaces) resettles truth to this geometry, and by the proposed solution which is

applicable to any point M , not on line AB , answers to the temporary settled age-old question for this problem.
Mathematical interpretation and all the relative Philosophical reflections based on the non-Euclid geometry theories must be properly revised and resettled in the truth one.

6.3. The sum of angles on any triangle is 180▫

Since the two dimensional Spaces exists on Space and Subspace (F.5) then this problem of angles must be on the boundaries of the two Spaces i.e. on the circumference of the circle and on any tangent of the circle and also to that point where Concentrated Logic of geometry exists for all units, as straight lines etc. It was prooved at first that, the triangle with hypotenuse the diameter of the circle is a right angled triangle and then the triangle of the Plane of the three vertices and that of the closed area of the circle (the Subspace), measured on the circumference is 180 ▫ .

6.4. Angles with Perpendicular sides are Equal or Supplementary.

In Proofed Succession (4.5.1), is referred that two angles with perpendicular sides are equal (or supplementary). To avoid any pretext, a clear proof is given to this presupposition showing that,
Any angle between the diameter AB of a circle is right angle (90▫).
Any two angles with vertices the points A , B of a diameter AB , have perpendicular sides and are also equal or supplementary.
Equal angles exist on equal arcs, and central angles are twice the inscribed angles.
The Sum of angles of any triangle is equal to two right angles. So ,
There is not any error in argument of proofs.
The 5th Postulate is Depended on (derived from) the prior four axioms. 7.a.

6.5. An Application to Spaces . [17]

De Moivre´s formula for complex numbers states that the multiplication of any two complex numbers say z1,z2, or [z1=x1+i.y1 , z2=x2+i.y2] where x=Re[z] the real part and y=Im[z] the Imaginary part of z , is the multiplication of their moduli r1,r2 , where moduli ,r, is the magnitude $[r=|r|=\sqrt{x^2+y^2}]$ and the addition of their angles ,φ1,φ2, where φ=argz=atan2(y,x) and so , z1.z2 = (x1+i.y1).(x2+i.y2) = r1..r2[cos(φ1+φ2)+i.sin(φ1+φ2)] and when z1=z2=z and φ1=φ2=φ then z=x+iy and $z.z = z^2 = r^2(\cos 2\varphi+i.\sin 2\varphi)$ and for , w , complex numbers $z^w= r^w$. [cos(wφ)+isin(wφ)]….(a) , and so for r=1 then $z^w= 1^w$. $[\cos\varphi+i\sin\varphi]^w$ = [cos.wφ+i.sin.wφ] ……..(a1)

The n.th root of any number z is a number b ($\sqrt[n]{z}$=b) such that b^n =z and when z is a point on the unit circle, for r=1 , the first vertex of the polygon where φ=0 , is then $[b=(\cos\varphi+i\sin\varphi)]^n = b^n = z = \cos(n\varphi)+i.\sin(n\varphi) = [\cos(360/n)+i.\sin(360/n)]^n = \cos 360^o + i.\sin 360^o = 1+0.i =1$ .. (b) , i.e. the w spaces which are the repetition of any unit complex number z (multiplication by itself) is equivalent to the addition of their angle and the mapping of the regular polygons on circles with unit sides , while the n spaces which are the different roots of unit 1 and are represented by the unit circle and have the points z=1 as one of their vertices are mapped as these regular polygons inscribed the unit circle.

Complex number , z , the first dimentional unit AB = z , is such that either repeated by itself as monad (z^w = z.z.z.z. w-times) or repeated by itself in monad ($\sqrt[n]{z} = z^{1/n} = z^w$, $z^{1/n}.z^{1/n}.z^{1/n}$….w = 1/n-times , or the nth roots of z equal to w = 1/n) remains unaltered forming Spaces Anti-spaces {z^w, $-z^w$}and the inversing Sub-spaces {$\sqrt[n]{z}$}, meaning that ,unit circle is mapped on itself simultaneously on the two bases , 1 and n=1/w , where w.n = 1. Let us see how this coexistence is happening by the operation of exponentiation . The logarithm of a number x with respect to base b is the exponent by which b must be raised to yield x . In other words , the logarithm of x to base b is the solution w to the equation b^w = x ,{eg.→ Log2(8) = 3 , since 2^3=2x2x2=8} and in case of the reciprocal (1/x) then b^w = (1/x) ,{eg. → Log3(1/3) = -1, since 3^{-1} = 1/3 } . If w or n , is any natural and real number then we refer to natural logarithm else to logarithm .

