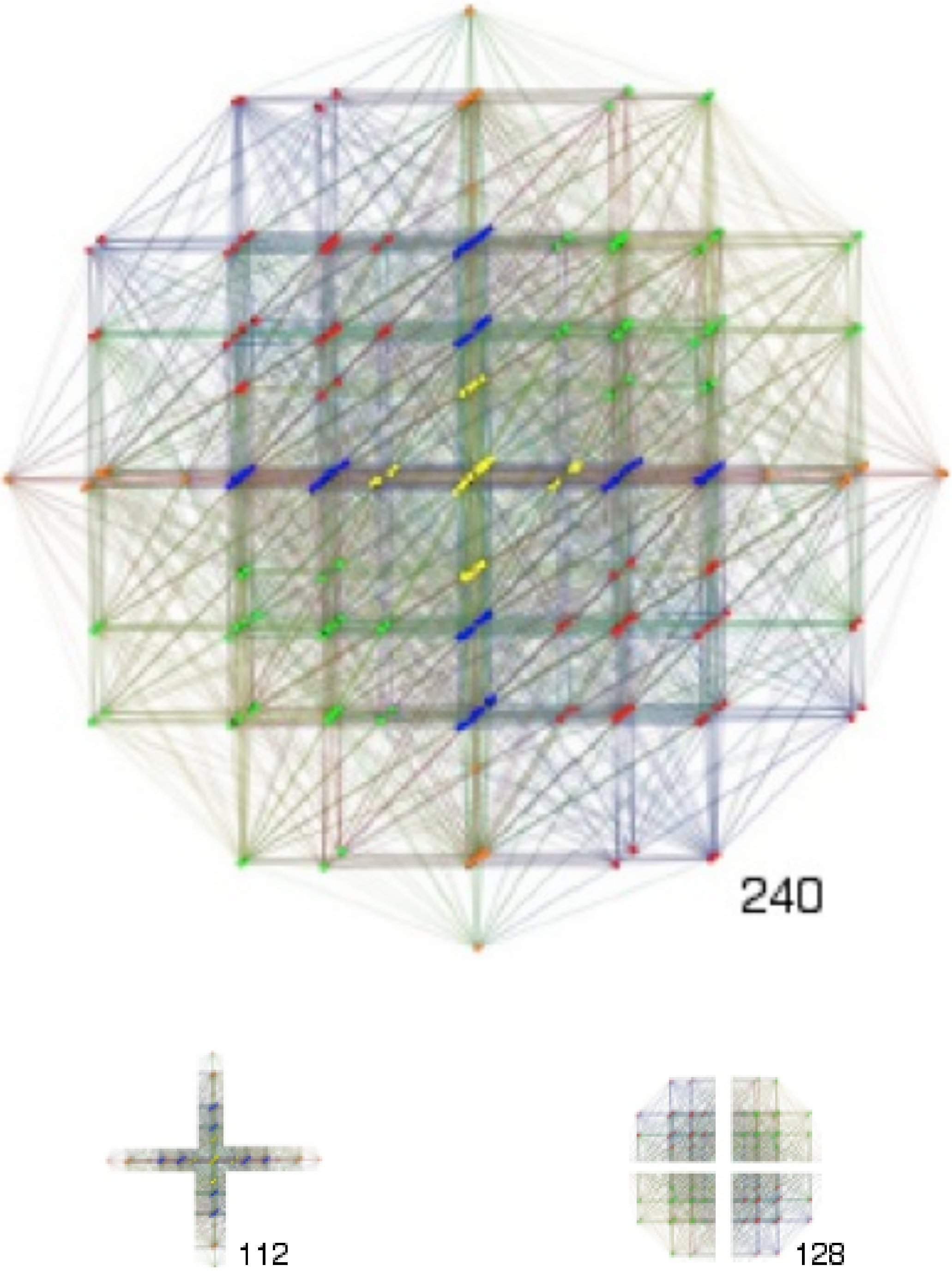
E8 Physics 2013



Frank Dodd (Tony) Smith, Jr. – Georgia – 2013 - viXra 1310.0182

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Here is how E8 Physics emerges from fundamental spinor fermions

to condense into a 26-dim String structure with strings as fermion World-Lines
with each fundamental fermion being surrounded by a Quantum Cloud
that has Kerr-Newman physical structure
corresponding to a Schwinger Source region
with complex harmonic Wyler/Hua Green's function propagator.
The Wyler/Hua complex bounded domain structure allows
realistic calculation of force strength constants and particle masses.

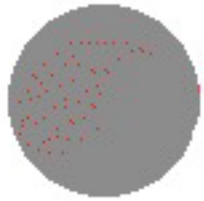
Here are some historical speculation questions:

Could Wyler's Green's function based on harmonic analysis of complex domains
have been used by Schwinger
to give more detailed models of his finite-region sources ?

Could Wyler's rejection at IAS Princeton under Dyson in the 1970s
have been at least in part due to Dyson's Feynman-type view
of point particles as fundamental ?

If Wyler had gone to see Schwinger at UCLA instead of Dyson at IAS Princeton
could Wyler and Schwinger together have developed source theory
in great enough detail that its advantages (no renormalization etc)
would have been clear to most physicists ?

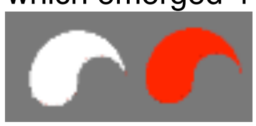
In the beginning there was Cl(0) spinor fermion void



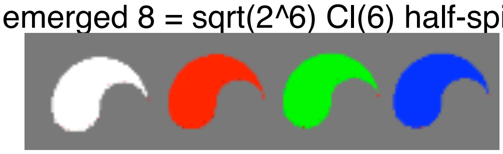
from which emerged 2 = sqrt(2^2) Cl(2) half-spinor fermions

 and its mirror 

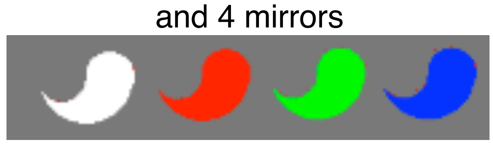
from which emerged 4 = sqrt(2^4) Cl(4) half-spinor fermions

 and 2 mirrors 

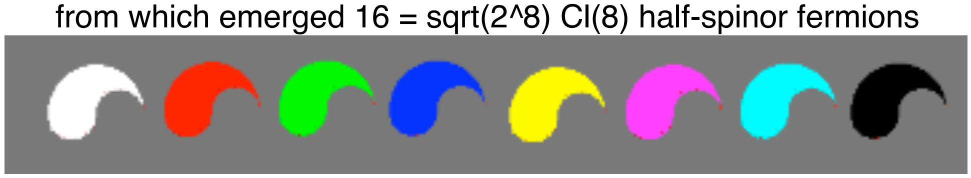
from which emerged 8 = sqrt(2^6) Cl(6) half-spinor fermions



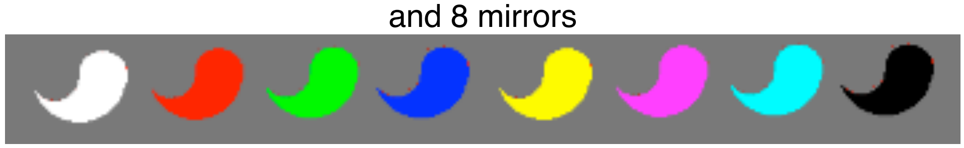
and 4 mirrors



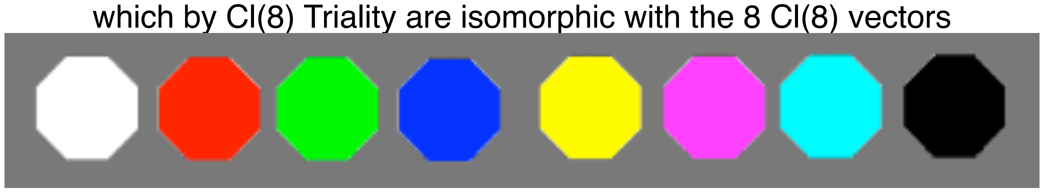
from which emerged 16 = sqrt(2^8) Cl(8) half-spinor fermions



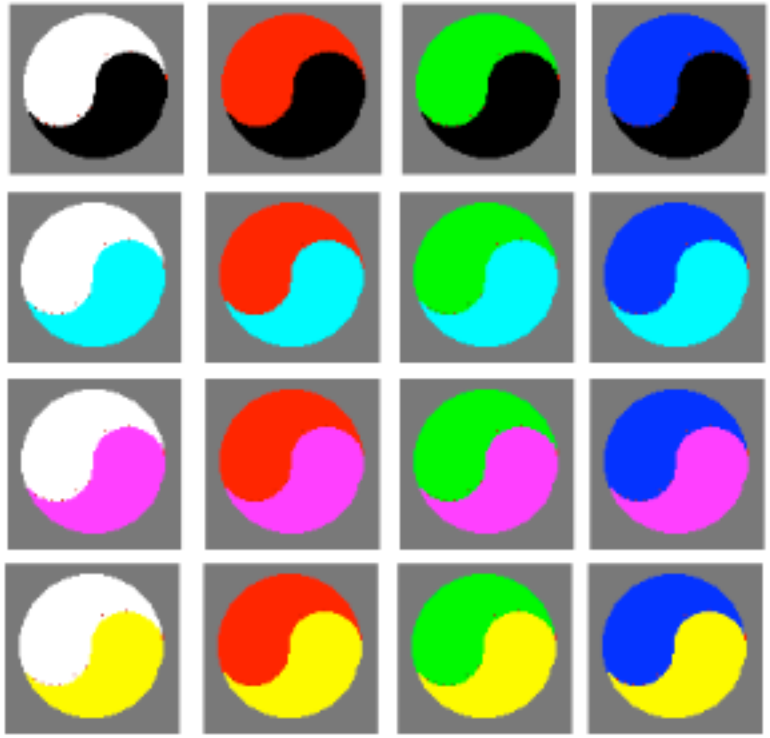
and 8 mirrors



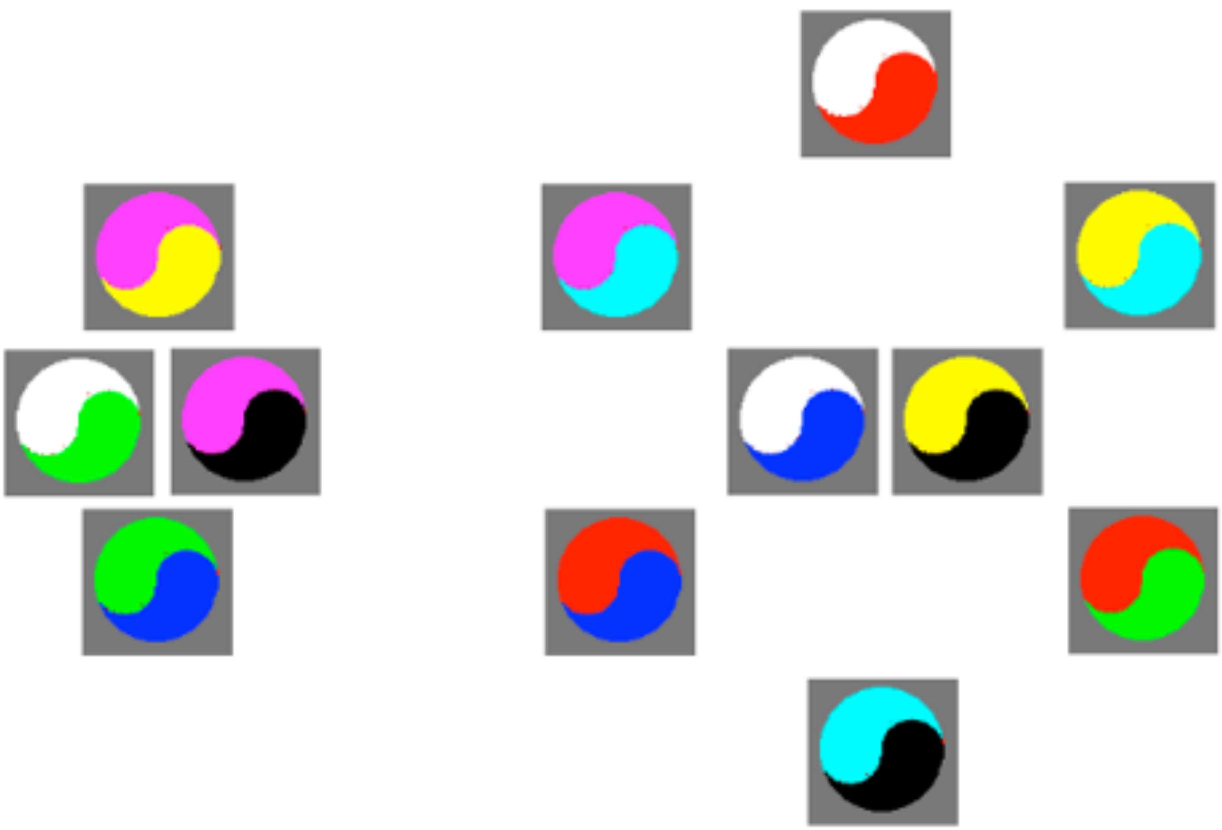
which by Cl(8) Triality are isomorphic with the 8 Cl(8) vectors



so that the 28 antisymmetric pairs of half-spinors and their mirrors are the 28 Cl(8) bivectors of the Lie Algebra of Gauge Groups:



16 of U(2,2) = U(1)xSU(2,2) = U(1)xSpin(2,4) for Conformal Gravity



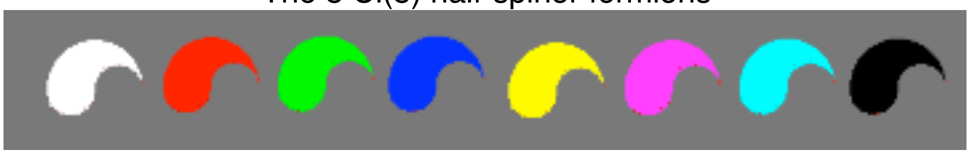
4 of U(2) = U(1)xSU(2) and 8 of SU(3) for the Standard Model.

As fermion particles the 8 Cl(8) half-spinors

represent
neutrino; red down quark, green down quark, blue down quark;
blue up quark, green up quark, red up quark; electron
(yellow, magenta, cyan, black are used for blue, green, red up quarks and electron)

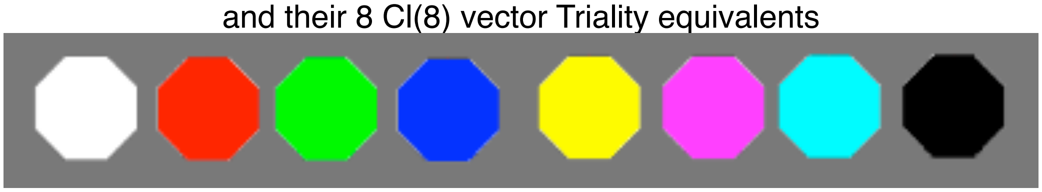
The 8 mirror Cl(8) half-spinors represent the corresponding fermion antiparticles.

The 8 Cl(8) half-spinor fermions

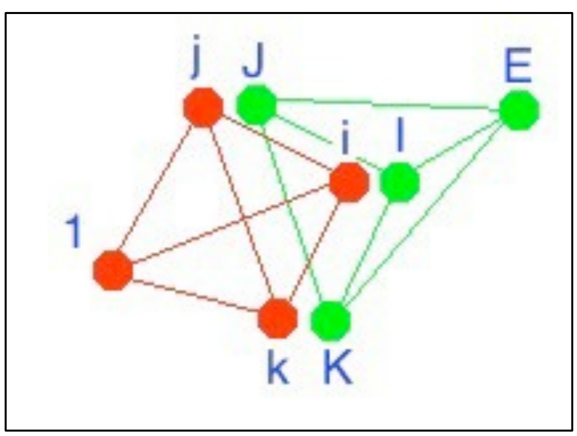


and their 8 mirror Triality equivalents

and their 8 Cl(8) vector Triality equivalents



correspond to
the Octonion basis elements {1,i,j,k,K,J,**I**,E}
and
can be represented as a pair of tetrahedra

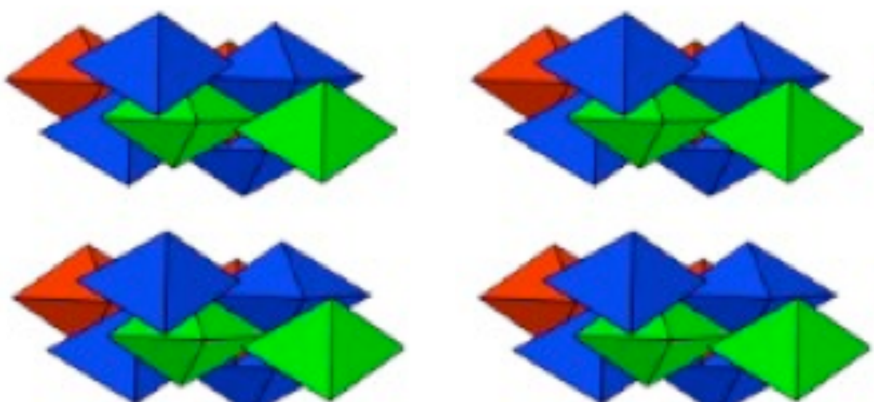


By Real Clifford Algebra 8-periodicity any large spinor space can be embedded in a tensor product of a number copies of the 16-dim full spinors of Cl(8) representable as a pair of a pair of tetrahedra

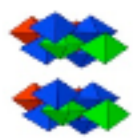
the tensor product of two of which

 x 

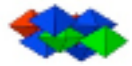
form the 128+128 = 256-dim full spinors of Cl(8)xCl(8) = Cl(16)



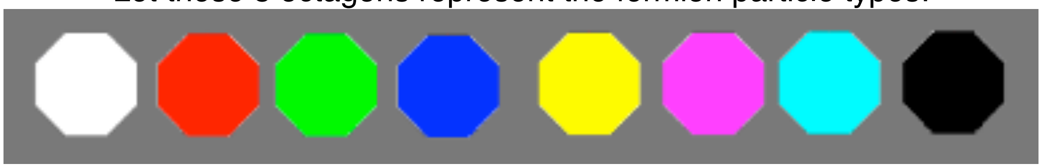
+

One set of 128-dim Cl(16) half-spinors  is the spinor/fermion part of the 248-dim Lie algebra E8 = 120-dim Spin(16) + 128-dim half-spinor of Spin(16)
and is also a representation of
the 128-dim spinor space denoted as T2 by Geoffrey Dixon who says in his paper "Matter Universe: Message in the Mathematics":
"... the 128-dimensional hyperspinor space T2 ...[is]... the doubling of T ...
The algebra T = C x H x O ...(complex algebra, quaternions, and octonions)... is 2x4x8 = 64-dimensional ... noncommutative, nonassociative, and nonalternative ...".

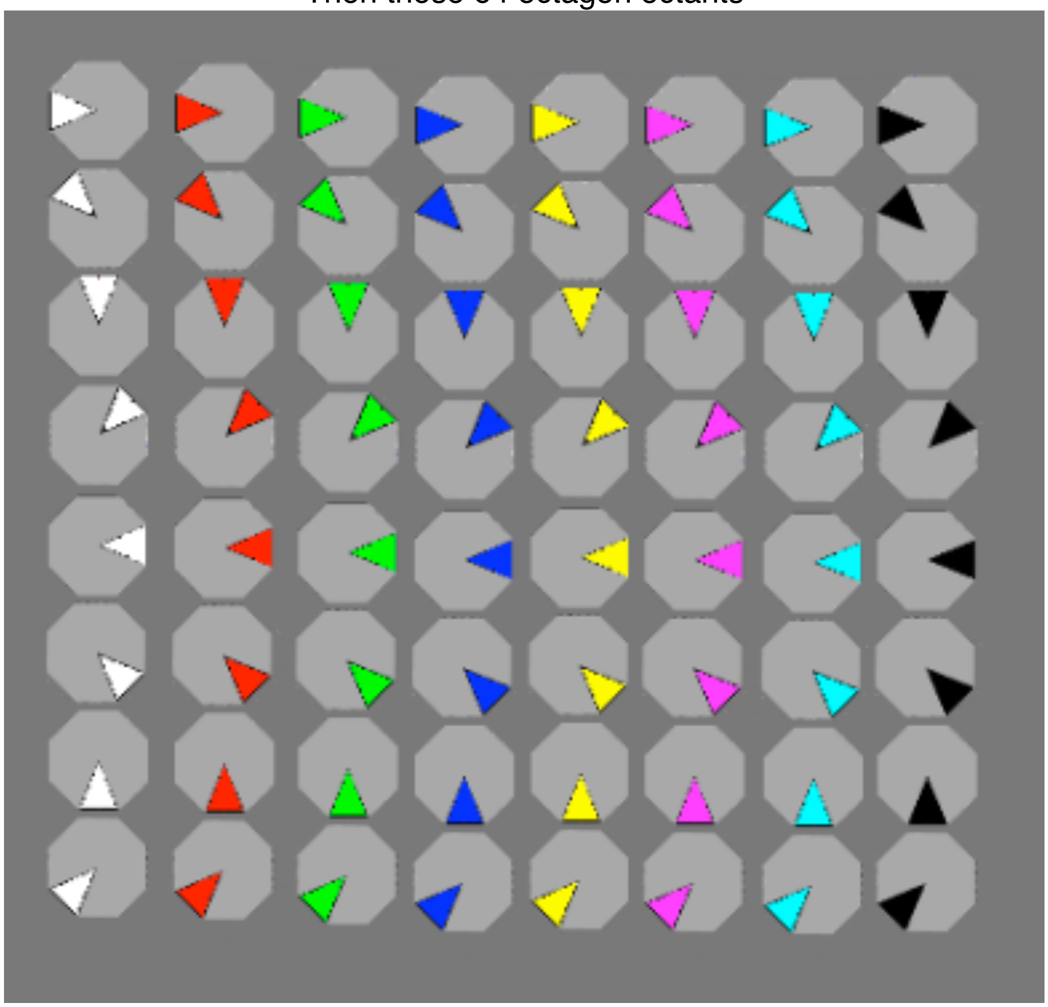
Within 128-dim T2,

each 64-dim factor T is represented by  half of the Spin(16) half-spinor space.
One 64-dim T represents fermion spinor particles
while the other T of T2 represents fermion spinor antiparticles.

Let these 8 octagons represent the fermion particle types:



Then these 64 octagon octants



represent the 8x8 = 64 covariant components of the fermion particles.
With respect to Cl(16) and E8 the Cl(8) Triality induces
Triality isomorphsim between the two 64-dim factors T
that represent fermion particles and antiparticles
and also of both of them
with the 64-dim D8 / D4xD4 space representing 8-dim position and momentum.

Construct a **Lagrangian** from E8 = D8 + D8-half-spinor
by **integration over 8-dim Spacetime** from D8/D4xD4 of E8 of
the Gravity and the Standard Model from the two D4 subalgebras of E8 and
a Dirac Fermion Particle-AntiParticle term from D8 half-spinors of E8.

This Lagrangian differs from conventional Gravity plus Standard Model in four respects:

1 - 8-dimensional spacetime with NonUnitary Octonionic Inflation
2 - no Higgs
3 - two D4 producing gauge groups
4 - 1 generation of fermions

These differences can be reconciled by freezing out at lower-than-Planck energies a preferred Quaternionic 4-dim subspace of the original (high-energy) 8-dim spacetime, thus forming an **8-dim Kaluza-Klein spacetime M4xCP2** where
M4 is 4-dim Minkowski Physical Spacetime and
CP2 is a 4-dim Internal Symmetry Space.

This Octonionic to Quaternionic symmetry breaking makes the Lagrangian consistent with experimental observations:

1 and 2 - The Octonionic to Quaternionic symmetry breaking from 8-dim Spacetime with NonUnitary Octonionic Inflation of our Universe to Unitary Quaternionic Post-Inflation M4 Minkowski Physical Spacetime produces the Higgs by a Mayer-Trautman mechanism.

3 - The CP2 = SU(3)/U(2) structure of Internal Symmetry Space allows one D4 to act with respect to M4 as the Conformal Group to produce Gravity by a MacDowell-Mansouri mechanism and the other D4 to act as the Standard Model with respect to CP2 by a Batakis mechanism.

4 - The 4+4 dimensional structure of M4xCP2 Kaluza-Klein produces the Second and Third Generations of Fermions and
accurate calculation of the Truth Quark mass for the Middle State of a 3-State Higgs-Tquark system with Higgs as Tquark Condensate by a model of Yamawaki et al.

The resulting structure has a Bosonic Gauge Term of dimensionality 28x1 = 28

and a Fermionic Spinor Fermion Term also of dimensionality 8 x 7/2 = 28

248 E8 = 120 D8 ⊕ 128 D8 Half Spinor

28 D4

D4 contains U(2,2)
U(2,2) = Spin(2,4)
Conformal Gravity

28 D4'

D4` contains U(4)
U(4) contains SU(3)

CP2 = SU(3) / U(2)
U(2) = SU(2) x U(1)

(8v , 8v)

M4 x CP2

(8s , 8s)

{ nu, ruq, guq, buq,
e, rdq, gdq, bdq }

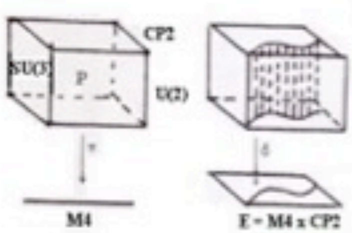
(8c , 8c)

{ $\overline{\text{nu}}$, $\overline{\text{ruq}}$, $\overline{\text{guq}}$, $\overline{\text{buq}}$,
$\overline{\text{e}}$, $\overline{\text{rdq}}$, $\overline{\text{gdq}}$, $\overline{\text{bdq}}$ }

Generations:	Propagator Origin	Propagator Destination
1	M4	M4
2	M4 CP2	CP2 M4
3	CP2	CP2

Lagrangian: $\int_{\text{KKspacetime}}$ gauge term + fermion term

Higgs-Mayer:



Kobayashi-Nomizu:

Theorem 11.7. *Assume in Theorem 11.5 that* $\mathfrak{k}$ *admits a subspace* $\mathfrak{m}$ *such that* $\mathfrak{k} = \mathfrak{j} + \mathfrak{m}$ *(direct sum) and* $\text{ad}(J)(\mathfrak{m}) = \mathfrak{m}$, *where* $\text{ad}(J)$ *is the adjoint representation of J in* $\mathfrak{k}$. *Then*

(1) *There is a 1:1 correspondence between the set of K-invariant connections in P and the set of linear mappings* $\Lambda_{\mathfrak{m}}: \mathfrak{m} \to \mathfrak{g}$ *such that*

$$\Lambda_{\mathfrak{m}}(\text{ad}(j)(X)) = \text{ad}(\lambda(j))(\Lambda_{\mathfrak{m}}(X)) \quad \text{for } X \in \mathfrak{m} \text{ and } j \in J;$$

the correspondence is given via Theorem 11.5 by

$$\Lambda(X) = \begin{cases} \lambda(X) & \text{if } X \in \mathfrak{j}, \\ \Lambda_{\mathfrak{m}}(X) & \text{if } X \in \mathfrak{m}. \end{cases}$$

(2) *The curvature form* Ω *of the K-invariant connection defined by* $\Lambda_{\mathfrak{m}}$ *satisfies the following condition:*

$$2\Omega_{u_0}(\tilde{X}, \tilde{Y}) = [\Lambda_{\mathfrak{m}}(X), \Lambda_{\mathfrak{m}}(Y)] - \Lambda_{\mathfrak{m}}([X, Y]_{\mathfrak{m}}) - \lambda([X, Y]_{\mathfrak{j}}) \quad \text{for } X, Y \in \mathfrak{m},$$

The Higgs and the T-quark form a system in which the Higgs is effectively a T-quark condensate.

How does T2 represent the first-generation fermions seen in experiments ?
Using basis {c1,ci} for C and {q1,qi,qj,qk} for H and {1,i,j,k,E,I,J,K} for O
each T can be decomposed as follows:
{q1,qi,qj,qk} represent { lepton , red quark , green quark , blue quark }
{c1,ci} represent { neutrino / down quark , electron / up quark }
{1,i,j,k,E,I,J,K} represent 8 covariant components of each fermion
with respect to 4+4 = 8-dim Kaluza-Klein Spacetime M4xCP2
with {1,i,j,k} representing 4-dim M4 Minkowski Physical Spacetime
and (E,I,J,K} representing 4-dim CP2 Internal Symmetry Space.

How do T2 fermions interact with each other ?
Consider fermionic 128-dim T2 as the spinor part of E8.
Construct a Local Lagrangian using the 120-dim Spin(16) part of E8
which can be decomposed into
two copies of the 28-dim Spin(8) Lie algebra
plus 64-dim of 8-dim spacetime position x 8-dim spacetime momentum
so that **the Lagrangian density has**
a fermionic term from the T2 spinor space and
gauge boson terms from the two copies of Spin(8)
which are integrated over the 8-dim spacetime as base manifold.

How does the Local Lagrangian Physics extend Globally ?
Since the E8 Lagrangian is Local, it is necessary to patch together Local Lagrangian Regions to form a Global Structure describing a Global E8 Algebraic Quantum Field Theory (AQFT). Each E8 of each region is embedded into Cl(16) and the completion of the union of all tensor products of all the Cl(16) are taken thus producing a generalized **Hyperfinite II1 von Neumann factor Algebraic Quantum Field Theory**.

What is the Physics of World-Line Histories of Particles/Antiparticles ?

8 + 8 + 8 = 24-dim of fermion particles and antiparticles and of spacetime can be represented by a Leech lattice underlying 26-dim String Theory in which strings represent World-Lines in the E8 Physics model.

The automorphism group of a single 26-dim String Theory cell modulo the Leech lattice is the Monster Group of order about 8 x 10^53.

A fermion particle/antiparticle does not remain a single Planck-scale entity because Tachyons create a cloud of particles/antiparticles.

The cloud is one Planck-scale Fundamental Fermion Valence Particle plus an effectively neutral cloud of particle/antiparticle pairs forming a Kerr-Newman black hole whose structure comes from the 24-dim Leech lattice part of the Monster Group which is 2^(1+24) times the double cover of Co1, for a total order of about 10^26.

(Since a Leech lattice is based on copies of an E8 lattice and since there are 7 distinct E8 integral domain lattices there are 7 (or 8 if you include a non-integral domain E8 lattice) distinct Leech lattices, and the physical Leech lattice is a superposition of them, effectively adding a factor of 8 to the order.)

The volume of the Kerr-Newman Cloud should be on the order of 10^27 x Planck scale, and **the Kerr-Newman Cloud should contain on the order of 10^27 particle/antiparticle pairs** and **its size should be** somewhat larger than, but roughly similar to, 10^(27/3) x 1.6 x 10^(-33) cm = roughly **10^(-24) cm**.

Kerr-Newman Clouds as Schwinger Sources:
Green's Function Propagators

Schwinger, in Nottingham hep-ph/9310283, said:
"... in the phenomenological **source theory ...**
there are no divergences, and no renormalization ...
the source concept ... is abstracted from the physical possibility
of creating or annihilating any particle in a suitable collision. ...
The basic physical act begins with the creation of a particle by a source,
followed by the propagation ... of that particle
between the neighborhoods of emission and detection,
and is closed by the source annihilation of the particle.
Relativistic requirements largely constrain the structure of
the propagation function - Green's function ...".

Wyler/Hua Complex Domain Structure of Schwinger Sources:
Bergman Kernels and Green's functions

Armand Wyler, in "The Complex Light Cone, Symmetric Space of the Conformal Group" (IAS Princeton, June 1972), said:
"... define the Bergman metric, the invariant differential operators
and their elementary solutions (Green functions) in the bounded realization
Dn of SO(n,2) / (SO(n) x SO(2) with Silov boundary Qn ...
the value of the structure constant alpha is obtained
as coefficient of the Green function of the Dirac equation in D5 ...".

AQFT:

Since the E8 classical Lagrangian is Local, it is necessary to patch together Local Lagrangian Regions to form a Global Structure describing a Global E8 Algebraic Quantum Field Theory (AQFT).

Mathematically, this is done by using Clifford Algebras
to embed E8 into Cl(16) and
using a copy of Cl(16) to represent each Local Lagrangian Region.
A Global Structure is then formed
by taking the tensor products of the copies of Cl(16).
Due to Real Clifford Algebra 8-periodicity, Cl(16) = Cl(8)xCl(8)
and any Real Clifford Algebra, no matter how large, can be embedded in a tensor product of factors of Cl(8), and therefore of Cl(8)xCl(8) = Cl(16).
Just as the completion of the union of all tensor products
of 2x2 complex Clifford algebra matrices produces
the usual Hyperfinite II1 von Neumann factor
that describes creation and annihilation operators
on the fermionic Fock space over C^(2n) (see John Baez's Week 175),
we can take
the completion of the union of all tensor products of Cl(16) = Cl(8)xCl(8)
to produce a generalized Hyperfinite II1 von Neumann factor
that gives a natural Algebraic Quantum Field Theory structure to the E8 model.

In each tensor product Cl(16) x ... x Cl(16)
each of the Cl(16) factors represents a distinct Local Lagrangian Region.
Since each Region is distinguishable from any other,
each factor of the tensor product is distinguishable
so that the AQFT has Maxwell-Boltzmann Statistics.

Within each Local Lagrangian Region Cl(16) lives its own E8.
Each 248-dim E8 has indistinguishable boson and fermion particles.
The 120-dim bosonic part has commutators and Bose Statistics
and
the 128-dim fermionic part has anticommutators and Fermi Statistics.

EPR:

For the E8 model AQFT to be realistic, it must be consistent with EPR entanglement relations. Joy Christian in arXiv 0904.4259 "Disproofs of Bell, GHZ, and Hardy Type Theorems and the Illusion of Entanglement" said: "... a [geometrically] correct local-realistic framework ... provides exact, deterministic, and local underpinnings for at least the Bell, GHZ-3, GHZ-4, and Hardy states. ... The alleged non-localities of these states ... result from misidentified [geometries] of the EPR elements of reality. ... The correlations are ... the classical correlations among the points of a 3 or 7-sphere ... S3 and S7 ... are ... parallelizable ...
The correlations ... can be seen most transparently in the elegant language of Clifford algebra ...".

To go beyond the interesting but not completely physically
realistic Bell, GHZ-3, GHZ-4, and Hardy states,
we must consider more complicated spaces than S3 and S7,
but still require that they be parallelizable
and be related to Clifford algebra structure.

As Martin Cederwall said in hep-th/9310115: "... The only
simply connected compact parallelizable manifolds are
the Lie groups [including S3 = SU(2)] and S7 ...".

We know that S3 = SU(2) = Spin(4) / SU(2) so that it has global
symmetry of Spin(4) transformations
and that 6-dimensional Spin(4) is the grade-2 part of the
16-dimensional Cl(4) Clifford algebra with graded structure
16 = 1 + 4 + 6 + 4 + 1 (where grades are 0,1,2, ...).

We also know that S7 = Spin(8) / Spin(7) so that it has global
symmetry of Spin(8) transformations
and that 28-dimensional Spin(8) is the grade-2 part of the
256-dimensional Cl(8) Clifford algebra with graded structure
256 = 1 + 8 + 28 + 56 + 70 + 56 + 28 + 8 + 1.

To get a Clifford algebra related parallelizable Lie group large enough to represent a realistic physics model, take the tensor product Cl(8) x Cl(8)
which by the 8-periodicity property of Real Clifford algebras is 256x256 = 65,536-dimensional Cl(16) with graded structure
(1x1) + (1x8+8x1) + (1x28+28x1+8x8) + ... = 1 + 16 + 120 + ...
whose 28+28+64 = 120-dimensional grade-2 part is Spin(16)
and
whose spinor representation has 256 = 128+128 dimensions.

Spin(16) has Cl(16) Clifford algebra structure and is a Lie group, and therefore parallelizable,
but it has grade-2 bivector bosonic structure and so can only represent physical things like gauge bosons and vector spacetime, and cannot represent physical things like fermions with spinor structure.

However, if we add one of the two 128-dimensional Cl(16) half-spinor representations to the bosonic adjoint 120-dimensional representation of Spin(16) ,
we get the 120+128 = 248-dimensional exceptional Lie group E8.

248-dimensional E8 has a 7-grading (due to Thomas Larsson)
8 + 28 + 56 + 64 + 56 + 28 + 8
(where grades are -3,-2,-1,0,1,2,3)

If 8 of the 64 central grade-0 elements are assigned to an 8-dimensional Cartan subalgebra of E8, the remaining 248-8 = 240 elements are the 240 Root Vectors of E8 which have a graded structure
8 28 56 56 56 28 8
that is consistent with the physical interpretations of my E8 model described earlier in this paper.

Since E8 is a Lie Group and therefore parallelizable
and lives in Clifford Algebra Cl(16)
my E8 Physics model should be consistent with EPR.

Chirality:

E8 = adjoint D8 + half-spinor D8 so D8 lives in Cl(16). Consider D8 representations
120-dim adjoint - denoted by D8adj
128-dim +half-spinor - denoted by D8s+
128-dim -half-spinor - denoted by D8s-
with physical interpretations
D8adj as gauge bosons plus spacetime
D8s+ as one generation of fermion particles and antiparticles
Ds- as one antigeneration of fermion particles and antiparticles
then
if you try to form a Lie algebra from D8adj + Ds+ + Ds- it does not work,
but if you try to form a Lie algebra from D8adj + Ds+ you succeed
and get E8 with the 64+64 = 128-dim Ds+ representing
one generation of fermion particles (one 64 of Ds+)
and one generation of fermion antiparticles (the other 64 of Ds+).
There is no physical D8s- antigeneration of fermions,
and one generation of D8s+ fermions lives inside E8.
The Atiyah-Singer index gives the net number of generations.
10-dim superstring theory uses the Euler index of the compact manifold (6-dim)
that reduces 10-dim spacetime to physical 4-dim.
E8 Physics the 4-dim compact manifold CP2 reduces 8-dim spacetime to M4 x CP2
(or reduces 10-dim spacetime to 6-dim Conformal spacetime that leads to M4 x CP2).

The index structure of the CP2 has Euler number 2+1 = 3 and Atiyah-Singer index -1/8
which is not the net number of generations because CP2 has no spin structure
so use a generalized spin structure (Hawking and Pope (Phys. Lett. 73B (1978) 42-44))
to get (for integral m) the generalized CP2 index n_R - n_L = (1/2) m (m+1)
Prior to Dimensional Reduction: m = 1, n_R - n_L = (1/2)x1x2 = 1 for 1 generation
After Reduction to 4+4 Kaluza-Klein: m = 2, n_R - n_L = (1/2)x2x3 = 1 for 3 generations
(second and third generations emerge as effective composites of the first)

Hawking and Pope say: "Generalized Spin Structures in Quantum Gravity ...
what happens in CP2 … is a two-surface K which cannot be shrunk to zero.
Parallel propagation of tetrads around K produces a curve in SO(4)
which cannot be shrunk to zero … i.e. it correspond[s] to a rotation through 2 pi …
Thus one could not define spinors consistently over such a space … In ... CP2 there is
a covariant constant two-form which can be taken as the electromagnetic field …
The index theorem then gives nR - nL = (1/2) m (m+1). This is always an integer
…

For an electromagnetic generalized spin structure [U(1) on CP2]
the fermions would have to carry half the electric charge of any bosons.
This obviously does not correspond with the real universe.
However, one could replace the electromagnetic field by
a Yang-Mills field whose group G had a double covering G~.

The fermion field would have to occur in representations which changed sign
under the non-trivial element of the kernel of the projection … G~ -> G
while
the bosons would have to occur in representations which did not change sign …".

In E8 Physics gauge bosons are in the 28+28 = 56-dim D4 + D4 subalgebra of E8.

One D4 acts on the M4 part of M4 x CP2 through its SU(2,2) = Spin(2,4) Conformal Subalgebra to give MacDowell-Mansouri Gravity

The other D4 = SO(8) acts on the CP2 part of M4 x CP2 through its SU(4) subalgebra that contains color SU(3). Electroweak SU(2)xU(1) comes from CP2 = SU(3) / U(2).
This D4 coupling to the 8-dim fundamental fermion particles comes from
the way that 28-dim Spin(8) couples to 8-dim D4-half-spinors based on Triality.
This D4 = SO(8) is the Hawking-Pope G which has double covering G~ = Spin(8).

The 8 fermion particles / antiparticles are D4 half-spinors represented within E8
by anti-commutators and so do change sign
while
the 28 gauge bosons are D4 adjoint represented within E8 by commutators
and so do not change sign.

The Octonionic structure of the 8-dim D4 half-spinors gives all the correct properties
(quantum numbers = electric charge, color charge, helicity).

This establishes what Hawking and Pope described as
"… the interesting possibility that there may be a connexion between
the topology of space-time and the spectrum of elementary particles …".

Coleman-Mandula:

Steven Weinberg said at pages 382-384 of his book
The Quantum Theory of Fields, Vol. III (Cambridge 2000):
"... The proof of the Coleman-Mandula theorem ... makes it clear
that the list of possible bosonic symmetry generators is essentially the same
in d greater than 2 spacetime dimensions as in four spacetime dimensions:
...
there are only the momentum d-vector Pu, a Lorentz generator Juv = -Jvu
(with u and v here running over the values 1, 2, ... , d-1, 0), and various
Lorentz scalar 'charges' ...
the fermionic symmetry generators furnish a representation of the
homogeneous Lorentz group ... or, strictly speaking, of its covering group
Spin(d-1,1). ...
The anticommutators of the fermionic symmetry generators with each other
are bosonic symmetry generators, and therefore must be a linear
combination of the Pu, Juv, and various conserved scalars. ...
the general fermionic symmetry generator must transform according to the
fundamental spinor representations of the Lorentz group ...
and not in higher spinor representations,
such as those obtained by adding vector indices to a spinor. ...".

In short, the important thing about Coleman-Mandula is that fermions in a unified model must "... transform according to the fundamental spinor representations of the Lorentz group ... or, strictly speaking, of its covering group Spin(d-1,1). ..." where d is the dimension of spacetime in the model.

In my E8 Physics model, E8 is the sum of
the adjoint representation and a half-spinor representation of Spin(16),
and
the Spin(16) structure (since Cl(16) = Cl(8) x Cl(8)) leads
to Spin(8) or Spin(1,7) structure with Triality automorphisms among
8-dim spacetime vectors and the two 8-dim half-spinors
and
the fermionic fundamental spinor representations of the E8 model are
therefore built with respect to Lorentz, spinor, etc representations based on
Spin(1,7) spacetime consistently with Weinberg's work,
so
the E8 model is consistent with Coleman-Mandula.

Mayer-Trautman Mechanism:

The objective is to reduce the integral over the 8-dim Kaluza-Klein M4 x CP2 to an integral over the 4-dim M4.

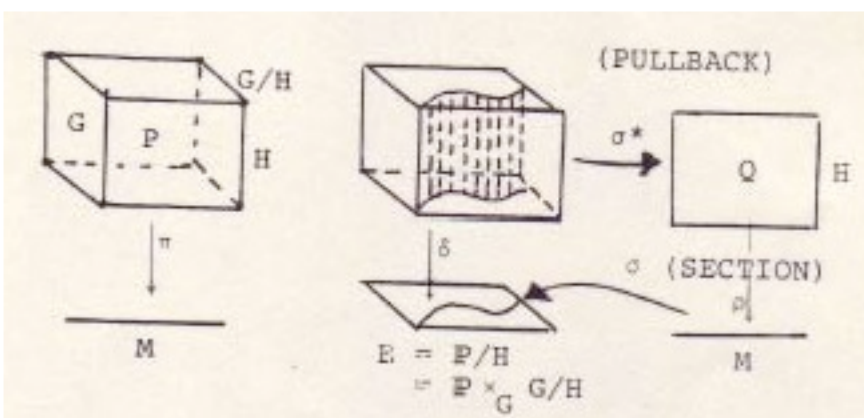
Since the D4 = U(2,2) acts on the M4, there is no problem with it.
Since the CP2 = SU(3) / U(2) has global SU(3) action,
the SU(3) can be considered as a local gauge group acting on the M4,
so there is no problem with it.

However, the U(2) acts on the CP2 = SU(3) / U(2) as little group, and so has local action on CP2 and then on M4, so the local action of U(2) on CP2 must be integrated out to get the desired U(2) local action directly on M4.

Since the U(1) part of U(2) = U(1) x SU(2) is Abelian, its local action on CP2 and then M4 can be composed to produce a single U(1) local action on M4, so there is no problem with it.

That leaves non-Abelian SU(2) with local action on CP2 and then on M4, and the necessity to integrate out the local CP2 action to get something acting locally directly on M4.

This is done by a mechanism due to Meinhard Mayer and A. Trautman in
"A Brief Introduction to the Geometry of Gauge Fields" and
"The Geometry of Symmetry Breaking in Gauge Theories",
Acta Physica Austriaca, Suppl. XXIII (1981) where they say:
"...



... We start out from ... four-dimensional M [M4] ...[and]... R ...[that is]... obtained from ... G/H [CP2 = SU(3) / U(2)] ... the physical surviving components of A and F, which we will denote by A and F, respectively, are a one-form and two form on

M [M4] with values in H [SU(2)] ... the remaining components will be subjected to symmetry and gauge transformations, thus reducing the Yang-Mills action ...[on M4 x CP2]... to a Yang-Mills-Ginzburg-Landau action on M [M4] ... Consider the Yang-Mills action on R ...

S_YM = Integral Tr (F /\ *F)

... We can ... split the curvature F into components along M [M4] (spacetime) and those along directions tangent to G/H [CP2] .
We denote the former components by F_!! and the latter by F_?? , whereas the mixed components (one along M, the other along G/H) will be denoted by F_!? ...
Then the integrand ... becomes

Tr(F_!! F^!! + 2 F_!? F^!? + F_?? F^??)

...
The first term .. becomes the [SU(2)] Yang-Mills action for the reduced [SU(2)] Yang-Mills theory
...
the middle term .. becomes, symbolically,

Tr Sum D_! PHI(?) D^! PHI(?)

where PHI(?) is the Lie-algebra-valued 0-form corresponding to the invariance of A with respect to the vector field ? , in the G/H [CP2] direction
...
the third term ... involves the contraction F_?? of F with two vector fields lying along G/H [CP2] ... we make use of the equation [from Mayer-Trautman, Acta Physica Austriaca, Suppl. XXIII (1981) 433-476, equation 6.18]

2 F_?? = [PHI(?) , PHI(?)] - PHI([?,?])

... Thus,
the third term ... reduces to what is essentially a Ginzburg-Landau potential in the components of PHI:

Tr F_?? F^?? = (1/4) Tr ([PHI , PHI] - PHI)^2

... special cases which were considered show that ...[the equation immediately above]... has indeed the properties required of a Ginzburg_Landau-Higgs potential, and moreover the relative signs of the quartic and quadratic terms are correct, and only one overall normalization constant ... is needed. ...".

See S. Kobayashi and K. Nomizu, Foundations of Differential Geometry, Volume I, Wiley (1963), especially section II.11:
"...

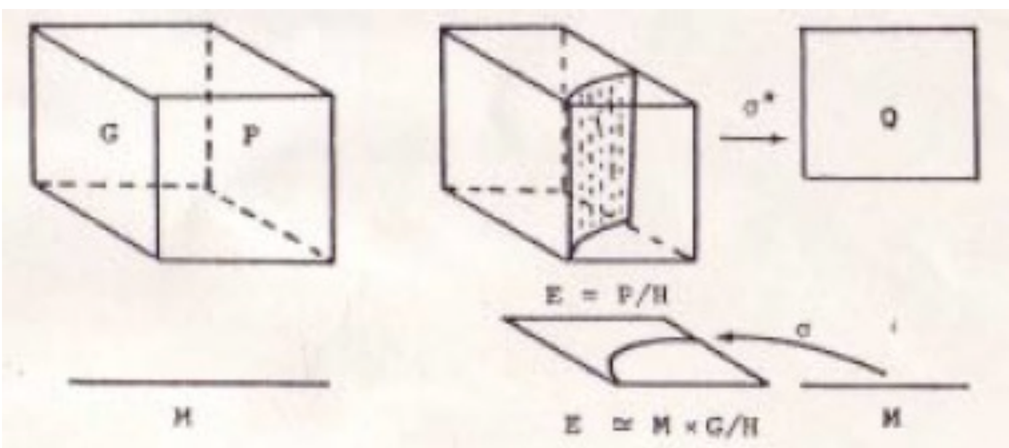
THEOREM 11.7. *Assume in Theorem* 11.5 *that* $\mathfrak{k}$ *admits a subspace* $\mathfrak{m}$ *such that* $\mathfrak{k} = \mathfrak{j} + \mathfrak{m}$ (*direct sum*) *and* $\mathrm{ad}(J)(\mathfrak{m}) = \mathfrak{m}$, *where* $\mathrm{ad}(J)$ *is the adjoint representation of* J *in* $\mathfrak{k}$. *Then* ...