This duality of coexistance on complex number z = AB (***the* w*.th power and the* n*.th root of z where w.n =1***) presupposes a common base ,m, which creates this unit polynomial exponentiation on all these Spaces the Anti-Spaces and the Sub-Spaces for which happens $m^w=r^w$.[cos(wφ)+i.sin(wφ)]=$r^{1/n}$.[cos.(φ+2λπ)/n+i.sin.(φ+2λπ)/n] where λ is an integer from 0 to n-1 , and for r=1 , m^w= $[\cos\varphi+i.\sin\varphi]^w$ or m^w= [cos(φ+2λπ)/n+i.sin(φ+2λπ)/n = $m^{1/n}$ = [cos.(φ+2λπ)/n + i.sin.(φ+2λπ)/n] = ... (c) i.e.

Since Spaces are composed of monads (the entities AB) which are the harmonic repetition in them {{ all the regular , for w > 2 polygons with monad AB as side limit to straight line AB for w = ±∞ , the (+) Space and the equilibrium (-) anti-Space of AB , to the complex plane for w = 2 , and to the circle with diameter AB for w=1 where on it exist all the roots of monad AB and are the circumscribed regular polygons in this Sub-Space }} and since monads are composed of purely real , |AB| , and purely Imaginary parts , d.∇i , therefore , all these to exist as regular polygons must be maped (sited) with natural and real numbers only and thus all Spaces , anti-Spaces and Subspaces of unit monad AB are represented as the polygonal exponentiation ,on this common base m .

Since Spaces , anti-Spaces {z^w, $-z^w$} and Subspaces {$\sqrt[n]{z}$}, which are the similar regular polygons , on and in unit monad AB = z , are both simultaneously created by the Summation of the exponentially unified in monad AB complex exponential dualities w, n , *where w. n= 1* , so repetition (rotation) of monads AB exist as this constant Summation on this common base **m ,** which is according to one of the four basic properties of logs as →
log.w(**1**=w.n)=log.w(w)+log.w(n=1/w)=1+1/w=1+n .(c1) which is the base of natural logarithms e and since 1= w.n then → $(1+n)^w = (1+1/w)^w$ = constant **= m = e** ←....(d) meaning that Spaces , anti-Spaces (the conjugates) and Subspaces , all as regular polygons represent the mapping (to any natural real and complex number as power w = 1/n) of any unit AB which is a complex number **z** , on the constant base **m** , where then is m^ ±(ª+ᵈ.i) = $(x+i.y)^w$ = $|z|^w$.[cos.wφ+i.sin.wφ] , a multi valued function where , sinφ = $y/\sqrt{x^2+y^2}$,cosφ =$x/\sqrt{x^2+y^2}$, |z|= $\sqrt{x^2+y^2}$,and for |z|=1

$m^\wedge \pm(ª+ᵈ.i) = (x+i.y)^w = [\cos.w\varphi + i.\sin.w\varphi]$ …..(d1)
Equation (d1) represents the general interconnection of Spaces and Subspaces , on and in all monad units AB.

For $w >= \pm 1$ then we have the Spaces and Anti-spaces.
For $w = \pm 1$ then we have Spaces and Anti-spaces with unit circles $r = \pm 1$ and the Sub-Spaces on these circles .
For $w = a \neq 0$ and $d = 0$ then $m^\wedge \pm(ª) = (x+i.y)^w = |z|^w.[\cos.\varphi+ i.\sin.\varphi]^w = |z|^w. [\cos.(w\varphi) + i.\sin.(w\varphi)]$, and it is De Moivre´s formula for exponentiation .
For x=0 then $m^\wedge \pm(ª+ᵈ.i) = (i.y)^w= |z|^w.[\cos.w\varphi+i.\sin.w\varphi]$ and on unit circle , $|z|^w= 1$, the common base $m^\wedge \pm(ª+ᵈ.i) =[\cos.w\varphi+i.\sin.w\varphi]$ and for a=0 the non-regular Polygons ,$d.i \equiv d.\nabla i$, then $m^\wedge\pm d.[\nabla i] = [\cos.w\varphi+ i.\sin.w\varphi]$ ....(d2) i.e. **(d2) is a quaternions exponentiation ,** a system that extends Imaginary part of complex numbers which products of (**i**) is not commutative where the order only of the variables follow the standard right hand rule, and for parallel vectors their quotient is scalar and tensor (Tz) of a unit vector **z** is one, then, base **m** becomes $m^\wedge \pm [\nabla i]^{d} = [\cos.d + i .\sin.d]$, which is for unit d .[∇i] = i , the Euler's formula which is [e^ (i.ᵈ) = cos.d + i .sin.d] and thus , this is the geometrical interpretation of **m and e .**
For $\varphi = 180^o = \pi$, $\cos.w\varphi = -1$, $\sin.w\varphi = 0$, (d2) becomes $m^\wedge\pm d.[\nabla i] = -1$, which is Euler's identity in general form and for Ellipsoid of axes y1,y2,y3 $m^\wedge \pm \pi.[y1+y2+y3]/\sqrt{3} = -1$, (a=0) the known Euler's identity for quaternions.