The curvature form Ω *of the K-invariant connection defined by* $\Lambda_{\mathfrak{m}}$ *satisfies the following condition:*

$$2\Omega_{u_0}(\tilde{X}, \tilde{Y}) = [\Lambda_{\mathfrak{m}}(X), \Lambda_{\mathfrak{m}}(Y)] - \Lambda_{\mathfrak{m}}([X, Y]_{\mathfrak{m}}) - \lambda([X, Y]_{\mathfrak{j}}) \quad \text{for } X, Y \in \mathfrak{m},$$

...".

Along the same lines,
Meinhard E. Mayer said (Hadronic Journal 4 (1981) 108-152):
"...



... each point of ... the ... fibre bundle ... E consists of a four- dimensional spacetime point x [in M4] to which is attached the homogeneous space G / H [SU(3) / U(2) = CP2] ... the components of the curvature lying in the homogeneous space G / H [= SU(3) / U(2)] could be reinterpreted as Higgs scalars (with respect to spacetime [M4]) ...
the Yang-Mills action reduces to a Yang-Mills action for the h-components [U(2) components] of the curvature over M [M4] and a quartic functional for the "Higgs scalars", which not only reproduces the Ginzburg-Landau potential, but also gives the correct relative sign of the constants, required for the BEHK ... Brout-Englert-Higgs-Kibble ... mechanism to work. ...".

MacDowell-Mansouri Mechanism:

Rabindra Mohapatra (in section 14.6 of Unification and Supersymmetry, 2nd edition, Springer-Verlag 1992) says:

§14.6. Local Conformal Symmetry and Gravity

Before we study supergravity, with the new algebraic approach developed, we would like to discuss how gravitational theory can emerge from the gauging of conformal symmetry. For this purpose we briefly present the general notation for constructing gauge covariant fields. The general procedure is to start with the Lie algebra of generators X_A of a group

$$[X_A, X_B] = f_{AB}^{C} X_C. \tag{14.6.1}$$

where f_{AB}^{C} are structure constants of the group. We can then introduce a gauge field connection h_μ^A as follows:

$$h_\mu = h_\mu^A X_A. \tag{14.6.2}$$

Let us denote the parameter associated with X_A by ε^A. The gauge transformations on the fields h_μ^A are given as follows:

$$\delta h_\mu^A = \partial_\mu \varepsilon^A + h_\mu^B \varepsilon^C f_{CB}^{A} = (D_\mu \varepsilon)^A. \tag{14.6.3}$$

We can then define a covariant curvature

$$R_{\mu\nu}^A = \partial_\nu h_\mu^A - \partial_\mu h_\nu^A + h_\nu^B h_\mu^C f_{CB}^{A}. \tag{14.6.4}$$

Under a gauge transformation

$$\delta_{\text{gauge}} R_{\mu\nu}^A = R_{\mu\nu}^B \varepsilon^C f_{CB}^{A}. \tag{14.6.5}$$

We can then write the general gauge invariant action as follows:

$$I = \int d^4x \, Q_{AB}^{\mu\nu\rho\sigma} R_{\mu\nu}^A R_{\rho\sigma}^B. \tag{14.6.6}$$

Let us now apply this formalism to conformal gravity. In this case

$$h_\mu = P_m e_\mu^m + M_{mn} \omega_\mu^{mn} + K_m f_\mu^m + D b_\mu. \tag{14.6.7}$$

The various $R_{\mu\nu}$ are

$$R_{\mu\nu}(P) = \partial_\nu e_\mu^m - \partial_\mu e_\nu^m + \omega_\mu^{mn} e_\nu^n - \omega_\nu^{mn} e_\mu^n - b_\mu e_\nu^m + b_\nu e_\mu^m, \tag{14.6.8}$$

$$R_{\mu\nu}(M) = \partial_\nu \omega_\mu^{mn} - \partial_\mu \omega_\nu^{mn} - \omega_\nu^{mp} \omega_{\mu,p}^{n} - \omega_\mu^{mp} \omega_{\nu,p}^{n} - 4(e_\mu^m f_\nu^n - e_\nu^m f_\mu^n), \tag{14.6.9}$$

$$R_{\mu\nu}(K) = \partial_\nu f_\mu^m - \partial_\mu f_\nu^m - b_\mu f_\nu^m + b_\nu f_\mu^m + \omega_\mu^{mn} f_\nu^n - \omega_\nu^{mn} f_\mu^n, \tag{14.6.10}$$

$$R_{\mu\nu}(D) = \partial_\nu b_\mu - \partial_\mu b_\nu + 2 e_\mu^m f_\nu^m - 2 e_\nu^m f_\mu^m. \tag{14.6.11}$$

The gauge invariant Lagrangian for the gravitational field can now be written down, using eqn. (14.6.6), as

$$S = \int d^4x \, \varepsilon_{mnrs} \varepsilon^{\mu\nu\rho\sigma} R_{\mu\nu}^{mn}(M) R_{\rho\sigma}^{rs}(M). \tag{14.6.12}$$

We also impose the constraint that

$$R_{\mu\nu}(P) = 0, \tag{14.6.13}$$

which expresses ω_μ^{mn} as a function of (e, b). The reason for imposing this constraint has to do with the fact that P_m transformations must be eventually identified with coordinate transformation. To see this point more explicitly let us consider the vierbein e_μ^m. Under coordinate transformations

$$\delta_{GC}(\xi^\nu)e_\mu^m = \partial_\mu \xi^\lambda e_\lambda^m + \xi^\lambda \partial_\lambda e_\mu^m. \tag{14.6.14}$$

Using eqn. (14.6.8) we can rewrite

$$\delta_{GC}(\xi^\nu)e_\mu^m = \delta_P(\xi e^n)e_\mu^m + \delta_M(\xi\omega^{mn})e_\mu^m + \delta_D(\xi b)e_\mu^m + \xi^\nu R_{\mu\nu}^m(P),$$

where

$$\delta_P(\xi^n)e_\mu^m = \partial_\mu \xi^m + \xi^n \omega_\mu^{mn} + \xi^m b_\mu. \tag{14.6.15}$$

If $R^{\mu\nu}(P) = 0$, the general coordinate transformation becomes related to a set of gauge transformations via eqn. (14.6.15).

At this point we also wish to point out how we can define the covariant derivative. In the case of internal symmetries $D_\mu = \partial_\mu - iX_A h_\mu^A$; now since momentum is treated as an internal symmetry we have to give a rule. This follows from eqn. (14.6.15) by writing a redefined translation generator $\bar{P}$ such that

$$\delta_{\bar{P}}(\xi) = \delta_{GC}(\xi^\nu) - \sum_{A'} \delta_{A'}(\xi^m h_m^A), \tag{14.6.16}$$

where A' goes over all gauge transformations excluding translation. The rule is

$$\delta_{\bar{P}}(\xi^m)\phi = \xi^m D_m^C \phi. \tag{14.6.17}$$

We also wish to point out that for fields which carry spin or conformal charge, only the intrinsic parts contribute to D_m^C and the orbital parts do not play any rule.

Coming back to the constraints we can then vary the action with respect to f_μ^m to get an expression for it, i.e.,

$$e_\nu^m f_{\mu m} = -\tfrac{1}{4}[e_m^\lambda e_{n\nu} R_{\mu\lambda}^{mn} - \tfrac{1}{6} g_{\mu\nu} R], \tag{14.6.18}$$

where f_μ^m has been set to zero in R written in the right-hand side.

This eliminates (from the theory the degrees of freedom) ω_μ^{mn} and f_μ^m and we are left with e_μ^m and b_μ. Furthermore, these constraints will change the transformation laws for the dependent fields so that the constraints do not change.

Let us now look at the matter coupling to see how the familiar gravity theory emerges from this version. Consider a scalar field ϕ. It has conformal weight $\lambda = 1$. So we can write a convariant derivative for it, eqn. (14.6.17)

$$D_\mu^C \phi = \partial_\mu \phi - \phi b_\mu. \tag{14.6.19}$$

We note that the conformal charge of ϕ can be assumed to be zero since $K_m = x^2\partial$ and is the dimension of inverse mass. In order to calculate $\Box^c\phi$ we start with the expression for d'Alambertian in general relativity

$$\frac{1}{e}\partial_\nu(g^{\mu\nu} e D^C_\mu \phi). \qquad (14.6.20)$$

The only transformations we have to compensate for are the conformal transformations and the scale transformations. Since

$$\delta b_\mu = -2\xi^m_k e_{m\mu}, \qquad \delta(\phi b_\mu) = \phi\delta b_\mu = -2\phi f^m_\mu e^\mu_m = +\tfrac{2}{12}\phi R, \qquad (14.6.21)$$

where, in the last step, we have used the constraint equation (14.6.18). Putting all these together we find

$$\Box^C\phi = \frac{1}{e}\partial_\nu(g^{\mu\nu} e D^c_\mu \phi) + b_\mu D^c_\mu \phi + \tfrac{2}{12}\phi R. \qquad (14.6.22)$$

Thus, the Lagrangian for conformal gravity coupled to matter fields can be written as

$$S = \int e\, d^4x\, \tfrac{1}{2}\phi\Box^C\phi. \qquad (14.6.23)$$

Now we can use conformal transformation to gauge $b_\mu = 0$ and local scale transformation to set $\phi = \kappa^{-1}$ leading to the usual Hilbert action for gravity. To summarize, we start with a Lagrangian invariant under full local conformal symmetry and fix conformal and scale gauge to obtain the usual action for gravity. We will adopt the same procedure for supergravity. An important technical point to remember is that, $\Box^C$, the conformal d'Alambertian contains R, which for constant ϕ, leads to gravity. We may call ϕ the auxiliary field.

After the scale and conformal gauges have been fixed, the conformal Lagrangian becomes a de Sitter Lagrangian. Einstein-Hilbert gravity can be derived from the de Sitter Lagrangian, as was first shown by **MacDowell and Mansouri (Phys. Rev. Lett. 38 (1977) 739)**. (Note that Frank Wilczek, in hep-th/9801184, says that the MacDowell-Mansouri "... approach to casting gravity as a gauge theory was initiated by MacDowell and Mansouri ... S. MacDowell and F. Mansouri, Phys. Rev. Lett. 38 739 (1977) ... , and independently Chamseddine and West ... A. Chamseddine and P. West Nucl. Phys. B 129, 39 (1977); also quite relevant is A. Chamseddine, Ann. Phys. 113, 219 (1978). ...".

The minimal group required to produce Gravity, and therefore the group that is used in calculating Force Strengths, is the de Sitter group, as is described by Freund in chapter 21 of his book Supersymmetry (Cambridge 1986)
(Note that chapter 21 is a Non-Supersymmetry chapter leading up
to a Supergravity description in the following chapter 22):
"... Einstein gravity as a gauge theory ... we expect a set of gauge fields w^ab_u for the Lorentz group and a further set e^a_u for the translations, ...
Everybody knows though, that Einstein's theory contains but one spin two field, originally chosen by Einstein as g_uv = e^a_u e^b_v n_ab

(n_ab = Minkowski metric).
What happened to the w^ab_u ?
The field equations obtained from the Hilbert-Einstein action by varying the w^ab_u are algebraic in the w^ab_u ... permitting us to express the w^ab_u in terms of the e^a_u
...
The w do not propagate ...
We start from the four-dimensional de-Sitter algebra ... so(3,2).
Technically this is the anti-de-Sitter algebra ...
We envision space-time as a four-dimensional manifold M.
At each point of M we have a copy of SO(3,2) (a fibre ...) ...
and we introduce the gauge potentials (the connection) h^A_mu(x)
A = 1,..., 10 , mu = 1,...,4. Here x are local coordinates on M.
From these potentials h^A_mu we calculate the field-strengths
(curvature components) [let @ denote partial derivative]
R^A_munu = @_mu h^A_nu - @_nu h^A_mu + f^A_BC h^B_mu h^C_nu
...[where]...
the structure constants f^C_AB ...[are for]... the anti-de-Sitter algebra ....
We now wish to write down the action S as an integral over
the four-manifold M ... S(Q) = INTEGRAL_M R^A /\ R^B Q_AB
where Q_AB are constants ... to be chosen ... we require
... the invariance of S(Q) under local Lorentz transformations
... the invariance of S(Q) under space inversions ...
...[AFTER A LOT OF ALGEBRA THAT I WON'T TYPE HERE]...
we shall see ...[that]... the action becomes invariant under all local [anti]de-Sitter transformations ...[and]... we recognize ... the familiar
Hilbert-Einstein action with cosmological term in vierbein notation ...
Variation of the vierbein leads to the Einstein equations with cosmological term.
Variation of the spin-connection ... in turn ... yield the torsionless Christoffel connection ... the torsion components ... now vanish.
So at this level full sp(4) invariance has been checked.
... Were it not for the assumed space-inversion invariance ...
we could have had a parity violating gravity. ...
Unlike Einstein's theory ...[MacDowell-Mansouri].... does not require Riemannian invertibility of the metric. ... the solution has torsion ... produced by an interference between parity violating and parity conserving amplitudes.
Parity violation and torsion go hand-in-hand.
Independently of any more realistic parity violating solution of the gravity equations this raises the cosmological question whether
the universe as a whole is in a space-inversion symmetric configuration. ...".

At this stage, we have reconciled the first 3 of the 4 differences between our E8 Physics Model and conventional Gravity plus the Standard Model. Now we turn attention to

Second and Third Fermion Generations:

As to the existence of 3 Generations of Fermions,
note that the 8 First Generation Fermion Particles
and the 8 First Generation Fermion AntiParticles
can each be represented by the 8 basis elements of the Octonions O,
and
that the **Second and Third Generations** can be represented by
Pairs of Octonions OxO
and
Triples of Octonions OxOxO
respectively.

When the non-unitary Octonionic 8-dim spacetime is reduced to the Kaluza-Klein M4 x CP2 at the End of Inflation,
there are 3 possibilities for a fermion propagator from point A to point B:

1 - A and B are both in M4, so its path can be represented by the single O;

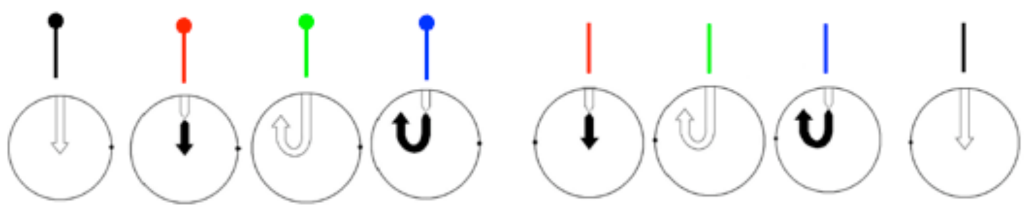
2 - Either A or B, but not both, is in CP2, so its path must be augmented by one projection from CP2 to M4, which projection can be represented by a second O, giving a second generation OxO;

3 - Both A and B are in CP2, so its path must be augmented by two projections from CP2 to M4, which projections can be represented by a second O and a third O,
giving a third generation OxOxO.

3 Generation Fermion Combinatorics

First Generation (8)

electron	red up quark	green up quark	blue up quark	red down quark	green down quark	blue down quark	neutrino
e	ie	je	ke	i	j	k	1

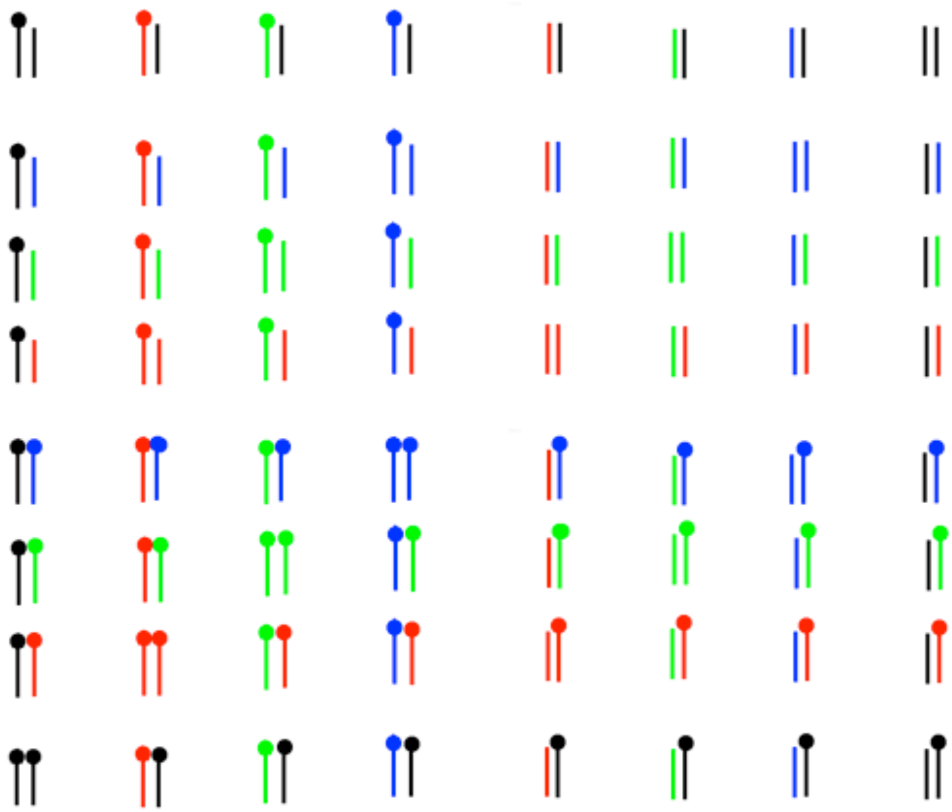


L	Li	Lj	Lk	i	j	k	1

The geometric representation of Octonions is from arXiv 1010.2979 by Jonathan Hackett and Louis H. Kauffman,
who say: "... we review the topological model for the quaternions based upon the Dirac string trick. We then extend this model, to create a model for the octonions - the non-associative generalization of the quaternions. ...
To construct this model of the quaternions using belt and buckle, we consider a belt that has been fixed to a wall with the non-buckle end. We consider rotations of the belt buckle about the three standard cartesian axes which we correspond to the three quaternionic roots of 1: i,j, and k. ... We ... get that carrying out any operation twice yields a belt that is twisted around by a full 2 pi ... if we perform 1 twice - giving us a 4 pi rotation - we can remove all of the twisting without rotating the belt buckle. ... We note that the operations are performed from left to right along a string of elements. ...
We construct our model for the octonions in a similar manner to the model for the quaternions. Rather than using a belt,
we will instead use a two toned ribbon (black on the back, and white on the front) with an arrowhead attached to one end (much as our belt had a buckle). The other end is then attached to the interior of a ring (much as our belt was attached to a wall). Lastly on the side of the ring we affix a flag that allows us to keep track of the orientation of the ring. ...
The operation L is defined by switching the side of the hoop that the flag is attached to, and performing a full 2 pi rotation of the hoop (or - alternately - the arrowhead) if the arrowhead is pointing up or if the state is flag-right, but not for both. ...
The original belt model of the quaternions is strongly related to the quaternions being a representation of SU(2), and SU(2) being a double cover of the rotation group SO(3).
The fact that this model of the octonions is an extension of the quaternionic model leads to the question of whether an analogue to the relationship with SU(2) and SO(3) exists. ...".

Perhaps relevant to that question is the fact that SU(4) is the double cover of SO(6)
and the relationship to the Conformal Group SU(2,2) = Spin(4,2).

Second Generation (8x8 = 64)



Mu Neutrino (1)

Rule: a Pair belongs to the Mu Neutrino if:
All elements are Colorless (black)
and all elements are Associative (that is, is 1 which is the only Colorless Associative element) .

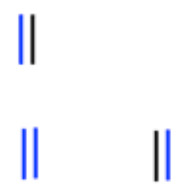
||

Muon (3)

Rule: a Pair belongs to the Muon if:
All elements are Colorless (black)
and at least one element is NonAssociative (that is, is e which is the only Colorless NonAssociative element).

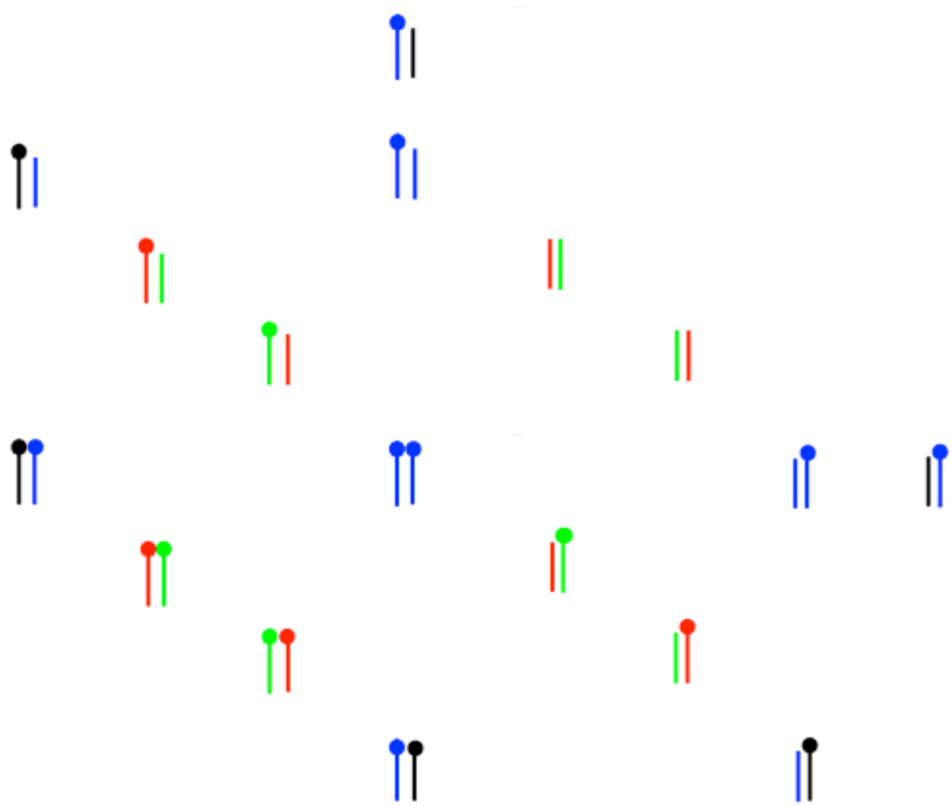
Blue Strange Quark (3)

Rule: a Pair belongs to the Blue Strange Quark if:
There is at least one Blue element and the other element is Blue or Colorless (black) and all elements are Associative (that is, is either 1 or i or j or k).

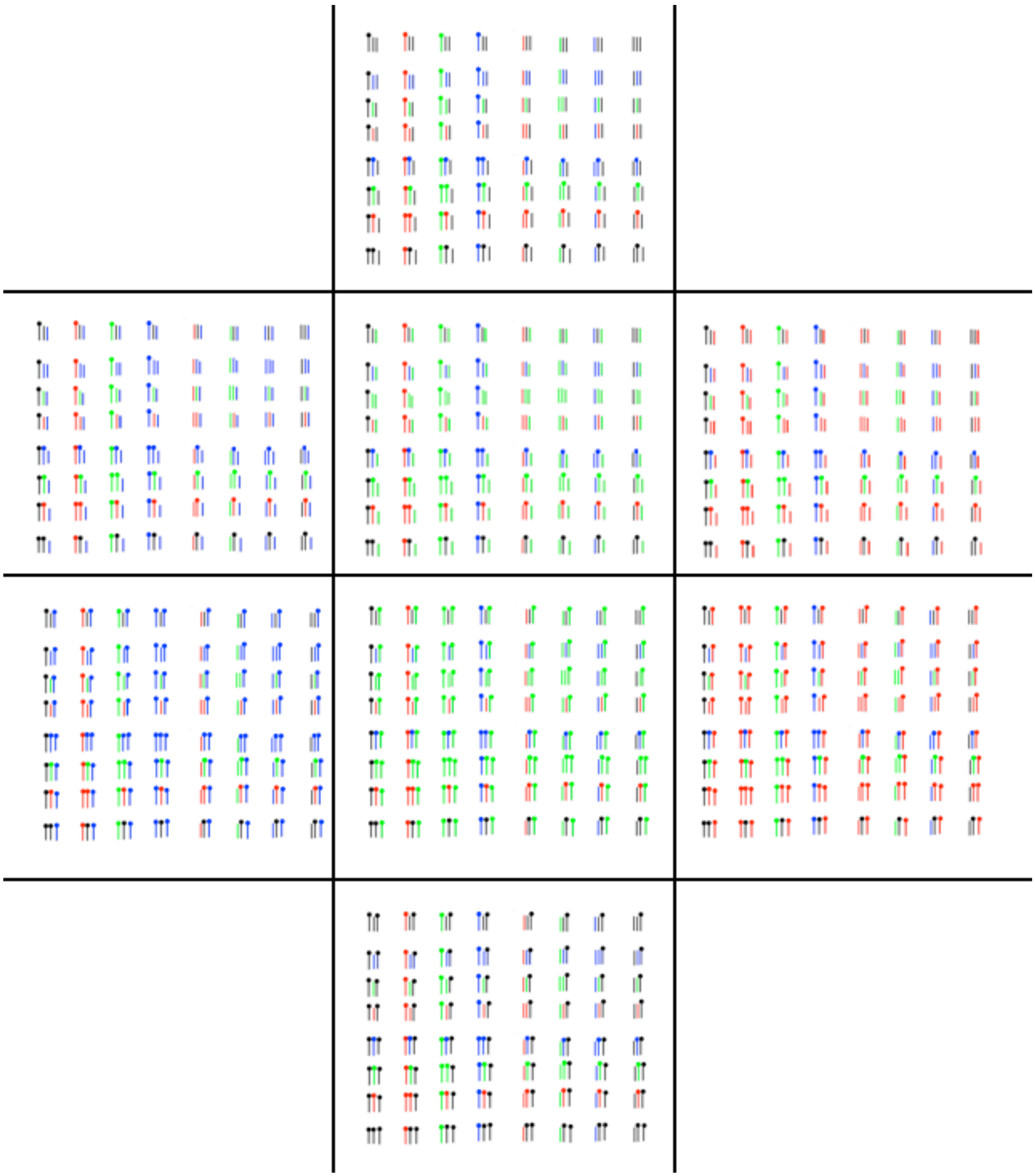


Blue Charm Quark (17)

Rules: a Pair belongs to the Blue Charm Quark if:
1 - There is at least one Blue element and the other element is Blue or Colorless (black) and at least one element is NonAssociative (that is, is either e or ie or je or ke)
2 - There is one Red element and one Green element (Red x Green = Blue).



Third Generation (8x8x8 = 512)

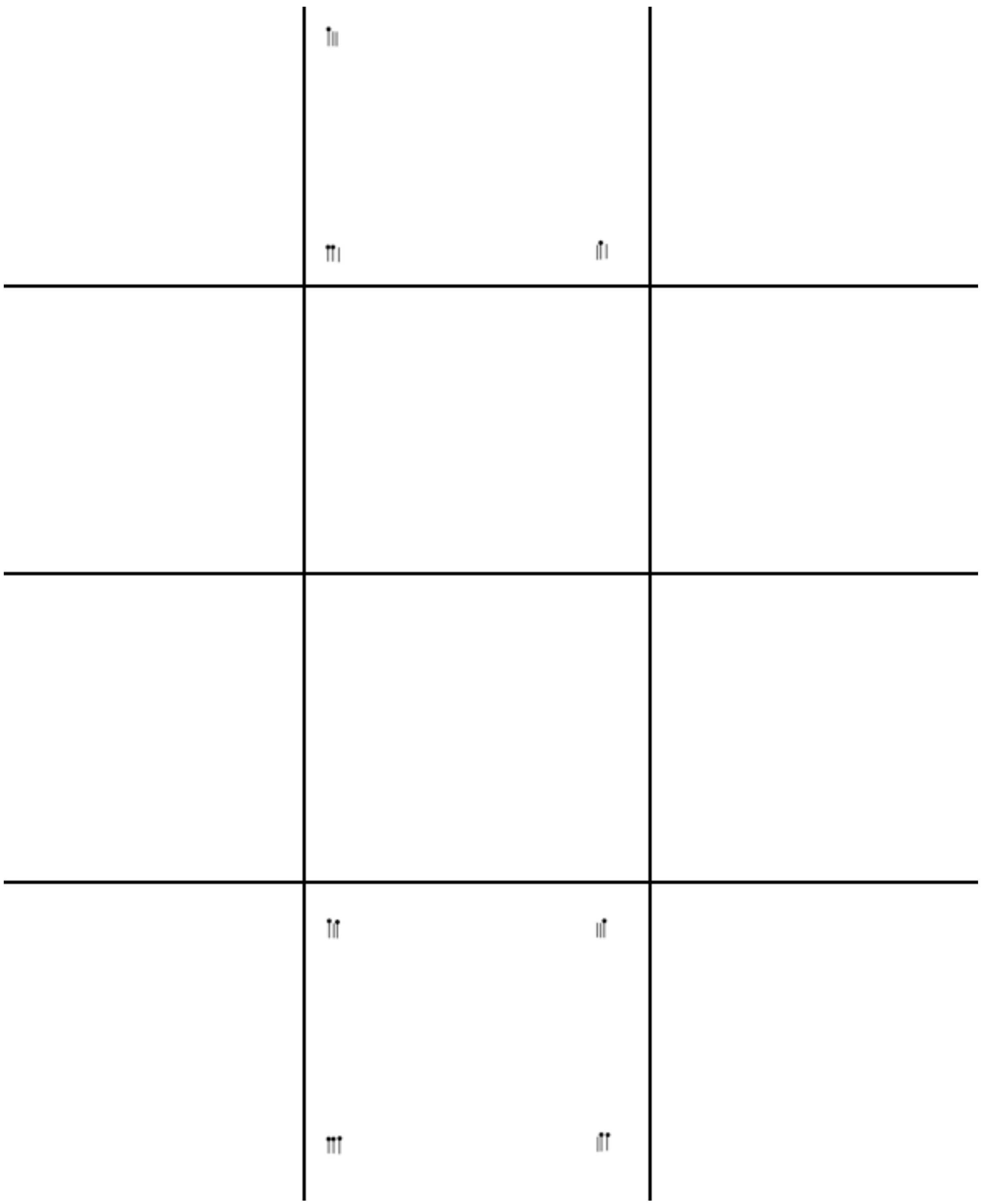


Tau Neutrino (1)

Rule: a Triple belongs to the Tau Neutrino if:
All elements are Colorless (black)
and all elements are Associative (that is, is 1 which is the only Colorless Associative element) .

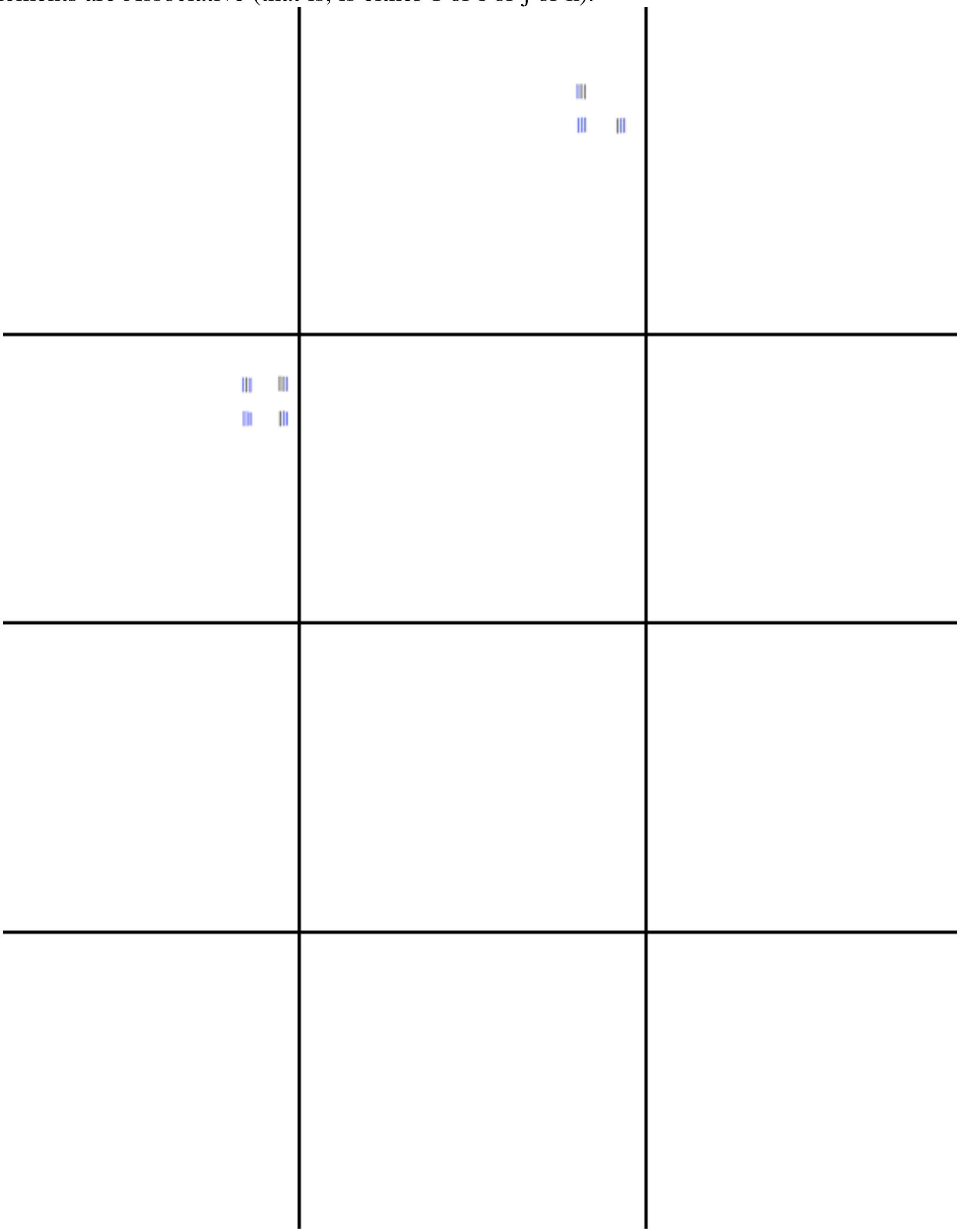
Tauon (7)

Rule: a Triple belongs to the Tauon if:
All elements are Colorless (black)
and at least one element is NonAssociative (that is, is e which is the only Colorless NonAssociative element).



Blue Beauty Quark (7)

Rule: a Triple belongs to the Blue Beauty Quark if:
There is at least one Blue element and all other elements are Blue or Colorless (black)
and all elements are Associative (that is, is either 1 or i or j or k).



Blue Truth Quark (161)

Rules: a Triple belongs to the Blue Truth Quark if:

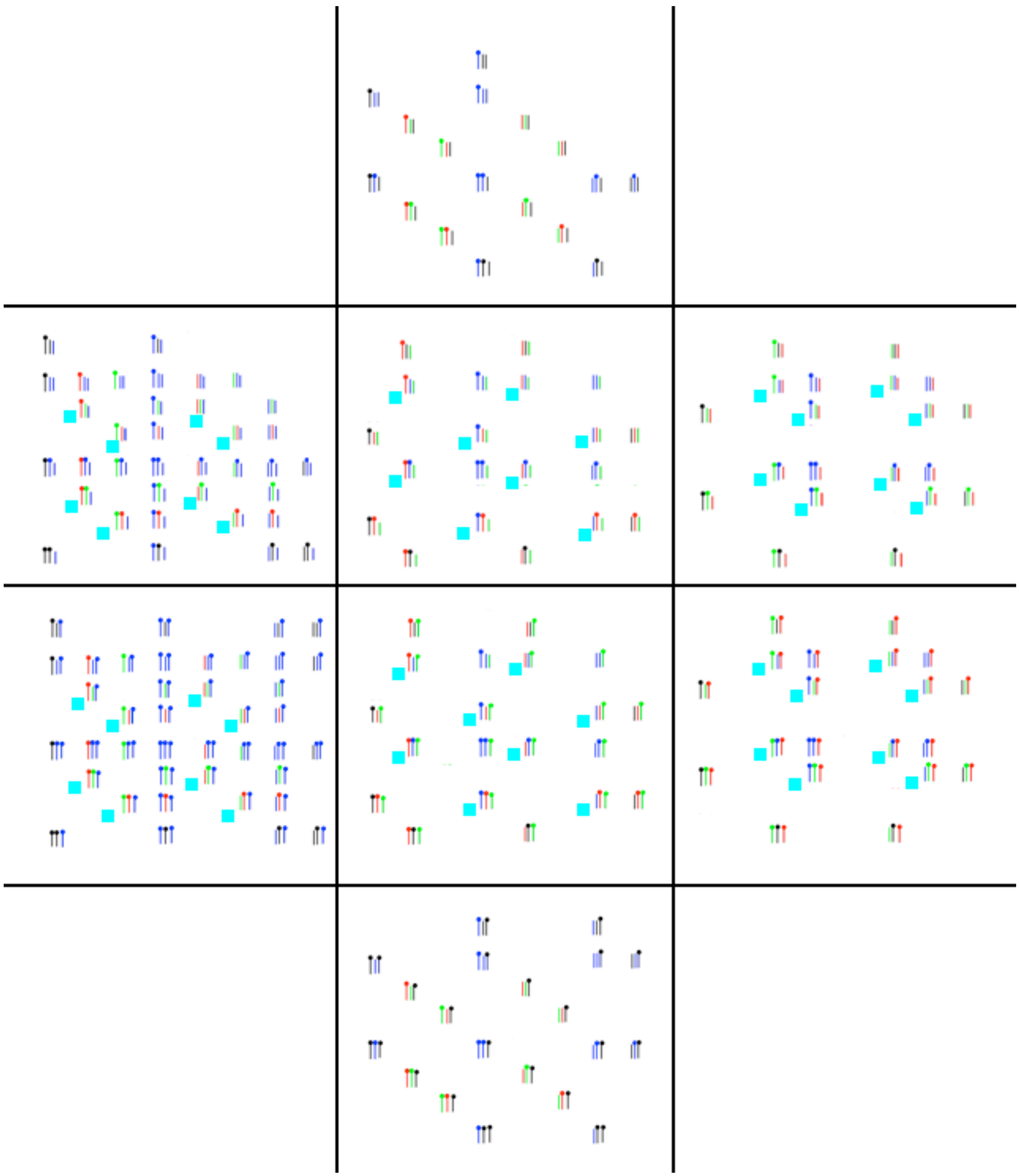
1 - There is at least one Blue element and all other elements are Blue or Colorless (black) and at least one element is NonAssociative (that is, is either e or ie or je or ke)

2 - There is one Red element and one Green element and the other element is Colorless (Red x Green = Blue)

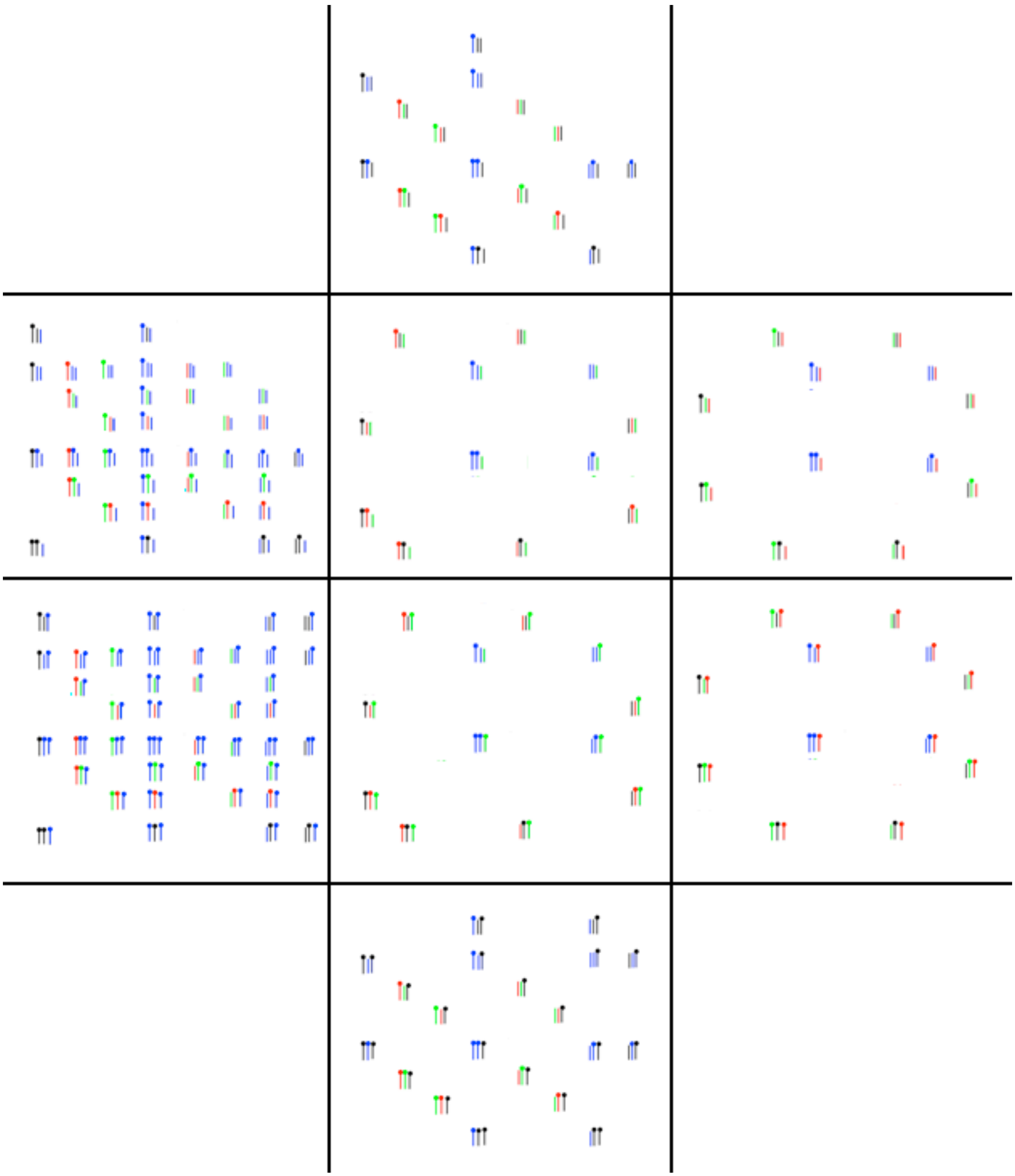
3 - The Triple has one element each that is Red, Green, or Blue, in which case the color of the Third element (for Third Generation) is determinative and must be Blue.

Candidates for Blue Truth Quark before application of Rule 3 (193)

with the 48 Rule 3 Candidates marked by cyan square:



Blue Truth Quark (161)



E8 Physics Fermions: 3 Conformal Generations

The E8 Lie Algebra of the E8 Physics Model contains two D4 Lie subalgebras:
248-dim E8 = 120-dim D8 + 128-dim half-spinor of D8
120-dim D8 = 28-dim D4 + 28-dim D4 + 64-dim D8 / D4xD4

One of the D4 contains an A2 = SU(3) Lie subalgebra that represents the Color Force of the Standard Model.
The Weak and Electromagnetic Forces are produces by a Batakis mechanism (see Class. Quantum Grav. 3 (1986) L99-L105 by N. A. Batakis) in which spacetime is 8-dimensional Kaluza-Klein M4 x CP2.
Color Force SU(3) acts globally on CP2 = SU(3) / SU(2)xU(1) and, due to Kaluza-Klein structure, acts as local gauge group on M4 Minkowski spacetime.
Local gauge group action of Weak SU(2) and Electromagnetic U(1) Forces comes from their being local isotropy groups of the symmetric space CP2.

Casimir Operators describe some physical properties of the Standard Model Forces:

A0 Lie algebra U(1) has trivial Weyl Group 1
and trivial Casimir of degree 1
so that the Photon carries no charge.

A1 Lie algebra SU(2) has Weyl Group S2 of order 2! = 2
and quadratic Casimir of degree 2 representing isospin
so that SU(2) Weak Bosons can carry Electromagnetic Charge.

A2 Lie algebra SU(3) has Weyl Group S3 of order 3! = 6
and two Casimir Operators of degrees 2 and 3:
a quadratic Casimir representing 2 { + , - } isospin charge states and
a cubic Casimir representing 3 { red, green, blue } colors
so that SU(3) Gluons can carry Electromagnetic Charge and Color Charge.

The other D4 contains an A3 = D3 Conformal Lie subalgebra that represents Gravity by a generalized MacDowell-Mansouri mechanism (see section 14.6 of Rabindra Mohapatra's book "Unification and Supersymmetry", 2nd edition, Springer-Verlag 1992).
The Conformal Group in the form SU(2,2) = Spin(2,4) is described by Robert Gilmore in his books "Lie Groups, Lie Algebras, and Some of Their Applications", Wiley 1974, and "Lie Groups, Physics, and Geometry", Cambridge 2008.

The Conformal Group has a Weyl Group of 2^2 x 3! = 24 elements
and has 3 Casimir Operators of degrees 2 and 4 and 6/2 = 3.

The Conformal degree 3 Casimir represents the 3 Generations of Fermions
(instead of the 3 colors as in the case of the Standard Model D4 of E8).

In its D3 Spin(2,4) form the Conformal Lie algebra can be represented
as a 6x6 antisymmetric real matrix:

0	J1	J2	M1	A1	G1
-J1	0	J3	M2	A2	G2
-J2	-J3	0	M3	A3	G3
-M1	-M2	-M3	0	A4	G4
-A1	-A2	-A3	-A4	0	G5
-G1	-G2	-G3	-G4	-G5	0

{J1,J2,J3} form a Spin(0,3) subalgebra of Spin(2,4) and produce
a quadratic Casimir Operator that represents an Angular Momentum Operator.

Adding {M1,M2,M3} forms a Spin(1,3) subalgebra of Spin(2,4) and produces
a second quadratic Casimir Operator that represents a Laplace-Runge-Lenz Operator.

Adding {A1,A2,A3} and {A4} forms a Spin(2,3) AntiDeSitter subalgebra of Spin(2,4)
with a quartic Casimir Operator that is a combination of {M1,M2,M3} and {A1,A2,A3}.
{A1,A2,A3} represent Momentum and {A4} represents Energy/Mass of Poincare Gravity
and its Dark Matter Primordial Black Holes.

Adding {G1,G2,G3} and {G4} and {G5} forms the full Spin(2,4) and produces
a cubic Casimir Operator for representation of 3 Generations of Fermions.
The {G1,G2,G3} represent 3 Higgs components giving mass to 3 Weak Bosons.
and {G4} represents massive Higgs Scalar as Fermion Condensate.
As Special Conformal and Scale degrees of freedom they also represent
the Momentum of Expansion of the Universe and its Dark Energy.

Adding {G5} represents HIggs/Fermion mass of Ordinary Matter.

The Higgs as a Fermionic Condensate gives mass to Fermions.
The fundamental Fermion Particles are those of the First Generation:

{neutrino, red down quark, green down quark, blue down quark;
electron , red up quark , green up quark , blue up quark}

They can be represented as basis elements {1,i,j,k,E,I,J,K} of Octonions O.

Each of {A4} and {G4} and {G5} can represent the mass of Fundamental Fermions.

The {A4} Conformal substructure

0	A4
-A4	0

represents First Generation Fermion Particles as Octonion basis elements O.

The {A4} plus {G5} Conformal substructure

0	A4	
-A4	0	G5
	-G5	0

represents Second Generation Fermion Particles as Octonion Pairs OxO.

The {A4} and {G5} plus {G4} Conformal substructure

0	A4	G4
-A4	0	G5
-G4	-G5	0

represents Third Generation Fermion Particles as Octonion Triples OxOxO.

Fermion AntiParticles are represented in a similar way.

Combinatorics of O and OxO and OxOxO produce realistic Fermion masses.

The Third Generation Truth Quark (Tquark) is by far the most massive Fermion so the Higgs as a Fermionic Condensate is effectively a Tquark Condensate.