De Moivre´s formula for n_{th} roots of a quaternion where $q = k.[\cos.\varphi+ [\nabla i].\sin.\varphi]$ is for $w = 1/n$, $q^w = k^w.[\cos.w\varphi + \varepsilon.\sin.w\varphi]$ where $q = z = \pm (x+y.i)$, decomposed into its scalar (x) and vector part (y.i) and this because all inscribed regular polygons in the unit circle have this first vertex at points 1 or at -1 (for real part $\varphi = 0$, $\varphi = 2\pi$) and all others at imaginary part where , k = Tz = Tensor (the length) of vector z in Euclidean coordinates which is $k = Tz = \sqrt{x^2+y1^2+y2^2+ yn^2}$, and for imaginary unit vector ã (a_1,a_2,a_3,a_n..w) , the unit vector **ε** of imaginary part → $\varepsilon=(y.i/Ty)=[y.\nabla i]/[Ty]=\pm(y_1.a_1+y_2.a_2+)/(\sqrt{+y1^2+y2^2+yn^2})$ the rotation angle $0 <= \varphi < 2\pi$, $\varphi = \pm \sin^{-1}Ty/Tz$, $\cos\varphi = x/Tz$, which follow Pythagoras theorem for them and for all their reciprocal quaternions ã' (ã.ã' =1) . Since also the directional derivative of the scalar field $y(y_1 , y_2 , y_n ..)$ in the direction i is→ $i (y_1 , y_2 , y_n .)= i_1.y_1+ i_2.y_2 .+ i_n .y_n$ and defined as i.grad y = $i_1.(\partial y/\partial 1) + i_2.(\partial y/\partial 2)+.= [i.\nabla].y$ which gives the change of field y in the direction → i, and [i.∇] is the single coherent unit , so coexistance between Spaces Antispaces and Sub-Spaces of any monad z= x+y.i = AB is happening through general equation (d) and (e) →

$m^\wedge\pm(ª+ᵈ.\nabla i) = q^w =(Tq)^w.[\cos.w\varphi + \varepsilon.\sin.w\varphi]$ ..(e) where

$m = \lim(1+1/w)^w$ for $w = 1\to\infty$, $q = z = \pm (x+y.i)$
$\sin\varphi = y/\sqrt{x^2+y^2}$, $\cos\varphi = x/\sqrt{x^2+y^2}$, $|z| = \sqrt{x^2+y^2}$,
$Tq = \sqrt{x^2+y1^2+y2^2+ \ldots.yn^2}$, $Ty = \sqrt{y1^2+y2^2+ \ldots.yn^2}$
$\varepsilon=(y.i/Ty)=[y.\nabla i] / [Ty]=(y_1.a_1+y_2.a_2+.)/(\sqrt{y1^2+y2^2+yn^2})$

and when ,
1… $q = \pm x \to$ then
$q^w = (Tq)^w.[\cos.w\varphi + \varepsilon.\sin.w\varphi] = \pm x^w.[\cos.w\varphi]$
since $Tq = \sqrt{x^2} = \pm x$ and $\varphi = 0$
$\varepsilon = (y.i/Ty) = [y.\nabla i] / [Ty] = 0$ and
$m^\wedge \pm(ª+ᵈ.\nabla i) = q^w = \pm x^w.\cos.x\varphi = \pm x^w$ ….(e1)

represents the mapping of monad AB=z in the real domain ,which is straight line -∞ , 0 , + ∞ ,and simultaneously all real roots of monad , z , in unit circle |z|=1 on the base of natural logarithms and when ,

2… $q = \pm y . i \to$ then
z , $q^w=(Tq)^w.[\cos.w\varphi+\varepsilon.\sin.w\varphi]=(Tq)^w.[1+\boldsymbol{\varepsilon}.\sin.w\varphi]$
$Tq = \sqrt{1+ y1^2+y2^2+ \ldots.yn^2}$, $Ty = \sqrt{y1^2+y2^2+ \ldots.yn^2}$
$\varepsilon = (y.i/Ty) = [y.\nabla i] / [Ty]$ and
$m^\wedge \pm (ª+ᵈ.\nabla i) = q^w = \pm (Tq)^w.[1+ \boldsymbol{\varepsilon}.\sin.w\varphi] =$
$\pm (Tq)^w/(Ty) . [(Ty)+y.\nabla i]. \sin.w\varphi$ …...(e2)

represents the mapping of monad AB=z in Imaginary domain ,which are all regular polygons with side monad AB , with the first vertice on line AB and simultaneously all roots on monad ,z, as the circumscribed in unit circle regular polygons on the base of natural logarithms , i.e. $m^\wedge \pm(ª+ᵈ.\nabla i) = m^\wedge (\pm ᵈ.\nabla i) = [\cos.w\varphi + i.\sin.w\varphi]$, and as $q^w = (Tq)^w. [1+ [y.\nabla i].\sin.w\varphi / (Ty)]$ then this is De Moivre's formula for quaternion and for for y.[∇i] = i $m^\wedge \pm(ª+ᵈ.\nabla i) = m^\wedge (\pmᵈ.i) = [\cos.w\varphi + i.\sin.w\varphi]$ it is Euler´s number [e^ (i.ᵈ) = cos.d + i .sin.d] .

3… $q = z = \pm (x+y.i) \to m^\wedge \pm(ª+ᵈ.\nabla i) = q^w$
Since Spaces , anti-Spaces $\{z^w, -z^w\}$ and Subspaces $\{\sqrt[n]{z}\}$, have q = AB as common base , so similar Polygons have all their sides parallel between them , therefore this parallel transport (formation) of the infinit bases it gets rotated into a mixure of exponential vectors ± (ª+ᵈ.∇i) . With this way is possible for geometry of spaces , to be in the presence of the infinite units A_pB_p , where $p = 0 \to\infty$ (the quantization of points A_p , B_p as monad A_pB_p , with the quantized Potential $dP_p = PA_p - PB_p$) , which are the vector bundle of matter , (the quanta of Points (A_pB_p) and the quanta of energy = $\varepsilon \to dP_p$ simultaneously on points A_p ,B_p, and on unit vector A_pB_p).

The addition of two quaternion **q1, q2** is equal to
$q1+q2 = (x1+y1.i.\nabla) + (x2+y2.i.\nabla) = (x1+x2) + (y1+ y2).i.\nabla) = x3 + y3 . i\nabla$ ………(e3)
a new quaternion , *i.e. a new space.*
The multiplication of two quaternion **q1, q2** is equal to
$q1.q2 = (x1+y1.i.\nabla) . (x2+y2.i.\nabla) = (x1x2–y1y2) + (y1x2+ .x1y2).i.\nabla) = (x3+y3.i.\nabla)$ …… (e4)
a new quaternion , *i.e. a new space* , not commutative where the order of q1,q2 only (*the variables*) follow the standard right hand rule and composed of the two rotations applied as a single rotation .

This property allows the coexistance of all quaternion [In ± Spaces as → $d\hat{s} = (x+i.y)^w$, and in ± Sub-spaces as → $d\hat{s} = (x+i.y)^{1/w}$ all rotated in (PNS)] as mutually exclusive and simultaneously Identical , and all carryng intrinally the known conservation laws of the two components . The equations of a Space (say Electromagnetic Force field) can be replaced by the one to three scalar spaces separatly ,
(Potential functions **) .**

In general differential geometry terms , parallel transport of a base vector (∂i) along a base vector (∂j) is expressed through Christoffel symbols $\nabla i\, g_j = \Gamma\, ij^n\, g_n$ where ,n, is a covariant derivative and i < n < j of base vector g , and the metric (*distance*) $g = ds^2 = |AB|^2 = g_{\mu\nu}.dx^{\wedge}{}_{\mu}.dx^{\wedge}{}_{\nu}$.