Note:
E8 has 8 Casimir Operators of degrees 2, 8, 12, 14, 18, 20, 24, 30
The Conformal quadratic 2 is in E8, the Conformal quartic 4 is in the 8 of E8,
and the Conformal cubic 6/2 = 3 is in the 12 of E8.
D8 has 8 Casimir Operators of degrees 2, 4, 6, 8, 10, 12, 14, 8
The Conformal quadratic 2 and quartic 4 are in D8 and the Conformal cubic 6/2 = 3 is in the 6 of D8.
D4 has 4 Casimir Operators of degrees 2, 4, 6, 4
The Conformal quadratic 2 and quartic 4 are in D4 and the Conformal cubic 6/2 = 3 is in the 6 of D4.

SUMMARY:

The Conformal Group in the form SU(2,2) = Spin(2,4) is described by Robert Gilmore in his book "Lie Groups, Physics, and Geometry", Cambridge 2008:
"... 8x8 matrices acting on the four coordinates and the four momenta ... satisfy an antisymmetric ... symplectic metric ... preserve[d by the] ... group ... Sp(8:R)
...[and a]...
symmetric metric ... with signature (+4,-4) ... preserve[d by the] ... group ... SO(4,4) ...

$$Sp(8;\mathbb{R}) \cap SO(4,4) = SU(2,2) \simeq SO(4,2)$$

... The fifteen-dimensional Lie algebra for the Dirac equation is ... summarized by the 6x6 matrix

$$\left[\begin{array}{cccc|cc} 0 & J_3 & -J_2 & M_1 & A_1 & \Gamma_1 \\ -J_3 & 0 & J_1 & M_2 & A_2 & \Gamma_2 \\ J_2 & -J_1 & 0 & M_3 & A_3 & \Gamma_3 \\ -M_1 & -M_2 & -M_3 & 0 & A_4 & \Gamma_4 \\ \hline A_1 & A_2 & A_3 & A_4 & 0 & \Gamma_5 \\ \Gamma_1 & \Gamma_2 & \Gamma_3 & \Gamma_4 & -\Gamma_5 & 0 \end{array}\right] \begin{array}{c} + \\ + \\ + \\ + \\ - \\ - \end{array}$$

... three ... operators A4 , G4 , G5 close under commutation and span ... so(2,1) ...
The Casimir operator for this [sub]algebra is C^2 = G5^2 - G4^2 - A4^2 ...[It can be]... used to determine eigenstates and energy eigenvalues ...".

{J1,J2,J3} represent Angular Momentum. {M1,M2,M3} represent LaPlace-Runge-Lenz.
{A1,A2,A3} represent Momentum.
{G1,G2,G3} represent Higgs for W-Bosons and Momentum of Universe Expansion.
{A4} and {G4} and {G5} represent Energy/Mass including Higgs mass for Fermions.

The {A4} Conformal substructure

0 A4
-A4 0

represents First Generation Fermion Particles as Octonion basis elements O.

The {A4} plus {G5} Conformal substructure

0 A4
-A4 0 G5
-G5 0

represents Second Generation Fermion Particles as Octonion Pairs OxO.

The {A4} plus {G5} plus {G4} Conformal substructure

0 A4 G4
-A4 0 G5
-G4 -G5 0

represents Third Generation Fermion Particles as Octonion Triples OxOxO.

Dark Energy - Dark Mattter - Ordinary Matter:

The Lorentz Group is represented by 6 generators

0	J1	J2	M1
-J1	0	J3	M2
-J2	-J3	0	M3
-M1	-M2	-M3	0

There are two ways to extend the Lorentz Group
(see arXiv gr-qc/9809061 by Aldrovandi and Peireira):

To the **Poincare Group with No Cosmological Constant** by adding 4 generators

0	J1	J2	M1	A1
-J1	0	J3	M2	A2
-J2	-J3	0	M3	A3
-M1	-M2	-M3	0	A4
-A1	-A2	-A3	-A4	0

{A1,A2,A3} represent Momentum and {A4} represents Energy/Mass of Poincare Gravity and its Dark Matter Primordial Black Holes.

and to the semidirect product of Lorentz and 4 Special Conformal generators
to get a **Non-Zero Cosmological Constant for Universe Expansion**

0	J1	J2	M1		G1
-J1	0	J3	M2		G2
-J2	-J3	0	M3		G3
-M1	-M2	-M3	0		G4
-G1	-G2	-G3	-G4		0

so that {G1,G2,G3} represent 3 Higgs components giving mass to 3 Weak Bosons
and {G4} represents massive Higgs Scalar as Fermion Condensate.
As Special Conformal and Scale Conformal degrees of freedom they also represent the Momentum of Expansion of the Universe and its Dark Energy.

One additional generator {G5} represents HIggs/Fermion mass of Ordinary Matter.

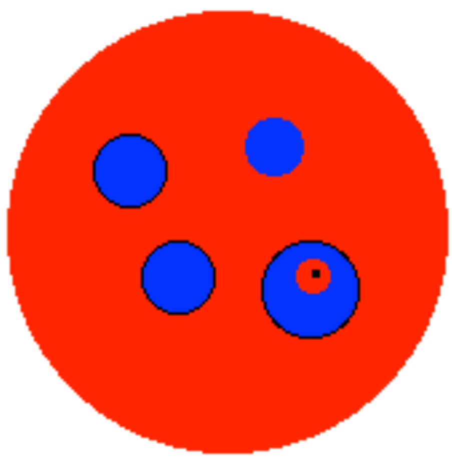
All 15 generators combine to make the full Conformal Lie Algebra SU(2,2) = Spin(2,4)

0	J1	J2	M1	A1	G1
-J1	0	J3	M2	A2	G2
-J2	-J3	0	M3	A3	G3
-M1	-M2	-M3	0	A4	G4
-A1	-A2	-A3	-A4	0	G5
-G1	-G2	-G3	-G4	-G5	0

Dark Energy - Dark Mattter - Ordinary Matter:

In E8 Physics, our 4-dimensional Physical SpaceTime Universe begins as a relatively small spatial volume in which all 15 generators of Conformal SU(2,2) = Spin(2,4) including all 4 Special Conformal and Scale Conformal generators are fully effective.

Rabindra Mohapatra (in section 14.6 of "Unification and Supersymmetry," 2nd edition, Springer-Verlag 1992) said: "... we start with a Lagrangian invariant under full local conformal symmetry and fix its conformal and scale gauge to obtain the usual action for gravity ... the conformal d'Alembartian contains ... curvature ... R, which for constant ... scalar field ... PHI, leads to gravity. We may call PHI the auxiliary field ...". I view PHI as corresponding to the Higgs 3 Special Conformal generators {G1,G2,G3} that are frozen fixed during expansion in some regions of our Universe to become Gravitationally Bound Domains (such as Galaxies) like icebergs in an ocean of water.



Since the Gravitationally Bound Domains (such as our Inner Solar System) have no Expansion Momentum we only see there the Poincare Part of Conformal Gravity plus the Higgs effects of {G4} and {G5} and the ElectroWeak Broken Symmetry caused by freezing-out fixing {G1,G2,G3}:

0	J1	J2	M1	A1	-
-J1	0	J3	M2	A2	-
-J2	-J3	0	M3	A3	-
-M1	-M2	-M3	0	A4	G4
-A1	-A2	-A3	-A4	0	G5
-	-	-	-G4	-G5	0

Here is a summary of E8 model calculation results. Since ratios are calculated, values for one particle mass and one force strength are assumed.
Quark masses are constituent masses. Some higher-order results are listed.

Dark Energy : Dark Matter : Ordinary Matter = 0.75 : 0.21 : 0.04

Particle/Force	Tree-Level	Higher-Order
e-neutrino	0	0 for nu_1
mu-neutrino	0	9 x 10^(-3) eV for nu_2
tau-neutrino	0	5.4 x 10^(-2) eV for nu_3
electron	0.5110 MeV	
down	312.8 MeV	charged pion = 139 MeV
up	312.8 MeV	proton = 938.25 MeV
		neutron - proton = 1.1 MeV
muon	104.8 MeV	106.2 MeV
strange	625 MeV	
charm	2090 MeV	
tauon	1.88 GeV	
beauty	5.63 GeV	
truth(low state)	130 GeV	
truth(middle state)	174 GeV	
truth(high state)	218 GeV	
W+	80.326 GeV	
W-	80.326 GeV	
W0	98.379 GeV	Z0 = 91.862 GeV
Higgs VEV	252.5 GeV (assumed)	Mplanck=1.217x10^19 GeV
Higgs(low state)	126 GeV	
Higgs(middle state)	182 GeV	
Higgs(high state)	239 GeV	
Gravity Gg	1(assumed)	
(Gg)(Mproton^2 / Mplanck^2)		5 x 10^(-39)
EM fine structure	1/137.03608	
Weak Gw	0.2535	
Gw(Mproton^2 / (Mw+^2 + Mw-^2 + Mz0^2))		1.05 x 10^(-5)
color force at 0.245 GeV	0.6286	0.106 at 91 GeV

Kobayashi-Maskawa parameters for W+ and W- processes are:

	d	s	b
u	0.975	0.222	0.00249 -0.00388i
c	-0.222 -0.000161i	0.974 -0.0000365i	0.0423
t	0.00698 -0.00378i	-0.0418 -0.00086i	0.999

The phase angle d13 is taken to be 1 radian.

neutrino mixing matrix:

	nu_1	nu_2	nu_3
nu_e	0.87	0.50	0
nu_m	-0.35	0.61	0.71
nu_t	0.35	-0.61	0.71

As to some higher-order and nonperturbative calculations, one motivation for my value of 245 MeV for the basic /\qcd of the color force is the paper of Shifman at hep-ph/9501222 in which Shifman said:
"... a set of data ("high-energy data") yield values of alpha_s(MZ) in the MSbar scheme which cluster around 0.125 ... with the error bars 0.005 ...
The corresponding value of LambdaQCD is about 500 MeV ... These numbers, accepted as the most exact results for the strong coupling constant existing at present, propagate further into a stream of papers ... devoted to various aspects of QCD. The question arises whether Quantum Chromodynamics can tolerate these numbers. I will argue below that the answer is negative.
... I believe that alpha_s(MZ) must be close to 0.11 and the corresponding value of LambdaQCD close to 200 MeV (or even smaller). ...
The value of alpha_s (M_Z) emerging from the so called global fits based mainly on the data at the Z peak (and assuming the standard model) is three standard deviations higher than the one stemming from the low-energy phenomenology. ...".
Patrascioiu and Seiler in hep-ph/9609292 said:
"... the running of alpha_s predicted by perturbation (PT) theory is not correctly describing the accelerator experiments at the highest energies. A natural explanation is provided by the authors' 1992 proposal that in fact the true running predicted by the nonperturbatively defined lattice QCD is different ...".
The Patrascioiu and Seiler paper indicates that my crude use of simple perturbative QCD running my not be correct. If you look at Figure 2 of their paper, you see that their "possible modified running of alpha_s" curve is at 100 GeV close to the 0.12 range, while their 2-loop PT curve is close to the 0.10 range of my crude perturbative calculation.
So, it may be that nonperturbative effects might bring calculations of my model closer to observations.
Further, it may be difficult to do very accurate nonperturbative QCD calculations, based in part on what Morozov and Niemi say in hep-th/0304178 :
"... The field theoretical renormalization group equations have many common features with the equations of dynamical systems. ... we propose that besides isolated fixed points, the couplings in a renormalizable field theory may also flow towards more general, even fractal attractors. This could lead to Big Mess scenarios ... ".

I am not contending that my tree-level calculations are in exact agreement with currently accepted observations.

I am contending that the overall approximate agreement of my calculations with observations of many parameters does indicate that the fundamental structure of my E8 physics model is sound.

My view of constituent quark masses is that they can be (and are in my model) meaningful, particularly in nonrelativistic quark models of light-quark hadrons (for heavier quarks, the percentage difference between current and constituent masses can be relatively small). For example, Guidry, in his book Gauge Field Theories, John Wiley (1991), says:
"... the current masses of the quarks ... are considerably smaller than the constituent masses for the lightest quarks Mu = 300 MeV Md = 300 MeV ...
... the masses of the constituent quarks presumably reflect a dressing by the confinement mechanism ... understanding of the relationship between current masses and constituent masses awaits a first-principles solution of the QCD bound-state problem. ... Nevertheless, nonrelativistic models of quark structure for hadrons have been found to work surprisingly well, even for light hadrons. ...".
As I said in quant-ph/9806023 :
"... The effectiveness of the NonRelativistic Quark Model of hadrons can be explained by Bohm's quantum theory applied to a fermion confined in a box, in which the fermion is at rest because its kinetic energy is transformed into PSI-field potential energy. ...".
Further, Georgi, in his book Weak Interactions and Modern Particle Theory, Benjamin-Cummings (1984), says:
"... Successes of the Nonrelativistic Quark Model ...
... The first striking success is that the baryon masses are given correctly by this picture ... The leading contribution to the baryon mass in the nonrelativistic limit is just the sum of the constituent quark masses. ... A good picture of the baryon masses is obtained if we take ... mu = md =...= 360 MeV ... ms = 540 MeV ...
... With these masses, the octet baryon magnetic moments are ...[in]... excellent ... agreement ... with the data ... The success ... in giving not only the ratios of the baryon magnetic moments, but even their overall scale, seems ... to be very significant. ... The mystery of the connection between QCD and the quark model remains ...".

My view is that the structure of my E8 model, in which constituent quark masses are calculated from volumes of bounded complex domains and their Shilov boundaries, may shed some light on the connection between QCD current masses and constituent masses. In particular, those geometric volumes may be related to effective summation over a lot of QCD states to produce a bound-state constituent result.

Two other higher-order calculations in my E8 model are:

1 - For the muon, my tree-level calculation is 104.8 MeV and the accepted observational value is about 105.6 MeV. All I have done is to note that the difference seems to me to be well within the range of radiative corrections. For example, following Bailin and Love, in their book Introduction to Gauge Field Theory, IOP (rev ed 1993):

Radiative corrections to order a for the muon decay rate using Sirlin's on-mass-shell renormalization scheme give a 7% increase35 in the muon decay rate compared to the tree graph prediction:

$$\Gamma(\mu) = \frac{G_F^2 m_\mu^5}{192\pi^3}\left(1 - 8\frac{m_e^2}{m_\mu^2} + 8\left(\frac{m_e^2}{m_\mu^2}\right)^3 - \left(\frac{m_e^2}{m_\mu^2}\right)^4 - 12\left(\frac{m_e^2}{m_\mu^2}\right)^2 \ln\left(\frac{m_e^2}{m_\mu^2}\right)\right)$$

Since the decay rate is directly proportional to mμ^5 , the increase can be considered to be an increase in the muon mass of about 1.36%, from the uncorrected theoretical value of 104.8 MeV to 106.2 MeV.

The experimental value is 105.7 MeV.

2 - For the proton-neutron mass difference (which is zero in my E8 model at tree level) further calculation involving connections between down valence quarks and virtual sea strange quarks gives a value of 1.1 MeV for the neutron mass excess over the proton mass, which is close to the accepted value of about 1.3 MeV.

Force Strengths:

The primary postulate for my E8 physics model is:

0 - I start with the emergence from the void of a binary choice, like Yin-Yang, which naturally gives a real Clifford algebra, so that physics is described by a very large real Clifford algebra (a generalized hyperfinite II1 von Neumann factor) that can be seen as a tensor product of a lot of Cl(16) Clifford Algebras, each of which contains an E8 Lie Algebra.

Then:

1 - Since Cl(16) = Cl(8)xCl(8) it is clear that Cl(8) describes physics locally and it is also clear that 248-dim E8 in Cl(16) can be described in terms of 256-dim Cl(8) which has an Octonionic 8-dim Vector Space.

2 - At low (after Inflation) energies a specific quaternionic submanifold freezes out, splitting the 8-dim spacetime into a 4+4 = 8-dim M4xCP2 Kaluza-Klein.

3 - Cl(8) bivector Spin(8) is the D4 Lie algebra two copies of which are in the E8 Physics Lagrangian that is integrated over a base manifold that is 8-dim M4xCP2 Kaluza-Klein. This shows that the **Force Strength is made up of two parts**:
the relevant spacetime manifold of gauge group global action
and
the relevant symmetric space manifold of gauge group local action.

4 -Roughly, the 4-dim spacetime Lagrangian gauge boson term is:
the integral over spacetime as seen by gauge boson acting globally of the gauge force term of the gauge boson acting locally for the gauge bosons of each of the four forces:

U(1) for electromagnetism
SU(2) for weak force
SU(3) for color force
Spin(5) - compact version of antiDeSitter Spin(2,3) for gravity by the MacDowell-Mansouri mechanism.

Look at the basic Lagrangian of a gauge theory model:

Integral over Spacetime of
Gauge Boson Force Term

In the conventional picture,
for each gauge force the gauge boson force term contains the force strength,
which in Feynman's picture is the amplitude to emit a gauge boson,
and can also be thought of as the probability = square of amplitude,
in an explicit (like $g |F|^2$) or an implicit (incorporated into the $|F|^2$) form.
Either way,
the conventional picture is that the force strength g is an ad hoc inclusion.

My E8 Physics model does not put in force strength g ad hoc,
but
constructs the integral such that the force strength emerges naturally from the geometry of each gauge force.

To do that, for each gauge force:

1 - make the spacetime over which the integral is taken be spacetime as it is seen by that gauge boson, that is, in terms of the symmetric space with global symmetry of the gauge boson:

the U(1) photon sees 4-dim spacetime as $T^4 = S1 \text{ x } S1 \text{ X } S1 \text{ x } S1$
the SU(2) weak boson sees 4-dim spacetime as S2 x S2
the SU(3) weak boson sees 4-dim spacetime as CP2
the Spin(5) of gravity sees 4-dim spacetime as S4.

2 - make the gauge boson force term have the volume of the Shilov boundary corresponding to the symmetric space with local symmetry of the gauge boson.
The nontrivial Shilov boundaries are:

for SU(2) Shilov = $RP^1 x S^2$
for SU(3) Shilov = S^5
for Spin(5) Shilov = $RP^1 x S^4$

The result is (ignoring technicalities for exposition) the geometric factor for force strength calculation.

Each force is related to a gauge group:

U(1) for electromagnetism
SU(2) for weak force
SU(3) for color force
Spin(5) - compact version of antiDeSitter Spin(2,3) for gravity by the MacDowell-Mansouri mechanism

Global:

Each gauge group is the global symmetry of a symmetric space
S1 for U(1)
S2 = SU(2)/U(1) = Spin(3)/Spin(2) for SU(2)
CP2 = SU(3)/SU(2)xU(1) for SU(3)
S4 = Spin(5)/Spin(4) for Spin(5)

Local:

Each gauge group is the local symmetry of a symmetric space
U(1) for itself
SU(2) for Spin(5) / SU(2)xU(1)
SU(3) for SU(4) / SU(3)xU(1)
Spin(5) for Spin(7) / Spin(5)xU(1)

The nontrivial local symmetry symmetric spaces correspond to bounded complex domains

SU(2) for Spin(5) / SU(2)xU(1) corresponds to IV3
SU(3) for SU(4) / SU(3)xU(1) corresponds to B^6 (ball)
Spin(5) for Spin(7) / Spin(5)xU(1) corresponds to IV5
The nontrivial bounded complex domains have Shilov boundaries
SU(2) for Spin(5) / SU(2)xU(1) corresponds to IV3 Shilov = RP^1xS^2
SU(3) for SU(4) / SU(3)xU(1) corresponds to B^6 (ball) Shilov = S^5
Spin(5) for Spin(7) / Spin(5)xU(1) corresponds to IV5 Shilov = RP^1xS^4

Global and Local Together:

Very roughly (see my web site tony5m17h.net and papers for details),
think of the force strength as

the integral over the global symmetry space of
the physical (ie Shilov Boundary) volume=strength of the force.

That is (again very roughly and intuitively):

the geometric strength of the force is given by the product of
the volume of a 4-dim thing with global symmetry of the force and
the volume of the Shilov Boundary for the local symmetry of the force.

When you calculate the product volumes (using some tricky normalization stuff),
you see that roughly:

Volume product for gravity is the largest volume
so since (as Feynman says) force strength = probability to emit a gauge boson
means that the highest force strength or probability should be 1
I normalize the gravity Volume product to be 1, and so roughly get:

Volume product for gravity = 1
Volume product for color = 2/3
Volume product for weak = 1/4
Volume product for electromagnetism = 1/137

There are two further main components of a force strength:

1 - for massive gauge bosons, a suppression by a factor of 1 / M^2
2 - renormalization running (important for color force).

Consider Massive Gauge Bosons:

I consider gravity to be carried by virtual Planck-mass black holes, so that the geometric strength of gravity should be reduced by 1/Mp^2
I consider the weak force to be carried by weak bosons, so that the geometric strength of gravity should be reduced by 1/MW^2
That gives the result:

gravity strength = G (Newton's G)
color strength = 2/3
weak strength = G_F (Fermi's weak force G)
electromagnetism = 1/137

Consider Renormalization Running for the Color Force::
That gives the result:

gravity strength = G (Newton's G)
color strength = 1/10 at weak boson mass scale
weak strength = G_F (Fermi's weak force G)
electromagnetism = 1/137

The use of compact volumes is itself a calculational device, because it would be more nearly correct, instead of

> the integral over the compact global symmetry space of
> the compact physical (ie Shilov Boundary) volume=strength of the force

to use

> the integral over the hyperbolic spacetime global symmetry space of
> the noncompact invariant measure of the gauge force term.

However, since the strongest (gravitation) geometric force strength is to be normalized to 1, the only thing that matters is ratios,
and the compact volumes (finite and easy to look up in the book by Hua)
have the same ratios as the noncompact invariant measures.

In fact, I should go on to say that continuous spacetime and gauge force geometric objects are themselves also calculational devices, and
that it would be even more nearly correct to do the calculations with respect to a discrete generalized hyperdiamond Feynman checkerboard.

Here are more details about the force strength calculations:

The force strength of a given force is

alphaforce = (1 / Mforce^2)
(Vol(MISforce))
(Vol(Qforce) / Vol(Dforce)^(1 / mforce))
where:

alphaforce represents the force strength;

Mforce represents the effective mass;

MISforce represents the part of the target
Internal Symmetry Space that is available for the gauge
boson to go to;

Vol(MISforce) stands for volume of MISforce,
and is sometimes also denoted by the shorter notation Vol(M);

Qforce represents the link from the origin
to the target that is available for the gauge
boson to go through;

Vol(Qforce) stands for volume of Qforce;

Dforce represents the complex bounded homogeneous domain
of which Qforce is the Shilov boundary;

mforce is the dimensionality of Qforce,
which is 4 for Gravity and the Color force,
2 for the Weak force (which therefore is considered to
have two copies of QW for each spacetime HyperDiamond link),
and 1 for Electromagnetism (which therefore is considered to
have four copies of QE for each spacetime HyperDiamond link)

Vol(Dforce)^(1 / mforce) stands for
a dimensional normalization factor (to reconcile the dimensionality
of the Internal Symmetry Space of the target vertex
with the dimensionality of the link from the origin to the target vertex).

The Qforce, Hermitian symmetric space,
and Dforce manifolds for the four forces are:

Gauge Group	Hermitian Symmetric Space	Type of Dforce	mforce	Qforce
Spin(5)	Spin(7) / Spin(5)xU(1)	IV5	4	RP^1xS^4
SU(3)	SU(4) / SU(3)xU(1)	B^6(ball)	4	S^5
SU(2)	Spin(5) / SU(2)xU(1)	IV3	2	RP^1xS^2
U(1)	-	-	1	-

The geometric volumes needed for the calculations are mostly taken from the book Harmonic Analysis of Functions of Several Complex Variables in the Classical Domains (AMS 1963, Moskva 1959, Science Press Peking 1958) by L. K. Hua [with unit radius scale].

Note that

Force	M	Vol(M)
gravity	S^4	8pi^2/3 - S^4 is 4-dimensional
color	CP^2	8pi^2/3 - CP^2 is 4-dimensional
weak	S^2 x S^2	2 x 4pi - S^2 is a 2-dim boundary of 3-dim ball 4-dim S^2 x S^2 = = topological boundary of 6-dim 2-polyball Shilov Boundary of 6-dim 2-polyball = S^2 + S^2 = = 2-dim surface frame of 4-dim S^2 x S^
e-mag	T^4	4 x 2pi - S^1 is 1-dim boundary of 2-dim disk 4-dim T^4 = S^1 x S^1 x S^1 x S^1 = = topological boundary of 8-dim 4-polydisk Shilov Boundary of 8-dim 4-polydisk = = S^1 + S^1 + S^1 + S^1 = = 1-dim wire frame of 4-dim T^4

Note (thanks to Carlos Castro for noticing this) that the volume lisrted for S5 is for a squashed S5, a Shilov boundary of the complex domain corresponding to the symmetric space SU(4) / SU(3) x U(1).

Note (thanks again to Carlos Castro for noticing this) also that the volume listed for CP2 is unconventional, but physically justified by noting that S4 and CP2 can be seen as having the same physical volume, with the only difference being structure at infinity.

Also note that for U(1) electromagnetism, whose photon carries no charge, the factors Vol(Q) and Vol(D) do not apply and are set equal to 1, and from another point of view, the link manifold to the target vertex is trivial for the abelian neutral U(1) photons of Electromagnetism, so we take QE and DE to be equal to unity.

Force	M	Vol(M)	Q	Vol(Q)	D	Vol(D)
gravity	S^4	8pi^2/3	RP^1xS^4	8pi^3/3	IV5	pi^5/2^4 5!
color	CP^2	8pi^2/3	S^5	4pi^3	B^6(ball)	pi^3/6
weak	S^2xS^2	2x4pi	RP^1xS^2	4pi^2	IV3	pi^3/24
e-mag	T^4	4x2pi	-	-	-	-

Using these numbers, the results of the calculations are the relative force strengths at the characteristic energy level of the generalized Bohr radius of each force:

Gauge Group	Force	Characteristic Energy	Geometric Force Strength	Total Force Strength
Spin(5)	gravity	approx 10^19 GeV	1	GGmproton^2 approx 5 x 10^-39
SU(3)	color	approx 245 MeV	0.6286	0.6286
SU(2)	weak	approx 100 GeV	0.2535	GWmproton^2 approx 1.05 x 10^-5
U(1)	e-mag	approx 4 KeV	1/137.03608	1/137.03608

The force strengths are given at the characteristic energy levels of their forces, because the force strengths run with changing energy levels.

The effect is particularly pronounced with the color force.

The color force strength was calculated using a simple perturbative QCD renormalization group equation at various energies, with the following results:

Energy Level	Color Force Strength
245 MeV	0.6286
5.3 GeV	0.166
34 GeV	0.121
91 GeV	0.106

Taking other effects, such as Nonperturbative QCD, into account, should give a Color Force Strength of about 0.125 at about 91 GeV

Fermion Masses:

The primary postulate for my E8 physics model is:

0 - I start with the emergence from the void of a binary choice, like Yin-Yang, which naturally gives a real Clifford algebra, so that physics is described by a very large real Clifford algebra (a generalized hyperfinite II1 von Neumann factor) that can be seen as a tensor product of a lot of Cl(16) Clifford Algebras, each of which contains an E8 Lie Algebra.

Then:

1 - Since Cl(16) = Cl(8)xCl(8) it is clear that Cl(8) describes physics locally and it is also clear that 248-dim E8 in Cl(16) can be described in terms of 256-dim Cl(8) which has two Octonionic 8-dim half-spinor spaces with physical interpretation by which first-generation fermion particles correspond to octonion basis
of Spin(8) +half-spinors

1 to e-neutrino
i to red down quark
j to green down quark
k to blue down quark
E to electron
I to red up quark
J to green up quark
K to blue up quark

and first-generation fermion antiparticles correspond to octonion basis
of Spin(8) -half-spinors

1 to e-antineutrino
i to red down antiquark
j to green down antiquark
k to blue down antiquark
E to positron
I to red up antiquark
J to green up antiquark
K to blue up antiquark

2 - The two Spin(8) 8-dim half-spinors and the Spin(8) 8-dim vectors are all related to each other by Triality. Modifying Steven Weinberg's description of physics Lagrangians in his book "Elemetary Particles and the Laws of Physics: The 1986 Dirac Memorial Lectures" to apply to 8-dim spacetime gives this quote

All terms in the Lagrangian density must have units $[\text{mass}]^8$, because length and time have units of inverse mass and the Lagrangian density integrated over spacetime must have no units. From the $m\psi\psi$ term, we see that the electron field must have units $[\text{mass}]^{7/2}$, because $\frac{7}{2} + \frac{7}{2} + 1 = 8$

from which it is clear that at high (UltraViolet) energies in the E8 physics model gauge boson terms have dimension 1 in the Lagrangian and fermion terms have dimension 7/2 in the Lagrangian, so that the Triality gives a Subtle Supersymmetry whereby

Total Boson Lagrangian Dimensionality = 28 x 1 = 28

is exactly balanced by

Total Fermion Lagrangian Dimensionality = 8 x 7 / 2 = 28

thus

the Triality Subtle Supersymmetry shows UltraViolet Finiteness of the E8 model

3 - At low (after Inflation) energies a specific quaternionic submanifold freezes out, splitting the 8-dim spacetime into a 4+4 = 8-dim M4xCP2 Kaluza-Klein and creating second and third generation fermions that can live in the 4-dim internal symmetry space and correspond respectively to pairs and triples of octonion basis elements,

4 - Cl(8) bivector Spin(8) is the D4 Lie algebra two copies of which are in the E8 Physics Lagrangian that is integrated over a base manifold that is 8-dim M4xCP2 Kaluza-Klein.

5 - Roughly, the 4-dim spacetime Lagrangian fermion term is integral over spacetime of spinor fermion term

In the conventional picture, the spinor fermion term is of the form m S S* where m is the fermion mass and S and S* represent the given fermion.
Although the mass m is derived from the Higgs mechanism, the Higgs coupling constants are, in the conventional picture, ad hoc parameters, so that effectively the mass term is, in the conventional picuture, an ad hoc inclusion.

My E8 model does not put in the mass m as an ad hoc Higgs coupling value,
but
constructs the Lagrangian integral such that the mass m emerges naturally from the geometry of the spinor fermions.

To do that,
make the spinor fermion mass term have the volume of the Shilov boundary corresponding to
the symmetric space with LOCAL symmetry of the Spin(8) gauge group
with respect to which the first generation spinor fermions are
seen as +half-spinor and -half-spinor spaces.

Note that due to Triality,
Spin(8) can act on those 8-dimensional half-spinor spaces similarly to the way it acts on 8-dimensional vector spacetime prior to dimensional reduction.

Then, take the the spinor fermion volume to be the Shilov boundary corresponding to the same symmetric space on which Spin(8) acts as a local gauge group that is used to construct 8-dimensional vector spacetime:

> the symmetric space Spin(10) / Spin(8)xU(1)
> corresponds to a bounded domain of type IV8
> whose Shilov boundary is RP^1 x S^7

Since all the first generation fermions see the spacetime over which the integral is taken in the same way (unlike what happens for the force strength calculation), the only geometric volume factor relevant for calculating first generation fermion mass ratios is in the spinor fermion volume term.

Since fermions in this model correspond to Kerr-Newman Black Holes, the quark mass in this model is a constituent mass.

Consider a first-generation massive lepton (or antilepton, i.e., electron or positron).
For definiteness, consider an electron E (a similar line of reasoning applies to the positron).
Gluon interactions do not affect the colorless electron (E)
By weak boson interactions or decay, an electron (E) can only be taken into itself or a massless (at tree level) neutrino.
As the lightest massive first-generation fermion, the electron cannot decay into a quark.
Since the electron cannot be related to any other massive Dirac fermion,
its volume V(electron) is taken to be 1.

Consider a first-generation quark (or antiquark).
For definiteness, consider a red down quark I (a similar line of reasoning applies to the others of the first generation).
By gluon interactions, the red quark (I) can be interchanged with the blue and green down quarks (J and K).
By weak boson interactions, it can be taken into the red, blue, and green up quarks (i, j, and k).
Given the up and down quarks, pions can be formed from quark-antiquark pairs, and the pions can decay to produce electrons (E) and neutrinos (1).
Therefore first-generation quarks or antiquarks can by gluons, weak bosons, or decay occupy the entire volume of the Shilov boundary RP1 x S7, which volume is pi^5 / 3, so its volume V(quark) is taken to be pi^5 / 3.

Consider graviton interactions with first-generation fermions.
Since MacDowell-Mansouri gravitation comes from 10 Spin(5) gauge bosons,
8 of which are charged (carrying color or electric charge)
as shown in the root Spin(5) root vector diagram

```
        *

    *       *

o               o     Spin(5) root vector diagram

    *       *

        *
```

in which the 6 root vectors * correspond to color carrying gauge bosons act similarly to the action of the 6 color-charged SU(3) gluons shown in the SU(3) root vector diagram

```
        *

    *       *

                      SU(3) root vector diagram

    *       *

        *
```

The 2 charged Spin(5) gravitons denoted by o carry electric charge.

However, even though the electron carries electric charge,
the electric charge carrying Spin(5) gravitons can only change the electron into a (tree-level) massless neutrino,
so the Spin(5) gravitons do not enhance the electron volume factor,
which remains electron volume (taking gravitons into account) = V(electron) = 1

Since the quark carries color charge,
Spin(5) graviton action on its color charge multiplies its volume V(quark) by 6, giving
quark gravity-enhanced volume = 6 x V(quark) = 6 pi^5 / 3 = 2 pi^5
The 2 Spin(5) gravitons carrying electric charge only cannot change quarks into leptons, so they do not enhance the quark volume factor, so we have (where md is down quark mass, mu is up quark mass, and me is electron mass)
md / me = mu / me = 2 pi^5 / 1 = 2 pi^5 = 612.03937

The proton mass is calculated as the sum of the constituent masses of its constituent quarks
mproton = mu + mu + md = 938.25 MeV
which is close to the experimental value of 938.27 MeV.

In the first generation,
each quark corresponds to a single octonion basis element
and the up and down quark constituent masses are the same:
First Generation - 8 singletons - mu / md = 1
Down - corresponds to 1 singleton - constituent mass 312 MeV
Up - corresponds to 1 singleton - constituent mass 312 MeV

Second and third generation calculations are generally more complicated.
Combinatorics indicates that in higher generations the up-type quarks are heavier than the down-type quarks.
The third generation case,
in which the fermions correspond to triples of octonions,
is simple enough to be used here as an illustration of the combinatoric effect:
Third Generation
8^3 = 512 triples
mt / mb = 483 / 21 = 161 / 7 = 23
down-type (Beauty) - corresponds to 21 triples - constituent mass 5.65 GeV
up-type (Truth) - corresponds to 483 triples - constituent mass 130 GeV

Here are more details about the fermion mass calculations:

Fermion masses are calculated as a product of four factors:

> V(Qfermion) x N(Graviton) x N(octonion) x Sym
>
> V(Qfermion) is the volume of the part of the half-spinor fermion particle manifold S^7 x RP^1 that is related to the fermion particle by photon, weak boson, and gluon interactions.
>
> N(Graviton) is the number of types of Spin(0,5) graviton related to the fermion. The 10 gravitons correspond to the 10 infinitesimal generators of Spin(0,5) = Sp(2). 2 of them are in the Cartan subalgebra. 6 of them carry color charge, and may therefore be considered as corresponding to quarks. The remaining 2 carry no color charge, but may carry electric charge and so may be considered as corresponding to electrons. One graviton takes the electron into itself, and the other can only take the first-generation electron into the massless electron neutrino. Therefore only one graviton should correspond to the mass of the first-generation electron. The graviton number ratio of the down quark to the first-generation electron is therefore 6/1 = 6.
>
> N(octonion) is an octonion number factor relating up-type quark masses to down-type quark masses in each generation.
>
> Sym is an internal symmetry factor, relating 2nd and 3rd generation massive leptons to first generation fermions. It is not used in first-generation calculations.

The ratio of the down quark constituent mass to the electron mass is then calculated as follows:

Consider the electron, E.
By photon, weak boson, and gluon interactions, E can only be taken into 1, the massless neutrino. The electron and neutrino, or their antiparticles, cannot be combined to produce any of the massive up or down quarks. The neutrino, being massless at tree level, does not add anything to the mass formula for the electron. Since the electron cannot be related to any other massive Dirac fermion, its volume V(Qelectron) is taken to be 1.

Next consider a red down quark ie. By gluon interactions, ie can be taken into je and ke, the blue and green down quarks. By also using weak boson interactions, it can be taken into i, j, and k, the red, blue, and green up quarks. Given the up and down quarks, pions can be formed from quark-antiquark pairs, and the pions can decay to produce electrons and neutrinos. Therefore the red down quark (similarly, any down quark) is related to any part of S^7 x RP^1, the compact manifold corresponding to { 1, i, j, k, ie, ie, ke, e } and therefore a down quark should have a spinor manifold volume factor V(Qdown quark) of the volume of S^7 x RP^1.
The ratio of the down quark spinor manifold volume factor tothe electron spinor manifold volume factor is just

V(Qdown quark) / V(Qelectron) = V(S^7x RP^1)/1 = pi^5 / 3.

Since the first generation graviton factor is 6,

md/me = 6V(S^7 x RP^1) = 2 pi^5 = 612.03937

As the up quarks correspond to i, j, and k, which are the octonion transforms under e of ie, je, and ke of the down quarks, the up quarks and down quarks have the same constituent mass

mu = md.

Antiparticles have the same mass as the corresponding particles.

Since the model only gives ratios of massses, the mass scale is fixed so that the electron mass me = 0.5110 MeV.

Then, the constituent mass of the down quark is md = 312.75 MeV,
and the constituent mass for the up quark is mu = 312.75 MeV.

These results when added up give a total mass of first generation fermion particles:
Sigmaf1 = 1.877 GeV

As the proton mass is taken to be the sum of the constituent masses of its constituent quarks

mproton = mu + mu + md = 938.25 MeV

The theoretical calculation is close to the experimental value of 938.27 MeV.

The third generation fermion particles correspond to triples of octonions.
There are 8^3 = 512 such triples.
The triple { 1,1,1 } corresponds to the tau-neutrino.

The other 7 triples involving only 1 and E correspond to the tauon:

{ e, e, e }
{ e, e, 1 }
{ e, 1, e }
{ 1, e, e }
{ 1, 1, e }
{ 1, e, 1 }
{ e, 1, 1 }

The symmetry of the 7 tauon triples is the same as the symmetry of the 3 down quarks, the 3 up quarks, and the electron, so the tauon mass should be the same as the sum of the masses of the first generation massive fermion particles. Therefore the tauon mass is calculated at tree level as 1.877 GeV.

The calculated Tauon mass of 1.88 GeV is a sum of first generation fermion masses, all of which are valid at the energy level of about 1 GeV.

However, as the Tauon mass is about 2 GeV,
the effective Tauon mass should be renormalized from the energy level of 1 GeV (where the mass is 1.88 GeV) to the energy level of 2 GeV.

Such a renormalization should reduce the mass.
If the renormalization reduction were about 5 percent,
the effective Tauon mass at 2 GeV would be about 1.78 GeV.

The 1996 Particle Data Group Review of Particle Physics gives a Tauon mass of 1.777 GeV.

Note that all triples corresponding to the tau and the tau-neutrino are colorless.

The beauty quark corresponds to 21 triples.
They are triples of the same form as the 7 tauon triples, but for 1 and ie, 1 and je, and 1 and ke, which correspond to the red, green, and blue beauty quarks, respectively.
The seven triples of the red beauty quark correspond to the seven triples of the tauon, except that the beauty quark interacts with 6 Spin(0,5) gravitons while the tauon interacts with only two.
The beauty quark constituent mass should be the tauon mass times the third generation graviton factor 6/2 = 3, so the B-quark mass is
mb = 5.63111 GeV.

The calculated Beauty Quark mass of 5.63 GeV is a consitituent mass, that is, it corresponds to the conventional pole mass plus 312.8 MeV.

Therefore, the calculated Beauty Quark mass of 5.63 GeV corresponds to a conventional pole mass of 5.32 GeV.

The 1996 Particle Data Group Review of Particle Physics gives a lattice gauge theory Beauty Quark pole mass as 5.0 GeV.

The pole mass can be converted to an MSbar mass if the color force strength constant alpha_s is known. The conventional value of alpha_s at about 5 GeV is about 0.22.
Using alpha_s (5 GeV) = 0.22, a pole mass of 5.0 GeV gives an MSbar 1-loop Beauty Quark mass of 4.6 GeV, and
an MSbar 1,2-loop Beauty Quark mass of 4.3, evaluated at about 5 GeV.

If the MSbar mass is run from 5 GeV up to 90 GeV, the MSbar mass decreases by about 1.3 GeV, giving an expected MSbar mass of about 3.0 GeV at 90 GeV.
DELPHI at LEP has observed the Beauty Quark and found a 90 GeV MSbar Beauty Quark mass of about 2.67 GeV, with error bars +/- 0.25 (stat) +/- 0.34 (frag) +/- 0.27 (theo).

Note that the theoretical model calculated Beauty Quari mass of 5.63 GeV corresponds to a pole mass of 5.32 GeV, which is somewhat higher than the conventional value of 5.0 GeV.
However,
the theoretical model calculated value of the color force strength constant alpha_s at about 5 GeV is about 0.166,
while the conventional value of the color force strength constant alpha_s at about 5 GeV is about 0.216,
and the theoretical model calculated value of the color force strength constant alpha_s at about 90 GeV is about 0.106,
while the conventional value of the color force strength constant alpha_s at about 90 GeV is about 0.118.
The theoretical model calculations gives a Beauty Quark pole mass (5.3 GeV) that is about 6 percent higher than the conventional Beauty Quark pole mass (5.0 GeV),
and a color force strength alpha_s at 5 GeV (0.166)
such that 1 + alpha_s = 1.166 is about 4 percent lower
than the conventional value of 1 + alpha_s = 1.216 at 5 GeV.

Note particularly that triples of the type { 1, ie, je } , { ie, je, ke }, etc.,
do not correspond to the beauty quark, but to the truth quark.

The truth quark corresponds to the remaining 483 triples,
so the constituent mass of the red truth quark
is 161/7 = 23 times the red beauty quark mass,
and the red T-quark mass is
mt = 129.5155 GeV
The blue and green truth quarks are defined similarly.

All other masses than the electron mass
(which is the basis of the assumption of the value of the Higgs scalar field vacuum expectation value v = 252.514 GeV),
including the Higgs scalar mass and Truth quark mass,
are calculated (not assumed) masses in the E8 model.

These results when added up give a total mass of third generation fermion particles:
Sigmaf3 = 1,629 GeV

The second generation fermion particles correspond to pairs of octonions.

There are 8^2 = 64 such pairs. The pair { 1,1 } corresponds to the mu-neutrino.

The pairs { 1, e }, { e, 1 }, and { e, e } correspond to the muon.
Compare the symmetries of the muon pairs to the symmetries of the first generation fermion particles.
The pair { e, e } should correspond to the e electron.
The other two muon pairs have a symmetry group S2, which is 1/3 the size of the color symmetry group S3 which gives the up and down quarks their mass of 312.75 MeV.
Therefore the mass of the muon should be the sum of
the { e, e } electron mass and
the { 1, e }, { e, 1 } symmetry mass, which is 1/3 of the up or down quark mass.
Therefore, mmu = 104.76 MeV .
According to the 1998 Review of Particle Physics of the Particle Data Group,
the experimental muon mass is about 105.66 MeV.

Note that all pairs corresponding to the muon and the mu-neutrino are colorless.

The red, blue and green strange quark each corresponds
to the 3 pairs involving 1 and ie, je, or ke.

The red strange quark is defined as the three pairs 1 and i,
because i is the red down quark.
Its mass should be the sum of two parts:
the { i, i } red down quark mass, 312.75 MeV, and
the product of the symmetry part of the muon mass, 104.25 MeV,
times the graviton factor.
Unlike the first generation situation,
massive second and third generation leptons can be taken,
by both of the colorless gravitons that may carry electric charge,
into massive particles.
Therefore the graviton factor for the second and third generations is 6/2 = 3.
Therefore the symmetry part of the muon mass times the graviton factor 3 is 312.75 MeV,
and the red strange quark constituent mass is
ms = 312.75 MeV + 312.75 MeV = 625.5 MeV

The blue strange quarks correspond to the three pairs involving j,
the green strange quarks correspond to the three pairs involving k,
and their masses are determined similarly.

The charm quark corresponds to the other 51 pairs.
Therefore, the mass of the red charm quark should be the sum of two parts:
the { i, i }, red up quark mass, 312.75 MeV;
and
the product of the symmetry part of the strange quark mass, 312.75 MeV,
and the charm to strange octonion number factor 51/9,
which product is 1,772.25 MeV.
Therefore the red charm quark constituent mass is
mc = 312.75 MeV + 1,772.25 MeV = 2.085 GeV

The blue and green charm quarks are defined similarly,
and their masses are calculated similarly.

The calculated Charm Quark mass of 2.09 GeV is a consitituent mass,
that is, it corresponds to the conventional pole mass plus 312.8 MeV.

Therefore, the calculated Charm Quark mass of 2.09 GeV corresponds to a conventional pole mass of 1.78 GeV.

The 1996 Particle Data Group Review of Particle Physics gives a range for the Charm Quark pole mass from 1.2 to 1.9 GeV.

The pole mass can be converted to an MSbar mass if the color force strength constant alpha_s is known. The conventional value of alpha_s at about 2 GeV is about 0.39, which is somewhat lower than the teoretical model value. Using alpha_s (2 GeV) = 0.39, a pole mass of 1.9 GeV gives an MSbar 1-loop mass of 1.6 GeV, evaluated at about 2 GeV.
These results when added up give a total mass of second generation fermion particles:
Sigmaf2 = 32.9 GeV

Higgs:

As with forces strengths, the calculations produce ratios of masses,
so that only one mass need be chosen to set the mass scale.

In the E8 model, the value of the fundamental mass scale vacuum expectation value v = <PHI> of the Higgs scalar field is set to be the sum of the physical masses of the weak bosons, W+, W-, and Z0,
whose tree-level masses will then be shown by ratio calculations
to be 80.326 GeV, 80.326 GeV, and 91.862 GeV, respectively,
and so that the electron mass will then be 0.5110 MeV.

The relationship between the Higgs mass and v is given
by the Ginzburg-Landau term from the Mayer Mechanism as

(1/4) Tr ([PHI , PHI] - PHI)^2

or, in the notation of quant-ph/9806009 by Guang-jiong Ni

(1/4!) lambda PHI^4 - (1/2) sigma PHI^2

where the Higgs mass M_H = sqrt(2 sigma)

Ni says:

> "... the invariant meaning of the constant lambda in the Lagrangian is not the coupling constant, the latter will change after quantization ... The invariant meaning of lambda is nothing but the ratio of two mass scales:
> lambda = 3 (M_H / PHI)^2
> which remains unchanged irrespective of the order ...".

Since <PHI>^2 = v^2, and assuming that lambda = (cos(pi / 6))^2 = 0.866^2
(a value consistent with the Higgs-Tquark condensate model of Michio Hashimoto, Masaharu Tanabashi, and Koichi Yamawaki in their paper at hep-ph/0311165)
we have

M_H^2 / v^2 = (cos(pi / 6))^2 / 3

In the E8 model, the fundamental mass scale vacuum expectation value v of the Higgs scalar field is the fundamental mass parameter that is to be set to define all other masses by the mass ratio formulas of the model and

v is set to be 252.514 GeV

so that

M_H = v cos(pi / 6) / sqrt(1 / 3) = 126.257 GeV

As described above, in the E8 model

v is set to be 252.514 GeV

A Non-Condensate Higgs is represented by a Higgs at a point in M4
that is connected to a Higgs representation in CP2 ISS by a line whose length represents the Higgs mass

```
          Higgs                     Higgs in CP2 ISS
            |                           |
            |                           |
            |                           |
            | mass = 145                | Non-Condensate Higgs Mass = 145
            |                           |
            |                           |
            |                           |
          Higgs                     Higgs in M4 spacetime
```

and the value of lambda is 1 = 1^2 so that the Higgs mass would be M_H = v /sqrt(3) = 145.789 GeV

However, in my E8 Physics model, the Higgs has structure of a Tquark condensate

```
          mass = 145
      T ----------- Tbar          Effective Higgs in CP2 ISS
       \     |     /                    |
        \    |    /                     |
mass = 145 \ |   /   mass = 145         | Higgs Effective Mass =
          \  |  /                       | = 145 x cos(pi/6)= 145 x 0.866 = 126
           \ | /                        |
          Higgs                     Higgs in M4 spacetime
```

in which the Higgs at a point in M4 is connected to a T and Tbar in CP2 ISS
so that the vertices of the Higgs-T-Tbar system are connected by lines forming an equilateral triangle
composed of 2 right triangles
(one from the CP2 origin to the T and to the M4 Higgs
and
another from the CP2 origin to the Tbar and to the M4 Higgs).