Christoffel symbols are the connection coefficients induced by the metric which deriv is 0 (metric free) for parallel transports and for symmetric (torsion free) is generally holding , the Levi-Civita connection coefficients $g_{\mu\nu}$, $\mu = \nu \rightarrow 0$ το 3 induced by the metric **g** alone , without recourse to Cartesian coordinates .
Christoffel symbols are numerical arrays of real numbers that describe in coordinates the effects of parallel transport in curved surfaces , and more generally manifolds [5].
Since , tensor , is something which is spatially consistent and picks up one change , that of basis matrix for every index (one of each),basic vectors g, are resulting covariant derivatives which work in any coordinate system and represent to an operation in the abstract uncoordinated manifold which is in imaginary domain on a constant base **m** . For any vector r(x1,y1,z1) in cartesian coordinates (x,y,z) Christoffel symbols vanish and ,

$\Gamma xy,z(r)_{z1} = \Gamma yz,x(r)_{x1} = \Gamma zx,y(r)_{y1} = 0 \rightarrow \nabla \times \mathbf{r} = 0$
For spatial Spherical coordinates (r,θ,φ) the metric g = $ds^2 = dr^2 + r^2.d\theta^2 + r^2.\sin^2\theta.d\varphi^2$ and Christoffel symbols
$\Gamma_{21}{}^2 = \Gamma_{12}{}^2 = \Gamma_{13}{}^3 = \Gamma_{31}{}^3 = 1/r$, $\Gamma_{22}' = -r$
$\Gamma_{32}{}^3 = \Gamma_{23}{}^3 = \cot\theta$, $\Gamma_{33}{}^2 = -\sin\theta.\cos\theta$, $\Gamma_{33}' = -r.\sin^2\theta$
On any region of a Space , is possible to be placed a coordinate system (x,y,z,ε) such that x,y,z are spacelike for the three dimensions and **ε** to the potential at points (ε = energy from greek word ενέργεια) , where then the metric tensor can be written $ds^2 = \varepsilon^2 + dx^2 + dy^2 + dz^2$ and in matrix form g =ds²= [1,0,0,0 / 0,1,0,0 / 0,0,1,0 / 0,0,0,1]

1.. Monads = Quantum = ds = ĀB / (n = ∞ → 0) = [a ± b.i] = 0 → ∞, are simultaneously (*actual infinity*) and also (*potential infinity*) in Complex number form , and this defines , infinity which exists between all points which are not coinciding (ds > 0) , and because **ds** comprises any two edge points with Imaginary part then this property differs between the infinite points. Plank length is a Monad **$ds = 1{,}62\times10^{-35}$** **,**for two points A , B , and for the moment is accepted as the smallest possible size .This Monad is also infinitely divided because edge points A,B are not coinciding i.e. **$|ds| = 1\times10^{-N}$** < ∞ , where N is any number and this because **(-) ↔ (+) = ∓ P → [dš , - P , + P] .**

2.. Spaces of Unit ĀB are (in Plane) the Infinite (+) Regular Polygons inscribed in the circles ***with AB as Side*** , (repetition of Unit AB) , the Nth Space , the Nth Unit Tensor of the **N** equal finite Elements **dš** , and for the ∞ Spaces , the line AB . For three dimension are the regular Solids in sphere . The diameter of this circle extends to infinity (*it is of potential nature*) where then $dš = (a+i.b)^w = |z|^w.[\cos w\varphi + i.\sin w\varphi]$

3.. Anti - Spaces of Unit ĀB are (in the three dimentional space) the Symmetrically Infinite (-) Regular Solids inscribed in the Sphere ***with AB as side*** of the Solid , (The Harmonic Repetition of Unit BA , symmetrical to AB) , the Nth Anti-Space , the Nth Unit Tensor of the N equal finite Anti-Elements and for the ∞ Spaces , the Plane through line BA . The radius of Spheres extends to infinity (*it is of potential nature*)and $dš = (a+i.b)^w = -|z|^w.[\cos w\varphi + i.\sin w\varphi]$

4.. Sub- Spaces of Unit ĀB are (in Plane) the Infinite Regular Polygons inscribed in the circle ***with AB as diameter*** , (Harmonic Repetition of the Roots in Unit AB) and in Nth Sub-Space, the Nth Unit Tensor of the N finite Roots and in case of ∞ Elements are the points on the circle *, and for 3D Space , the points on Sphere* AB) .The Superposition of (+) Spaces , (-) Anti -Spaces , (±) Sub-Spaces of unit AB is happening for all unit magnitudes near zero (Point = 0 =Nothing) and the Infinite magnitude (↔ = ± ∞ = Infinite) meaning that all Spaces are in one Space and $dš = \pm(a+i.b)^{1/w} = \pm|z|^{1/w}.[\cos w\varphi + i.\sin w\varphi]$

5.. Remarks : Lagrange equation of motion for a single point (Primary Point **A** is the only Space) states that this point must move from the Initial Position **A** to another position say **B** . This Equilibrium for points A and B , presupposes in Mechanics the Principle of Virtual Displacements and the work done $W = \int P.ds = 0$, or when ds = distance AB then → [ds .($P_A + P_B$) = 0] …(1)
From equation (1) are self created all the , Spaces [S] the equilibrium Anti-Spaces [AS] and the Sub-Spaces [SS] with infinite points in them and with a finite work on it . Monad **dš** (dipole AB) is a complex number of the type [**z = x+∇i.y**] ..(1a) representing the real part (**x**) , *the distance AB* , and imaginary parts (∇i.y) which is the work done of equation ..(1) . Complex number ,**z**, ***the first dimentional unit AB*** , is such that either repeated by itself as monad (z^w = z.z.z.z. w-times) or repeated times itself in monad ($\sqrt[n]{z} = z^{1/n} = z^w$, $z^{1/n}.z^{1/n}.z^{1/n}$….w = 1/n- times , or the nth roots of z equal to w = 1 / n) remains unaltered forming Spaces ($\mathbf{z}^w$) , Anti-spaces(**-** $\mathbf{z}^w$) and the inversing Sub-spaces ($\sqrt[n]{\mathbf{z}}$) , meaning that , unit circle is mapped on itself simultaneously on the two bases , 1 and n=1/w , where w.n = 1. This duality of coexistance on AB [***the* w.*th power and the* n.*th root of z where* w.n =1**] presupposes a common base ,**m**, which creates this unit polynomial exponentiation . Analysing this exponentiation according to one of the four basic properties of logs then →
log.w(**1**= w.n) = log.w(w)+log.w(n=1/w) = 1+1/w = 1+ n and it is the base of natural logarithms **e** and since 1= w.n then → $(1+n)^w = (1+1/w)^w$ = constant **= m = e** ← ....(2)
Since the first dimentional unit **AB** is a complex number with many imaginary parts (and this because of the infinite variables) then this unit has the general type of quaternion .i.e.
$m\hat{}\pm(^a+{}^d.\nabla i) = q^w = (Tq)^w.[\cos w\varphi + \varepsilon.\sin w\varphi]$ ..(e) where

$m = \lim(1+1/w)^w$ for $w = 1 \rightarrow \infty$, $q = z = \pm(x+y.i)$
$\sin\varphi = y/\sqrt{x^2+y^2}$, $\cos\varphi = x/\sqrt{x^2+y^2}$, $|z| = \sqrt{x^2+y^2}$,
$Tq = \sqrt{x^2+y1^2+y2^2+\ldots.yn^2}$, $Ty = \sqrt{y1^2+y2^2+\ldots.yn^2}$
$\varepsilon=(y.i/Ty)=[y.\nabla i] / [Ty]=(y1.a1+y2.a2+.)/(\sqrt{y1^2+y2^2+yn^2})$

[PNS] ↔ ***quaternion*** ↔ [**dŝ** = **x**+∇i.**y**] is a vector with two components , ***the one* x** ,is the only space with Scalar Potential field **Φo** , which is only half lengths of Space Anti-Space , (*the longitudinal positions*) , (x) → (-x) straight line connecting Space [S] , Anti-Space [AS] in [PNS] and in it exist *,the initial Work, Impulse , bounded on points which cannot be created or destroyed which is analogous to the (x) magnitude* , ***and the other one* y** ,is the infinite local curl fields **So ,** due to the Spin which is the intrinsic rotation of the Space and Anti-Space .