In the T-quark condensate picture

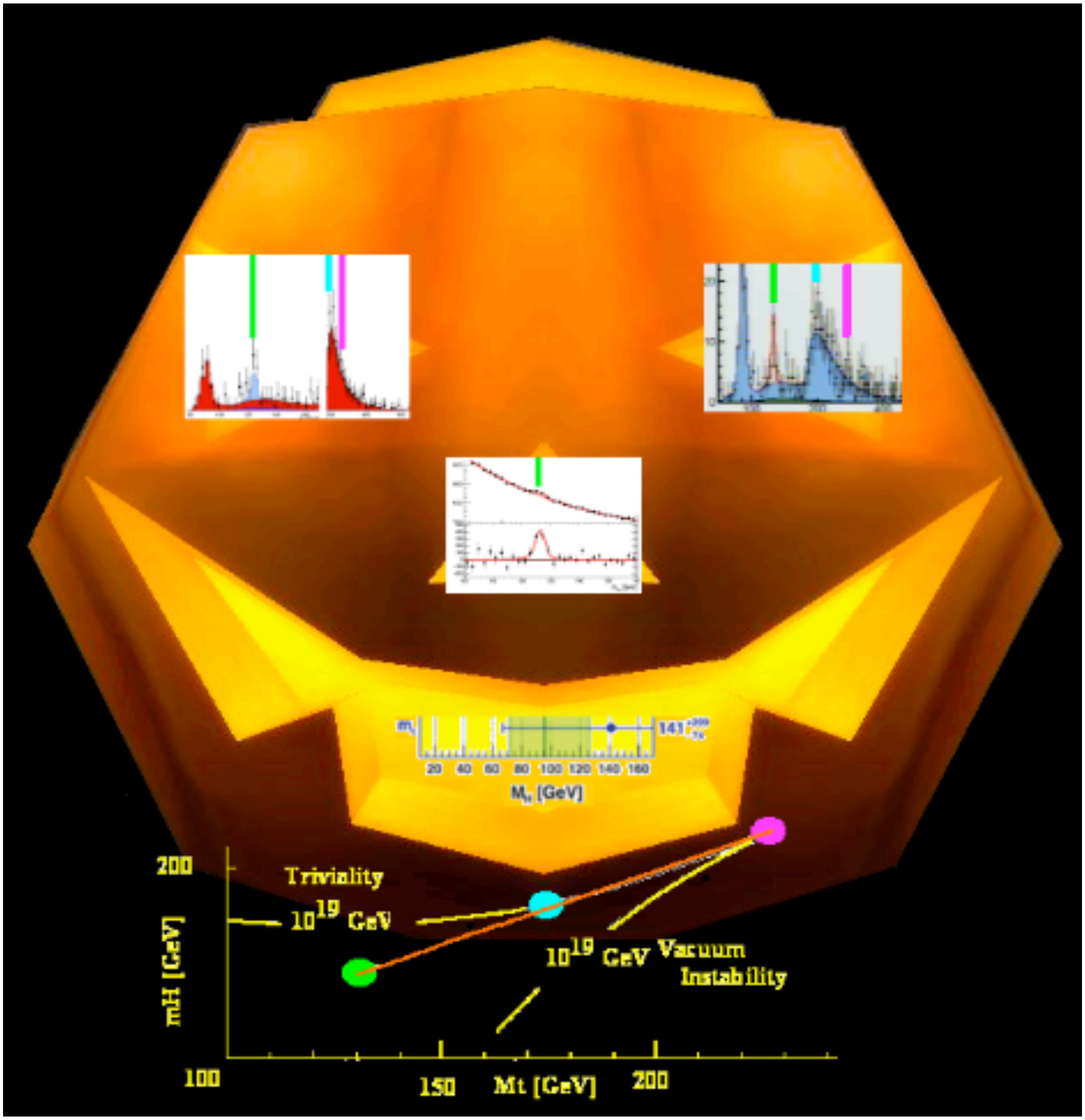
lambda = 1^2 = lambda(T) + lambda(H) = (sin(pi / 6))^2 + (cos(pi / 6))^2

and

lambda(H) = (cos(pi / 6))^2

Therefore:
The Effective Higgs mass observed by LHC is:

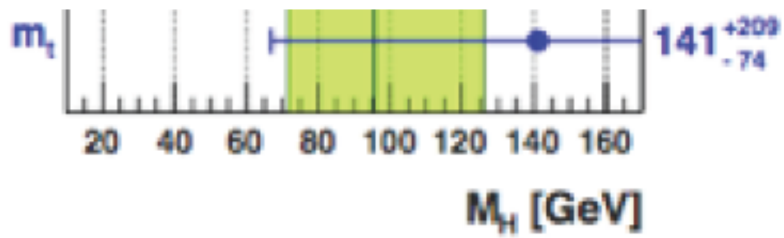
Higgs Mass = 145.789 x cos(pi/6) = 126.257 GeV.



LHC data (about 25/fb) from Halloween 2011 through Moriond 2013:

Using the ideas of - African IFA Divination; Clifford Algebra Cl(8)xCl(8) = Cl(16); Lie Algebra E8 ;
Hua Geometry of Bounded Complex Domains; Mayer Geometric Higgs Mechanism;
Batakis 8-dim Kaluza-Klein structure of hep-ph/0311165 by Hashimoto et al;
Segal Conformal Gravity version of the MacDowell-Mansouri Mechanism;
Real Clifford Algebra generalized Hyperfinite II1 von Neumannn factor AQFT; and
Joy Christian EPR Geometry - my E8 Physics model has been developed with a 3-state Higgs system
in which the Higgs is related to the Primitive Idempotents of the real Clifford Algebra Cl(8).

The Pumpkin Mouth Plot shows that the Electroweak Gfitter best fit for a floating Tquark mass as is required in my 3-State Higgs-Tquark System



is for a Higgs mass range that includes all three of its states: 126 GeV, around 200 GeV, and around 250 GeV.

Pumpkin Eye-Nose-Eye Plots are for LHC data (about 25/fb) up to the long shutdown at the end of 2012:

Left Eye: ATLAS Higgs ZZ-4l at Moriond 2013

Nose: ATLAS Higgs digamma at Moriond 2013

Right Eye: CMS Higgs ZZ-4l at Moriond 2013

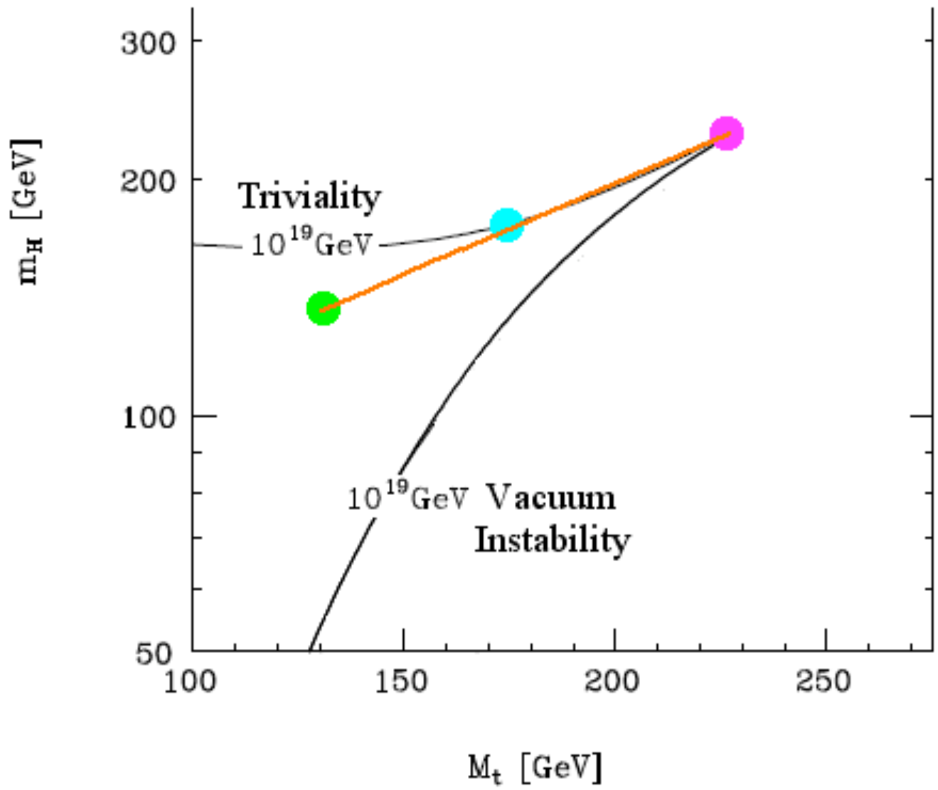
According to hep-ph/0307138 by C. D. Froggatt:
"... the top quark mass is the dominant term in the SM fermion mass matrix ... [so]... it is likely that its value will be understood dynamically ... the self-consistency of the pure SM up to some physical cut-off scale /\ imposes constraints on both the top quark and Higgs boson masses.

The first constraint is the so-called triviality bound: the running Higgs coupling constant lambda(mu) should not develop an Landau pole for mu < /\ .

The second is the vacuum stability bound: the running Higgs coupling constant lambda(mu) should not become negative leading to the instability of the usual SM vacuum.

These bounds are illustrated in Fig. 3 ... we shall be interested in the large cut-off scales /\ = 10^19 GeV, corresponding to the Planck scale [I have edited this sentence to restrict coverage to a Planck scale SM cut-off and have edited Fig. 3 and added material relevant to my E8 Physics model with 3 Higgs-Tquark states]
...
The upper part of ...[the]... curve corresponds to the triviality bound.

The lower part of ...[the]... curve coincides with the vacuum stability bound
and the point in the top right-hand corner, where it meets the triviality bound curve,
is the quasi-fixed infra-red fixed point for that value of /\ . ...



... Fig. 3: SM bounds in the (Mt , MH) plane ...”.

The Magenta Dot is the high-mass state of a 220 GeV Truth Quark and a 240 GeV Higgs.
It is at the critical point of the Higgs-Tquark System with respect to Vacuum Instability and Triviality.
It corresponds to the description in hep-ph/9603293 by Koichi Yamawaki of the Bardeen-Hill-Lindner model
That high-mass Higgs is around 250 GeV in the range of the Higgs Vacuum Instability Boundary
which range includes the Higgs VEV.

The Gold Line leading down from the Critical Point roughly along the Triviality Boundary line is based on Renormalization Group calculations with the result that MH / MT = 1.1 as described by Koichi Yamawaki in hep-ph/9603293 .

The Cyan Dot where the Gold Line leaves the Triviality Boundary to go into our Ordinary Phase is the middle-mass state of a 174 GeV Truth Quark and Higgs around 200 GeV. It corresponds to the Higgs mass calculated by Hashimoto, Tanabashi, and Yamawaki in hep-ph/0311165 where they show that
for 8-dimensional Kaluza-Klein spacetime with the Higgs as a Truth Quark condensate
$172 < MT < 175$ GeV and $178 < MH < 188$ GeV.
That mid-mass Higgs is around the 200 GeV range of the Higgs Triviality Boundary at which
the composite nature of the Higgs as T-Tbar condensate in (4+4)-dim Kaluza-Klein becomes manifest.

The Green Dot where the Gold Line terminates in our Ordinary Phase
is the low-mass state of a 130 GeV Truth Quark and a 126 GeV Higgs.

As to composite Higgs and the Triviality boundary, Pierre Ramond says in his book Journeys Beyond the Standard Model (Perseus Books 1999) at pages 175-176:
"... The Higgs quartic coupling has a complicated scale dependence. It evolves according to
d lambda / d t = (1 / 16 pi^2) beta_lambda
where the one loop contribution is given by
beta_lambda = 12 lambda^2 - ... - 4 H ...
The value of lambda at low energies is related [to] the physical value of the Higgs mass according to the tree level formula \
m_H = v sqrt(2 lambda)
while the vacuum value is determined by the Fermi constant
...
for a fixed vacuum value v, let us assume that the Higgs mass and therefore lambda is large. In that case, beta_lambda is dominated by the lambda^2 term, which drives the coupling towards its Landau pole at higher energies.
Hence the higher the Higgs mass, the higher lambda is and the close[r] the Landau pole to experimentally accessible regions.
This means that for a given (large) Higgs mass,
we expect the standard model to enter a strong coupling regime
at relatively low energies, losing in the process our ability to calculate.
This does not necessarily mean that the theory is incomplete,
only that we can no longer handle it ...
it is natural to think that this effect is caused by new strong interactions,
and that the Higgs actually is a composite ...
The resulting bound on lambda is sometimes called the **triviality bound**.
The reason for this unfortunate name (the theory is anything but trivial)
stems from lattice studies where the coupling is assumed to be finite everywhere;
in that case the coupling is driven to zero, yielding in fact a trivial theory.
In the standard model lambda is certainly not zero. ...".

Composite Higgs as Tquark condensate studies by Yamawaki et al have produced realistic models that are consistent with my E8 model with a 3-State System:

1 - My basic E8 Physic model state
with Tquark mass = 130 GeV and Higgs mass = 126 GeV

2 - Triviality boundary 8-dim Kaluza-Klein state described by Hashimoto, Tanabashi, and Yamawaki in hep-ph/0311165 where they say:
“... "... We perform the most attractive channel (MAC) analysis in the top mode standard model with TeV-scale extra dimensions, where the standard model gauge bosons and the third generation of quarks and leptons are put in D(=6,8,10,...) dimensions. In such a model, bulk gauge couplings rapidly grow in the ultraviolet region. In order to make the scenario viable, only the attractive force of the top condensate should exceed the critical coupling, while other channels such as the bottom and tau condensates should not. We then find that the top condensate can be the MAC for D=8 ... We predict masses of the top (m_t) and the Higgs (m_H) ... based on the renormalization group for the top Yukawa and Higgs quartic couplings with the compositeness conditions at the scale where the bulk top condenses ... for ...[Kaluza-Klein type]... dimension... D=8 ...
m_t = 172-175 GeV and m_H=176-188 GeV ...".

3 - Critical point BHL state
with Tquark mass = 218 +/- 3 GeV and Higgs mass = 239 +/- 3 GeV
As Yamawaki said in hep-ph/9603293: "... **the BHL formulation of the top quark condensate ... is based on the RG equation combined with the compositeness condition ... start[s] with the SM Lagrangian which includes explicit Higgs field at the Lagrangian level ... BHL is crucially based on the perturbative picture ...[which]... breaks down at high energy near the compositeness scale / \ ...[10^19 GeV]**... there must be a certain matching scale /_Matching such that the perturbative picture (BHL) is valid for mu < /_Matching, while only the nonperturbative picture (MTY) becomes consistent for mu > /_Matching ... However, **thanks to the presence of a quasi-infrared fixed point, BHL prediction is numerically quite stable against ambiguity at high energy region, namely, rather independent of whether this high energy region is replaced by MTY or something else**. ... Then we expect mt = mt(BHL) = ... = 1/(sqrt(2)) ybart v within 1-2%, where ybart is the quasi-infrared fixed point given by Beta(ybart) = 0 in ... the one-loop RG equation ... The composite Higgs loop changes ybart^2 by roughly the factor Nc/(Nc +3/2) = 2/3 compared with the MTY value, i.e., 250 GeV -> 250 x sqrt(2/3) = 204 GeV, while the electroweak gauge boson loop with opposite sign pulls it back a little bit to a higher value. **The BHL value is then**

given by mt = 218 +/- 3 GeV, at /\ = 10^19 GeV. The Higgs boson was predicted as a tbar-t bound state with a mass MH = 2mt based on the pure NJL model calculation1. Its mass was also calculated by BHL through the full RG equation ... the result being ... **MH / mt = 1.1) at /.\ = 10^19 GeV** ...".
... the top quark condensate proposed by Miransky, Tanabashi and Yamawaki (**MTY**) and by Nambu independently ... **entirely replaces the standard Higgs doublet by a composite one formed by a strongly coupled short range dynamics (four-fermion interaction) which triggers the top quark condensate**. The Higgs boson emerges as a tbar-t bound state and hence is deeply connected with the top quark itself. ... MTY introduced explicit four-fermion interactions responsible for the top quark condensate in addition to the standard gauge couplings. Based on the explicit solution of the ladder SD equation, MTY found that even if all the dimensionless four-fermion couplings are of O(1), only the coupling larger than the critical coupling yields non-zero (large) mass ... The model was further formulated in an elegant fashion by Bardeen, Hill and Lindner (BHL) in the SM language, based on the RG equation and the compositenes condition. BHL essentially incorporates 1/Nc sub-leading effects such as those of the composite Higgs loops and ... gauge boson loops which were disregarded by the MTY formulation. We can explicitly see that **BHL is in fact equivalent to MTY at 1/Nc-leading order**. Such effects turned out to reduce the above MTY value 250 GeV down to 220 GeV ...".

8-dim Kaluza-Klein spacetime physics as required by Hashimoto, Tanabashi, and Yamawaki for the Middle State of the 3-State System
was described by N. A. Batakis in Class. Quantum Grav. 3 (1986) L99-Ll05
in terms a M4xCP2 structure similar to that of my E8 Physics model.
Although spacetime and Standard Model gauge bosons worked well for Batakis,
he became discouraged by difficulties with fermions,
perhaps because he did not use Clifford Algebras with natural spinor structures for fermions.

Higgs Mass Calculations:

Low-Mass State

The calculations produce ratios of masses, so that only one mass need be chosen to set the mass scale.
In the E8 model, the value of the fundamental mass scale vacuum expectation value v = <PHI> of the Higgs scalar field is set to be the sum of the physical masses of the weak bosons, W+, W-, and Z0,
such that, in accord with ratios calculated in the E8 model, the electron mass will be 0.5110 MeV.
Effectively, the electron mass of 0.5110 MeV is the only input into the calculated particle masses.

The relationship between the Higgs mass and v is given
by the Ginzburg-Landau term from the Mayer Mechanism as
(1/4) Tr ([PHI , PHI] - PHI)^2
or, in the notation of quant-ph/9806009 by Guang-jiong Ni
(1/4!) lambda PHI^4 - (1/2) sigma PHI^2
where the Higgs mass M_H = sqrt(2 sigma)Ni says:
"... the invariant meaning of the constant lambda in the Lagrangian is not the
coupling constant, the latter will change after quantization ... The invariant
meaning of lambda is nothing but the ratio of two mass scales:
lambda = 3 (M_H / PHI)^2 which remains unchanged irrespective of the order ...".

Since <PHI>^2 = v^2, and assuming that lambda = (cos(pi / 6))^2 = 0.866^2
(a value consistent with the Higgs-Tquark condensate model of Michio Hashimoto,
Masaharu Tanabashi, and Koichi Yamawaki in their paper at hep-ph/0311165)
we have
M_H^2 / v^2 = (cos(pi / 6))^2 / 3
In the E8 model, the fundamental mass scale vacuum expectation value v of the
Higgs scalar field is the fundamental mass parameter that is to be set to define all
other masses by the mass ratio formulas of the model and
v is set to be 252.514 GeV
so that
M_H = v cos(pi / 6) / sqrt(1 / 3) = 126.257 GeV

A Non-Condensate Higgs is represented by a Higgs at a point in M4
that is connected to a Higgs representation in CP2 ISS by a line whose length represents the Higgs mass

Higgs | Higgs in CP2 ISS
| mass = 145 | Non-Condensate Higgs Mass = 145
Higgs | Higgs in M4 spacetime

and the value of lambda is 1 = 1^2
so that the Non-Condensate Higgs mass would be M_H = v /sqrt(3) = 145.789 GeV

However, in my E8 Physics model, the Higgs has beyond-tree-level structure due to a Tquark condensate

```
            mass = 145
        T ----------- Tbar          Effective Higgs in CP2 ISS
         \      |      /                 |
          \     |     /                  |
mass = 145 \    |    /   mass = 145      | Higgs Effective Mass =
            \   |   /                    | = 145 x cos(pi/6)= 145 x 0.866 = 126
             \  |  /                     |
             Higgs                     Higgs in M4 spacetime
```

in which the Higgs at a point in M4 is connected to a T and Tbar in CP2 ISS
so that the vertices of the Higgs-T-Tbar system are connected by lines forming an equilateral triangle
composed of 2 right triangles (one from the CP2 origin to the T and to the M4 Higgs
and
another from the CP2 origin to the T and to the M4 Higgs).

In the T-quark condensate picture

lambda = 1^2 = lambda(T) + lambda(H) = (sin(pi/6))^2 + (cos(pi6))^2
and
lambda(H) = (cos(pi / 6))^2

Therefore:
The effective Higgs mass observed by experiments such as the LHC is:

Higgs Mass = 145.789 x cos(pi/6) = 126.257

Mid-Mass State

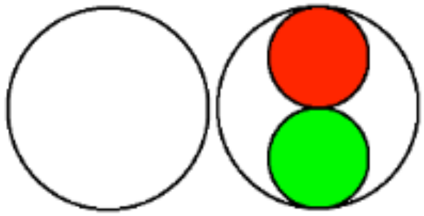
In my E8 Physics model, the Mid-Mass Higgs has structure is not restricted to Effective M4 Spacetime
as is the case with the Low-Mass Higgs Ground State
but extends to the full 4+4 = 8-dim structure of M4xCP2 Kaluza-Klein.

```
        T ----------- Tbar     in CP2 Internal Symmetry Space
         \           /
          \         /
           \       /
            \     /
             \   /
             Higgs             in M4 Physical Spacetime
```

Therefore the Mid-Mass Higgs looks like a 3-particle system of Higgs + T + Tbar.
The T and Tbar form a Pion-like state. Since Tquark Mid-Mass State is 174 GeV
the Mid-Mass T-Tbar that lives in the CP2 part of (4+4)-dim Kaluza-Klein
has mass (174+174) x (135 / (312+312) = 75 GeV.
The Higgs that lives in the M4 part of (4+4)-dim Kaluza-Klein
has, by itself, its Low-Mass Ground State Effective Mass of 125 GeV.

So, the total Mid-Mass Higgs lives in full 8-dim Kaluza-Klein with mass 75+125 = 200 GeV.
This is consistent with the Mid-Mass States of the Higgs and Tquark
being on the Triviality Boundary of the Higgs - Tquark System
and
with the 8-dim Kaluza-Klein model in hep-ph/0311165 by Hashimoto, Tanabashi, and Yamawaki.

As to the cross-section of the Mid-Mass Higgs compared to that of the Low-Mass Ground State



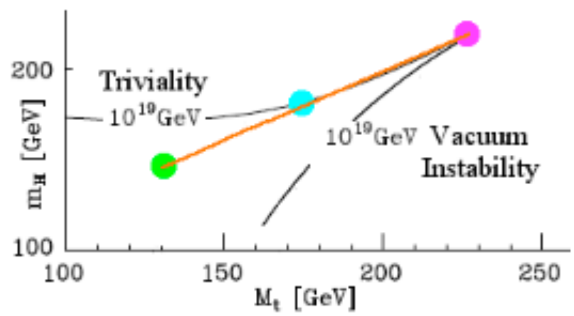
consider that the entire Ground State cross-section lives only in 4-dim M4 spacetime
(left white circle)
while for the Mid-Mass Higgs that cross-section lives in full 4+4 = 8-dim Kaluza-Klein spacetime
(right circle with red area only in CP2 ISS and white area partly in CP2 ISS
with only green area effectively living in 4-dim M4 spacetime)
so that our 4-dim M4 Physical Spacetime experiments only see for the Mid-Mass Higgs
a cross-section that is 25% of the full Ground State cross-section.
The 25% may also be visualized in terms of 8-dim coordinates {1,i,j,k,E,I,J,K}

	1	i	j	k	E	I	J	K
1	11	1i	1j	1k	1E	1I	1J	1K
i	i1	ii	ij	ik	iE	iI	iJ	iK
j	j1	ji	jj	jk	jE	jI	jJ	jK
k	k1	ki	kj	kk	kE	kI	kJ	kK
E	E1	Ei	Ej	Ek	EE	EI	EJ	EK
I	I1	Ii	Ij	Ik	IE	II	IJ	IK
J	J1	Ji	Jj	Jk	JE	JI	JJ	JK
K	K1	Ki	Kj	Kk	KE	KI	KJ	KK

in which {1,i,j,k} represent M4 and {E,I,J,K} represent CP2.

High-Mass State

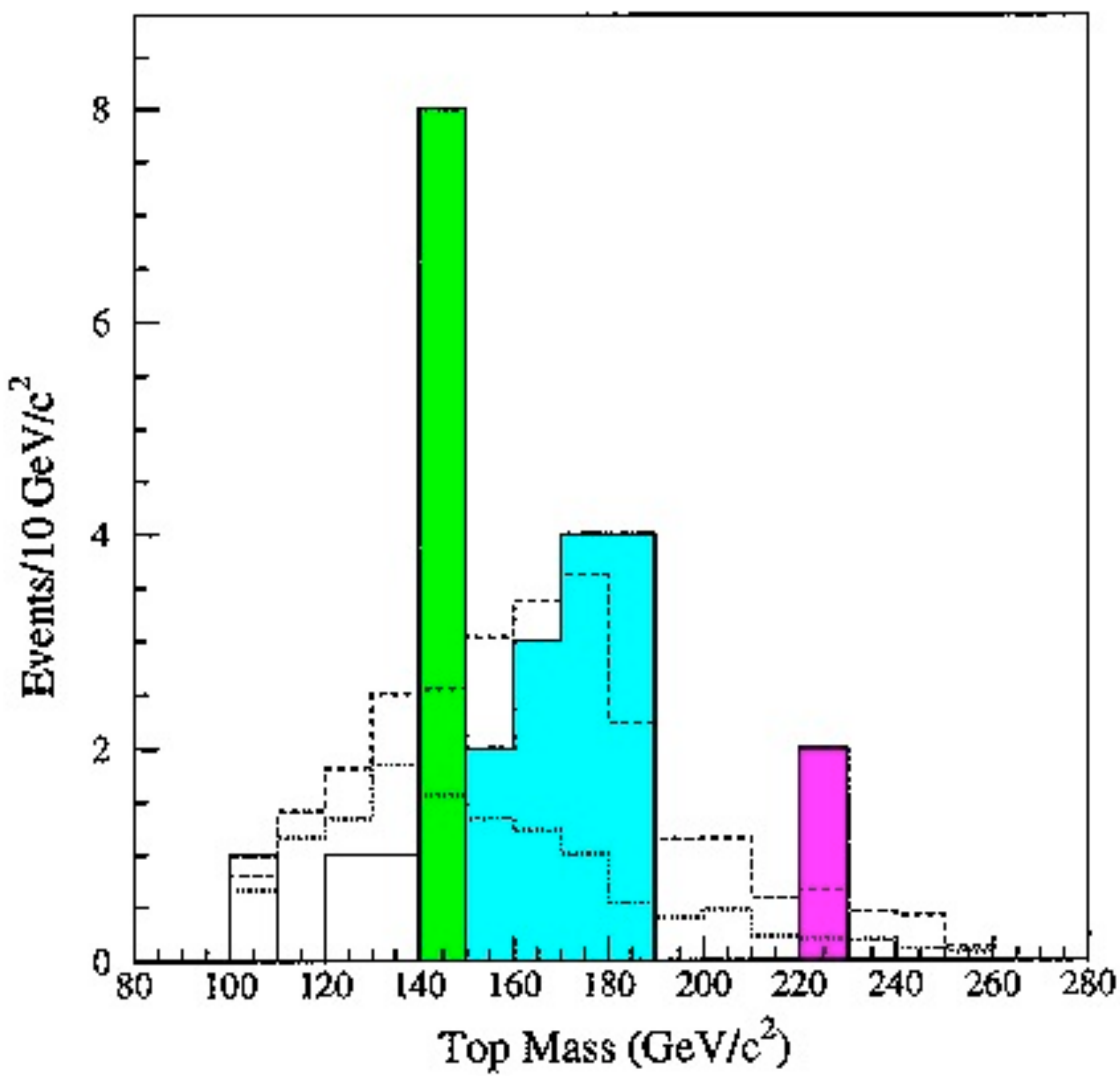
In my E8 Physics model, the High-Mass Higgs State is at the Critical Point of the Higgs-Tquark System



where the Triviality Boundary intersects the Vacuum Instability Boundary which is also at the Higgs Vacuum Expectation Value VEV around 250 GeV.

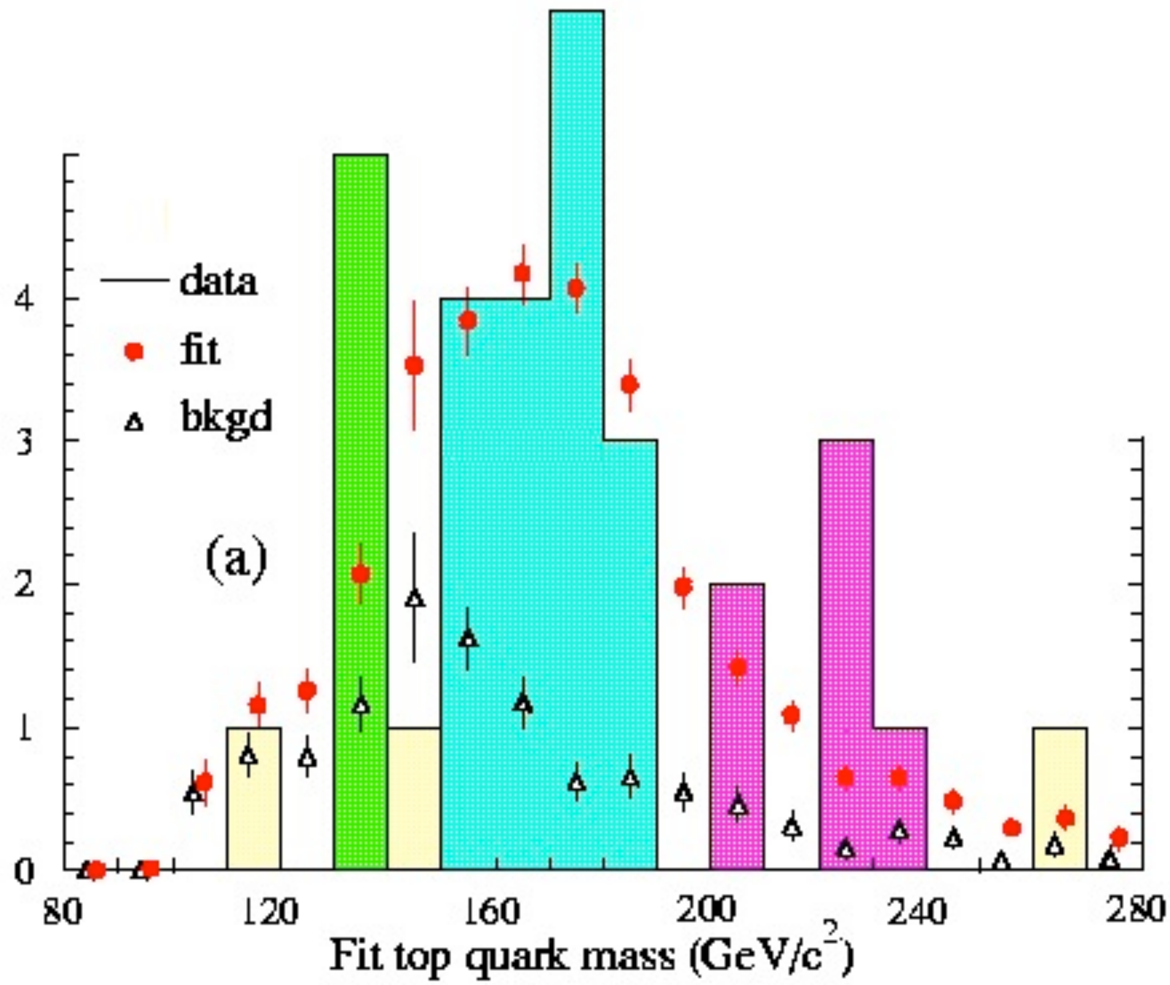
As with the Mid-Mass Higgs, the High-Mass Higgs lives in all 4+4 = 8 Kaluza-Klein dimensions and so has a cross-section that is 25% of the Higgs Ground State cross-section.

In 1994 a seimileptonic histogram from CDF



seems to me to show all three states of the T-quark.

In 1997 a semileptonic histogram from D0

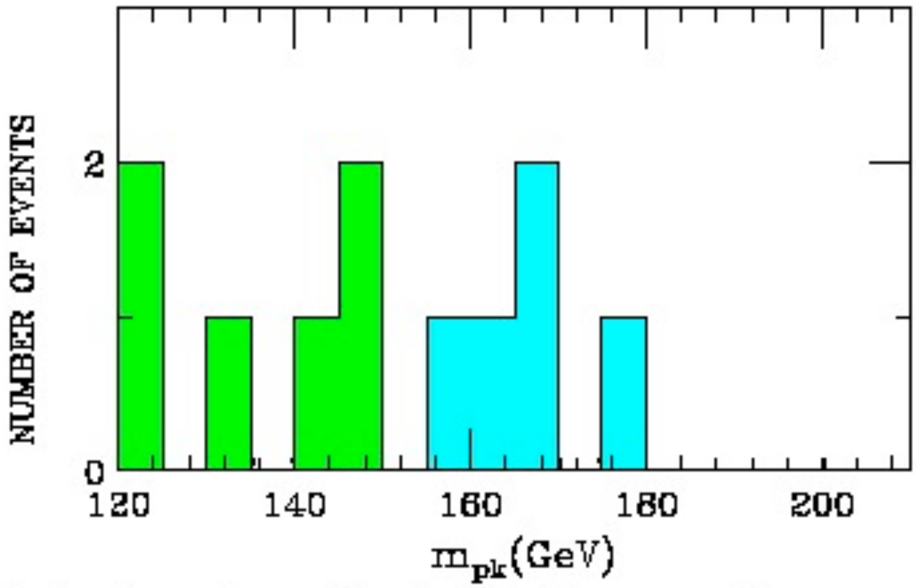


also seems to me to show all three states of the T-quark.

The fact that the low (green) state showed up in both independent detectors
indicates
a significance of 4 sigma.

Some object that the low (green) state peak should be as wide as the peak for the middle (cyan) state,
but
my opinion is that the middle (cyan) state should be wide because it is on the Triviality boundary where the composite nature of the Higgs as T-Tbar condensate becomes manifest and
the low (cyan) state should be narrow because it is in the usual non-trivial region where the T-quark acts more nearly as a single individual particle.

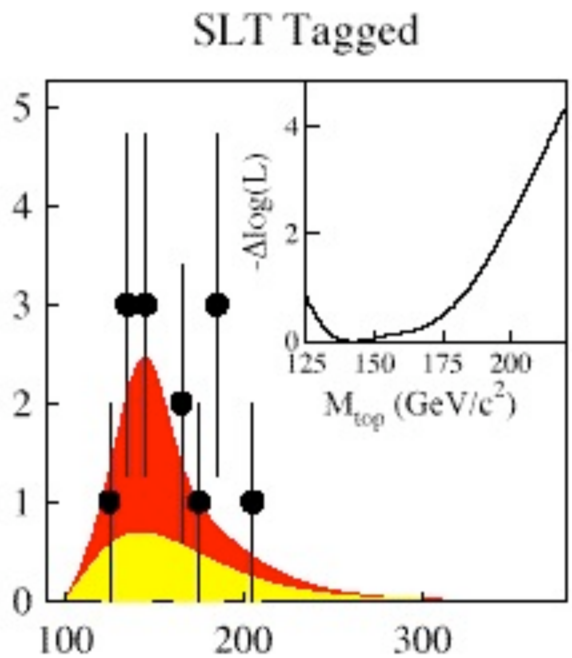
In 1998 a dilepton histogram from CDF



The distribution of m_{pk} values determined from 11 CDF dilepton events available empirically.

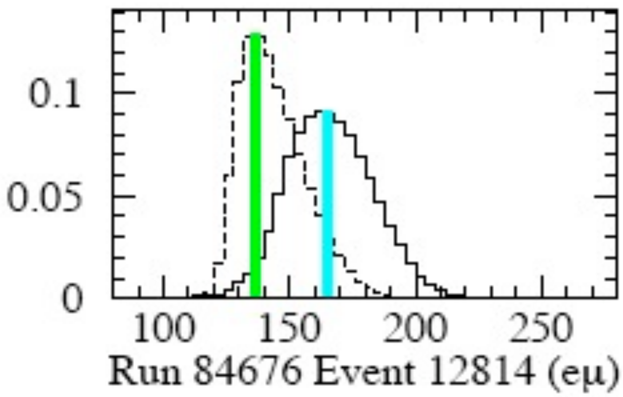
seems to me to show both the low (green) state and the middle (cyan) state of the T-quark.

In 1998 an analysis of 14 SLT tagged lepton + 4 jet events by CDF



showed a T-quark mass of 142 GeV (+33,-14) that seems to me to be consistent with the low (green) state of the T-quark.

In 1997 the Ph.D. thesis of Erich Ward Varnes (Varnes-fermilab-thesis-1997-28) at page 159 said:
"... distributions for the dilepton candidates. For events with more than two jets, the dashed curves show the results of considering only the two highest ET jets in the reconstruction ...



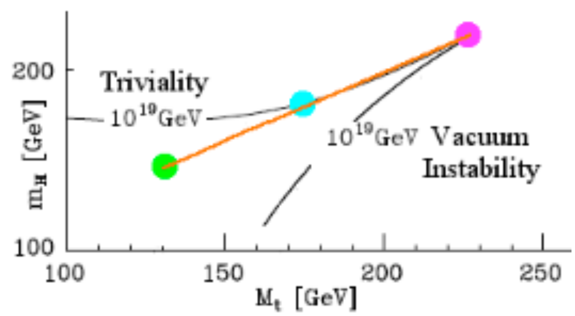
..." (colored bars added by me)

The event for all 3 jets (solid curve) seens to me to correspond to decay of a middle (cyan) T-quark state with one of the 3 jets corresponding to decay from the Triviality boundary down to the low (green) T-quark state, whose immediately subsequent decay is corresponds to the 2-jet (dashed curve) event at the low (green) energy level.
After 1998 until very recently Fermilab focussed its attention on detailed analysis of the middle (cyan) T-quark state, getting much valuable detailed information about it but **not producing much information about the low or high states**.

Standard Model Higgs: 126, 200, 250 GeV

In the 25/fb of data collected through the run ending with the long shutdown at the end of 2012, the LHC has observed a 126 GeV (about 133 proton masses) state of the Standard Model Higgs boson.

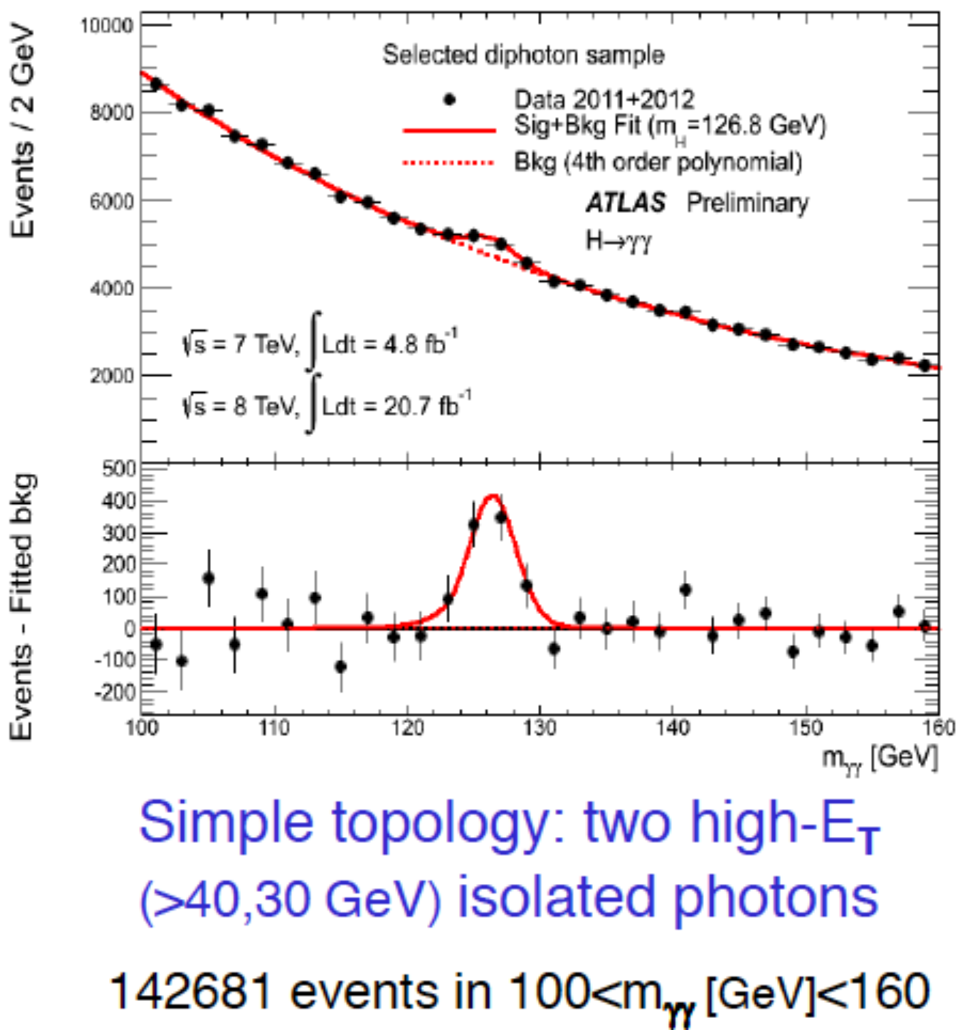
In my E8 Physics model the Higgs/Tquark system has 3 mass states



with the low-mass Higgs state calculated in my E8 Physics model to be 126.257 GeV.
The 3-state Higgs-Tquark system also has, near the Higgs Vacuum Expectation Value around 250 GeV, a high-mass state at a critical point with respect to Vacuum Instability and Triviality, as well as a mid-mass state around 200 GeV at which the system renormalization path enters conventional 4-dim Physical Spacetime, departing from the Triviality boundary at which an (4+4)-dim Klauza-Klein spacetime is manifested.

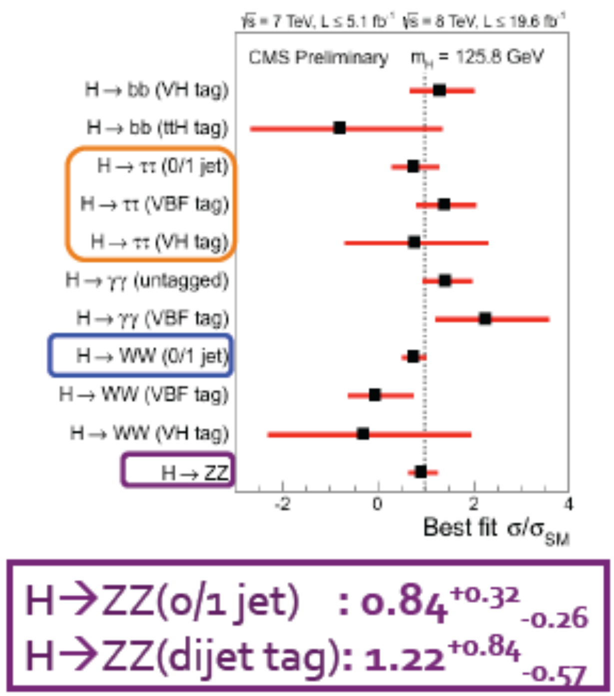
Here are some details about the LHC observation at 126 GeV and related results shown at Moriond 2013:

The digamma histogram for ATLAS



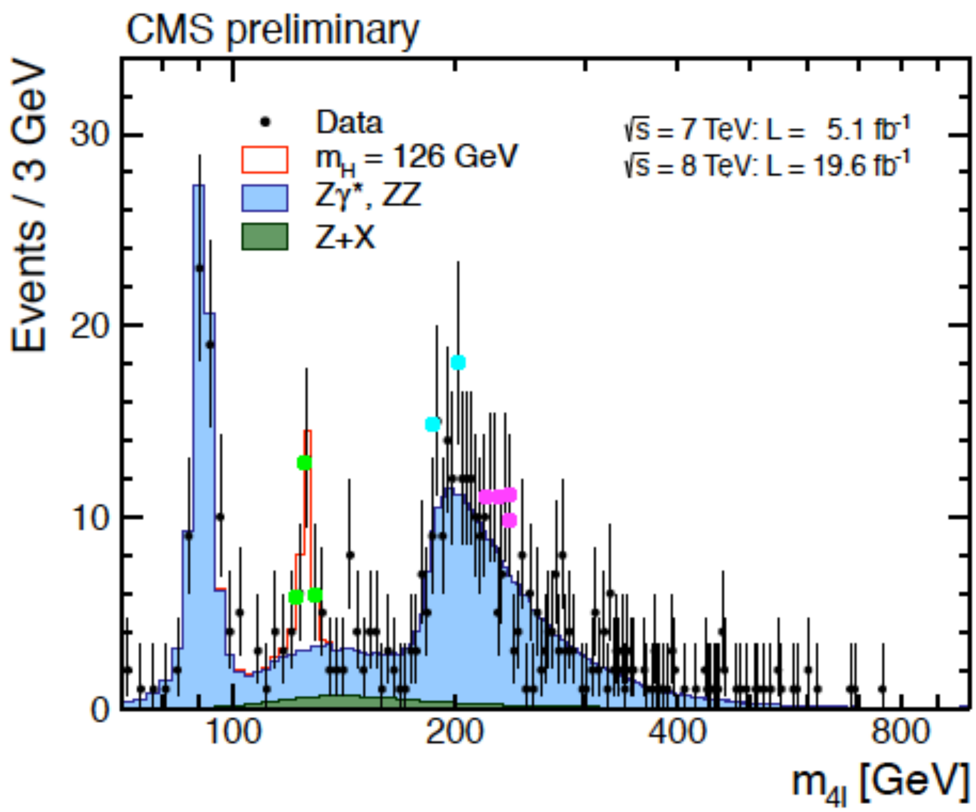
clearly shows only one peak below 160 GeV and it is around 126 GeV.

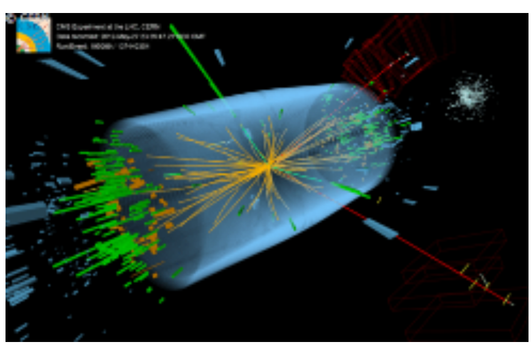
CMS shows the cross sections for Higgs at 125.8 GeV



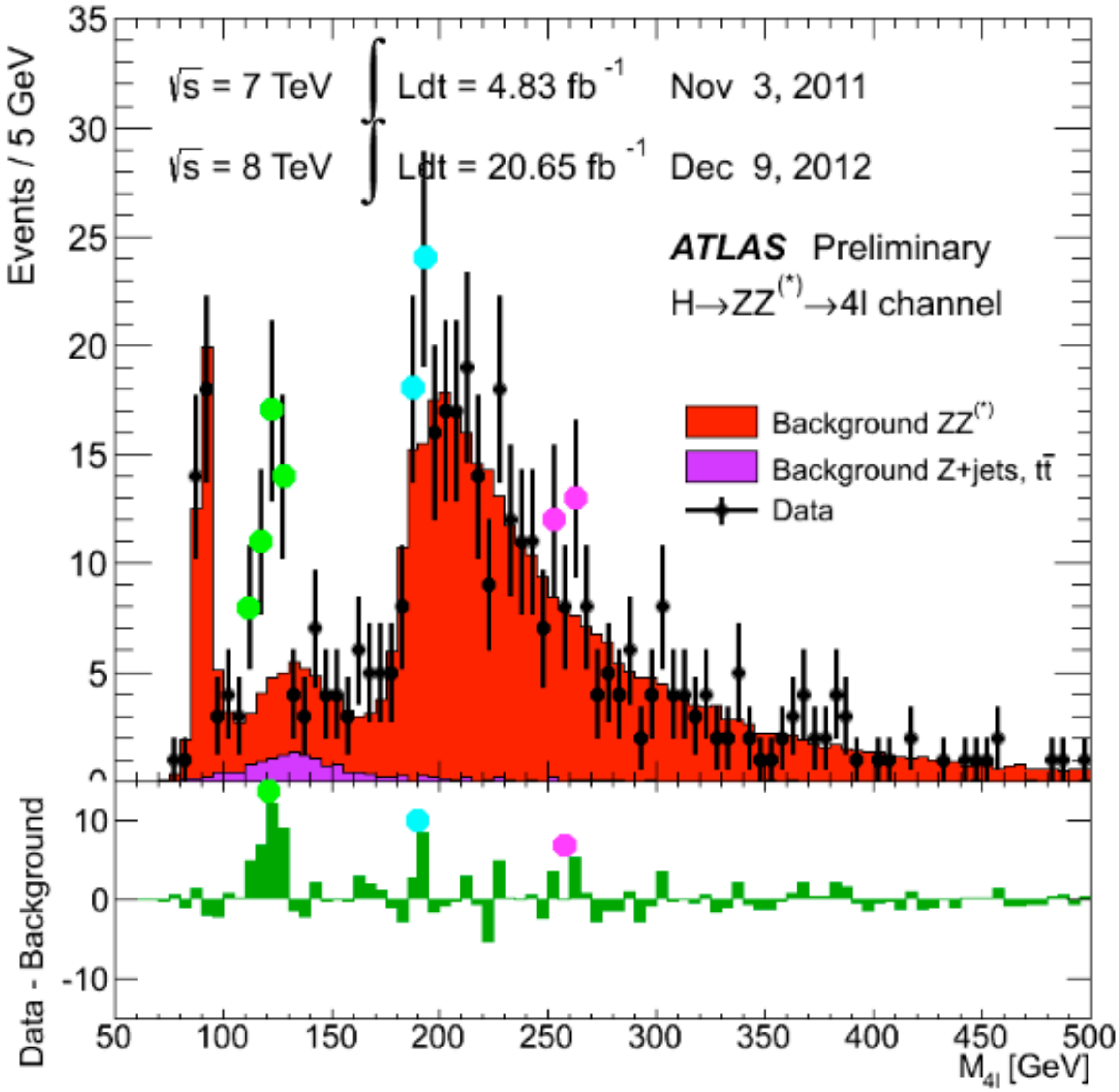
to be substantially consistent with the Standard Model for the WW and ZZ channels, a bit low for tau-tau and bb channels (but that is likely due to very low statistics there), and a bit high for the digamma channel (but that may be due to phenomena related to the Higgs as a Tquark condensate).

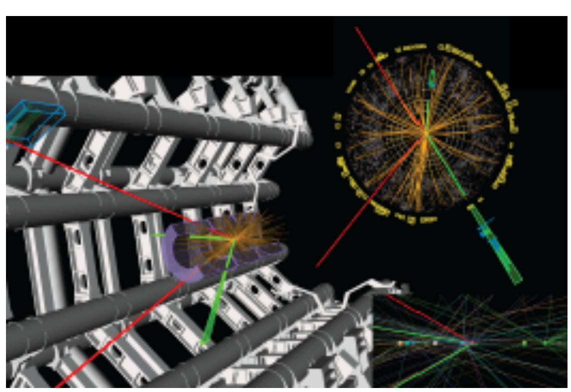
A CMS histogram (some colors added by me) for the Golden Channel Higgs to ZZ to 4l shows the peak around 126 GeV (green dots - lowHiggs mass state. The CMS histogram also indicates other excesses around 200 GeV (cyan dots - midHiggs mass state)
and around 250 GeV (magenta dots - highHiggs mass state).
An image of one of the events is shown below the histogram.



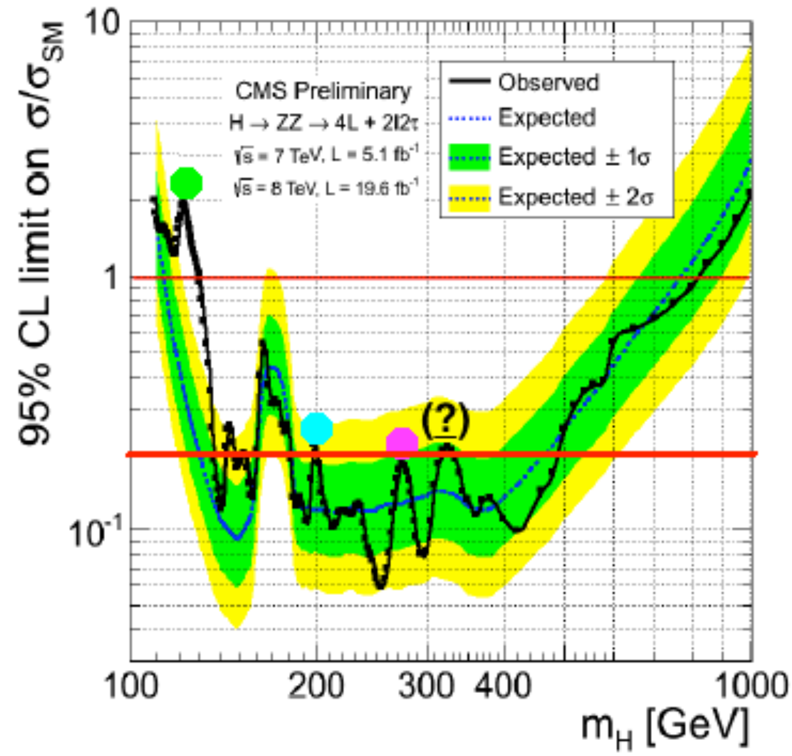


An ATLAS ZZ to 4l histogram (some colors added by me) show the peak around 126 GeV (green dots - lowHiggs mass state. The ATLAS histogram also indicates other excesses around 200 GeV (cyan dots - midHiggs mass state)
and around 250 GeV (magenta dots - highHiggs mass state) .
An image of one of the events is shown below the histogram.





CMS showed a Brazil Band Plot for the High Mass Higgs to ZZ to 4l/2l2tau channel
where the top red line represents the expected cross section of a single Standard Model Higgs
and the lower red line represents about 20% of the expected Higgs SM cross section.



The green dot peak is at the 126 GeV Low Mass Higgs state with expected Standard Model cross section.

The cyan dot peak is around the 200 GeV Mid Mass Higgs state expected to have about 25% of the SM cross section.

The magenta dot peak is around the 250 (+/- 20 or so) GeV High Mass Higgs state expected to have about 25% of the SM cross section.

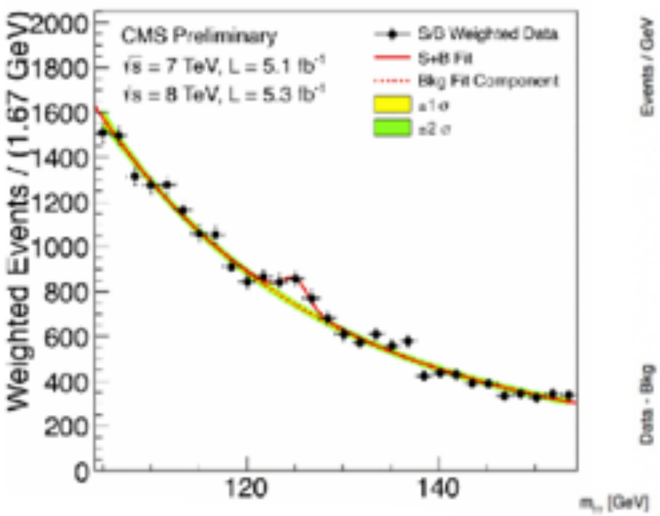
The **(?)** peak is around 320 GeV where I would not expect a Higgs Mass state
and I note that in fact it seems to have gone away in the full ATLAS ZZ to 4l histogram shown above
because between 300 and 350 GeV the two sort-of-high excess bins are adjacent to deficient bins .

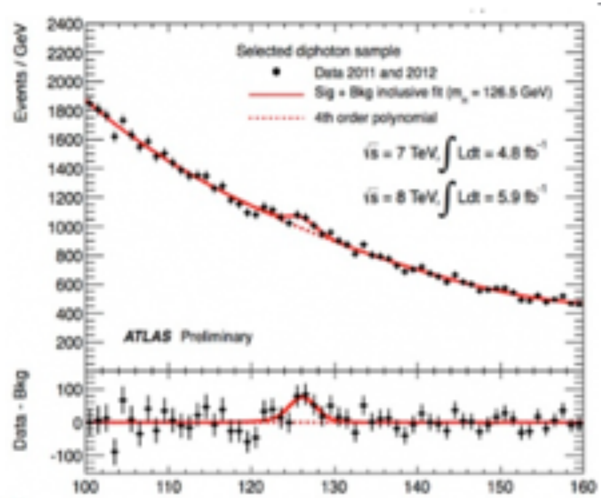
It will probably be no earlier than 2016 (after the long shutdown) that the LHC will produce substantially more data than the 25/fb available at Moriond 2013 and therefore no earlier than 2016 for the green and yellow Brazil Bands to be pushed down (throughout the 170 GeV to 500 GeV region) below 10 per cent (the 10^(-1) line) of the SM cross section
as is needed to show whether or not the cyan dot , magenta dot, and/or **(?)** peaks are real or statistical fluctuations.
My guess (based on E8 Physics) is that the cyan dot and magenta dot peaks will prove to be real
and that the **(?)** peak will go away as a statistical fluctuation
but whatever the result, it is now clear that Nature likes the plain vanilla Standard Model
(with or maybe without a couple of Little Brother Higgs states, where Little refers to cross section).

Sgr A* and Higgs = Tquark-Tantiquark Condensate:

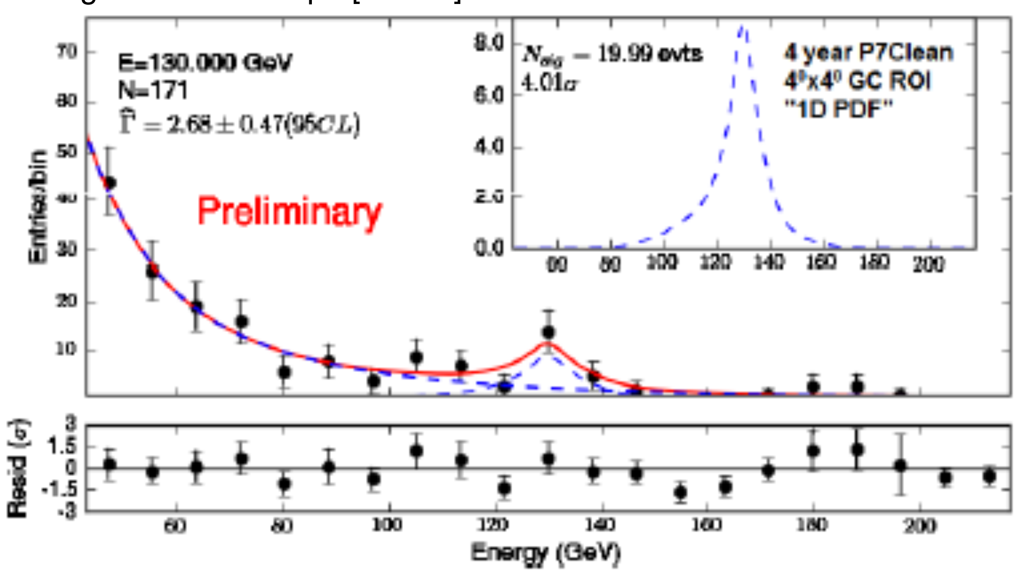
Sagittarius A* (Sgr A*) is a very massive black hole in the center of our Galaxy into which large amounts of Hydrogen fall. As the Hydrogen approaches Sgr A* it increases in energy, ionizing into protons and electrons, and eventually producing a fairly dense cloud of infalling energetic protons whose collisions with ambient protons are at energies similar to the proton-proton collisions at the LHC.

LHC diphoton histograms for ATLAS and CMS as of mid-2012 clearly show a peak that probably is evidence of a Higgs boson with mass around 125 GeV.





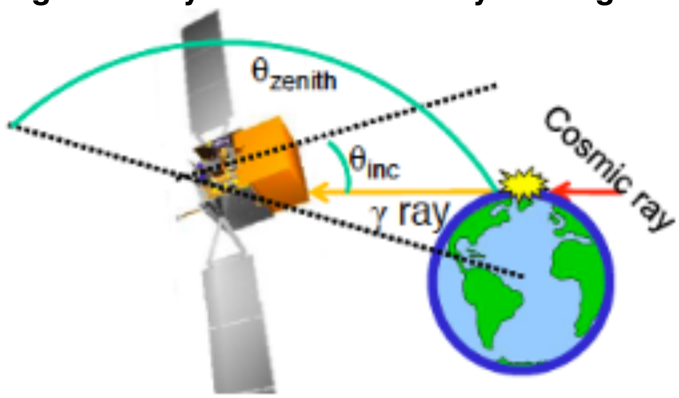
Andrea Albert at The Fermi Symposium 11/2/2012 said: "... gamma rays detectable by the Fermi Large Area Telescope [FLAT] ...



... Line-like Feature near 135 GeV ... localized in the galactic center ...".

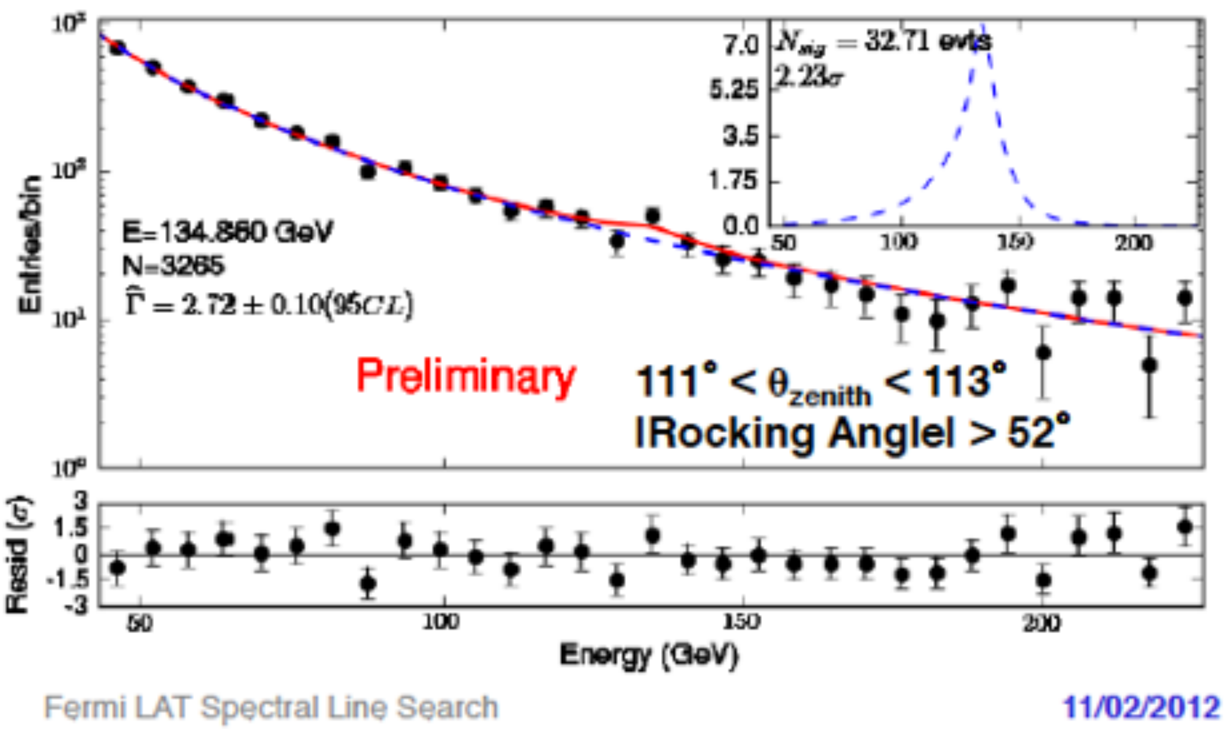
Sgr A* and Higgs = Tquark-Tantiquark Condensate:

In addition to the Galactic Center observations,
Fermi LAT looked at gamma rays from Cosmic Rays hitting Earth's atmosphere



by looking at the Earth Limb.

Andrea Albert at The Fermi Symposium 11/2/2012 also said: "... Earth Limb is a bright gamma-ray source ... From cosmic-ray interactions in the atmosphere ...



... Line-like feature ... at 135 GeV .. Appears when LAT is pointing at the Limb ...".

Since 90% of high-energy Cosmic Rays are Protons and since their collisions with Protons and other nuclei in Earth's atmosphere produce gamma rays,
the 135 GeV Earth Limb Line seen by Fermi LAT is also likely to be the Higgs produced by collisions analagous to those at the LHC.

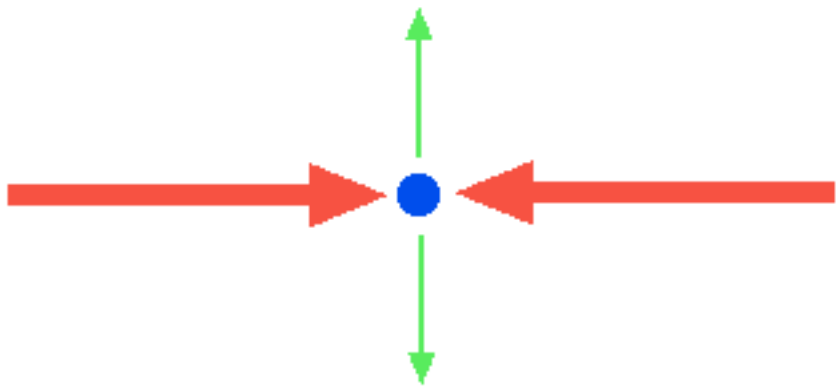
Sgr A* and Higgs = Tquark-Tantiquark Condensate:

Olivier K. in a comment in Jester's blog on 10 November 2012 said:
"... Could the 135GeV bump be related ... to current Higgs ... properties ? ...
The coincidence between GeV figures ...[for LHC] Higgs mass and this [Fermi LAT] bump is thrilling for an amateur like me...".

Jester in his resonaances blog on 17 April 2012 said, about Fermi LAT:
"... the plot shows the energy of *single* photons as measured by Fermi, not the invariant mass of photon pairs ...".

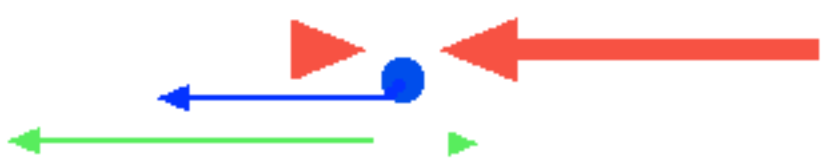
Since the LHC 125 GeV peak is for "invariant mass of photon pairs"
and the Fermi LAT 135 GeV peak is for ""single" photons"
how could both correspond to a Higgs mass state around 130 GeV ?

The LHC sees collisions of high-energy protons (red arrows) forming Higgs (blue dot)



with the Higgs at rest decaying into a photon pair (green arrows)
giving the observed Higgs peak (around 130 GeV) as **invariant mass of photon pairs**.

Fermi LAT at Galactic Center and Earth Limb sees
collisions of one high-energy proton with a low-energy (relatively at rest) proton forming Higgs



with Higgs moving fast from momentum inherited from the high-enrgy proton decaying into two photons: one with low energy not observed by Fermi LAT
and the other being observed by Fermi LAT as a high-energy gamma ray carrying almost all of the Higgs decay energy (around 130 GeV) as **a "single" photon**.

Therefore, **the coincidence noted by Olivier K. is probably a realistic phenomenon**.

Sgr A* and Higgs = Tquark-Tantiquark Condensate:

Jester, replying to the comment by Olivier K., dismissed the proposal that Fermi LAT may have seen the Higgs, saying on 11 November 2012:
"Afaik,
there's no model connecting the 130(5)GeV Fermi line to the 125 GeV Higgs."

so

I hereby propose a model:
Protons from Hydrogen infalling into Sgr A* acquire enough energy and density to produce proton-proton collisions similar to those at the LHC,
as could Cosmic Ray Protons hitting the Earth's atmosphere,
and
the 135 GeV Line observed by Fermi LAT is due to proton-proton collisions producing Higgs in the diphoton channel
and
the125 GeV Higgs-like evidence observed by ATLAS and CMS is also due to proton-proton collisions producing Higgs in the diphton channel
and
the difference between 135 GeV at Fermi LAT and 125 GeV at LHC
can be accounted for by comparing details of experimental setup and analysis-related assumptions.

Given that model,
I propose that Olivier K. be given credit for stating the possibility that both Fermi LAT and the LHC have indeed seen the Higgs,
which is an interesting example of
mutual confirmation of Collider Physics and Astrophysics observations.

The {G4} conformal generator that represents both Dark Energy of Universe Expansion and the Massive Higgs Scalar as Fermionic Condensate (dominated by third-generation Tquark-Tantiquark Condensate) may be involved in the Sgr A* Galactic Center Process.

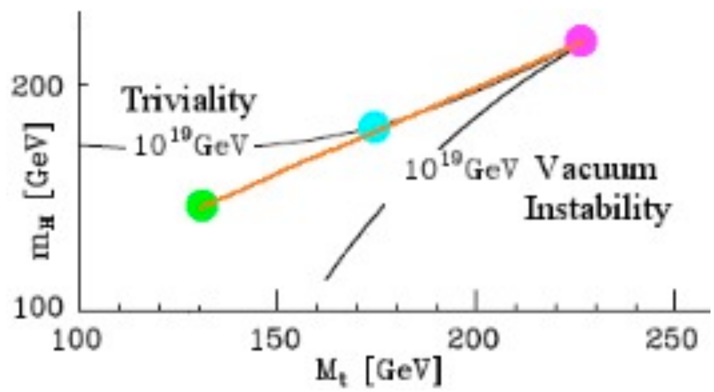
Sgr A* and Higgs = Tquark-Tantiquark Condensate:

Due to its relationship with the Higgs as Tquark-Tantiquark Condensate,

the Truth Quark might be related by {G4} to Dark Energy of Universe Expansion

as well as

by a 3-state mass system due to its interaction with the Higgs as Condensate to



a Strong Coupling / Composite-Higgs Regime (known as Triviality)

and

a Vacuum Instability Regime.

To get W-boson masses,
denote the 3 SU(2) high-energy weak bosons
(massless at energies higher than the electroweak unification)
by W+, W-, and W0,
corresponding to the massive physical weak bosons W+, W-, and Z0.

The triplet { W+, W-, W0 } couples directly with the T - Tbar quark-antiquark pair,
so that the total mass of the triplet { W+, W-, W0 } at the electroweak unification
is equal to the total mass of a T - Tbar pair, 259.031 GeV.

The triplet { W+, W-, Z0 } couples directly with the Higgs scalar,
which carries the Higgs mechanism by which the W0 becomes the physical Z0,
so that the total mass of the triplet { W+, W-, Z0 }
is equal to the vacuum expectation value v of the Higgs scalar field,
v = 252.514 GeV.

What are individual masses of members of the triplet { W+, W-, Z0 } ?

First, look at the triplet { W+, W-, W0 }
which can be represented by the 3-sphere S^3.
The Hopf fibration of S^3 as
S^1 --> S^3 --> S^2
gives a decomposition of the W bosons
into the neutral W0 corresponding to S^1
and the charged pair W+ and W- corresponding to S^2.

The mass ratio of the sum of the masses of W+ and W- to the mass of W0
should be the volume ratio of the S^2 in S^3 to the S^1 in S3.
The unit sphere S^3 in R^4 is normalized by 1 / 2.
The unit sphere S^2 in R^3 is normalized by 1 / sqrt(3).
The unit sphere S^1 in R^2 is normalized by 1 / sqrt(2).
The ratio of the sum of the W+ and W- masses to the W0 mass should then be
(2 / sqrt3) V(S^2) / (2 / sqrt2) V(S^1) = 1.632993

Since the total mass of the triplet { W+, W-, W0 } is 259.031 GeV, the total mass
of a T - Tbar pair, and the charged weak bosons have equal mass, we have
M_W+ = M_W- = 80.326 GeV and M_W0 = 98.379 GeV.

The charged W+/- neutrino-electron interchange must be symmetric with the electron-neutrino interchange, so that the absence of right-handed neutrino particles requires that the charged W+/- SU(2) weak bosons act only on left-handed electrons.

Each gauge boson must act consistently on the entire Dirac fermion particle sector, so that the charged W+/- SU(2) weak bosons act
only on left-handed fermion particles of all types.

The neutral W0 weak boson does not interchange Weyl neutrinos
with Dirac fermions, and so is not restricted to left-handed fermions,
but also has a component that acts on both types of fermions,
both left-handed and right-handed, conserving parity.

However, the neutral W0 weak bosons are related to the charged W+/- weak bosons by custodial SU(2) symmetry, so that
the left-handed component of the neutral W0 must be
equal to the left-handed (entire) component of the charged W+/-.

Since the mass of the W0 is greater than the mass of the W+/-,
there remains for the W0 a component acting on both types of fermions.

Therefore the full W0 neutral weak boson interaction
is proportional to (M_W+/-^2 / M_W0^2) acting on left-handed fermions
and
(1 - (M_W+/-^2 / M_W0^2)) acting on both types of fermions.

If (1 - (M_W+/-2 / M_W0^2)) is defined to be sin(theta_w)^2
and denoted by K,
and if the strength of the W+/- charged weak force
(and of the custodial SU(2) symmetry) is denoted by T,
then the W0 neutral weak interaction can be written as
W0L = T + K and W0LR = K.

Since the W0 acts as W0L with respect to the parity violating SU(2) weak force and as W0LR with respect to the parity conserving U(1) electromagnetic force of the U(1) subgroup of SU(2),
the W0 mass mW0 has two components:
the parity violating SU(2) part mW0L that is equal to M_W+/-
and the parity conserving part M_W0LR that acts like a heavy photon.

As M_W0 = 98.379 GeV = M_W0L + M_W0LR, and as M_W0L = M_W+/- = 80.326 GeV, we have M_W0LR = 18.053 GeV.

Denote by *alphaE = *e^2 the force strength of the weak parity conserving U(1) electromagnetic type force that acts through the U(1) subgroup of SU(2).

The electromagnetic force strength alphaE = e^2 = 1 / 137.03608 was calculated above using the volume V(S^1) of an S^1 in R^2, normalized by 1 / sqrt(2).

The *alphaE force is part of the SU(2) weak force whose strength alphaW = w^2 was calculated above using
the volume V(S^2) of an S^2 \subset R^3, normalized by 1 / sqrt(3).

Also, the electromagnetic force strength alphaE = e^2 was calculated above using a 4-dimensional spacetime with global structure of the 4-torus T^4 made up of four S^1 1-spheres,
while the SU(2) weak force strength alphaW = w^2 was calculated above using two 2-spheres S^2 x S^2,
each of which contains one 1-sphere of the *alphaE force.

Therefore
    *alphaE = alphaE (sqrt(2) / sqrt(3))(2 / 4) = alphaE / sqrt(6),
    *e = e / (4th root of 6) = e / 1.565 ,
and the mass mW0LR must be reduced to an effective value
    M_W0LReff = M_W0LR / 1.565 = 18.053/1.565 = 11.536 GeV
for the *alphaE force to act like an electromagnetic force in the E8 model:
    *e M_W0LR = e (1/5.65) M_W0LR = e M_Z0,
where the physical effective neutral weak boson is denoted by Z0.

Therefore, the correct E8 model values for weak boson masses and the Weinberg angle theta_w are:
    M_W+ = M_W- = 80.326 GeV;
    M_Z0 = 80.326 + 11.536 = 91.862 GeV;
Sin(theta_w)^2 = 1 - (M_W+/- / M_Z0)^2 = 1 - (6452.2663 / 8438.6270) = 0.235.

Radiative corrections are not taken into account here, and may change these tree-level values somewhat.

Kobayashi-Maskawa Mixing
Above and Below ElectroWeak Symmetry Breaking

Below the energy level of ElectroWeak Symmetry Breaking
the Higgs mechanism gives mass to particles.

According to a Review on the Kobayashi-Maskawa mixing matrix by Ceccucci, Ligeti, and Sakai in the 2010 Review of Particle Physics (note that I have changed their terminology of CKM matrix to the KM terminology that I prefer because I feel that it was Kobayashi and Maskawa, not Cabibbo, who saw that 3x3 was the proper matrix structure):
"... the charged-current W± interactions couple to the ... quarks with couplings given by ...

Vud	Vus	Vub
Vcd	Vcs	Vcb
Vtd	Vts	Vtb

This Kobayashi-Maskawa (KM) matrix is a 3 × 3 unitary matrix.
It can be parameterized by three mixing angles and the CP-violating KM phase ...
The most commonly used unitarity triangle arises from
Vud Vub∗ + Vcd Vcb∗ + Vtd Vtb∗ = 0, by dividing each side by the best-known one, Vcd Vcb∗
... $\bar{\rho} + i\bar{\eta}$ = −(Vud Vub∗)/(Vcd Vcb∗) is phase-convention- independent ...

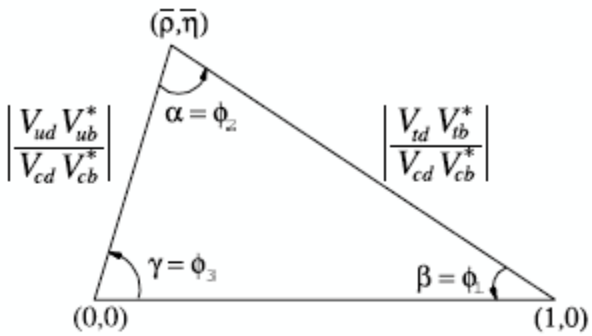


Figure 11.1: Sketch of the unitarity triangle.

... sin 2β = 0.673 ± 0.023 ... α = 89.0 +4.4 −4.2 degrees ... γ = 73 +22 −25 degrees ...

The sum of the three angles of the unitarity triangle, α + β + γ = (183 +22 −25) degrees,
is ... consistent with the SM expectation. ...

The area... of ...[the]... triangle...[is]... half of the Jarlskog invariant, J,
which is a phase-convention-independent measure of CP violation,
defined by Im Vij Vkl Vil∗ Vkj∗ = J SUM(m,n) ε_ikm ε_jln
...

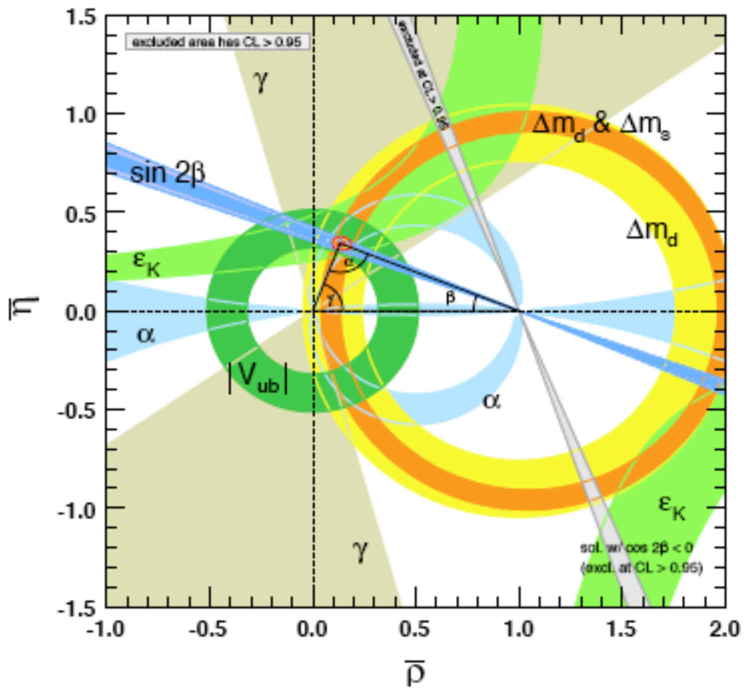


Figure 11.2: Constraints on the $\bar{\rho}, \bar{\eta}$ plane.
The shaded areas have 95% CL.

The fit results for the magnitudes of all nine KM elements are ...

0.97428 ± 0.00015	0.2253 ± 0.0007	0.00347 +0.00016 −0.00012
0.2252 ± 0.0007	0.97345 +0.00015 −0.00016	0.0410 +0.0011 −0.0007
0.00862 +0.00026 −0.00020	0.0403 +0.0011−0.0007	0.999152 +0.000030−0.000045

and the Jarlskog invariant is J = (2.91 +0.19-0.11) × 10−5. ...".

Above the energy level of ElectroWeak Symmetry Breaking particles are massless.

Kea (Marni Sheppeard) proposed that in the Massless Realm the mixing matrix might be democratic.

In Z. Phys. C - Particles and Fields 45, 39-41 (1989) Koide said: "...
the mass matrix ... MD ... of the type ... 1/3 x m x

$$\begin{matrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{matrix}$$

... has name... "democratic" family mixing ... the ... democratic ... mass matrix can be diagonalized by the transformation matrix A ...

$$\begin{matrix} 1/\mathrm{sqrt}(2) & -1/\mathrm{sqrt}(2) & 0 \\ 1/\mathrm{sqrt}(6) & 1/\mathrm{sqrt}(6) & -2/\mathrm{sqrt}(6) \\ 1/\mathrm{sqrt}(3) & 1/\mathrm{sqrt}(3) & 1/\mathrm{sqrt}(3) \end{matrix}$$

as A MD At =

$$\begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & m \end{matrix}$$

...".

Up in the Massless Realm you might just say that there is no mass matrix,
just a democratic mixing matrix of the form 1/3 x

$$\begin{matrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{matrix}$$

with no complex stuff and no CP violation in the Massless Realm.
When go down to our Massive Realm by ElectroWeak Symmetry Breaking
then you might as a first approximation use m = 1
so that all the mass first goes to the third generation as

$$\begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{matrix}$$

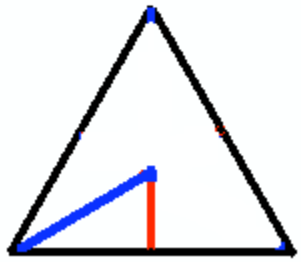
which is physically like the Higgs being a T-Tbar quark condensate.

Consider a 3-dim Euclidean space of generations:

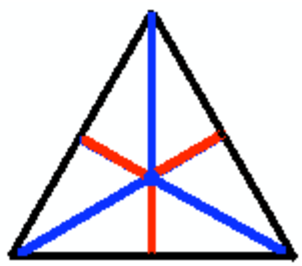
The case of mass only going to one generation
can be represented as a line or 1-dimensional simplex

in which the blue mass-line covers the entire black simplex line.

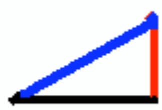
If mass only goes to one other generation
that can be represented by a red line extendng to a second dimension
forming a small blue-red-black triangle



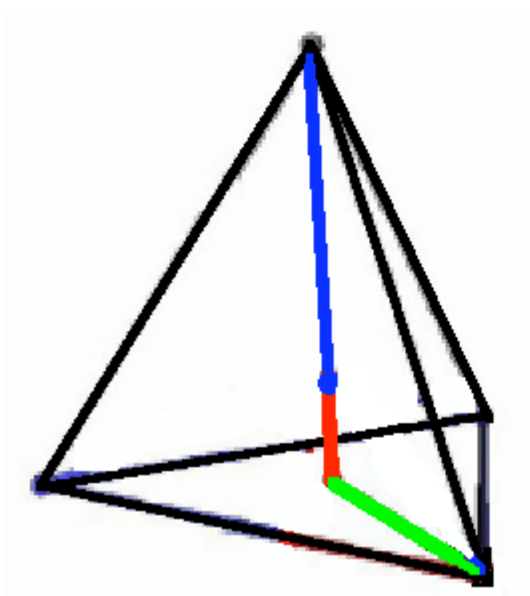
that can be extended by reflection to form six small triangles
making up a large triangle.



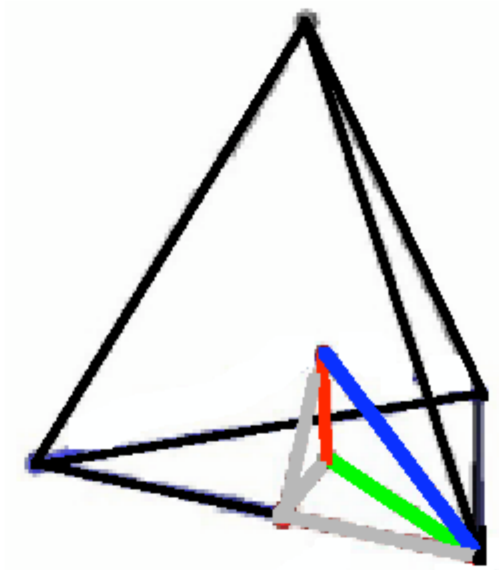
Each of the six component triangles has 30-60-90 angle structure:



If mass goes on further to all three generations
that can be represented by a green line extending to a third dimension

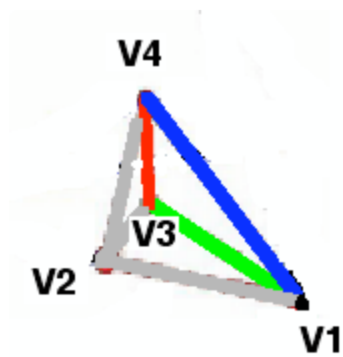


If you move the blue line from the top vertex to join the green vertex



you get a small blue-red-green-gray-gray-gray tetrahedron
that can be extended by reflection to form 24 small tetrahedra
making up a large tetrahedron.
Reflection among the 24 small tetrahedra corresponds
to the 12+12 = 24 elements of the Binary Tetrahedral Group.

The basic blue-red-green triangle of the basic small tetrahedron



has the angle structure of the K-M Unitary Triangle.

Using data from R. W. Gray's "Encyclopedia Polyhedra: A Quantum Module" with lengths

V1.V2 = (1/2) EL ≡ Half of the regular Tetrahedron's edge length.
V1.V3 = (1 / sqrt(3)) EL ≅ 0.577 350 269 EL
V1.V4 = 3 / (2 sqrt(6)) EL ≅ 0.612 372 436 EL
V2.V3 = 1 / (2 sqrt(3)) EL ≅ 0.288 675 135 EL
V2.V4 = 1 / (2 sqrt(2)) EL ≅ 0.353 553 391 EL

V3.V4 = 1 / (2 sqrt(6)) EL ≅ 0.204 124 145 EL

the Unitarity Triangle angles are:

β = V3.V1.V4 = arccos(2 sqrt(2) / 3) ≅ 19.471 220 634 degrees so sin 2β = 0.6285

α = V1.V3.V4 = 90 degrees

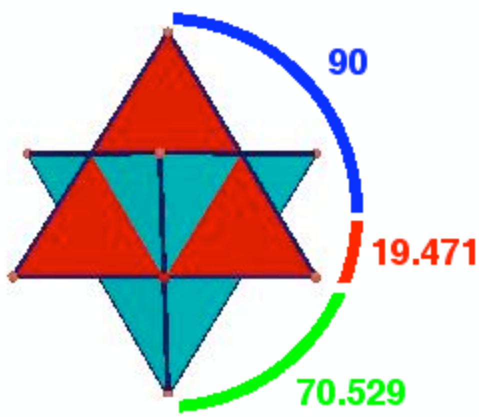
γ = V1.V4.V3 = arcsin(2 sqrt(2) / 3) ≅ 70.528 779 366 degrees

which is substantially consistent with the 2010 Review of Particle Properties

sin 2β = 0.673 ± 0.023 so β = 21.1495 degrees
α = 89.0 +4.4 −4.2 degrees
γ = 73 +22 −25 degrees

and so also consistent with the Standard Model expectation.

The constructed Unitarity Triangle angles can be seen on the Stella Octangula configuration of two dual tetrahedra (image from gauss.math.nthu.edu.tw):



In my E8 Physics model the Kobayashi-Maskawa parameters are determined in terms of the sum of the masses of the 30 first-generation fermion particles and antiparticles, denoted by
Smf1 = 7.508 GeV,

and the similar sums for second-generation and third-generation fermions, denoted
by Smf2 = 32.94504 GeV and Smf3 = 1,629.2675 GeV.

The reason for using sums of all fermion masses (rather than sums of quark masses only) is that all fermions are in the same spinor representation of Spin(8), and the Spin(8) representations are considered to be fundamental.

The following formulas use the above masses to calculate Kobayashi-Maskawa parameters:

phase angle d13 = gamma = 70.529 degrees

sin(theta12) = s12 = [me+3md+3mu]/sqrt([me^2+3md^2+3mu^2]+
+ [mmu^2+3ms^2+3mc^2]) = 0.222198

sin(theta13) = s13 = [me+3md+3mu]/sqrt([me^2+3md^2+3mu^2]+
+ [mtau^2+3mb^2+3mt^2]) = 0.004608

sin(*theta23 = [mmu+3ms+3mc]/sqrt([mtau^2+3mb^2+3mt^2]+
+ [mmu^2+3ms^2+3mc^2])

sin(theta23) = s23 = sin(*theta23) sqrt(Sigmaf2 / Sigmaf1) = 0.04234886

The factor sqrt(Smf2 /Smf1) appears in s23 because an s23 transition is to the second generation and not all the way to the first generation, so that the end product of an s23 transition has a greater available energy than s12 or s13 transitions by a factor of Smf2 / Smf1 .

Since the width of a transition is proportional to the square of the modulus of the relevant KM entry and the width of an s23 transition has greater available energy than the s12 or s13 transitions by a factor of Smf2 / Smf1
the effective magnitude of the s23 terms in the KM entries is increased by the factor sqrt(Smf2 /Smf1) .

The Chau-Keung parameterization is used, as it allows the K-M matrix to be represented as the product of the following three 3x3 matrices:

1	0	0
0	cos(theta23)	sin(theta23)
0	-sin(theta23)	cos(theta23)

cos(theta13)	0	sin(theta13)exp(-i d13)
0	1	0
-sin(theta13)exp(i d13)	0	cos(theta13)

cos(theta12)	sin(theta12)	0
-sin(theta12)	cos(theta12)	0
0	0	1

The resulting Kobayashi-Maskawa parameters for W+ and W- charged weak boson processes, are:

	d	s	b
u	0.975 0.222	0.00249	-0.00388i
c	-0.222 -0.000161i	0.974 -0.0000365i	0.0423
t	0.00698 -0.00378i	-0.0418 -0.00086i	0.999

The matrix is labelled by either (u c t) input and (d s b) output, or, as above, (d s b) input and (u c t) output.

For Z0 neutral weak boson processes, which are suppressed by the GIM mechanism of cancellation of virtual subprocesses, the matrix is labelled by either (u c t) input and (u'c't') output, or, as below, (d s b) input and (d's'b') output:

	d	s	b
d'	0.975 0.222	0.00249	-0.00388i
s'	-0.222 -0.000161i	0.974 -0.0000365i	0.0423
b'	0.00698 -0.00378i	-0.0418 -0.00086i	0.999

Since neutrinos of all three generations are massless at tree level, the lepton sector has no tree-level K-M mixing.

In hep-ph/0208080, Yosef Nir says: "... Within the Standard Model, the only source of CP violation is the Kobayashi-Maskawa (KM) phase ... The study of CP violation is, at last, experiment driven. ... The CKM matrix provides a consistent picture of all the measured flavor and CP violating processes. ...
There is no signal of new flavor physics. ...
Very likely, the KM mechanism is the dominant source of CP violation in flavor changing processes.
... The result is consistent with the SM predictions. ...".

Neutrino Masses and Mixing

Consider the three generations of neutrinos:
nu_e (electron neutrino); nu_m (muon neutrino); nu_t
and three neutrino mass states: nu_1 ; nu_2 : nu_3
and
the division of 8-dimensional spacetime into
4-dimensional physical Minkowski spacetime
plus
4-dimensional CP2 internal symmetry space.

The heaviest mass state nu_3 corresponds to a neutrino
whose propagation begins and ends in CP2 internal symmetry space,
lying entirely therein. According to the D4-D5-E6-E7-E8 VoDou
Physics Model the mass of nu_3 is zero at tree-level
but it picks up a first-order correction propagating
entirely through internal symmetry space by
merging with an electron through the weak and electromagnetic forces,
effectively acting not merely as a point
but
as a point plus an electron loop at both beginning and ending points
so
the first-order corrected mass of nu_3 is given by
M_nu_3 x (1/sqrt(2)) = M_e x GW(mproton^2) x alpha_E
where the factor (1/sqrt(2)) comes from the Ut3 component
of the neutrino mixing matrix
so that
M_nu_3 = sqrt(2) x M_e x GW(mproton^2) x alpha_E =
       = 1.4 x 5 x 10^5 x 1.05 x 10^(-5) x (1/137) eV =
       = 7.35 / 137 = 5.4 x 10^(-2) eV.
Note that the neutrino-plus-electron loop can be anchored
by weak force action through any of the 6 first-generation quarks
at each of the beginning and ending points, and that the
anchor quark at the beginning point can be different from
the anchor quark at the ending point,
so that there are 6x6 = 36 different possible anchorings.

The intermediate mass state nu_2 corresponds to a neutrino
whose propagation begins or ends in CP2 internal symmetry space
and ends or begins in physical Minkowski spacetime,
thus having only one point (either beginning or ending) lying
in CP2 internal symmetry space where it can act not merely
as a point but as a point plus an electron loop.
According to the D4-D5-E6-E7-E8 VoDou Physics Model the mass

of nu_2 is zero at tree-level
but it picks up a first-order correction at only one (but not both)
of the beginning or ending points
so that so that there are 6 different possible anchorings
for nu_2 first-order corrections, as opposed to the 36 different
possible anchorings for nu_3 first-order corrections,
so that
the first-order corrected mass of nu_2 is less than
the first-order corrected mass of nu_3 by a factor of 6,
so
the first-order corrected mass of nu_2 is
M_nu_2 = M_nu_3 / Vol(CP2) = 5.4 x 10^(-2) / 6
       = 9 x 10^(-3)eV.

The low mass state nu_1 corresponds to a neutrino
whose propagation begins and ends in physical Minkowski spacetime.
thus having only one anchoring to CP2 interna symmetry space.
According to E8 Physics the mass of nu_1 is zero at tree-level
but it has only 1 possible anchoring to CP2
as opposed to the 36 different possible anchorings for nu_3 first-order corrections
or the 6 different possible anchorings for nu_2 first-order corrections
so that
the first-order corrected mass of nu_1 is less than
the first-order corrected mass of nu_2 by a factor of 6,
so
the first-order corrected mass of nu_1 is
M_nu_1 = M_nu_2 / Vol(CP2) = 9 x 10^(-3) / 6
       = 1.5 x 10^(-3)eV.