Because in [S] and [AS] forces P_A - P_B are acting in the same straight line so moment lever is zero (0) , therefore Primary [S] and [AS] is ir-rotational and so it is possible to express this Primary field as a scalar function (Φo). This shows that [PNS] is a Space Work *or* Space-Spin *or* ***Space - Energy Existence*** where Time is not existing ,because **Φo** and **So** are not time-varying. The same also is holding for the infinite dipole **AnBn** which are also complex number with their all properties as that of quaternions . Because quaternion properties are wrapped in lower and in higher dimensions ***only by rotation*** , *this is the property of spaces* , so all dipole AnBn may have ***commons*** , which may bleed off in any Space , *a very useful device for Quantum mechanics* . Geometrically is stated that , this property of commons allows to the dipole AnBn or Spaces [**dŝ** = $x_n+\nabla i.y_n$] , to be also a Space-Time existence which is wrapped in the **Space - Energy Existence** and this because of , Operation ↔ ***Quaternion*** → Notation **m^±(ª+ᵈ.∇i) =** $\mathbf{q^w} \rightarrow$ **= (Tq)^{w}. [cos.wφ + ε.sin.wφ]** , scatters Part or all **Content** of the quaternion **q , in all Spaces and Sub-Spaces as q^{w}.** [i.e. The duality of coexistence ↔ of the Content of **dŝ** from **w.th power** to the **n.th root** of **q , where w.n =1 ,** is the measuring of , *drag areas* and other *trapped accumulators* , and ***by rotation to convert them*** in Spaces] . An extend analysis in [23] .

Since Natural logarithm of any complex number b, can be defined by any natural and real number as the power ,**w**, which represent the mapping to which a constant say e, would have to be raised to equal b , i.e. e^W= b and or e^ln(b) = e , [base e]^ ^{1}natural number w^1 = b ,Therefore , both fomula (d) and (e) represent the same mapping which is the regular polygonal exponentiation of unit complex monad AB = 1 on the two equal numbers , e and m , as the base of natural logarithms .

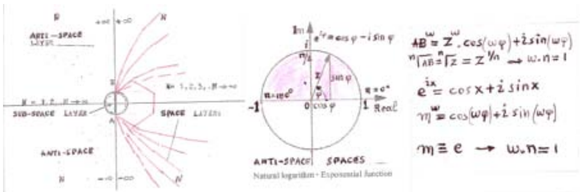


Fig. 14. Spaces Anti-Spaces on Monad AB

Natural logarithm – Exponential function

7. Criticism to Non-Euclid Geometries

The essential difference between Euclidean and non-Euclidean geometries is the nature of parallel lines.
Euclid's fifth postulate, the parallel postulate, states that, within a two-dimensional plane ABM for a given line AB and a point M, which is not on AB .F2(3), i.e. MA+MB > AB, there is exactly one line through M that does not intersect AB because if MA+MB = AB then point M is on line AB and then lines MA, MB coincide each one passing from two points only and thus is answered the why any line contains at least two points i.e.a quaternion
In Euclid geometry, in case of two straight lines that are both perpendicular to a third line, the lines remain at a constant distance from each other and are known as parallels. Now is proved that, a point M on the Nth Space, of any first dimensional Unit AB = 0 →∞, jointly exists, with all Sub-Spaces of higher than N Spaces, and with all Spaces of lower than N Subspaces. It is shown that this duality of coexistance converts the Content of any Space (complex number) to any other Space .

This is the Structure of Euclidean geometry. [F.5] As in fundamental theorem of Algebra Equations of Nth degree can be reduced into all N-a or N+a degree, by using the roots of the equations, in the same way Multi –Spaces are formed on AB. Nano-scale-Spaces, inorganic and organic, Cosmic-scale-Spaces are now unified in our world scale. Euclidean Empty Space is Homogenously Continues, but all first dimensional Unit-Spaces Heterogeneous and this because all Spaces constitute another Unit (the Nth Space Tensor is the boundaries of N points). All above referred and many others are springing from the first acceptance for point, and the approaching of Points. Spaces and Subspaces are the quaternion and by their multiplication is created another one quaternion , which is a very important logical notion for the laws concerning Continuity or not , the conservation laws , Continues Transformations in Space and in Time for Mechanics, Physics Chemistry and motions generally. From this logic yields that a limited and not an unlimited Universe can Spring anywhere.. Since Non-existence is found everywhere then Existence is found and is done everywhere on monad AB . (3.1)
If Universe follows Euclidean geometry, then this is not expanded indefinitely at escape velocity, ut is moving in Changeable Spaces with all types of motions , < a twin symmetrically axial –centrifugal rotation > into a Steady Space (This is System AB┴AB = 0→ AB → ∞), with all types of curvatures. (It is a Moving and Changeable Universe into a Steady Formation) [7]. It was proved that on every point in Euclid Spaces exist infinite Impulse P = 0 → P → ∞, and so is growing the idea that Matter was never concentrated at a point and also Energy was never high < very high energy > .[14 -17] , i.e. Bing Bang has never been existed, but it is a Space conservation Energy State → W = ∫A-B [P.ds] = Σ P.δ = 0.[21]