Therefore:

the mass-squared difference D(M23^2) = M_nu_3^2 - M_nu_2^2 =
                                     = (2916 - 81) x 10^(-6) eV^2 =
                                     = 2.8 x 10^(-3) eV^2

and

the mass-squared difference D(M12^2) = M_nu_2^2 - M_nu_1^2 =
                                     = (81 - 2) x 10^(-6) eV^2 =
                                     = 7.9 x 10^(-5) eV^2

The 3 × 3 unitary neutrino mixing matrix neutrino mixing matrix U

	nu_1	nu_2	nu_3
nu_e	Ue1	Ue2	Ue3
nu_m	Um1	Um2	Um3
nu_t	Ut1	Ut2	Ut3

can be parameterized (based on the 2010 Particle Data Book)
by 3 angles and 1 Dirac CP violation phase

```
         c12 c13                        s12 c13                        s13 e-id

 U =    - s12 c23 - c12 s23 s13 eid     c12 c23 - s12 s23 s13 eid      s23 c13

         s12 s23 - c12 c23 s13 eid    - c12 s23 - s12 c23 s13 eid      c23 c13
```

where cij = cos(theta_ij) , sij = sin(theta_ij)

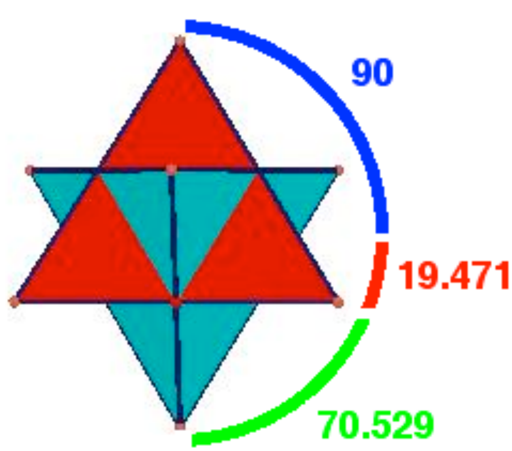
The angles are

theta_23 = pi/4 = 45 degrees
because
nu_3 has equal components of nu_m and nu_t so
that Um3 = Ut3 = 1/sqrt(2) or, in conventional
notation, mixing angle theta_23 = pi/4
so that cos(theta_23) = 0.707 = sqrt(2)/2 = sin(theta_23)

theta_13 = 9.594 degrees = asin(1/6)
and cos(theta_13) = 0.986
because sin(theta_13) = 1/6 = 0.167 = |Ue3| = fraction of nu_3 that is nu_e

theta_12 = pi/6 = 30 degrees
because
sin(theta_12) = 0.5 = 1/2 = Ue2 = fraction of nu_2 begin/end points
that are in the physical spacetime where massless nu_e lives
so that cos(theta_12) = 0.866 = sqrt(3)/2

d = 70.529 degrees is the Dirac CP violation phase
ei(70.529) = cos(70.529) + i sin(70.529) = 0.333 + 0.943 i
This is because the neutrino mixing matrix has 3-generation structure
and so has the same phase structure as the KM quark mixing matrix
in which the Unitarity Triangle angles are:
β = V3.V1.V4 = arccos(2 sqrt(2) / 3) ≅ 19.471 220 634 degrees so sin 2β = 0.6285
α = V1.V3.V4 = 90 degrees
γ = V1.V4.V3 = arcsin(2 sqrt(2) / 3) ≅ 70.528 779 366 degrees
The constructed Unitarity Triangle angles can be seen on the Stella Octangula
configuration of two dual tetrahedra (image from gauss.math.nthu.edu.tw):



Then we have for the neutrino mixing matrix:

	nu_1	nu_2	nu_3
nu_e	0.866 x 0.986	0.50 x 0.986	0.167 x e-id
nu_m	-0.5 x 0.707 -0.866 x 0.707 x 0.167 x eid	0.866 x 0.707 -0.5 x 0.707 x 0.167 x eid	0.707 x 0.986
nu_t	0.5 x 0.707 -0.866 x 0.707 x 0.167 x eid	-0.866 x 0.707 -0.5 x 0.707 x 0.167 x eid	0.707 x 0.986

	nu_1	nu_2	nu_3
nu_e	0.853	0.493	0.167 e-id
nu_m	-0.354 -0.102 eid	0.612 -0.059 eid	0.697
nu_t	0.354 -0.102 eid	-0.612 -0.059 eid	0.697

Since ei(70.529) = cos(70.529) + i sin(70.529) = 0.333 + 0.943 i
and .333e-i(70.529) = cos(70.529) - i sin(70.529) = 0.333 - 0.943 i

	nu_1	nu_2	nu_3
nu_e	0.853	0.493	0.056 - 0.157 i
nu_m	-0.354 -0.034 - 0.096 i	0.612 -0.020 - 0.056 i	0.697
nu_t	0.354 -0.034 - 0.096 i	-0.612 -0.020 - 0.056 i	0.697

for a result of

	nu_1	nu_2	nu_3
nu_e	0.853	0.493	0.056 - 0.157 i
nu_m	-0.388 - 0.096 i	0.592 - 0.056 i	0.697
nu_t	0.320 - 0.096 i	0.632 - 0.056 i	0.697

which is consistent with the approximate experimental values of mixing angles shown in the Michaelmas Term 2010 Particle Physics handout of Prof Mark Thomson if the matrix is modified by taking into account the March 2012 results from Daya Bay observing non-zero theta_13 = 9.54 degrees.

Proton-Neutron Mass Difference:

According to the 1986 CODATA Bulletin No. 63,
the experimental value of the neutron mass is 939.56563(28) Mev,
and the experimental value of the proton is 938.27231(28) Mev.

The neutron-proton mass difference 1.3 Mev is due to the fact that
the proton consists of two up quarks and one down quark,
while the neutron consists of one up quark and two down quarks.

The magnitude of the electromagnetic energy difference mN - mP is about 1 Mev, but the sign is wrong: mN - mP = -1 Mev, and the proton's electromagnetic mass is greater than the neutron's.

The difference in energy between the bound states, neutron and proton, is not due to a difference between the Pre-Quantum constituent masses of the up quark and the down quark, which are calculated in the E8 model to be equal.

It is due to the difference between the Quantum color force interactions of the up and down constituent valence quarks with the gluons and virtual sea quarks in the neutron and the proton.

An up valence quark, constituent mass 313 Mev, does not often swap places with a 2.09 Gev charm sea quark, but a 313 Mev down valence quark can more often swap places with a 625 Mev strange sea quark.

Therefore the Quantum color force constituent mass of the down valence quark is heavier by about
(ms - md) (md/ms)^2 a(w) |Vds| = 312 x 0.25 x 0.253 x 0.22 Mev = 4.3 Mev,

(where a(w) = 0.253 is the geometric part of the weak force strength and |Vds| = 0.22 is the magnitude of the K-M parameter mixing first generation down and second generation strange)
so that the Quantum color force constituent mass Qmd of the down quark is

Qmd = 312.75 + 4.3 = 317.05 MeV.

Similarly, the up quark Quantum color force mass increase is about
(mc - mu) (mu/mc)^2 a(w) |V(uc)| = 1777 x 0.022 x 0.253 x 0.22 Mev = 2.2 Mev,

(where |Vuc| = 0.22 is the magnitude of the K-M parameter mixing first generation up and second generation charm)
so that the Quantum color force constituent mass Qmu of the up quark is
Qmu = 312.75 + 2.2 = 314.95 MeV.

Therefore, the Quantum color force Neutron-Proton mass difference is
mN - mP = Qmd - Qmu = 317.05 Mev - 314.95 Mev = 2.1 Mev.
Since the electromagnetic Neutron-Proton mass difference is roughly
mN - mP = -1 MeV
the total theoretical Neutron-Proton mass difference is
mN - mP = 2.1 Mev - 1 Mev = 1.1 Mev,
an estimate that is fairly close to the experimental value of 1.3 Mev.

Note that in the equation (ms - md) (md/ms)^2 a(w) |Vds| = 4.3 Mev ,
Vds is a mixing of down and strange by a neutral Z0,
compared to the more conventional Vus mixing by charged W.
Although real neutral Z0 processes are suppressed by the GIM mechanism,
which is a cancellation of virtual processes,
the process of the equation is strictly a virtual process.

Note also that the K-M mixing parameter |Vds| is linear.
Mixing (such as between a down quark and a strange quark) is a two-step process, that goes approximately as the square of |Vds|:
First the down quark changes to a virtual strange quark,
producing one factor of |Vds|.
Then, second, the virtual strange quark changes back to a down quark,
producing a second factor of |Vsd|, which is approximately equal to |Vds|.

Only the first step (one factor of |Vds|) appears in the Quantum mass formula used to determine the neutron mass.

Measurement of a neutron mass includes a sum over histories of the valence quarks inside the neutron in some of which you will "see" some of the two valence down quarks in a virtual transition state or change from down to strange before the second action, or change back. Therefore, you should take into account those histories in the sum in which you see a strange valence quark, and you get the linear factor |Vds| in the above equation.

Pion Mass:

The quark content of a charged pion is a quark - antiquark pair: either Up plus antiDown or Down plus antiUp. Experimentally, its mass is about 139.57 MeV.

The quark is a Naked Singularity Kerr-Newman Black Hole, with electromagnetic charge e and spin angular momentum J and constituent mass M 312 MeV, such that e^2 + a^2 is greater than M^2 (where a = J / M).

The antiquark is a also Naked Singularity Kerr-Newman Black Hole, with electromagnetic charge e and spin angular momentum J and constituent mass M 312 MeV, such that e^2 + a^2 is greater than M^2 (where a = J / M).

According to General Relativity, by Robert M. Wald (Chicago 1984) page 338 [Problems] ... 4. ...:

> "... Suppose two widely separated Kerr black holes with parameters (M1 , J1) and (M2 , J2) initially are at rest in an axisymmetric configuration, i.e., their rotation axes are aligned along the direction of their separation.
>
> Assume that these black holes fall together and coalesce into a single black hole.
>
> Since angular momentum cannot be radiated away in an axisymmetric spacetime, the final black hole will have momentum J = J1 + J2. ...".

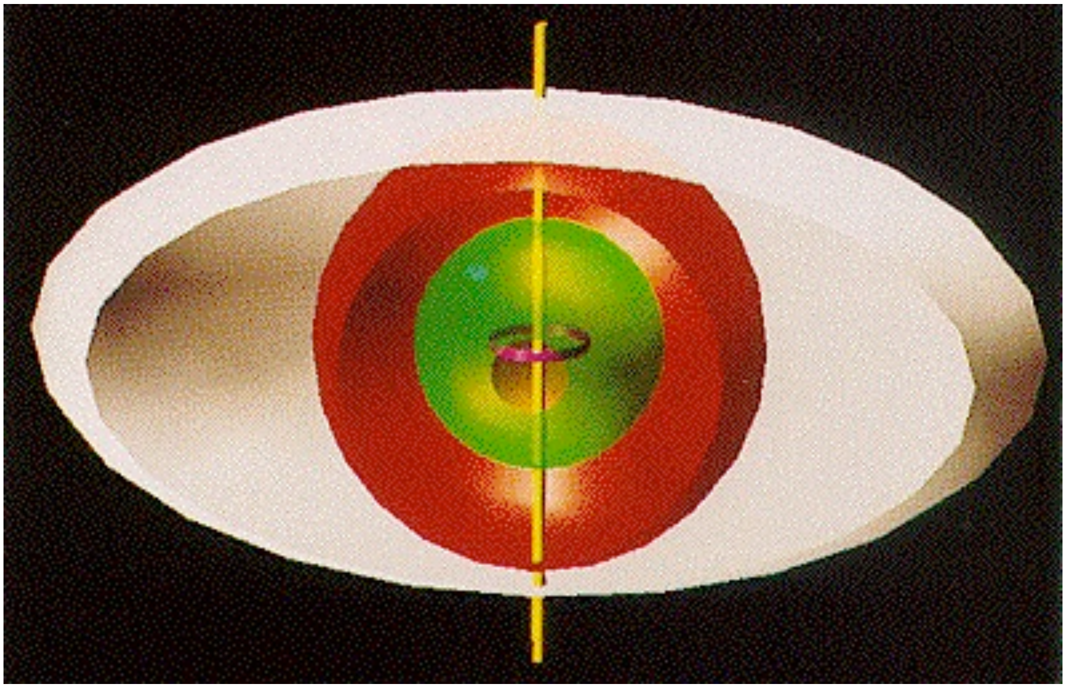
The neutral pion produced by the quark - antiquark pair would have zero angular momentum, thus reducing the value of e^2 + a^2 to e^2 .

For fermion electrons with spin 1/2, 1 / 2 = e / M (see for example Misner, Thorne, and Wheeler, Gravitation (Freeman 1972), page 883) so that M^2 = 4 e^2 is greater than e^2 for the electron. In other words, the angular momentum term a^2 is necessary to make e^2 + a^2 greater than M^2 so that the electron can be seen as a Kerr-Newman naked singularity.

Since the magnitude of electromagnetic charge of each quarks or antiquarks less than that of an electron, and since the mass of each quark or antiquark (as well as the pion mass) is greater than that of an electron, and since the quark - antiquark pair (as well as the pion) has angular momentum zero, the quark - antiquark pion has M^2 greater than e^2 + a^2 = e^2.

(Note that color charge, which is nonzero for the quark and the antiquark and is involved in the relation M^2 less than sum of spin-squared and charges-squared by which quarks and antiquarks can be see as Kerr-Newman naked singularities, is not relevant for the color-neutral pion.)

Therefore, the pion itself is a normal Kerr-Newman Black Hole with Outer Event Horizon = Ergosphere at r = 2M (the Inner Event Horizon is only the origin at r = 0) as shown in this image

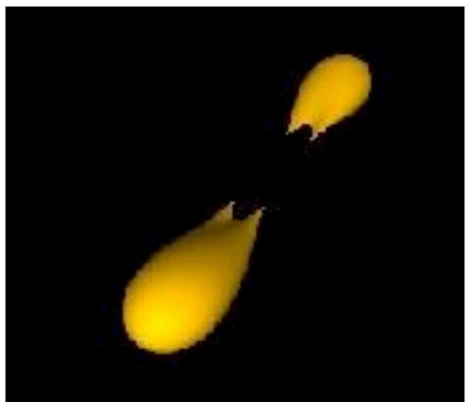


from Black Holes - A Traveller's Guide, by Clifford Pickover (Wiley 1996) in which the Ergosphere is white, the Outer Event Horizon is red, the Inner Event Horizon is green, and the Ring Singularity is purple. In the case of the pion, the white and red surfaces coincide, and the green surface is only a point at the origin.

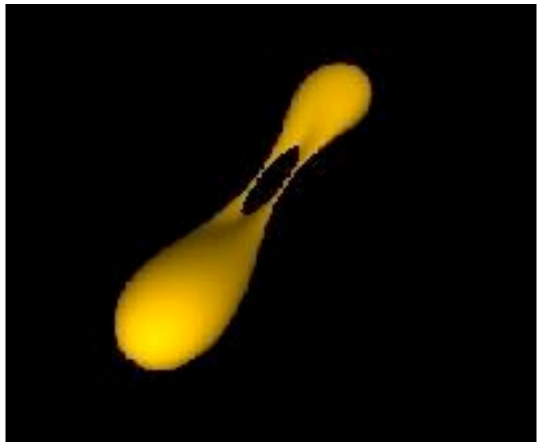
According to section 3.6 of Jeffrey Winicour's 2001 Living Review of the Development of Numerical Evolution Codes for General Relativity (see also a 2005 update):

> "... The black hole event horizon associated with ... slightly broken ... degeneracy [of the axisymmetric configuration]... reveals new features not seen in the degenerate case of the head-on collision ... If the degeneracy is slightly broken, the individual black holes form with spherical topology but as they approach, tidal distortion produces two sharp pincers on each black hole just prior to merger.
>
> ... Tidal distortion of approaching black holes ...

... Formation of sharp pincers just prior to merger ..



... toroidal stage just after merger ...



At merger, the two pincers join to form a single ... toroidal black hole.

The inner hole of the torus subsequently [begins to] close... up (superluminally) ... [If the closing proceeds to completion, it]... produce[s] first a peanut shaped black hole and finally a spherical black hole. ...".

In the physical case of quark and antiquark forming a pion, the toroidal black hole remains a torus. The torus is an event horizon and therefore is not a 2-spacelike dimensional torus, but is a (1+1)-dimensional torus with a timelike dimension.

The effect is described in detail in Robert Wald's book General Relativity (Chicago 1984). It can be said to be due to extreme frame dragging, or to timelike translations becoming spacelike as though they had been Wick rotated in Complex SpaceTime.

As Hawking and Ellis say in The LargeScale Structure of Space-Time (Cambridge 1973):

"... The surface r = r+ is ... the event horizon ... and is a null surface ...

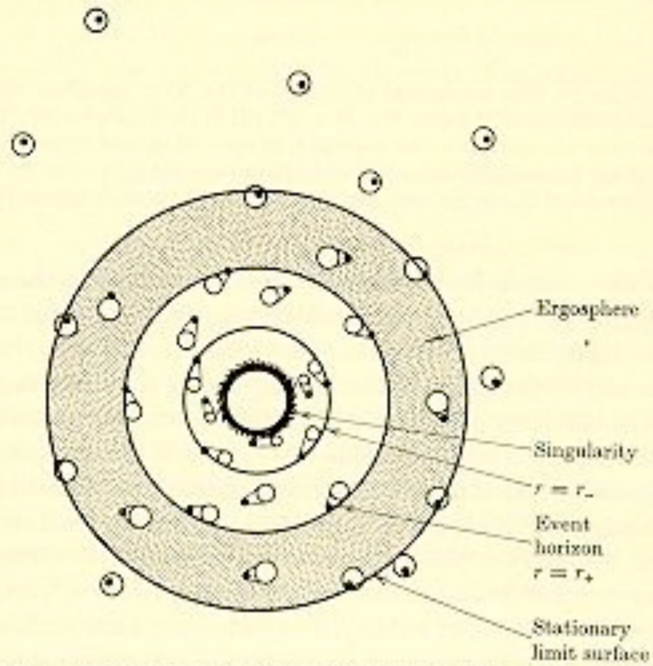


FIGURE 30. The equatorial plane of a Kerr solution with $m^2 > a^2$. The circles represent the position a short time later of flashes of light emitted by the points represented by heavy dots.

> ... On the surface r = r+ .... the wavefront corresponding to a point on this surface lies entirely within the surface. ...".

A (1+1)-dimensional torus with a timelike dimension can carry a Sine-Gordon Breather, and the soliton and antisoliton of a Sine-Gordon Breather correspond to the quark and antiquark that make up the pion.

Sine-Gordon Breathers are described by Sidney Coleman in his Erica lecture paper Classical Lumps and their Quantum Descendants (1975), reprinted in his book Aspects of Symmetry (Cambridge 1985), where Coleman writes the Lagrangian for the Sine-Gordon equation as (Coleman's eq. 4.3):

L = (1 / B^2) ((1/2) (df)^2 + A (cos(f) - 1))

and Coleman says:

> "... We see that, in classical physics, B is an irrelevant parameter: if we can solve the sine-Gordon equation for any non-zero B, we can solve it for any other B. The only effect of changing B is the trivial one of changing the energy and momentum assigned to a given soluition of the equation. This is not true in quantum physics, becasue the relevant object for quantum physics is not L but [eq. 4.4]

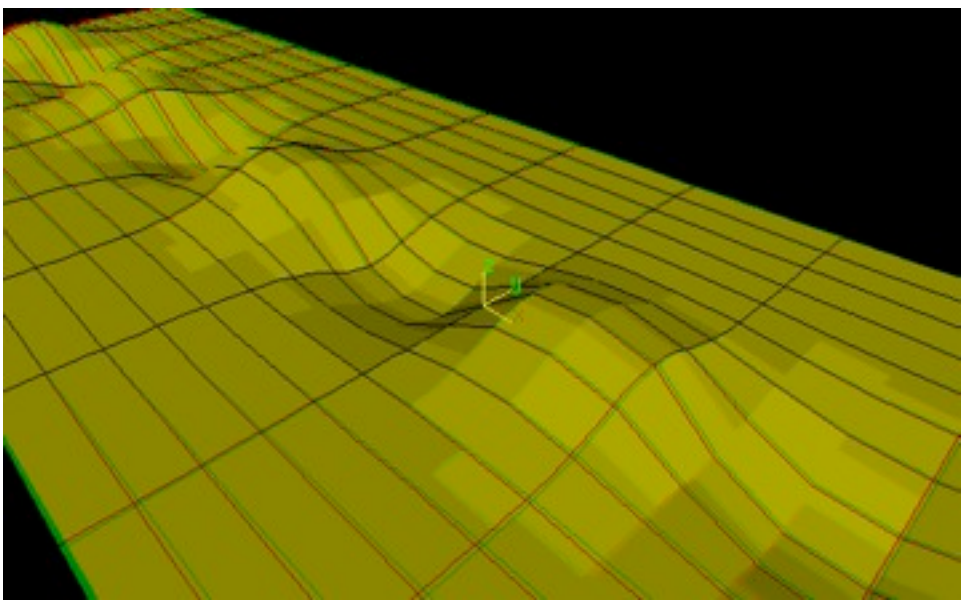
L / hbar = (1 / (B^2 hbar)) ((1/2) (df)^2 + A (cos(f) - 1))

An other way of saying the same thing is to say that in quantum physics we have one more dimensional constant of nature, Planck's constant, than in classical physics. ... the classical limit, vanishingf hbar, is exactly the same as the small-coupling limit, vanishing B ... from now on I will ... set hbar equal to one. ...

... the sine-Gordon equation ...[has]... an exact periodic solution ... [eq. 4.59]...

f(x, t) = (4 / B) arctan((n sin(w t) / cosh(n w x))

where [eq. 4.60] n = sqrt(A - w^2) / w and w ranges from 0 to A. This solution has a simple physical interpretation ... a soliton far to the left ...[and]... an antisoliton far to the right. As sin(w t) increases, the soliton and antisoliton mover farther apart from each other. When sin(w t) passes thrpough one, they turn around and begin to approach one another. As sin(w t) comes down to zero ... the soliton and antisoliton are on top of each other ... when sin(w t) becomes negative .. the soliton and antisoliton have passed each other. ...[



This stereo image of a Sine-Gordon Breather was generated by the program 3D-Filmstrip for Macintosh by Richard Palais. You can see the stereo with red-green or red-cyan 3D glasses. The program is on the WWW at http://rsp.math.brandeis.edu/3D-Filmstrip. The Sine-Gordon Breather is confined in space (y-axis) but periodic in time (x-axis), and therefore naturally lives on the (1+1)-dimensional torus with a timelike dimension of the Event Horizon of the pion. ...]

... Thus, Eq. (4.59) can be thought of as a soliton and an antisoliton oscillation about their common center-of-mass. For this reason, it is called 'the doublet [or Breather] solution'. ... the energy of the doublet ...[eq. 4.64]

E = 2 M sqrt(1 - (w^2 / A))

where [eq. 4.65] M = 8 sqrt(A) / B^2 is the soliton mass. Note that the mass of the doublet is always less than twice the soliton mass, as we would expect from a soltion-antisoliton pair. ... Dashen, Hasslacher, and Neveu ... Phys. Rev. D10, 4114; 4130; 4138 (1974). A pedagogical review of these methods has been written by R. Rajaraman (Phys. Reports 21, 227 (1975 ... Phys. Rev. D11, 3424 (1975) ...[Dashen, Hasslacher, and Neveu found that]... there is only a single series of bound states, labeled by the integer N ... The energies ... are ... [eq. 4.82]

E_N = 2 M sin(B'^2 N / 16)

where N = 0, 1, 2 ... < 8 pi / B'^2 , [eq. 4.83]

B'^2 = B^2 / (1 - (B^2 / 8 pi))

and M is the soliton mass. M is not given by Eq. (4.675), but is the soliton mass corrected by the DHN formula, or, equivalently, by the first-order weak coupling expansion. ... I have written the equation in this form .. to eliminate A, and thus avoid worries about renormalization conventions. Note that the DHN formula is identical to the Bohr-Sommerfeld formula, except that B is replaced by B'. ... Bohr and Sommerfeld['s] ... quantization formula says that if we have a one-parameter family of periodic motions, labeled by the period, T, then an energy eigenstate occurs whenever [eq. 4.66]

[Integral from 0 to T](dt p qdot = 2 pi N,

where N is an integer. ... Eq.(4.66) is cruder than the WKB formula, but it is much more general; it is always the leading approximation for any dynamical system ... Dashen et al speculate that Eq. (4.82) is exact. ...

the sine-Gordon equation is equivalent ... to the massive Thirring model. This is surprising, because the massive Thirring model is a canonical field theory whose Hamiltonian is expressedin terms of fundamental Fermi fields only. Even more surprising, when B^2 = 4 pi , that sine-Gordon equation is equivalent to a free massive Dirac theory, in one spatial dimension. ... Furthermore, we can identify the

mass term in the Thirring model with the sine-Gordon interaction,
[eq. 5.13]

$$M = - (A / B^2) N_m \cos(B f)$$

.. to do this consistently ... we must say [eq. 5.14]

$$B^2 / (4 \pi) = 1 / (1 + g / \pi)$$

....[where]... g is a free parameter, the coupling constant [for the Thirring model]... Note that if $B^2 = 4 \pi$, $g = 0$, and the sine-Gordon equation is the theory of a free massive Dirac field. ... It is a bit surprising to see a fermion appearing as a coherent state of a Bose field. Certainly this could not happen in three dimensions, where it would be forbidden by the spin-statistics theorem. However, there is no spin-statistics theorem in one dimension, for the excellent reason that there is no spin. ... the lowest fermion-antifermion bound state of the massive Thirring model is an obvious candidate for the fundamental meson of sine-Gordon theory. ... equation (4.82) predicts that all the doublet bound states disappear when B^2 exceeds 4 pi . This is precisely the point where the Thirring model interaction switches from attractive to repulsive. ... these two theories ... the massive Thirring model .. and ... the sine-Gordon equation ... define identical physics. ... I have computed the predictions of ...[various]... approximation methods for the ration of the soliton mass to the meson mass for three values of B^2 : 4 pi (where the qualitative picture of the soliton as a lump totally breaks down), 2 pi, and pi . At 4 pi we know the exact answer

... I happen to know the exact answer for 2 pi, so I have included this in the table. ...

Method	B^2 = pi	B^2 = 2 pi	B^2 = 4 pi
Zeroth-order weak coupling expansion eq2.13b	2.55	1.27	0.64
Coherent-state variation	2.55	1.27	0.64
First-order weak coupling expansion	2.23	0.95	0.32
Bohr-Sommerfeld eq4.64	2.56	1.31	0.71
DHN formula eq4.82	2.25	1.00	0.50
Exact	?	1.00	0.50

...[eq. 2.13b] E = 8 sqrt(A) / B^2 ...[is the]... energy of the lump ... of sine-Gordon theory ... frequently called 'soliton...' in the literature ... [Zeroth-order is the classical case, or classical limit.] ...

... Coherent-state variation always gives the same result as the ... Zeroth-order weak coupling expansion ... .

The ... First-order weak-coupling expansion ... explicit formula ... is (8 / B^2) - (1 / pi). ...".

Note that, using the VoDou Physics constituent mass of the Up and Down quarks and antiquarks, about 312.75 MeV, as the soliton and antisoliton masses, and setting B^2 = pi and using the DHN formula, the mass of the charged pion is calculated to be

(312.75 / 2.25) MeV = 139 MeV

which is in pretty good agreement with the experimental value of about 139.57 MeV.

Why is the value B^2 = pi (or, using Coleman's eq. (5.14), the Thirring coupling constant g = 3 pi) the special value that gives the pion mass ?

Because B^2 = pi is where the First-order weak coupling expansion substantially coincides with the (probably exact) DHN formula.

In other words, the physical quark - antiquark pion lives where the first-order weak coupling expansion is exact.

Near the end of his article, Coleman expressed "Some opinions":

> "... This has been a long series of physics lectures with no reference whatsoever to experiment. This is embarrassing.
>
> ... Is there any chance that the lump will be more than a theoretical toy in our field? I can think of two possiblities.
>
> One is that there will appear a theory of strong-interaction dynamics in which hadrons are thought of as lumps, or, ... as systems of quarks bound into lumps. ... I am pessimistic about the success of such a theory. ... However, I stand ready to be converted in a moment by a convincing computation.
>
> The other possibility is that a lump will appear in a realistic theory ... of weak and electromagnetic interactions ... the theory would have to imbed the U(1)xSU(2) group ... in a larger group without U(1) factors ... it would be a magnetic monopole. ...".

This description of the hadronic pion as a quark - antiquark system governed by the sine-Gordon - massive Thirring model should dispel Coleman's pessimism about his first stated possibility and relieve his embarrassment about lack of contact with experiment.

As to his second stated possibility, very massive monopoles related to SU(5) GUT are still within the realm of possible future experimental discoveries.

Further material about the sine-Gordon doublet Breather and the massive Thirring equation can be found in the book Solitons and Instantons (North-Holland 1982,1987) by R. Rajaraman, who writes:

> "... the doublet or breather solutions ... can be used as input into the WKB method. ... the system is ... equivalent to the massive Thirring model, with the SG soliton state identifiable as a fermion. ... Mass of the quantum soliton ... will consist of a classical term followed by quantum corrections. The energy of the classical soliton ... is ... [eq. 7.3]
>
> E_cl[f_sol] = 8 m^3 / L
>
> The quantum corrections ... to the 'soliton mass' ... is finite as the momentum cut-off goes to infinity and equals (- m / pi). Hence the quantum soliton's mass is [eq. 7.10]
>
> M_sol =(8 m^3 / L) - (m / pi) +O(L).

The mass of the quantum antisoliton will be, by ... symmetry, the same as M_sol. ...

The doublet solutions ... may be quantised by the WKB method. ... we see that the coupling constant (L / m^2) has been replaced by a 'renormalised' coupling constant G ... [eq. 7.24]

G = (L / m^2) / (1 - (L / 8 pi m^2))

... as a result of quantum corrections. ... the same thing had happened to the soliton mass in eq. (7.10). To leading order, we can write [eq. 7.25]

M_sol = (8 m^3 / L) - (m / pi) = 8 m / G

... The doublet masses ... bound-state energy levels ... E = M_N, where ... [eq. 7.28]

M_N = (16 m / G) sin(N G / 16) ; N = 1, 2, ... < 8 pi / G

Formally, the quantisation condition permits all integers N from 1 to oo , but we run out of classical doublet solutions on which these bound states are based when N > 8 pi / G . ... The classical solutions ... bear the same relation to the bound-state wavefunctionals ... that Bohr orbits bear to hydrogen atom wavefunctions. ...

Coleman ... show[ed] explicitly ... the SG theory equivalent to the charge-zero sector of the MT model, provided ... L / 4 pi m^2 = 1 / (1 + g / pi)

...[where in Coleman's work set out above such as his eq. (5.14) , B^2 = L / m^2]...

Coleman ... resurrected Skyrme's conjecture that the quantum soliton of the SG model may be identified with the fermion of the MT model. ... ".

What about the Neutral Pion?

The quark content of the charged pion is u_d or d_u , both of which are consistent with the sine-Gordon picture. Experimentally, its mass is 139.57 Mev.

The neutral pion has quark content (u_u + d_d)/sqrt(2) with two components, somewhat different from the sine-Gordon picture, and a mass of 134.96 Mev.

The effective constituent mass of a down valence quark increases (by swapping places with a strange sea quark) by about

DcMdquark = (Ms - Md) (Md/Ms)2 aw V12 =

= 312x0.25x0.253x0.22 Mev = 4.3 Mev.

Similarly, the up quark color force mass increase is about

DcMuquark = (Mc - Mu) (Mu/Mc)2 aw V12 =

= 1777x0.022x0.253x0.22 Mev = 2.2 Mev.

The color force increase for the charged pion DcMpion± = 6.5 Mev.

Since the mass Mpion± = 139.57 Mev is calculated from a color force sine-Gordon soliton state, the mass 139.57 Mev already takes DcMpion± into account.

For pion0 = (u_u + d_d)/ sqrt 2 , the d and _d of the the d_d pair do not swap places with strange sea quarks very often because it is energetically preferential for them both to become a u_u pair.

Therefore, from the point of view of calculating DcMpion0, the pion0 should be considered to be only u_u , and DcMpion0 = 2.2+2.2 = 4.4 Mev.

If, as in the nucleon, DeM(pion0-pion±) = -1 Mev, the theoretical estimate is

DM(pion0-pion±) = DcM(pion0-pion±) + DeM(pion0-pion±) =

= 4.4 - 6.5 -1 = -3.1 Mev,

roughly consistent with the experimental value of -4.6 Mev.

Planck Mass:

In the E8 model, a Planck-mass black hole is not a tree-level classical particle such as an electron or a quark, but a quantum entity resulting from the Many-Worlds quantum sum over histories at a single point in spacetime.

Consider an isolated single point, or vertex in the lattice picture of spacetime. In the E8 model, fermions live on vertices, and only first-generation fermions can live on a single vertex. (The second-generation fermions live on two vertices that act at our energy levels very much like one, and the third-generation fermions live on three vertices that act at our energy levels very much like one.)

At a single spacetime vertex, a Planck-mass black hole is the Many-Worlds quantum sum of all possible virtual first-generation particle-antiparticle fermion pairs permitted by the Pauli exclusion principle to live on that vertex.
Once a Planck-mass black hole is formed, it is stable in the E8 model. Less mass would not be gravitationally bound at the vertex. More mass at the vertex would decay by Hawking radiation.

In the E8 model, a Planck-mass black hole can be formed:
as the end product of Hawking radiation decay of a larger black hole;
by vacuum fluctuation;
or perhaps by using a pion laser.

Since Dirac fermions in 4-dimensional spacetime can be massive
(and are massive at low enough energies for the Higgs mechanism to act),
the Planck mass in 4-dimensional spacetime is the sum of masses
of all possible virtual first-generation particle-antiparticle fermion pairs
permitted by the Pauli exclusion principle.

There are 8 fermion particles and 8 fermion antiparticles
for a total of 64 particle-antiparticle pairs.

A typical combination should have several quarks, several antiquarks,
a few colorless quark-antiquark pairs that would be equivalent to pions,
and some leptons and antileptons.
Due to the Pauli exclusion principle, no fermion lepton or quark could be present at the vertex more than twice unless they are in the form of boson pions, colorless first-generation quark-antiquark pairs not subject to the Pauli exclusion principle.
Of the 64 particle-antiparticle pairs, 12 are pions.

A typical combination should have about 6 pions.

If all the pions are independent,
the typical combination should have a mass of about .14x6 GeV = 0.84 GeV.

However, just as the pion mass of .14 GeV is less than
the sum of the masses of a quark and an antiquark,
pairs of oppositely charged pions may form a bound state of less mass
than the sum of two pion masses.

If such a bound state of oppositely charged pions has a mass as small as .1 GeV,
and
if the typical combination has one such pair and 4 other pions, then the typical combination could have a mass in the range of 0.66 GeV.

Summing over all 2^64 combinations,
the total mass of a one-vertex universe should give a Planck mass roughly around 0.66 x 2^64 = 1.217 x 10^19 GeV.

Since each fermion particle has a corresponding antiparticle,
a Planck-mass Black Hole is neutral with respect to electric and color charges.

The value for the Planck mass given in the Particle Data Group's 1998 review is 1.221 x 10^19 GeV.

How did our Universe evolve in its first 10^(-11) seconds ?

Our Universe began as a Quantum Fluctuation from a Parent Universe whereby
our Universe initially had Planck Scale Temperature / Energy
10^32 K = 1.22 x 10^19 GeV.
Its physics was then described by a Lagrangian with:

Gauge Boson term of 28-dimensional adjoint Spin(8)
that eventually produces 16-dim U(2,2) Conformal Gravity/Higgs
and the 12-dim SU(3)xSU(2)xU(1) Standard Model;

Fermion term of 8-dimensional half-spinor Spin(8)
corresponding to first-generation fermion particles and antipartices
(electron, RGB Up quarks; neutrino, RGB down quarks);

Base Manifold of 8-dimensional Octonionic Spacetime.

With respect to 8-dimensional Spacetime
the dimensionality of the Gauge Boson term is 28 x 1 = 28
and
the dimensionality of the Fermion term is 8 x 7/2 = 28
(see Weinberg's 1986 Dirac Memorial Lecture at page 88
and note that 7/2 + 7/2 + 1 = 8)
so
the E8 Physics Lagranigian is clearly Ultraviolet Finite at the Planck Scale
due to Triality-based cancellations, an effective Subtle Supersymmetry.
Since the lower energy forms of E8 Physics are derived from
the Planck Scale Lagrangian, they also benefit from the cancellations.

As Our Universe began to cool down below the Planck Scale
Inflationary Expansion started due to Octonionic Quantum Non-Unitarity
(see Adler's book "Quaternionic Quantum Mechanics ..." at pages 50-52 and 561).
Paola Zizzi describes the Octonionic Inflationary Era in terms of Clifford Algebras in gr-qc/0007006 and related papers. In short, the 64 doublings of Zizzi Inflation produce about 10^77 fermion particles.

NonUnitary Octonionic Inflation:

In his book Quaternionic Quantum Mechanics and Quantum Fields ((Oxford 1995), Stephen L. Adler says at pages 50-52, 561:

> "... If the multiplication is associative, as in the complex and quaternionic cases, we can remove parentheses in ... Schroedinger equation dynamics ... to conclude that ... the inner product < f(t) | g(t) > ... is invariant ... this proof fails in the octonionic case, and hence one cannot follow the standard procedure to get a unitary dynamics. ...[so there is a]...
> **failure of unitarity in octonionic quantum mechanics**...".

The non-associativity and non-unitarity of octonions might account for particle creation without the need for tapping the energy of an inflaton field.
The non-associative structure of octonions manifests itself in interesting ways:

The 7-sphere S7   EXPANDS TO   S7 x G2 x S7 = D4 Lie Algebra.

The 480 Octonion multiplications double-cover the 240 Root Vectors of E8.

There are 7 independent E8 lattices, each corresponding to an integral domain, differing in the configuration of the 240 E8 Root Vectors that are the innermost shell surrounding the origin of the lattice at unit distance (also sometimes normalized as 2) from the origin. Here is a list of them with points on the line with iE8, jE8 notation being common points with the iE8 and jE8 lattices):

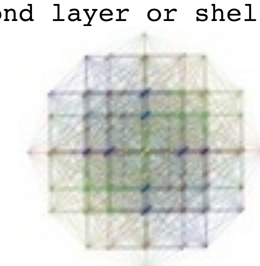
```
1E8:   ±1,  ±i,  ±j,  ±k,  ±e,  ±ie,  ±je,  ±ke,
    (±1 ±je ±i  ±j)/2                 (±k ±e ±ie ±ke)/2
    (±1 ±j  ±ie  ±ke)/2    5E8, 6E8    (±i ±k ±e ±je)/2
    (±1 ±ke ±k  ±i)/2                 (±j ±e ±ie ±je)/2
    (±1 ±i  ±e  ±ie)/2     7E8, 3E8    (±j ±k ±je ±ke)/2
    (±1 ±ie ±je ±k)/2      2E8, 4E8    (±i ±j ±e ±ke)/2
    (±1 ±k  ±j  ±e)/2                 (±i ±ie ±je ±ke)/2
    (±1 ±e  ±ke ±je)/2                (±i ±j ±k ±ie)/2
```

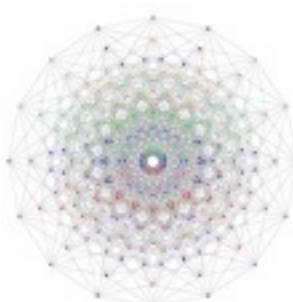
```
2E8:  ±1,  ±i,  ±j,  ±k,  ±e,  ±ie,  ±je,  ±ke,
     (±1  ±i  ±k  ±e)/2                (±j ±ie ±je ±ke)/2
     (±1 ±e  ±je ±j)/2    7E8, 6E8     (±i ±k ±ie ±ke)/2
     (±1 ±j  ±ke ±k)/2                 (±i ±e ±ie ±je)/2
     (±1 ±k  ±ie ±je)/2   1E8, 4E8     (±i ±j ±e ±ie)/2
     (±1 ±je  ±i  ±ke)/2  3E8, 5E8     (±j ±k ±e ±ie)/2
     (±1 ±ke  ±e  ±ie)/2               (±i ±j ±k ±je)/2
     (±1  ±ie  ±j  ±i)/2               (±k ±e ±je ±ke)/2

3E8:   ±1,  ±i,  ±j,  ±k,  ±e,  ±ie,  ±je,  ±ke,
     (±1  ±k  ±ke ±ie)/2               (±i ±j ±e ±je)/2
     (±1 ±ie  ±i  ±e)/2    E8, 1E8     (±j ±k ±je ±ke)/2
     (±1 ±e  ±j  ±ke)/2                (±i ±k ±ie ±je)/2
     (±1 ±ke ±je ±i)/2     2E8, 5E8    (±j ±k ±e ±ie)/2
     (±1 ±i  ±k   ±j)/2    4E8, 6E8   (±e ±ie ±je ±ke)/2
     (±1 ±j  ±ie ±je)/2                (±i ±k ±e ±ke)/2
     (±1 ±je  ±e ±k)/2                 (±i ±j ±ie ±ke)/2

 4E8:   ±1,  ±i,  ±j,  ±k,  ±e,  ±ie,  ±je,  ±ke,
     (±1  ±ke ±j  ±je)/2                (± i±k ±e ±ie)/2
     (±1 ±je  ±k  ±ie)/2    1E8, 2E8   (±i ±j ±e ±ke)/2
     (±1 ±ie ±e  ±j)/2                  (±i ±k ±je ±ke)/2
     (±1 ±j   ±i ±k)/2      3E8, 6E8   (±e ±ie ±je ±ke)/2
     (±1 ±k  ±ke  ±e)/2     7E8, 5E8   (±i ±j ±ie ±je)/2
     (±1  ±e  ±je  ±i)/2                (±j ±k ±ie ±ke)/2
     (±1 ±i  ±ie ±ke)/2                 (±j ±k ±e ±je)/2

 5E8:  ±1,  ±i,  ±j,  ±k,  ±e,  ±ie,  ±je,  ±ke,
     (±1  ±j  ±e  ±i)/2                (±k ±ie ±je±ke)/2
     (±1  ±i  ±ke ±je)/2   2E8, 3E8    (±j ±k ±e ±ie)/2
     (±1 ±je ±ie ±e)/2                 (±i ±j ±k ±ke)/2
     (±1 ±e  ±k  ±ke)/2    7E8, 4E8    (± i± j± ie ±je)/2
     (±1  ±ke  ±j  ±ie)/2  1E8, 6E8   (±i ±k ±e ±je)/2
     (±1 ±ie ±i  ±k)/2                 (±j ±e ±je ±ke)/2
     (±1 ±k  je  ±j)/2                 (±i ±e ±ie ±ke)/2

6E8:   ±1,  ±i,  ±j,  ±k,  ±e,  ±ie,  ±je,  ±ke,
     (±1  ±e  ±ie ±k)/2                (±i ±j ±je ±ke)/2
     (±1  ±k  ±j  ±i)/2    3E8, 4E8   (±e ±ie ±je ±ke)/2
     (±1  ±i ±je ±ie)/2                (±j ±k ±e ±ke)/2
     (±1  ±ie ±ke ±j)/2    5E8, 1E8   (±i ±k ±e ±je)/2
     (±1  ±j  ±e  ±je)/2   7E8, 2E8   (±i ±k ±ie± ke)/2
     (±1  ±je ±k  ±ke)/2               (±i ±j ±e ±ie)/2
     (±1 ±ke  ±i   ±e)/2               (±j ±k ±ie ±je)/2

 7E8:  ±1,  ±i,  ±j,  ±k,  ±e,  ±ie,  ±je,  ±ke,
     (±1 ±ie ±je ±ke)/2                (±e  ±i  ±j   ±k)/2
     (±1 ±ke ±e  ±k)/2     5E8, 4E8     (±i  ±j  ±ie  ±je)/2
     (±1 ±k  ±i  ±je)/2                (±j  ±ie ±ke ±e)/2
     (±1 ±je ±j  ±e)/2     6E8, 2E8   (±ie ±ke  ±k  ±i)/2
     (±1 ±e  ±ie ±i)/2     3E8, 1E8   (±ke ±k  ±je ±j)/2
     (±1 ±i  ±ke ±j)/2                 (±k  ±je ±e  ±ie)/2
     (±1 ±j  ±k  ±ie)/2                (±je  ±e  ±i  ±ke)/2
```

The vertices that appear in more than one lattice are:

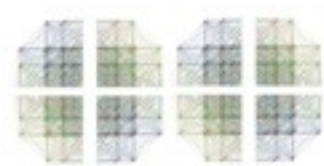
±1, ±i, ±j, ±k, ±e, ±ie, ±je, ±ke	in	all of them;
(±1±i±j±k)/2 and (±e±ie±je±ke)/2	in	3E8, 4E8, and 6E8 ;
(±1±i±e±ie)/2 and (±j±k±je±ke)/2	in	7E8, 1E8, and 3E8 ;
(±1±j±e±je)/2 and (±i±k±ie±ke)/2	in	7E8, 2E8, and 6E8 ;
(±1±k±e±ke)/2 and (±i±j±ie±je)/2	in	7E8, 4E8, and 5E8 ;
(±1±i±je±ke)/2 and (±j±k±e±ie)/2	in	2E8, 3E8, and 5E8 ;
(±1±j±ie±ke)/2 and (±i±k±e±je)/2	in	1E8, 5E8, and 6E8 ;
(±1±k±ie±je)/2 and (±i±j±e±ke)/2	in	1E8, 2E8, and 4E8 .

The unit vertices in the E8 lattices do not include any of the 256 E8 light cone vertices, of the form (±1 ±i ±j ±k ±e ±ie ±je ±ke)/2.
They appear in the next layer out from the origin, at radius sqrt 2, which layer contains in all 2160 vertices:
2160 = 112 + 256 + 1792 = 112 + (128+128) + 7(128+128)
the 112 = root vectors of D8
the (128+128) = 8-cube = two mirror image D8 half-spinors
the 7(128+128) = 7 copies of 8-cube for 7 independent E8 lattices,
each 8-cube = two mirror image D8 half-spinors related by triality
to the 112 and thus to the (128+128) and thus to each other.
All 7 E8 lattices have the same second layer or shell. In the image below,

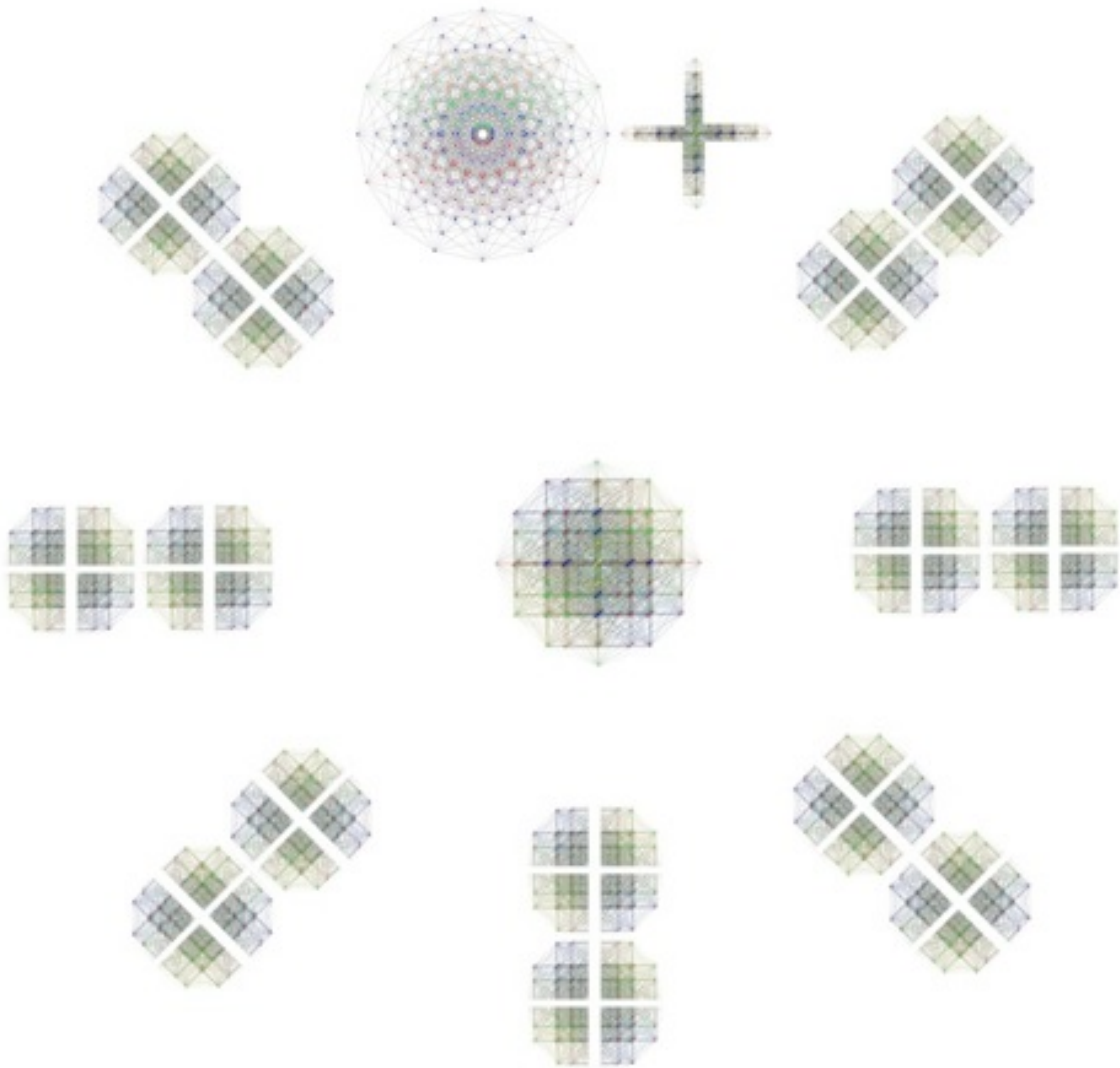
the 240 in the first layer look like  the 112 look like



the 256 look like in the second the 1792 look like



(7 copies of 128+128).