Gravity is particle also, in Space-Energy level which is beyond Plank's length level which needs a new type of light to see , with wave length smaller than that of our known visible light and thus can enter our wave length of light ,and thus the Euclidean geometry describes all these physical Spaces . Analysing [6-5] is seen that gravity is a quaternion and as such is a space as all others . An extend analysis in [23].

Hyperbolic geometry, by contrast, states that there are infinitely many lines through M, not intersecting AB. In Hyperbolic geometry, the two lines " curve away " from each other, increasing in distance as one moves further from the points of intersection with the common perpendicular, which have been called ultra-parallels. The simplest model for Hyperbolic geometry is the pseudo-sphere of Beltrami-Klein, which is a portion of the appropriate curvature of Hyperbolic Space, and the Klein model, by contrast, calls a segment as line and the disk as Plane.??? In hyperbolic geometry the three angles of a triangle add less than 180 ▫ , without referring that triangle is not in Plane but on Sphere < Spherical triangle F2(1) > This omission created the wrong hyperbolic geometry. Mobius strip and Klein bottle (complete one-sided objects of three and four dimensions) transfers the parallel Postulate to a problem of one point M and a Plane, because all curves and other curve lines are not lines (For any point on a straight line exists < the whole is equal to the parts which is an equality > and not the inequality of the three points) because contradict to the three points only and anywhere. Einstein's theory of general relativity is bounded in deviation Plank's length level where the proposition of the constancy of speed of light exists as Space-time . Euclid geometry is extended to zero length level where Gravity exists as particle with wavelength near zero and infinite Energy , a different phenomenon than Space-time. The Energy and the Momentum of particles may increase without limit and Speed also can exceeds c , that of light . Since all Spaces ± Spaces , ± Sub-spaces are quaternion which have the property that allows the coexistance of all quaternion as mutually exclusive and simultaneously Identical and this is succeeded also by rotation only , then is needed a new insight for the objective reality .

In this way is proved that propositions are true only then , they follow objective logic of nature which is the meter of all logics. Answers also to those who compromise incompatibility by addition or mixure.

If our Universe follows Hyperbolic geometry then this is expanded indefinitely, which contradicts to the homogenous and isotropic Empty Spaces and also to the laws of conservation of Energy.

This guides to a concentrated at a point matter and Energy < very high energy > , Bing Bang event .Elliptic geometry, by contrast, states that , all lines through point M, intersect AB. In Elliptic geometry the two lines “curve toward” each other and eventually intersect. The simplest model for Elliptic geometry is a sphere, where lines are “great circles”??? For any great circle (which is not a straight line ???) and a point M which is not on the circle all circles (not lines ???) through point M will intersect the circle. In elliptic geometry the three angles of a triangle add greater than 180°, without referring that triangle is not in Plane, but in the Sphere, < in Spherical triangle F2(3) >. This omission created the wrong elliptic geometry. If Universe follows Elliptic geometry then this is expanded to a halt and then this will stark to shrink possibly not to explode as is said, but to change the axial-centrifugal motion to the initial Rectilinear.

7.. Conclutions

A line is not a great circle, so anything is built on this logic is a mislead false.

The fact that the sum of angles on any triangle is 180° is springing for the first time, in this article (Rational Figured numbers or Figures) . This admission of two or more than two parallel lines, instead of one of Euclid’s, does not proof the truth of the admission . The same to Euclid’s also , until the present proved method . Euclidean geometry does not distinguish , Space from time because time exists only in its deviation -Plank's length level-, neither Space from Energy because –Space and Energy exists as different quanta on any first dimensional Unit AB - which connects the only two fundamental elements of Universe, that of Points and that of Energy. The way of their transformation in [23] .

The proposed Method in this article, based on the prior four axioms only, proofs, (not using any admission but a pure geometric logic under the restrictions imposed to seek the solution) that , through point M on any Plane ABM (three points only which consist the Plane), passes only one line of which all points equidistant from AB as point M , i.e. the right is to Euclid Geometry.

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