The real 4_21 Witting polytope of the E8 lattice in R8 has
240 vertices;
6,720 edges;
60,480 triangular faces;
241,920 tetrahedra;
483,840 4-simplexes;
483,840 5-simplexes 4_00;
138,240 + 69,120 6-simplexes 4_10 and 4_01; and
17,280 7-simplexes 4_20 and 2,160 7-cross-polytopes 4_11.

The E8 lattice in R8 has a counterpart in complex C4,
the self-reciprocal honeycomb of Witting polytopes,
a lattice of all points whose 4 coordinates are Eisenstein integers with the equivalent congruences
u1 + u2 + u3 = u2 - u3 + u4 = 0 (mod i sqrt(3)) and
u3 - u2 = u1 - u3 = u2 - u1 = u4 (mod i sqrt(3)).

The self-reciprocal Witting polytope in C4 has
240 vertices,
2,160 edges,
2,160 faces, and
240 cells.

It has 27 edges at each vertex.
Its symmetry group has order 155,520 = 3 x 51,840.
It is 6-symmetric, so its central quotient group has order 25,920.
It has 40 diameters orthogonal to which are 40 hyperplanes of symmetry,each of which contains 72 vertices.
It has a van Oss polygon in C2, its section by a plane joining an edge to the center, that is the 3{4}3 in C2, with 24 vertices and 24 edges.

The 7 Imaginary Octonions correspond to the 7 independent E8 lattices and therefore to the 7 Onarhedra/Heptavertons:

```
                                  E
                                 / \
                                /   \
                               J-----j           E j
        J       I---j        / |     | \         |/
I  --> / \  --> |   | -->  i   |  I  |  i   = J--I--k
      i---K     k---E        \ |     | /        /|
                               K-----k         K i
                                \   /
                                 \ /
                                  E
```

```
                                  E
                                 / \
                                /   \
                               K-----k           E k
        j       J---i        / |     | \         |/
J  --> / \  --> |   | -->  j   |  J  |  j   = K--J--i
      I---K     k---E        \ |     | /        /|
                               I-----i         I i
                                \   /
                                 \ /
                                  E
```

```
                                     E
                                    / \
                                   /   \
                                  I-----i             E i
         J        K---i         / |     | \           |/
K  -->  / \  -->  |   | -->   k   |  K  |   k   =  I--K--j
       I---k      j---E         \ |     | /          /|
                                  J-----j           J k
                                   \   /
                                    \ /
                                     E
                                     k
                                    / \
                                   /   \
                                  I-----J             k J
         I        J---j         / |     | \           |/
i  -->  / \  -->  |   | -->   j   |  i  |   j   =  I--i--E
       E---i      K---k         \ |     | /          /|
                                  K-----E           K j
                                   \   /
                                    \ /
                                     k
                                     k
                                    / \
                                   /   \
                                  J-----I             k I
         J        K---k         / |     | \           |/
j  -->  / \  -->  |   | -->   i   |  j  |   i   =  J--j--E
       E---j      I---i         \ |     | /          /|
                                  K-----E           K i
                                   \   /
                                    \ /
                                     k
                                     i
                                    / \
                                   /   \
                                  K-----J             i J
         K        I---i         / |     | \           |/
k  -->  / \  -->  |   | -->   j   |  k  |   j   =  K--k--E
       E---k      J---j         \ |     | /          /|
                                  I-----E           I j
                                   \   /
                                    \ /
                                     i
                                     I
                                    / \
                                   /   \
                                  J-----k             I k
         j        I---J         / |     | \           |/
E  -->  / \  -->  |   | -->   i   |  E  |   i   =  J--E--j
       i---k      K---E         \ |     | /          /|
                                  K-----j           K i
                                   \   /
                                    \ /
                                     I
```

```
Just as each of the 7 imaginary octonions correspond, in my E8 physics
model,to the 7 types of charged fermions (electron; red, blue, green up
quarks; red, blue, green down quarks), each Onarhedron/Heptaverton
corresponds to a charge-neutral set of all 7 charged fermions. Consider that
the initial Big Bang produced a particle-antiparticle pair of the 7 charged
fermions, plus the 8th fermion (neutrino) corresponding to the real number 1.
```

As 8-dimensional Spacetime remains Octonionic throughout Inflation, the paper gr-qc/0007006 by Paola Zizzi shows that

> "... during inflation, the universe can be described as a superposed state of quantum ... [qubits]. The self-reduction of the superposed quantum state is ... reached at the end of inflation ...[at]... the decoherence time ...
> [Tdecoh = 10^9 Tplanck = 10^(-34) sec] ... and corresponds to
> a superposed state of ... [10^19 = 2^64 qubits]. ...
> This is also the number of superposed tubulins-qubits in our brain ...
> leading to a conscious event. ...".

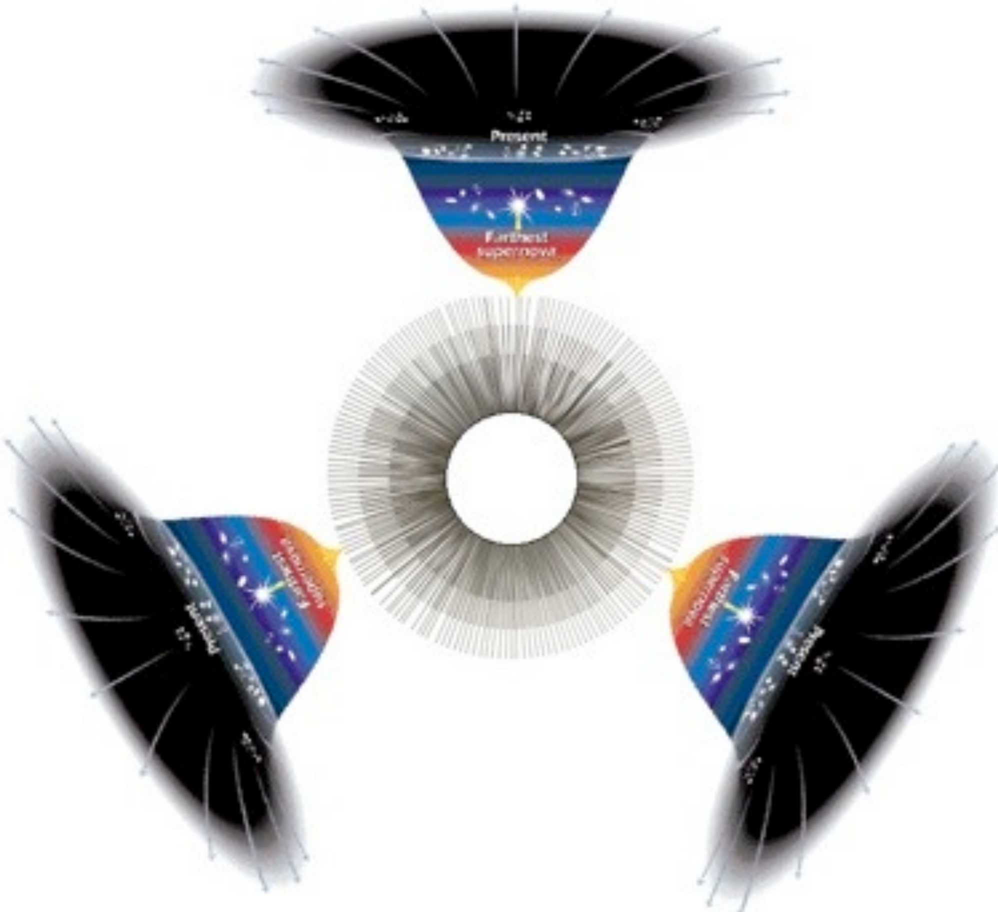
The number of doublings (also known as e-foldings) is estimated in astro-ph/0107459 by Banks and Fischler, who say:

> "... If the present acceleration of the universe is due to an asymptotically deSitter universe with small cosmological constant, then the number of e-foldings during inflation is bounded. ... The essential ingredient is that because of the UV-IR connection, entropy requires storage space. The existence of a small cosmological constant restricts the available storage space. ... We obtain the upper bound ... N_e = 85 ... where we took [the cosmological constant] /\ to be of O(10^(-3) eV). For the sake of comparison, the case k = 1/3 [corresponding to the equation of state for a radiation-dominated fluid, such as the cosmic microwave background] yields ... N_e= 65 ... This value for the maximum number of e-foldings is close to the value necessary to solve the "horizon problem".

If at each of the 64 doubling stages of Zizzi inflation the 2 particles of a pair produced 8+8 = 16 fermions,
then at the end of inflation such a non-unitary octonionic process would have produced about 2 x 16^64 = 4 x (2^4)^64 = 4 x 2^256 = 4 x 10^77 fermion particles. The figure of 4 x 10^77 is similar number of particles estimated by considering the initial fluctuation to be a Planck mass Black Hole
and the 64 doublings to act on such Black Holes.
Roger Penrose, in his book The Emperor's New Mind (Oxford 1989, pages 316-317) said:

> "... in our universe ... Entropy ... increases ... Something forced the entropy to be low in the past. ... the low-entropy states in the past are a puzzle. ...".

The Zizzi Inflation phase of our universe ends with decoherence "collapse" of the 2^64 Superposition Inflated Universe into Many Worlds of the Many-Worlds Quantum Theory, only one of which Worlds is our World.



In this image:
the central white circle is the Inflation Era in which everything is in Superposition; the boundary of the central circle marks the decoherence/collapse at the End of Inflation; and each line radiating from the central circle corrresponds to one decohered/collapsed Universe World (of course, there are many more lines than actually shown), only three of which are explicitly indicated in the image, and only one of which is Our Universe World. Since our World is only a tiny fraction of all the Worlds, it carries only a tiny fraction of the entropy of the 2^64 Superposition Inflated Universe, thus solving Penrose's Puzzle.

At the End of Inflation Our Universe had Temperature / Energy
10^27 K = 10^14 GeV

A consequence of the end of Octonionic Inflation
was the freezing out of a preferred Quaternionic Subspace
so that 8-dim Octonionic Spacetime was converted into
(4+4)-dim Kaluza-Klein spacetime M4 x CP2
where M4 is Minkowski Physical 4-dim spacetime and
CP2 = SU(3) / SU(2)xU(1) is a Batakis 4-dim Internal Symmetry Space.
The geometry of that splitting of spacetime produces a Higgs mechanism.
(see Meinhard Mayer and A. Trautman in “A Brief Introduction to the Geometry of Gauge Fields” and“The Geometry of Symmetry Breaking in Gauge Theories”, Acta Physica Austriaca, Suppl. XXIII (1981))

Since each of the 10^77 fermions had energy of 10^14 GeV
collisions among them would for each of the 10^77 fermions
produce jets containing about 10^12 particles of energy 100 GeV or so
so that the total number of such particles is about 10^89.

According to Weinberg's book "Cosmology":
"... above 10^13 K, nucleons would not yet have formed from their three constituent quarks, and there would have been roughly as many quark-antiquark pairs in thermal equilibrium as photons ... before annihilation there must have been a slight excess ... of quarks over antiquarks, so that some quarks would survive to form nucleons when all the antiquarks had annihilated with quarks. There was also a slight excess of electrons over positrons, to maintain charge neutrality of the universe ...".
Therefore, in the interval
between **the End of Inflation** and **ElectroWeak Symmetry Breaking**
most of the quarks in 10^89 fermions formed quark-antiquark pairs that
produced as a condensate the Higgs that is needed for Mayer-Higgs.
The quark-antiquark condensate Higgs then

Breaks ElectroWeak Symmety at Temperature / Energy
3 x 10^15 K = 300 GeV

and gives mass to particles and at age 10^-(11) seconds
ends the Massless Phase of the history of Our Universe.

Dark Energy : Dark Matter : Ordinary Matter:

Gravity and the Cosmological Constant come from the MacDowell-Mansouri Mechanism and the 15-dimensional Spin(2,4) = SU(2,2) Conformal Group, which is made up of:

3 Rotations;
3 Boosts;
4 Translations;
4 Special Conformal transformations; and
1 Dilatation.

According to gr-qc/9809061 by R. Aldrovandi and J. G. Peireira:
"... If the fundamental spacetime symmetry of the laws of Physics is that given by the de Sitter instead of the Poincare group, the P-symmetry of the weak cosmological-constant limit and the Q-symmetry of the strong cosmological-constant limit can be considered as limiting cases of the fundamental symmetry. ...
... N ...[is the space]... whose geometry is gravitationally related to an infinite cosmological constant ...[and]... is a 4-dimensional cone-space in which ds = 0, and whose group of motion is Q. Analogously to the Minkowski case, N is also a homogeneous space, but now under the kinematical group Q, that is, N = Q/L [where L is the Lorentz Group of Rotations and Boosts]. In other words, the point-set of N is the point-set of the special conformal transformations. Furthermore, the manifold of Q is a principal bundle P(Q/L,L), with Q/L = N as base space and L as the typical fiber. The kinematical group Q, like the Poincare group, has the Lorentz group L as the subgroup accounting for both the isotropy and the equivalence of inertial frames in this space. However, the special conformal transformations introduce a new kind of homogeneity. Instead of ordinary translations, all the points of N are equivalent through special conformal transformations. ...
... Minkowski and the cone-space can be considered as dual to each other, in the sense that their geometries are determined respectively by a vanishing and an infinite cosmological constants. The same can be said of their kinematical group of motions: P is associated to a vanishing cosmological constant and Q to an infinite cosmological constant.
The dual transformation connecting these two geometries is the spacetime inversion x^u -> x^u / sigma^2 . Under such a transformation, the Poincare group P is transformed into the group Q, and the Minkowski space M becomes the cone-space N. The points at infinity of M are concentrated in the vertex of the cone-space N, and those on the light-cone of M becomes the infinity of N. It is

interesting to notice that, despite presenting an infinite scalar curvature, the concepts of space isotropy and equivalence between inertial frames in the cone-space N are those of special relativity. The difference lies in the concept of uniformity as it is the special conformal transformations, and not ordinary translations, which act transitively on N. ..."

Since the Cosmological Constant comes from
the 10 Rotation, Boost, and Special Conformal generators
of the Conformal Group Spin(2,4) = SU(2,2),
the fractional part of our Universe of the Cosmological Constant
should be about 10 / 15 = 67%.

Since Black Holes, including Dark Matter Primordial Black Holes, are curvature singularities in our 4-dimensional physical spacetime,
and since Einstein-Hilbert curvature comes from the 4 Translations
of the 15-dimensional Conformal Group Spin(2,4) = SU(2,2)
through the MacDowell-Mansouri Mechanism (in which the generators corresponding to the 3 Rotations and 3 Boosts do not propagate),
the fractional part of our Universe of Dark Matter Primordial Black Holes
should be about 4 / 15 = 27%.

Since Ordinary Matter gets mass from the Higgs mechanism
which is related to the 1 Scale Dilatation
of the 15-dimensional Conformal Group Spin(2,4) = SU(2,2),
the fractional part of our universe of Ordinary Matter
should be about 1 / 15 = 6%.

Therefore, our Flat Expanding Universe should, according to the cosmology of the model, have (without taking into account any evolutionary changes with time) roughly:
67% Cosmological Constant
27% Dark Matter - possilbly primordial stable Planck mass black holes
6% Ordinary Matter

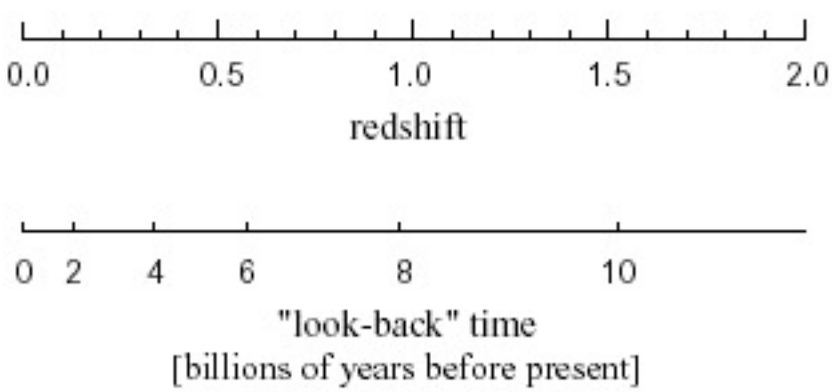
As Dennnis Marks pointed out to me,
since density rho is proportional to (1+z)^3(1+w) for red-shift factor z
and a constant equation of state w:

w = -1 for /\ and the average overall density of /\ Dark Energy remains constant with time and the expansion of our Universe;
and
w = 0 for nonrelativistic matter so that the overall average density of Ordinary Matter declines as 1 / R^3 as our Universe expands;
and
w = 0 for primordial black hole dark matter - stable Planck mass black holes - so that Dark Matter also has density that declines as 1 / R^3 as our Universe expands;
so that the ratio of their overall average densities must vary with time, or scale factor R of our Universe, as it expands.

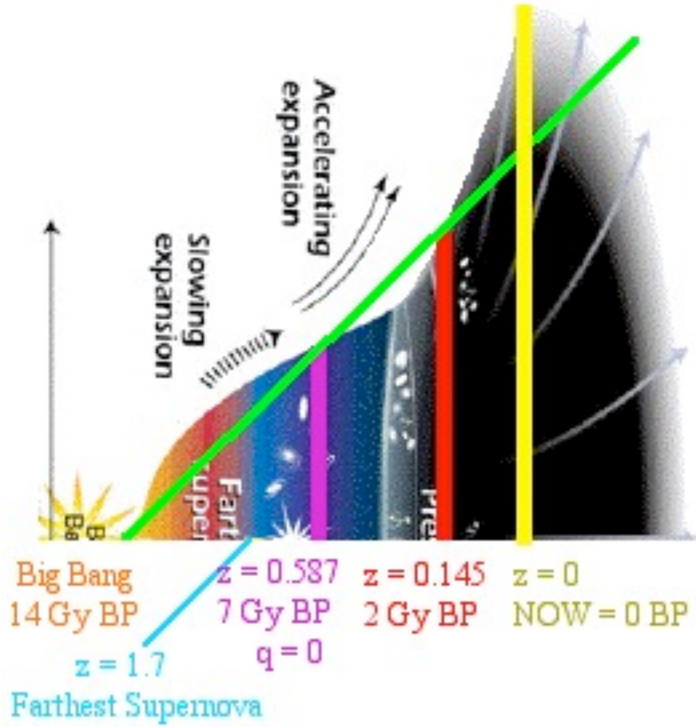
Therefore, the above calculated ratio 0.67 : 0.27 : 0.06 is valid
only for a particular time, or scale factor, of our Universe.

When is that time ? Further, what is the value of the ratio now ?

Since WMAP observes Ordinary Matter at 4% NOW,
the time when Ordinary Matter was 6% would be
at redshift z such that
1 / (1+z)^3 = 0.04 / 0.06 = 2/3 , or (1+z)^3 = 1.5 , or 1+z = 1.145 , or z = 0.145.
To translate redshift into time,
in billions of years before present, or Gy BP, use this chart



from a www.supernova.lbl.gov file SNAPoverview.pdf to see that
the time when Ordinary Matter was 6%
would have been a bit over 2 billion years ago, or 2 Gy BP.



In the diagram, there are four Special Times in the history of our Universe:
the Big Bang Beginning of Inflation (about 13.7 Gy BP);

1 - the End of Inflation = Beginning of Decelerating Expansion
(beginning of green line also about 13.7 Gy BP);

2 - the End of Deceleration (q=0) = Inflection Point =
= Beginning of Accelerating Expansion
(purple vertical line at about z = 0.587 and about 7 Gy BP).
According to a hubblesite web page credited to Ann Feild, the above diagram "... reveals changes in the rate of expansion since the universe's birth 15 billion years ago. The more shallow the curve, the faster the rate of expansion. The curve changes noticeably about 7.5 billion years ago, when objects in the universe began flying apart as a faster rate. ...".
According to a CERN Courier web page: "... Saul Perlmutter, who is head of the Supernova Cosmology Project ... and his team have studied altogether some 80 high red-shift type Ia supernovae. Their results imply that the universe was decelerating for the first half of its existence, and then began accelerating approximately 7 billion years ago. ...".
According to astro-ph/0106051 by Michael S. Turner and Adam G. Riess: "... current supernova data ... favor deceleration at z > 0.5 ... SN 1997ff at z = 1.7

provides direct evidence for an early phase of slowing expansion if the dark energy is a cosmological constant ...".

3 - the Last Intersection of the Accelerating Expansion of our Universe
of Linear Expansion (green line) with the Third Intersection
(at red vertical line at z = 0.145 and about 2 Gy BP),
which is also around the times of the beginning of the Proterozoic Era and Eukaryotic Life, Fe_2O_3 Hematite ferric iron Red Bed formations, a Snowball Earth, and the start of the Oklo fission reactor. 2 Gy is also about 10 Galactic Years for our Milky Way Galaxy and is on the order of the time for the process of a collision of galaxies.

4 - Now.

Those four Special Times define four Special Epochs:

The Inflation Epoch, beginning with the Big Bang and ending with the End of Inflation. The Inflation Epoch is described by Zizzi Quantum Inflation ending with Self-Decoherence of our Universe (see gr-qc/0007006).

The Decelerating Expansion Epoch, beginning with the Self-Decoherence of our Universe at the End of Inflation. During the Decelerating Expansion Epoch, the Radiation Era is succeeded by the Matter Era, and the Matter Components (Dark and Ordinary) remain more prominent than they would be under the "standard norm" conditions of Linear Expansion.

The Early Accelerating Expansion Epoch, beginning with the End of Deceleration and ending with the Last Intersection of Accelerating Expansion with Linear Expansion. During Accelerating Expansion, the prominence of Matter Components (Dark and Ordinary) declines, reaching the "standard norm" condition of Linear Expansion at the end of the Early Accelerating Expansion Epoch at the Last Intersection with the Line of Linear Expansion.

The Late Accelerating Expansion Epoch, beginning with the Last Intersection of Accelerating Expansion and continuing forever, with New Universe creation happening many times at Many Times. During the Late Accelerating Expansion Epoch, the Cosmological Constant /\ is more prominent than it would be under the "standard norm" conditions of Linear Expansion.
Now happens to be about 2 billion years into the Late Accelerating Expansion Epoch.

What about Dark Energy : Dark Matter : Ordinary Matter now ?

As to how the Dark Energy /\ and Cold Dark Matter terms have evolved during the past 2 Gy, a rough estimate analysis would be:

/\ and CDM would be effectively created during expansion in their natural ratio 67 : 27 = 2.48 = 5 / 2, each having proportionate fraction 5 / 7 and 2 / 7, respectively;

CDM Black Hole decay would be ignored; and

pre-existing CDM Black Hole density would decline by the same 1 / R^3 factor as Ordinary Matter, from 0.27 to 0.27 / 1.5 = 0.18.

The Ordinary Matter excess 0.06 - 0.04 = 0.02 plus the first-order CDM excess 0.27 - 0.18 = 0.09 should be summed to get a total first-order excess of 0.11, which in turn should be distributed to the /\ and CDM factors in their natural ratio 67 : 27, producing, for NOW after 2 Gy of expansion:

CDM Black Hole factor = 0.18 + 0.11 x 2/7 = 0.18 + 0.03 = 0.21

for a total calculated Dark Energy : Dark Matter : Ordinary Matter ratio for now of

0.75 : 0.21 : 0.04

so that the present ratio of 0.73 : 0.23 : 0.04 observed by WMAP seems to me to be substantially consistent with the cosmology of the E8 model.

2013 Planck Data (arxiv 1303.5062) showed "... anomalies ... previously observed in the WMAP data ... alignment between the quadrupole and octopole moments ... asymmetry of power between two ... hemispheres ... Cold Spot ... are now confirmed at ... 3 sigma ... but a higher level of confidence ...".

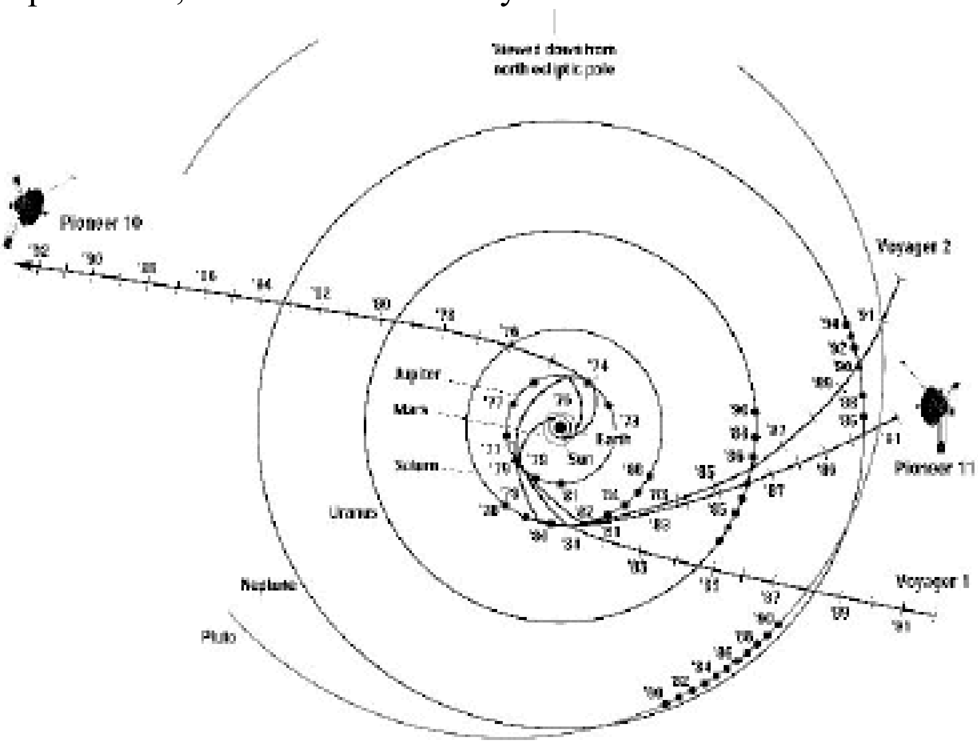
rough evolution E8 calculation: DE : DM : OM = 75 : 20 : 05
WMAP: DE : DM : OM = 73 : 23 : 04
Planck: DE : DM : OM = 69 : 26 : 05
basic E8 Conformal calculation: DE : DM : OM = 67 : 27 : 06

Since uncertainties are substantial, I think that there is reasonable consistency.

Pioneer Anomaly:

After the Inflation Era and our Universe began its current phase of expansion,
some regions of our Universe become Gravitationally Bound Domains
(such as, for example, Galaxies)
in which the 4 Conformal GraviPhoton generators are frozen out,
forming domains within our Universe like IceBergs in an Ocean of Water.

On the scale of our Earth-Sun Solar System, the region of our Earth, where we do our local experiments, is in a Gravitationally Bound Domain.



Pioneer spacecraft are not bound to our Solar System and are experiments beyond the Gravitationally Bound Domain of our Earth-Sun Solar System.
In their Study of the anomalous acceleration of Pioneer 10 and 11 gr-qc/0104064 John D. Anderson, Philip A. Laing, Eunice L. Lau, Anthony S. Liu, Michael Martin Nieto, and Slava G. Turyshev say: "... The latest successful precession maneuver to point ...[Pioneer 10]... to Earth was accomplished on 11 February 2000, when Pioneer 10 was at a distance from the Sun of 75 AU. [The distance from the Earth was [about] 76 AU with a corresponding round-trip light time of about 21 hour.] ... The next attempt at a maneuver, on 8 July 2000, was unsuccessful ... conditions will again be favorable for an attempt around July, 2001. ... At a now nearly constant velocity relative to the Sun of 12.24 km/s, Pioneer 10 will continue its motion into interstellar space, heading generally for the red star Aldebaran ... about

68 light years away ... it should take Pioneer 10 over 2 million years to reach its neighborhood....
[the above image is] Ecliptic pole view of Pioneer 10, Pioneer 11, and Voyager trajectories. Digital artwork by T. Esposito. NASA ARC Image # AC97-0036-3.
... on 1 October 1990 ... Pioneer 11 ... was [about] 30 AU away from the Sun ...
The last communication from Pioneer 11 was received in November 1995, when the spacecraft was at distance of [about] 40 AU from the Sun. ... Pioneer 11 should pass close to the nearest star in the constellation Aquila in about 4 million years ...
... Calculations of the motion of a spacecraft are made on the basis of the range time-delay and/or the Doppler shift in the signals. This type of data was used to determine the positions, the velocities, and the magnitudes of the orientation maneuvers for the Pioneer, Galileo, and Ulysses spacecraft considered in this study. ... The Pioneer spacecraft only have two- and three-way S-band Doppler. ... analyses of radio Doppler ... data ... indicated that an apparent anomalous acceleration is acting on Pioneer 10 and 11 ... The data implied an **anomalous, constant acceleration with a magnitude a_P = 8 x 10^(-8) cm/cm/s^2, directed towards the Sun** ...
... **the size of the anomalous acceleration is of the order c H, where H is the Hubble constant** ...
... Without using the apparent acceleration, CHASMP shows a steady frequency drift of about -6 x 10^(-9) Hz / s, or 1.5 Hz over 8 years (one-way only). ... This equates to a clock acceleration, -a_t, of -2.8 x 10^(-18) s / s^2 . The identity with the apparent Pioneer acceleration is a_P = a_t c. ...
... Having noted the relationships
a_P = c a_t
and that of ...
a_H = c H -> 8 x 10^(-8) cm / s^2
if H = 82 km / s / Mpc ...
we were motivated to try to think of any ... "time" distortions that might ... fit the CHASMP Pioneer results ... In other words ...
Is there any evidence that some kind of "time acceleration" is being seen?
... In particular we considered ... Quadratic Time Augmentation. This model adds a quadratic-in-time augmentation to the TAI-ET (International Atomic Time - Ephemeris Time) time transformation, as follows
ET -> ET + (1/2) a_ET ET^2
The model fits Doppler fairly well ...

... There was one [other] model of the ...[time acceleration]... type that was especially fascinating. This model adds a quadratic in time term to the light time as seen by the DSN station:
delta_TAI = TAI_received - TAI_sent ->
-> delta_TAI + (1/2) a_quad (TAI_received^2 - TAI_sent^2)

It mimics a line of sight acceleration of the spacecraft, and could be thought of as an expanding space model.

Note that a_quad affects only the data. This is in contrast to the a_t ... that affects both the data and the trajectory. ... This model fit both Doppler and range very well. Pioneers 10 and 11 ... the numerical relationship between the Hubble constant and a_P ... remains an interesting conjecture. ...".

In his book Mathematical Cosmology and Extragalactic Astronomy (Academic Press 1976) (pages 61-62 and 72), Segal says:
"... Temporal evolution in ... Minkowski space ... is
H -> H + s I
... unispace temporal evolution ... is ...
H -> (H + 2 tan(a/2)) / (1 - (1/2) H tan(a/2)) = H + a I + (1/4) a H^2 + O(s^2)
...".

Therefore,
the Pioneer Doppler anomalous acceleration is an experimental observation of a system that is not gravitationally bound in the Earth-Sun Solar System, and its results are consistent with Segal's Conformal Theory.

Rosales and Sanchez-Gomez say, at gr-qc/9810085:
"... the recently reported anomalous acceleration acting on the Pioneers spacecrafts should be a consequence of the existence of some local curvature in light geodesics when using the coordinate speed of light in an expanding spacetime. This suggests that the Pioneer effect is nothing else but the detection of cosmological expansion in the solar system. ... the ... problem of the detected misfit between the calculated and the measured position in the spacecrafts ... this quantity differs from the expected ... just in a systematic "bias" consisting on an effective residual acceleration directed toward the center of coordinates;
its constant value is ... H c ...
This is the acceleration observed in Pioneer 10/11 spacecrafts. ... a periodic orbit does not experience the systematic bias but only a very small correction ... which is not detectable ... in the old Foucault pendulum experiment ... the motion of the

pendulum experiences the effect of the Earth based reference system being not an inertial frame relatively to the "distant stars". ... Pioneer effect is a kind of a new cosmological Foucault experiment, the solar system based coordinates, being not the true inertial frame with respect to the expansion of the universe, mimics the role that the rotating Earth plays in Foucault's experiment ...".

The Rosales and Sanchez-Gomez idea of a 2-phase system in which objects bound to the solar system (in a "periodic orbit") are in one phase (non-expanding pennies-on-a-balloon) while unbound (escape velocity) objects are in another phase (expanding balloon) that "feels" expansion of our universe is very similar to my view of such things as described on this page.

The Rosales and Sanchez-Gomez paper very nicely unites:
the physical 2-phase (bounded and unbounded orbits) view;
the Foucault pendulum idea; and the cosmological value H c.

My view, which is consistent with that of Rosales and Sanchez-Gomez,
can be summarized as a 2-phase model based on Segal's work
which has two phases with different metrics:

a metric for outside the inner solar system, a dark energy phase in which gravity is described in which all 15 generators of the conformal group are effective, some of which are related to the dark energy by which our universe expands;
and
a metric for where we are, in regions dominated by ordinary matter, in which the 4 special conformal and 1 dilation degrees of freedom of the conformal group are suppressed and the remaining 10 generators (antideSitter or Poincare, etc) are effective, thus describing ordinary matter phenomena.

If you look closely at the difference between the metrics in those two regions, you see that the full conformal dark energy region gives an "extra acceleration" that acts as a "quadratic in time term" that has been considered as an explanation of the Pioneer effect by John D. Anderson, Philip A. Laing, Eunice L. Lau, Anthony S. Liu, Michael Martin Nieto, and Slava G. Turyshev in their paper at gr-qc/0104064.

Jack Sarfatti has a 2-phase dark energy / dark matter model that can give a similar anomalous acceleration in regions where c^2 /\ dark energy / dark matter is effectively present. If there is a phase transition (around Uranus at 20 AU)
whereby ordinary matter dominates inside that distance from the sun
and exotic dark energy / dark matter appears at greater distances,
then Jack's model could also explain the Pioneer anomaly
and it may be that Jack's model with ordinary and exotic phases
and my model with deSitter/Poincare and Conformal phases
may be two ways of looking at the same thing.

As to what might be the physical mechanism of the phase transition, Jack says "... Rest masses of [ordinary matter] particles ... require the smooth non-random Higgs Ocean ... which soaks up the choppy random troublesome zero point energy ...".

In other words in a region in which ordinary matter is dominant, such as the Sun and our solar system, the mass-giving action of the Higgs mechanism "soaks up" the Dark Energy zero point conformal degrees of freedom that are dominant in low-ordinary mass regions of our universe (which are roughly the intergalactic voids that occupy most of the volume of our universe).
That physical interpretation is consistent with my view.

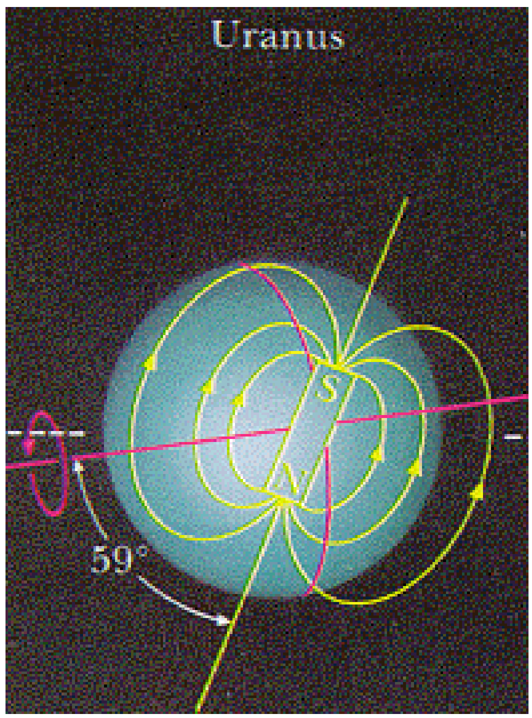
Transition at Orbit of Uranus:

It may be that the observation of the Pioneer phase transition at Uranus from ordinary to anomalous acceleration is an experimental result that gives us a first look at dark energy / dark matter phenomena that could lead to energy sources that could be even more important than the nuclear energy discovered during the past century.

In gr-qc/0104064 Anderson et al say:

"... Beginning in 1980 ... at a distance of 20 astronomical units (AU) from the Sun ... we found that the largest systematic error in the acceleration residuals was a constant bias, aP, directed toward the Sun. Such anomalous data have been continuously received ever since. ...",

so that the transition from inner solar system Minkowski acceleration to outer Segal Conformal acceleration occurs at about 20 AU, which is about the radius of the orbit of Uranus. That phase transition may account for the unique rotational axis of Uranus,



which lies almost in its orbital plane.

The most stable state of Uranus may be with its rotational axis pointed toward the Sun, so that the Solar hemisphere would be entirely in the inner solar system Minkowski acceleration phase and the anti-Solar hemisphere would be in entirely in the outer Segal Conformal acceleration phase.

Then the rotation of Uranus would not take any material from one phase to the other, and there would be no drag on the rotation due to material going from phase to phase.
Of course, as Uranus orbits the Sun, it will only be in that most stable configuration twice in each orbit, but an orbit in the ecliptic containing that most stable configuration twice (such as its present orbit) would be in the set of the most stable ground states, although such an effect would be very small now.
However, such an effect may have been been more significant on the large gas/dust cloud that was condensing into Uranus and therefore it may have caused Uranus to form initially with its rotational axis pointed toward the Sun.
In the pre-Uranus gas/dust cloud, any component of rotation that carried material from one phase to another would be suppressed by the drag of undergoing phase transition, so that, after Uranus condensed out of the gas/dust cloud, the only remaining component of Uranus rotation would be on an axis pointing close to the Sun, which is what we now observe.
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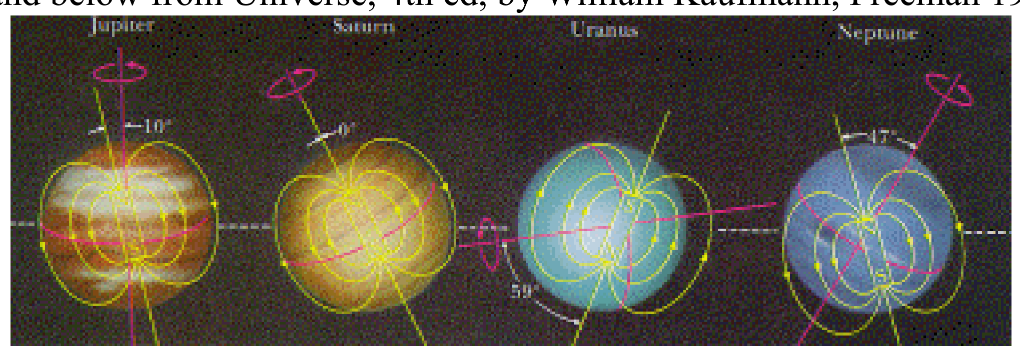
Much of the perpendicular (to Uranus orbital plane) angular momentum from the original gas/dust cloud may have been transferred (via particles "bouncing" off the phase boundary) to the clouds forming Saturn (inside the phase boundary) or Neptune (outside the phase boundary, thus accounting for the substantial (relative to Jupiter) deviation of their rotation axes from exact perpendicularity (see images above and below from Universe, 4th ed, by William Kaufmann, Freeman 1994).



According to Utilizing Minor Planets to Assess the Gravitational Field in the Outer Solar System, astro-ph/0504367, by Gary L. Page, David S. Dixon, and John F. Wallin:
"... the great distances of the outer planets from the Sun and the nearly circular orbits of Uranus and Neptune makes it very difficult to use them to detect the

Pioneer Effect. ... The ratio of the Pioneer acceleration to that produced by the Sun at a distance equal to the semimajor axis of the planets is 0.005, 0.013, and 0.023 percent for Uranus, Neptune, and Pluto, respectively. ... Uranus' period shortens by 5.8 days and Neptune's by 24.1, while Pluto's period drops by 79.7 days. ... an equivalent change in aphelion distance of 3.8 x 10^10, 1.2 x 10^11, and 4.3 x 10^11 cm for Uranus, Neptune, and Pluto. In the first two cases, this is less than the accepted uncertainty in range of 2 x 10^6 km [or 2 x 10^11 cm] (Seidelmann 1992). ... Pluto['s] ... orbit is even less well-determined ... than the other outer planets. ... . [C]omets ... suffer ... from outgassing ... [and their nuclei are hard to locate precisely] ...".

According to a google cache of an Independent UK 23 September 2002 article by Marcus Chown:
"... The Pioneers are "spin-stabilised", making them a particularly simple platform to understand. Later probes ... such as the Voyagers and the Cassini probe ... were stabilised about three axes by intermittent rocket boosts. The unpredictable accelerations caused by these are at least 10 times bigger than a small effect like the Pioneer acceleration, so they completely cloak it. ...".

Conformal Gravity Dark Energy:

I. E. Segal proposed a MInkowski-Conformal 2-phase Universe
and
Beck and Mackey proposed 2 Photon-GraviPhoton phases:

Minkowski/Photon phase locally Minkowski with ordinary Photons and
Gravity weakened by 1 / (M_Planck)^2 = 5 x 10^(-39).

so that we see Dark Energy as only 3.9 GeV/m^3

Conformal/GraviPhoton phase with GraviPhotons and Conformal symmetry
(like the massless phase of energies above Higgs EW symmetry breaking)
With massless Planck the 1 / M_Planck^2 Gravity weakening goes away
and the Gravity Force Strength becomes the strongest possible = 1
so Conformal Gravity Dark Energy should be enhanced by M_Planck^2
from the Minkowski/Photon phase value of 3.9 GeV/m^3.

The Energy Gap of our Universe as superconductor condensate spacetime
is from
from 3 x 10^(-18) Hz (radius of universe) to 3 x 10^43 Hz (Planck length)
and
its RMS amplitude is 10^13 Hz = 10 THz = energy of neutrino masses =
= critical temperature Tc of BSCCO superconducting crystals.

Neutrino masses are involved because their mass is zero at tree level
and their masses that we observe come from
virtual graviphotons becoming virtual neutrino-antineutrino pairs.

BSCCO superconducting crystals are by their structure natural Josephson
Junctions. Dark Energy accumulates (through graviphotons) in the
superconducting layers of BSCCO.

Josephson Junction control voltage acts as a valve for access to the
BSCCO Dark Energy, an idea due to Jack Sarfatti.

In E8 Physics Dark Energy comes from the Conformal/GraviPhoton phase. The geometry of the Conformal Sector is closely related to the Penrose Paradise of Twistors. Yu. Manin in his 1981 book "Mathematics and Physics" said: "... In a world of light there are neither points nor moments of time; beings woven from light would live "nowhere" and "nowhen" ... the whole life history of a free photon [is] the smallest "event" that can happen to light. ...".

Here is how the Conformal/GraviPhoton phase of Gravity works:
The Lorentz Group is represented by 6 generators

```
 0   J1  J2  M1
-J1   0  J3  M2
-J2 -J3   0  M3
-M1 -M2 -M3   0
```

There are two ways to extend the Lorentz Group:

(see arXiv gr-qc/9809061 by Aldrovandi and Peireira):

to the Poincare Group of Minkowski Space with No Cosmological Constant of the Minkowski/Photon phase where ordinary Photons usually live by adding 4 generators

```
 0   J1  J2  M1       A1
-J1   0  J3  M2       A2
-J2 -J3   0  M3       A3
-M1 -M2 -M3   0       A4

-A1 -A2 -A3 -A4       0
```

{A1,A2,A3} represent Momentum
and {A4} represents Energy/Mass of Poincare Gravity
and its Dark Matter Primordial Black Holes

and

to the semidirect product of Lorentz and 4 Special Conformal generators
to get a Non-Zero Cosmological Constant for Universe Expansion
of the Conformal/GraviPhoton phase where GraviPhotons usually live

0	J1	J2	M1	G1
-J1	0	J3	M2	G2
-J2	-J3	0	M3	G3
-M1	-M2	-M3	0	G4
-G1	-G2	-G3	-G4	0

so that {G1,G2,G3} represent 3 Higgs components
giving mass to 3 Weak Bosons
and {G4} represents massive Higgs Scalar as Fermion Condensate.
As Special Conformal and Scale Conformal degrees of freedom they also represent the Momentum of Expansion of the Universe and Dark Energy.

One more generator {G5} represents Higgs mass of Ordinary Matter.
All 15 generators combine
to make the full Conformal Lie Algebra SU(2,2) = Spin(2,4) of
the universal Conformal Space with a Non-Zero Cosmological Constant
for Universe Expansion

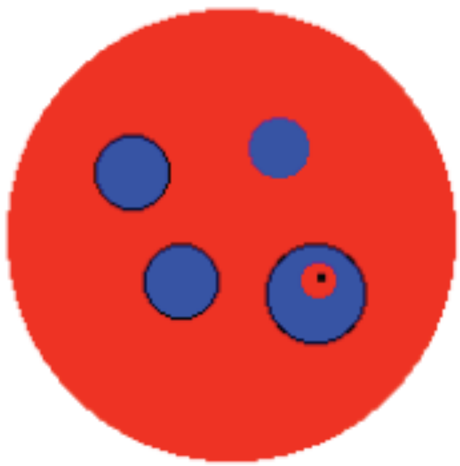
0	J1	J2	M1	G1	A1
-J1	0	J3	M2	G2	A2
-J2	-J3	0	M3	G3	A3
-M1	-M2	-M3	0	G4	A4
-G1	-G2	-G3	-G4	0	G5
-A1	-A2	-A3	-A4	-G5	0

10 generators in the 5x5 upper diagonal correspond to Dark Energy (DE)
the 4 upper generators of the 6th column correspond to Dark Matter (DM)
the 5th generator of the 6th column corresponds to Ordinary Matter (OM)

The basic 10 : 4 : 1 = 67 : 27 : 06 ratio of DE : DM : OM
has evolved over the history of Our Universe to its present value of

DE : DM : OM = 75 : 20 : 05 (rough evolution calculation)
DE : DM : OM = 73 : 23 : 04 (measured by WMAP)
DE : DM : OM = 69 : 26 : 05 (measured by Planck)

Rabindra Mohapatra in section 14.6 of his book "Unification and Supersymmetry" said:
"... we start with a Lagrangian invariant under full local conformal symmetry and fix its conformal and scale gauge to obtain the usual action for gravity ... the conformal d'Alembartian contains ... curvature ... R, which for constant ...
scalar field ... PHI, leads to gravity. We may call PHI the auxiliary field ...".
I view PHI as corresponding to the Higgs 3 Special Conformal generators {G1,G2,G3} that are frozen fixed during expansion in some regions of our Universe to become Gravitationally Bound Domains (such as Galaxies) like icebergs in an ocean of water.



Since the Gravitationally Bound Domains (such as our Inner Solar System) have no Expansion Momentum we only see there the Poincare Part of Conformal Gravity plus the Higgs effects of {G4} and {G5} and the ElectroWeak Broken Symmetry caused by freezing-out fixing {G1,G2,G3}:

0	J1	J2	M1	-	A1
-J1	0	J3	M2	-	A2
-J2	-J3	0	M3	-	A3
-M1	-M2	-M3	0	G4	A4
-	-	-	-G4	0	G5
-A1	-A2	-A3	-A4	-G5	0

Irving Ezra Segal in his book "Mathematical Cosmology and Extragalactic Astronomy" said: "... Minkowski space [is] the set of all 2 x 2 Hermitian matrices ... H(2)
...

```
                    t + x        y + iz
(t,x,y,z) ->
                    y - iz       t - x
```

...
universal [Conformal] space [is] the unitary 2 x 2 group, denoted by U(2) ... [which corresponds to S1 x S3] by
...

```
(t,p) -> e^(it) u
```

where
[U(2) = U(1) x SU(2) and u is the point of SU(2) corresponding to p in S3]
...
[There is] a local causality-preserving transformation between Minkowski [R1xR3] space ... and universal [Conformal RP1 x S3] space ...
...[with]... two-fold covering space S1 x S3 ... oo=fold covering R1 x S3
[the coverings may be considered equivalent in cosmology discussion]
...
Any element of [the 15-dimensional Conformal Group] SU(2,2) can be represented in the form

```
A    B
C    D
```

where A, B, C, D are ... 2x2 matrices ...[with]... the standard action

U -> (AU + B) (CU + D)^(-1)

[of Linear Fractional Mobius Transformations on unispace U(2)]...
...
Maxwell's equations and the wave equation are conformally invariant [so] the properties of solutions are basically independent of whether they are analyzed from a flat [Minkowski] or curved [Conformal] standpoint
...

the ... 15 ...[su(2,2)] generators of symmetries of [Conformal] unispace ...
differ from the 11 generators of ... transformations in Minkowski space
by terms of order 1 / R^2 , as R -> oo
... R being the radius of the universe [S3 in unispace Conformal U(2)]
...
the fundamental local dynamical variables of the chronometric [Conformal] theory, energy, momenta, etc., differ from those of special relativistic [Mikowski] field theory by terms of the order R^(-1) or less,
where R is the radius of the universe ... The square of the mass is ... represented by the [Conformal] D'Alembartian ...[or]...
the Casimir operator for O(2,3) which differs from [the Conformal D'Alembartian] only by terms of order R^(-1) ...".

Irving Ezra Segal in his paper for "Proceedings of the Summer Research Institute on the legacy of John von Neumann" (AMS 1990) said: "... The Einstein energy H [is] the sum H0 + H1 of the conventional relativistic ... Minkowski energy H0 and the super-relativistic [Conformal Dark] energy H1. ...
H0 and H1 are respectively scale-covariant and scale-contravariant i.e., transform like r and 1/r respectively.
... the decomposition H = H0 + H1 is Lorentz-covariant ..
H0 and H1 correspond to effective potentials of the form Lr and -G/r where r is the Euclidean distance ...".

Aubert Daigneault and Atruro Sangalli in Notices of the AMS 48 (2001) 9-16 said: "...
Irving Ezra Segal ... proposed ... chronometric cosmology (CC) ...
conformal immersion of Minkowski space M = R x R3 ... into ... R x S3
... time coordinates x0 [flat Minkowski] and t [curved Conformal]
are related by ... x0 = 2r tan(t/2r)
from which the relation ... redshift ... z = tan^2(t/2r) may be derived ...
the curvature of space is the reason for the ... redshift ...
x0 tends to t as r tends to infinity. The ... differences ... can ... be established from the series expansion of x0 in powers of t:

x0 = t + t^3 / (12r^2) + t^5 / (120r^4) + ...

... a cosmological constant ... Λ ...[is]... related
to the radius r of [the unispace Conformal] S3 by

r = Λ^(-1/2)

...".

Christian Beck and Michael C. Mackey in astro-ph/0703364 said: "... Electromagnetic dark energy .... is based on a Ginzburg-Landau ... phase transition for the gravitational activity of virtual photons ... in two different phases:
gravitationally active [GraviPhotons] ...
and gravitationally inactive [Photons] ...
Let IPI^2 be the number density of gravitationally active photons ...
start from a Ginzburg-Landau free energy density ...

F = a IPI^2 + (1/2) b IPI^4

... The equilibrium state Peq is ... a minimum of F ... for T > Tc ...

Peq = 0 [and] Feq = 0

... for T< Tc

IPeqI^2 = - a / b [and] Fdeq = -(1/2) a^2 / b

... temperature T [of] virtual photons underlying dark energy ... is ..

h v = ln3 k T

... dark energy density ...[is]...

rho_dark = (1/2) (pi h / c^3) (v_c)^4

... The currently observed dark energy density in the universe of about 3.9 GeV/m^3 implies that the critical frequency v_c is ...

v_c = 2.01 THz

...
BCS Theory yields ... for Fermi energy ... in copper ... 7.0 eV
and the critical temperature of ... YBCO ... around 90 K ...

h v_c = 8 x 10^(-3) eV

... Solar neutrino measurements provide evidence fo a neutrino mass of about m_v c^2 = 9 x 10^-3 eV ...

[E8 Physics has first-order masses for the 3 generations of neutrinos as
1 x 10^(-3) and 9 x 10^(-3) and 5.4 x 10(-2) eV]

... in solid state physics the critical temperature is essentially determined by the energy gap of the superconductor ... (i.e. the energy obtained when a Cooper pair forms out of two electrons) ...
for [graviphotons] ... at low temperatures (frequencies) Cooper-pair like states [of neutrino-antineutrino pairs] can form in the vacuum ... the ... energy gap would be of the order of typical neutrino mass differences ...".

Clovis Jacinto de Matos and Christian Beck in arXiv 0707.1797 said: "...
Tajmar's experiments ... at Austrian Research Centers Gmbh-ARC ...
with ... rotating superconducting rings ... demonstrated ...
a clear azimuthal acceleration ... directly proportional to the
superconductive ring angular acceleration, and
an angular velocity orthogonal to the ring's equatorial plane ...
In 1989 Cabrera and Tate, through the measurement of the London moment magnetic trapped flux, rekported an anomalous Cooper pair mass excess in thin rotating Niobium supeconductive rings ...
A non-vanishing cosmological constant (CC) /\ can be interpreted in terms of a non-vanishing vacuum energy density

rho_vac = (c^4 / 8 pi G) /\

which corresponds to dark energy with equation of state w = -1.
The ... astronomically observed value [is]... /\ = 1.29 x 10^(-52) [1/m^2] ...
Graviphotons can form weakly bounded states with Cooper pairs,
increasing their mass slightly from m to m' .
The binding energy is Ec = u c^2 :

m' = m + my - u

... Since the graviphotons are bounded to the Cooper pairs,
their zeropoint energies form a condensate capable of the
gravitoelectrodynamic properties of superconductive cavities. ...
Beck and Mackey's Ginzburg-Landau-like theory leads to a finite dark energy density dependent on the frequency cutoff v_c of vacuum fluctuations:

rho* = (1/2) (pi h / c^3) (v_c)^4

in vacuum one may put rho* = rho_vac from which the cosmological cutoff frequency v_cc is estimated as

v_cc = 2.01 THz

The corresponding "cosmological" quantum of energy is:

Ecc = h v_cc = 8.32 MeV

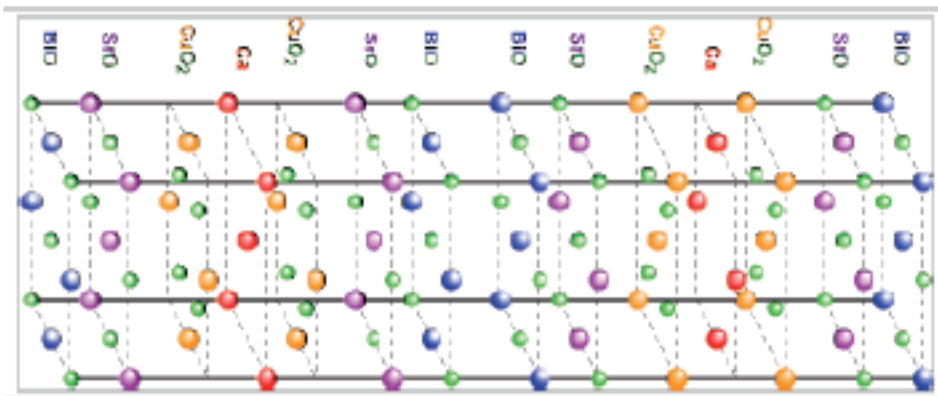
... In the interior of superconductors ... the effective cutoff frequency can be different ... h v = ln 3 k T ... we find the cosmological critical temprature Tcc

Tcc = 87.49 K

This temperature is characteristic of the BSCCO High-Tc superconductor.
...".

Xiao Hu and Shi-Zeng Lin in arXiv 0911.5371 said: "... The Josephson effect is a phenomenon of current flow across two weakly linked superconductors separated by a thin barrier, i.e. Josephson junction, associated with coherent quantum tunneling of Cooper pairs. ... The Josephson effect also provides a unique way to generate high-frequency electromagnetic (EM) radiation by dc bias voltage ... The discovery of cuprate high-Tc superconductors accelerated the effort to develop novel source of EM waves based on a stack of atomically dense-packed intrinsic Josephson junctions (IJJs), since the large superconductivity gap covers the whole terahertz (THz) frequency band. Very recently, strong and coherent THz radiations have been successfully generated from a mesa structure of Bi2Sr2CaCu2O8+d single crystal ...[

BSCCO image from Wikipedia



]...

which works both as the source of energy gain and as the cavity for resonance. This experimental breakthrough posed a challenge to theoretical study on the phase dynamics of stacked IJJs, since the phenomenon cannot be explained by the known solutions of the sine-Gordon equation so far. It is then found theoretically that, due to huge inductive coupling of IJJs produced by the nanometer junction separation and the large London penetration depth ... of the material,
a novel dynamic state is stabilized in the coupled sine-Gordon system, in which +/- pi kinks in phase differences are developed responding to the standing wave of Josephson plasma and are stacked alternately in the c-axis. This novel solution of the inductively coupled sine-Gordon equations captures the important features of experimental observations.
The theory predicts an optimal radiation power larger than the one observed in recent experiments by orders of magnitude ...".

What are some interesting BSCCO JJ Array configurations ?

Christian Beck and Michael C. Mackey in astro-ph/0605418 describe

"... the AC Josephson effect ...
a Josephson junction consists of two superconductors with an insulator sandwiched in between. In the Ginzburg-Landau theory each superconductor is described by a complex wave function whose absolute value squared yields the density of superconducting electrons. Denote the phase difference between the two wave functions ... by P(t).
...
at zero external voltage a superconductive current given by Is = Ic sin(P) flows between the two superconducting electrodes ... Ic is the maximum superconducting current the junction can support.
...
if a voltage difference V is maintained across the junction, then the phase difference P evolves according to

d P / dt = 2 e V / hbar

i.e. the current ... becomes an oscillating curent
with amplitude Ic and frequency v = 2 e V / h
This frequency is the ... Josephson frequency ... The quantum energy h v ... can be interpreted as the energy change of a Cooper pair that is transferred across the junction ...".

Xiao Hu and Shi-Zeng Lin in arXiv 1206.516 said:

"... to enhance the radiation power in teraherz band based on the intrinsic Josephson Junctions of Bi2Sr2CaCu2O8+d single crystal ...
we focus on the case that the Josephson plasma is uniform along a long crystal as established by the cavity formed by the dielectric material. ...
A ... pi kink state ... is characterized by static +/- pi phase kinks in the lateral directions of the mesa, which align themselves alternatingly along the c -axis. The pi phase kinks provide a strong coupling between the uniform dc current and the cavity modes, which permits large supercurrent flow into the system at the cavity resonances, thus enhances the plasma oscillation and radiates strong EM wave ...
The maximal radiation power ... is achieved when the length of BSCCO single crystal at c-axis equals the EM wave length. ...".

Each long BSCCO single crystal looks geometrically like a line so configure the JJ Array using BSCCO crystals as edges.

The simplest polytope, the Tetrahedron, is made of 6 edges:
Feigelman, Ioffe, Geshkenbein, Dayal, and Blatter in cond-mat/0407663 said:
"... Superconducting tetrahedral quantum bits ...

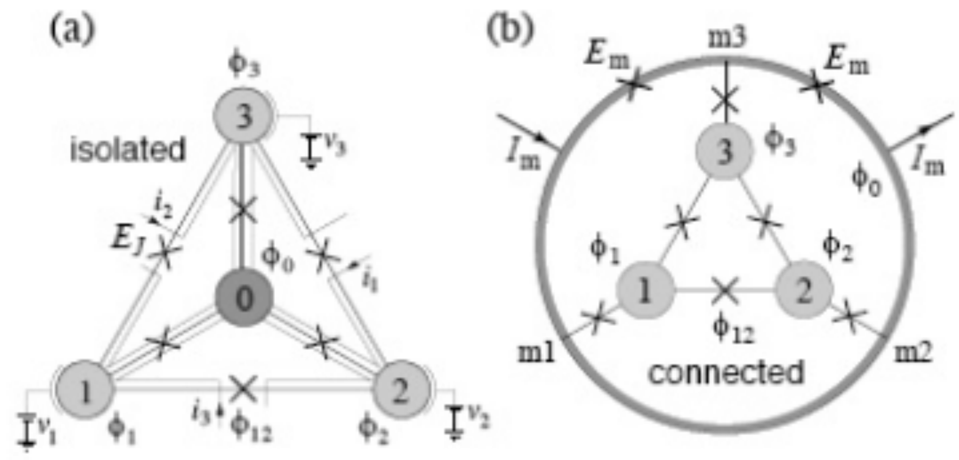
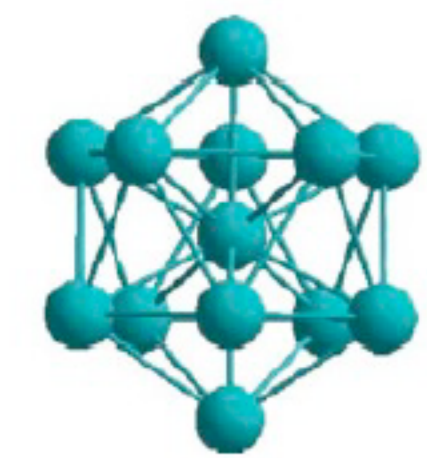


FIG. 1: (a) Tetrahedral superconducting qubit involving four islands and six junctions (with Josephson coupling E_J and charging energy E_C); all islands and junctions are assumed to be equal and arranged in a symmetric way. The islands are attributed phases ϕ_i, $i = 0, \ldots, 3$. The qubit is manipulated via bias voltages v_i and bias currents i_i. In order to measure the qubit's state it is convenient to invert the tetrahedron as shown in (b) — we refer to this version as the 'connected' tetrahedron with the inner dark-grey island in (a) transformed into the outer ring in (b). The measurement involves additional measurement junctions with couplings $E_m \gg E_J$ on the outer ring which are driven by external currents I_m (schematic, see Fig. 6 for details); the large coupling E_m effectively binds the ring segments into one island.

... tetrahedral qubit design ... emulates a spin-1/2 system in a vanishing magnetic field, the ideal starting point for the construction of a qubit. Manipulation of the tetrahedral qubit through external bias signals translates into application of magnetic fields on the spin;
the application of the bias to different elements of the tetrahedral qubit corresponds to rotated operations in spin space. ...".

42 edges make an Icosahedron plus its center

(image from Physical Review B 72 (2005) 115421 by Rogan et al)

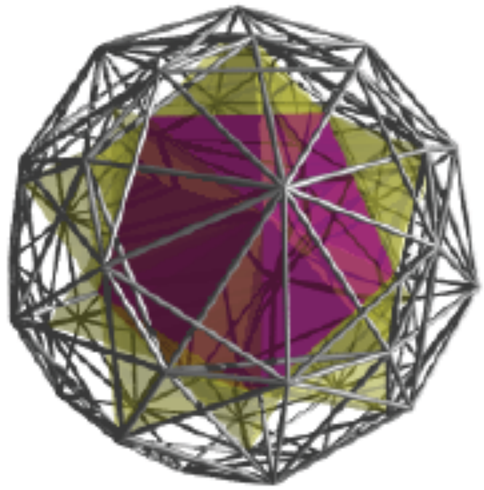


with 30 exterior edges and 12 edges from center to vertices.
It has 20 cells which are approximate Tetrahedra in flat 3-space but become exact regular Tetrahedra in curved 3-space.

Could an approximate-20Tetrahedra-Icosahedron configuration of 42 BSCCO JJ tap into Dark Energy so that the Dark Energy might regularize the configuration to exact Tetrahedra and so curve/warp spacetime from flat 3-space to curved 3-space ?

720 edges make a 4-dimensional 600-cell

(image from Wikipedia)

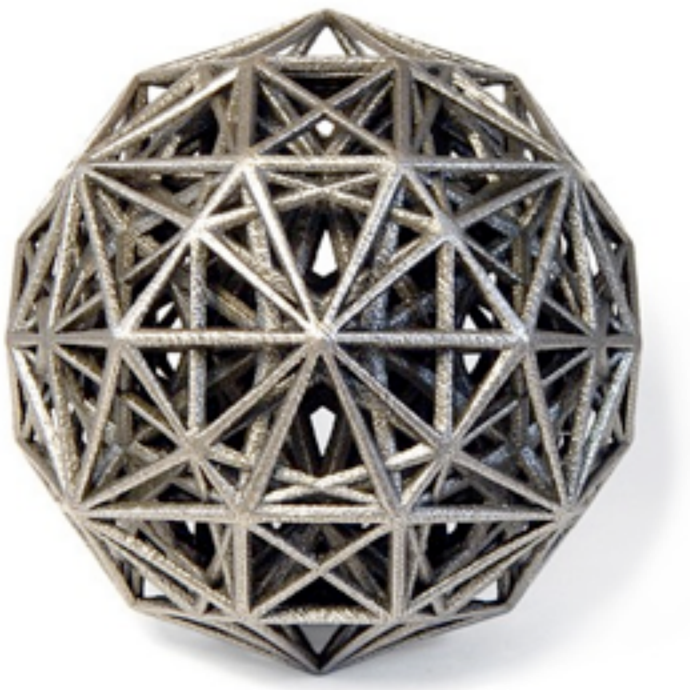


At each vertex 20 Tetrahedral faces meet forming an Icosahedron
which is exact because the 600-cell lives on a curved 3-shere in 4-space.
It has 600 Tetrahedral 3-dim faces and 120 vertices

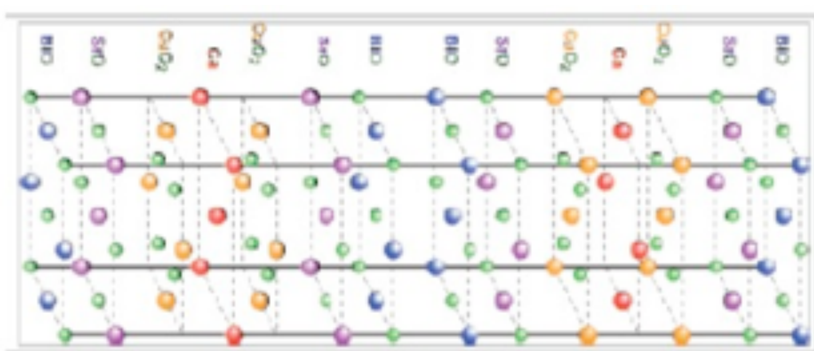
Could a 600 approximate-Tetrahedra configuration of 720 BSCCO JJ
approximating projection of a 600-cell into 3-space
tap into Dark Energy so that the Dark Energy might regularize
the configuration to exact Tetrahedra and an exact 600-cell
and so curve/warp spacetime from flat 3-space to curved 3-space ?

The basic idea of Dark Energy from BSCCO Josephson Junctions is based on the 600-cell as follows: Consider 3-dim models of 600-cell such as metal sculpture from Bathsheba Grossman who says:

"… for it I used an orthogonal projection rather than the Schlegel diagrams of the other polytopes I build.
… In this projection all cells are identical, as there is no perspective distortion. …".



For the Dark Energy experiment each of the 720 lines would be made of a single BSCCO crystal



whose layers act naturally to make the BSCCO crystal an intrinsic Josepson Junction. (see Wikipedia and arXiv 0911.5371)

Each of the 600 tetrahedral cells of the 600-cell
has 6 BSCCO crystal JJ edges.
Since the 600-cell is in flat 3D space the tetrahedra are distorted.

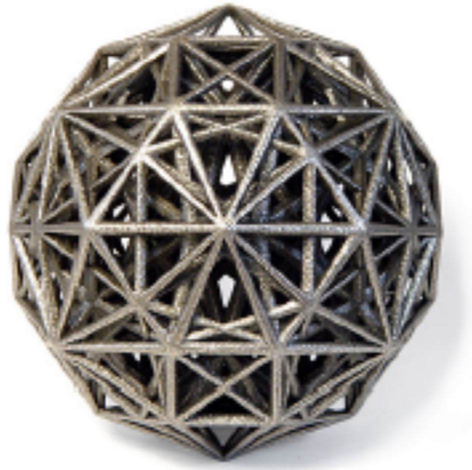
According to the ideas of Beck and Mackey (astro--ph/0703364) and of Clovis Jacinto de Matos (arXiv 0707.1797) the superconducting Josephson Junction layers of the 720 BSCCO crystals will bond with Dark Energy GraviPhotons that are pushing our Universe to expand.

My idea is that the Dark Energy GraviPhotons will not like being configured as edges of tetrahedra that are distorted in our flat 3D space and
they will use their Dark Energy to make all 600 tetrahedra to be exact and regular by curving our flat space (and space-time).

My view is that the Dark Energy Graviphotons will have enough strength to do that because their strength will NOT be weakened by the (1 / M_Planck)^2 factor that makes ordinary gravity so weak.

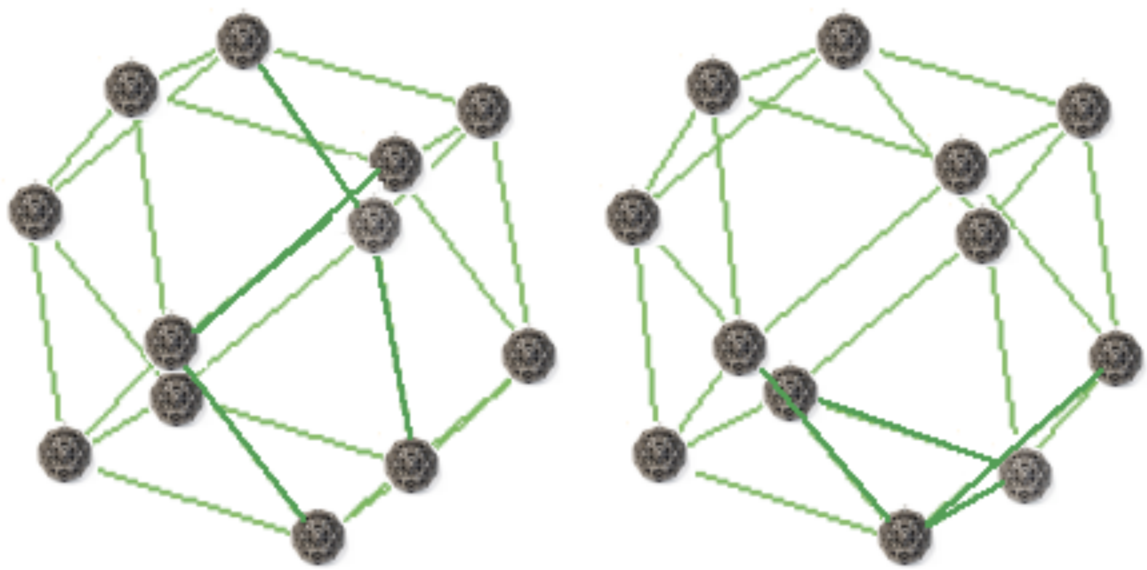
It seems to me to be a clearly designed experiment that will either
1- not work and show my ideas to be wrong or
2 - work and open the door for humans to work with Dark Energy.

Consider BSCCO JJ 600-cells

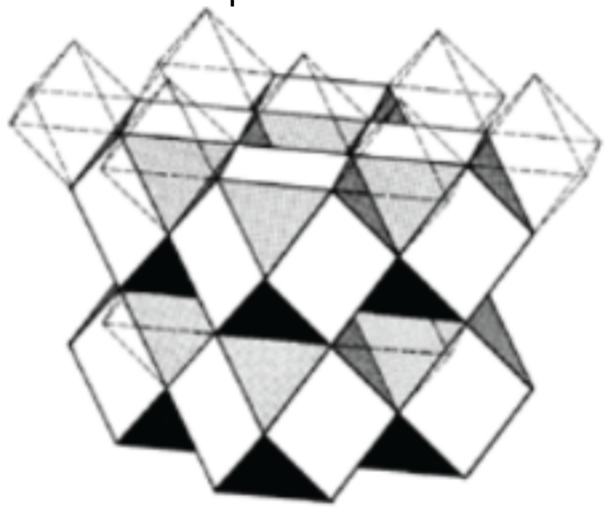


in this configuration:

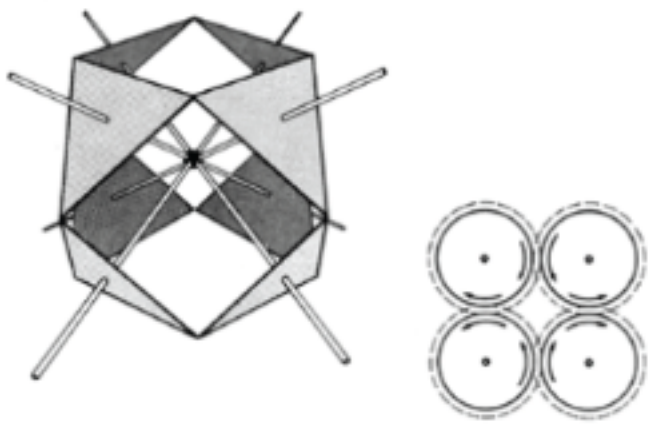
First put 12 of the BSCCO JJ 600-cells at the vertices of a cuboctahedron shown here as a 3D stereo pair:



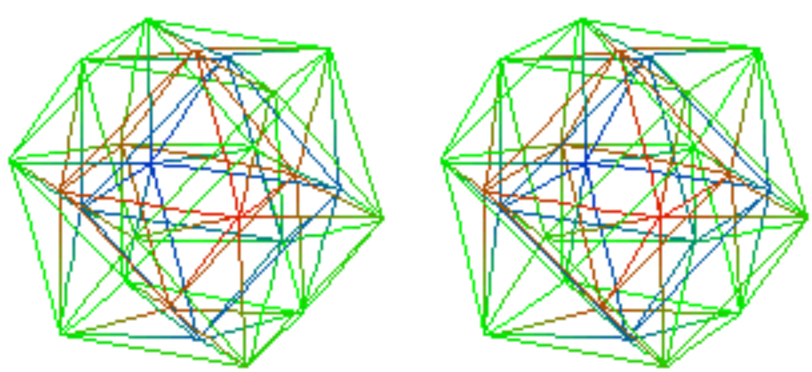
Cuboctahedra do not tile 3D flat space without interstitial octahedra



but BSCCO JJ 600-cell cuboctahedra can be put together
square-face-to-square-face in flat 3D configurations including flat sheets.
As Buckminster Fuller described, the 8 triangle faces of a cuboctahedron



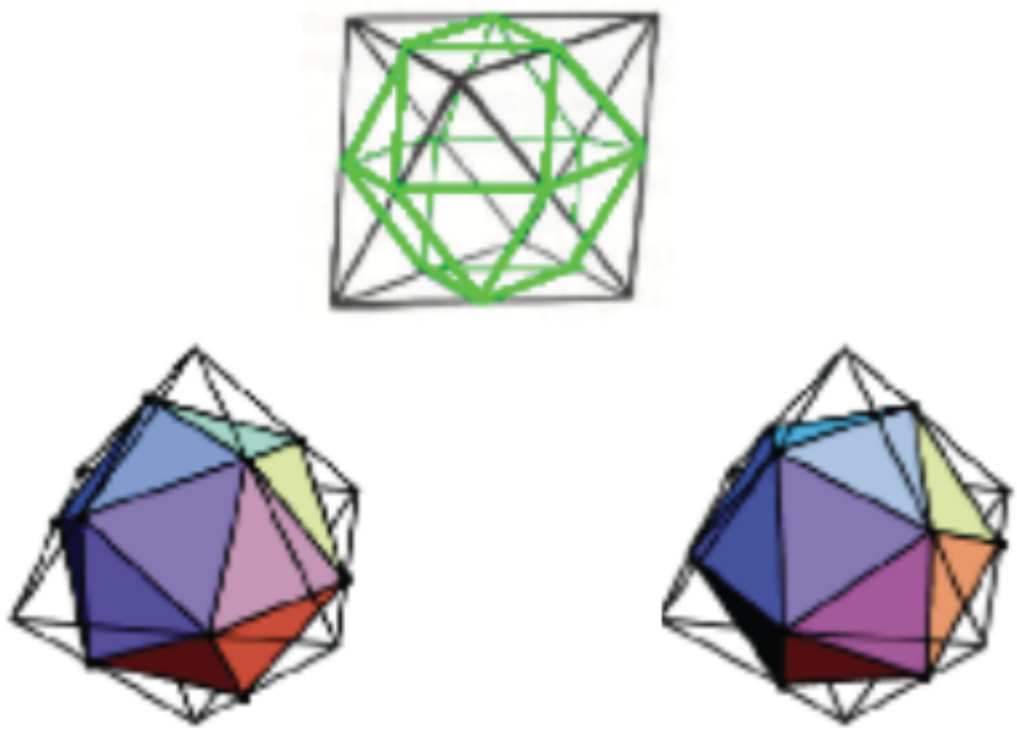
give it an inherently 4D structure consistent with the green cuboctahedron



central figure of a 24-cell (3D stereo 4thD blue-green-red color)
that tiles flat Euclidean 4D space.

So, cuboctahedral BSCCO JJ 600-cell structure likes flat 3D and 4D space
but
if BSCCO JJ Dark Energy act to transform flat space into curved space
like a 720-edge 600-cell with 600 regular tetrahedra
then
Dark Energy should transform cuboctahedral BSCCO JJ 600-cell structure into
a 720-edge BSCCO JJ 600-cell structure that likes curved space.

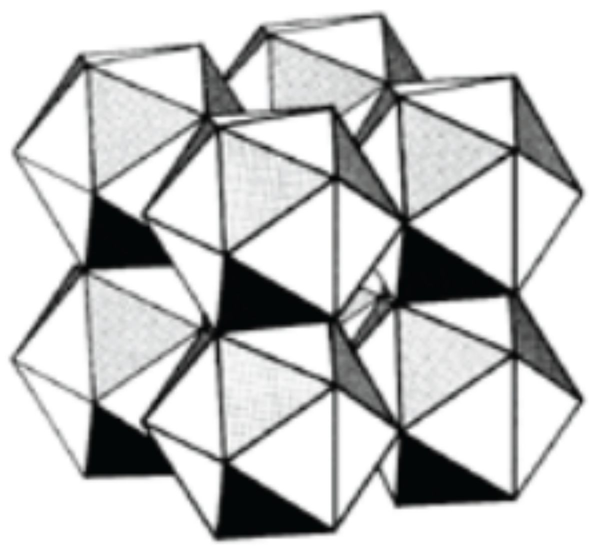
There is a direct JItterbug transformation of
the 12-vertex cuboctahedron to the 12-vertex icosahedron



whereby the 12 cuboctahedron vertices as midpoints of octahedral edges are mapped to
12 icosahedron vertices as Golden Ratio points of octahedral edges.
There are two ways to map a midpoint to a Golden Ratio point.
For the Dark Energy experiment the same choice of mapping
should be made consistently throughout the BSCCO JJ 600-cell structure.

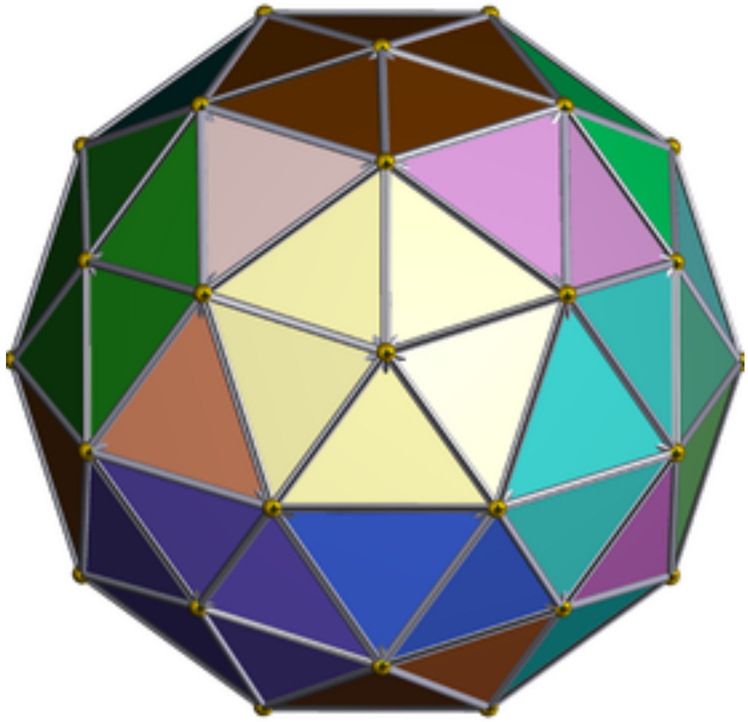
The result of the Jitterbug mapping is that
each cuboctahedron in the BSCCO JJ 600-cell structure
with its 12 little BSCCO JJ 600-cells at its 12 vertices
is mapped to
an icosahedron with 12 little BSCCO JJ 600-cells at its 2 vertices:

and the overall cuboctahedral BSCCO JJ 600-cell structure
is transformed into
an overall icosahedral BSCCO JJ 600-cell structure



does not fit in flat 3D space in a naturally characteristic way
(This is why icosahedral QuasiCrystal structures do not extend as simply throughout flat 3D space as do cuboctahedral structures).

However,
the BSCCO JJ 600-cell structure JItterbug icosahedra
do live happily in 3-sphere curved space within the icosahedral 120-cell
(see Wikipedia)



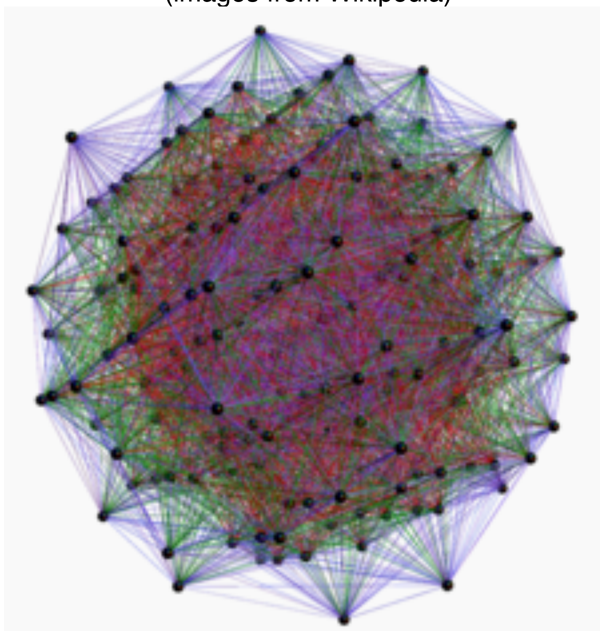
which has the same 720-edge arrangement as the 600-cell.

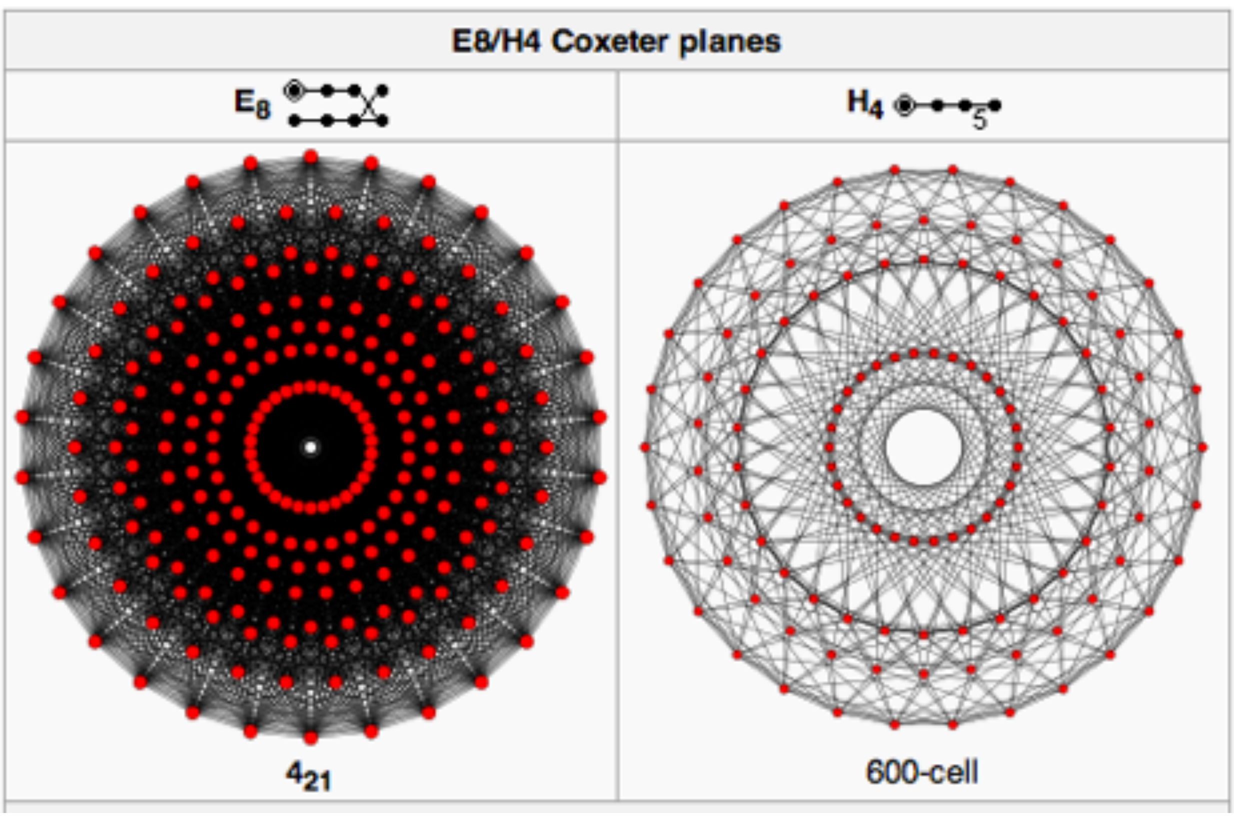
The icosahedral 120-cell is constructed by 5 icosahedra around each edge.
It has:

cells - 120 {3,5}
faces - 1200 {3}
edges - 720
vertices - 120
vertex figure - {5,5/2}
symmetry group H4,[3,3,5]
dual - small stellated 120-cell

6720 edges make an 8-dimensional Witting 4_21 Polytope

(images from Wikipedia)





Wikipedia said "... The 4_21 is related to the 600-cell by a geometric folding of the Coxeter-Dynkin diagrams. This can be seen in the E8/H4 Coxeter plane projections. The 240 vertices of the 4_21 polytope are projected into 4-space as two copies of the 120 vertices of the 600-cell, one copy smaller than the other [by the Golden Ratio] with the same orientation. Seen as a 2D orthographic projection in the E8/H4 Coxeter plane, the 120 vertices of the [larger] 600-cell are projected in the same four rings as seen in the 4_21. The other 4 rings of the 4_21graph ... match ... the four rings of the ... smaller ... 600-cell.

...

The 4_21 ... is the vertex figure for a uniform tessellation of 8-dimensional space, represented by symbol 5_21

...

The vertex arrangement of 521 is called the E8 lattice ... the E8 lattice can ... be constructed as a union of the vertices of two 8-demicube honeycombs (called a D82 or D8+ lattice)

...

Each point of the E8 lattice is surrounded by 2160 8-orthoplexes and 17280 8-simplices. The 2160 deep holes near the origin are exactly the halves of the norm 4 lattice points. The 17520 norm 8 lattice points fall into two classes (two orbits under the action of the E8 automorphism group): 240 are twice the norm 2 lattice points while 17280 are 3 times the shallow holes surrounding the origin

...".

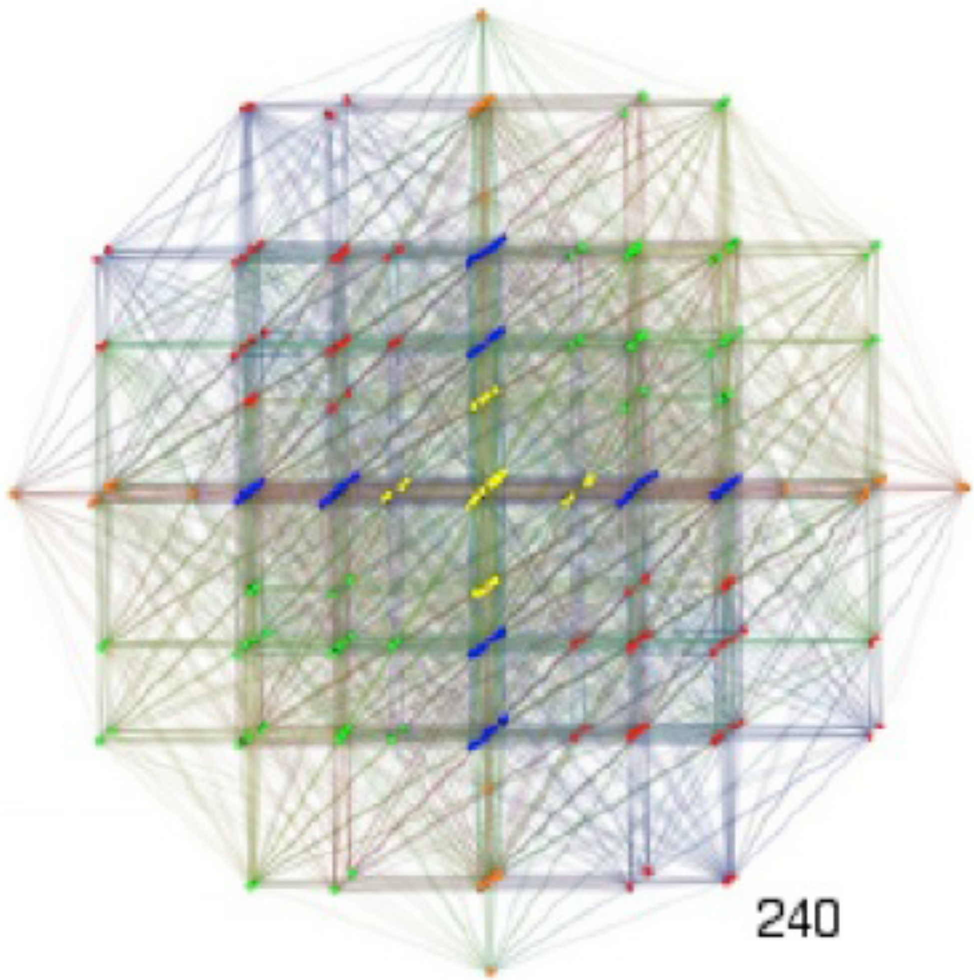
An E8 lattice represents an Integral Domain of the Octonions.
There are 480 Octonion Multiplications and
there are 7 E8 lattice Octonion Integral Domains.

The structures of E8 Physics (including Conformal Gravity Dark Energy) are naturally compatible with a 6720-edge configuration.

Klee Irwin is also working on projections of E8 lattices
into 3-space QuasiCrystal structures that might be relevant
in designing useful configurations of BSCCO JJ.

E8 Root Vector Physical Interpretations

Here is an explicit enumeration of the E8 Root Vector vertices with coordinates for a specific E8 lattice and my physical interpretation of each with illustrations using a cube-type projection of the 240 E8 Root Vector vertices:



E8 248 generators: 240 Root Vectors + 8 in Cartan Subalgebra

220 generators are used to construct a CG + SM Lagrangian
CG = Conformal Gravity U(2,2)   SM = Standard Model SU(3)xU(2).

All 248 = 28 + 220 are used to construct a Quantum Heisenberg-type algebra that arises from the maximal contraction of E8:

E8 -> SL(8) + h_92

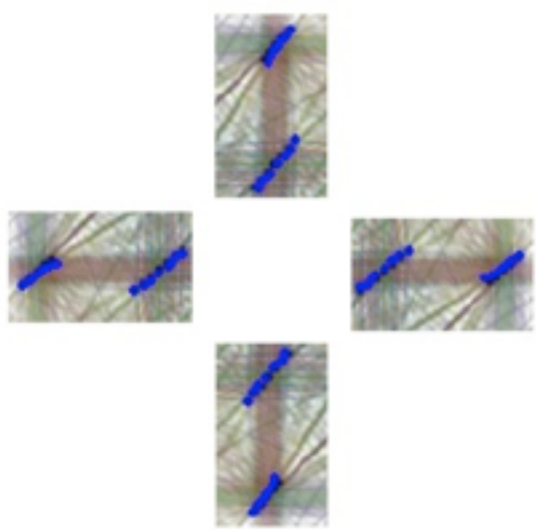
SL(8) is 63-dimensional and h_92 is 92+1+92 = 185-dimensional.
First 92: 64 fermion particle + 16 CG + 12 h92DualSM
Dual 92: 64 fermion antiparticle + 12 SM + 16 h92DualCG

Spacetime 64 of the 248

±1, ±i, ±j, ±k, ±e, ±ie, ±je, ±ke

(±1	±i			±e	±ie			)/2
(±1		±j		±e		±je		)/2
(±1			±k	±e			±ke	)/2

The 64 correspond to 8 position x 8 momentum coordinates
in a 4+4 = 8-dim Kaluza-Klein spacetime
with 4-dim Minkowski physical spacetime
plus 4-dim Internal Symmetry Space



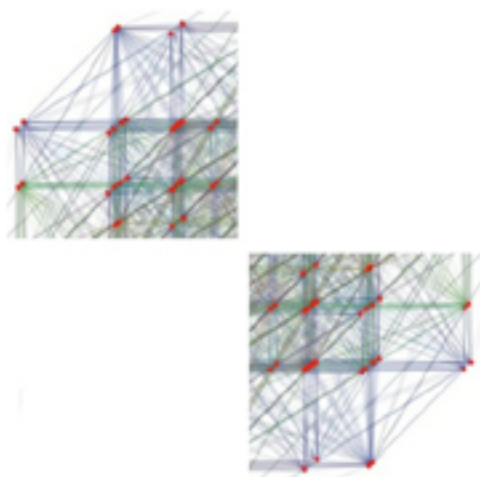
Fermion Particles   64 of the 248
(first generation)

(-1					±ie	±je	±ke	)/2	electron	8 components
(-1		±j	±k		±ie			)/2	red up quark	8 components
(-1	±i		±k			±je		)/2	green up quark	8 components
(-1	±i	±j					±ke	)/2	blue up quark	8 components
(	±i	±j	±k	-e				)/2	neutrino	8 components
(	±i			-e		±je	±ke	)/2	red down quark	8 components
(		±j		-e	±ie		±ke	)/2	green down quark	8 components
(			±k	-e	±ie	±je		)/2	blue down quark	8 components

The 64 correspond to 8 spacetime components of 8 fundamental fermion particles. The 8 components of each fermion are determined by the signs of the i/ie and j/je and k/ke as follows:

+++	1-component	---	e-component
++-	i-component	--+	ie-component
+-+	j-component	-+-	je-component
-++	k-component	+--	ke-component

All fermion particles are fundamentally left-handed. Right-handed states only emerge due to massive states moving slower than the speed of light. Second and third generations of fermions emerge dynamically from the splitting of 8-dim Octonion spacetime into 4+4 = 8-dim Kaluza-Klein.



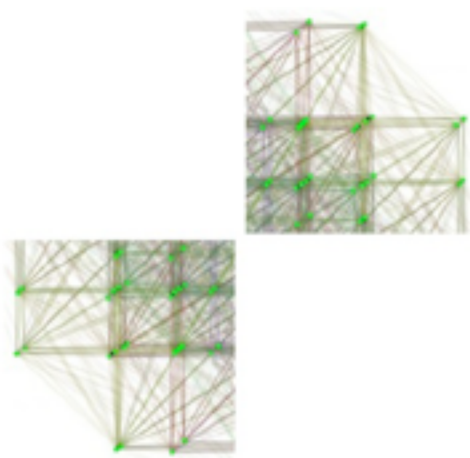
Fermion AntiParticles 64 of the 248
(first generation)

(-1					±ie	±je	±ke	)/2	positron	8 components
(-1		±j	±k		±ie			)/2	red up antiquark	8 components
(-1	±i		±k			±je		)/2	green up antiquark	8 components
(-1	±i	±j					±ke	)/2	blue up antiquark	8 components
(	±i	±j	±k	-e				)/2	antineutrino	8 components
(	±i			-e		±je	±ke	)/2	red down antiquark	8 components
(		±j		-e	±ie		±ke	)/2	green down antiquark	8 components
(			±k	-e	±ie	±je		)/2	blue down antiquark	8 components

The 64 correspond to 8 spacetime components of 8 fundamental fermion antiparticles. The 8 components of each fermion are determined by the signs of the i/ie and j/je and k/ke as follows:

+++	1-component	---	e-component
++-	i-component	--+	ie-component
+-+	j-component	-+-	je-component
-++	k-component	+--	ke-component

All fermion particles are fundamentally left-handed. Right-handed states only emerge due to massive states moving slower than the speed of light. Second and third generations of fermions emerge dynamically from the splitting of 8-dim Octonion spacetime into 4+4 = 8-dim Kaluza-Klein.

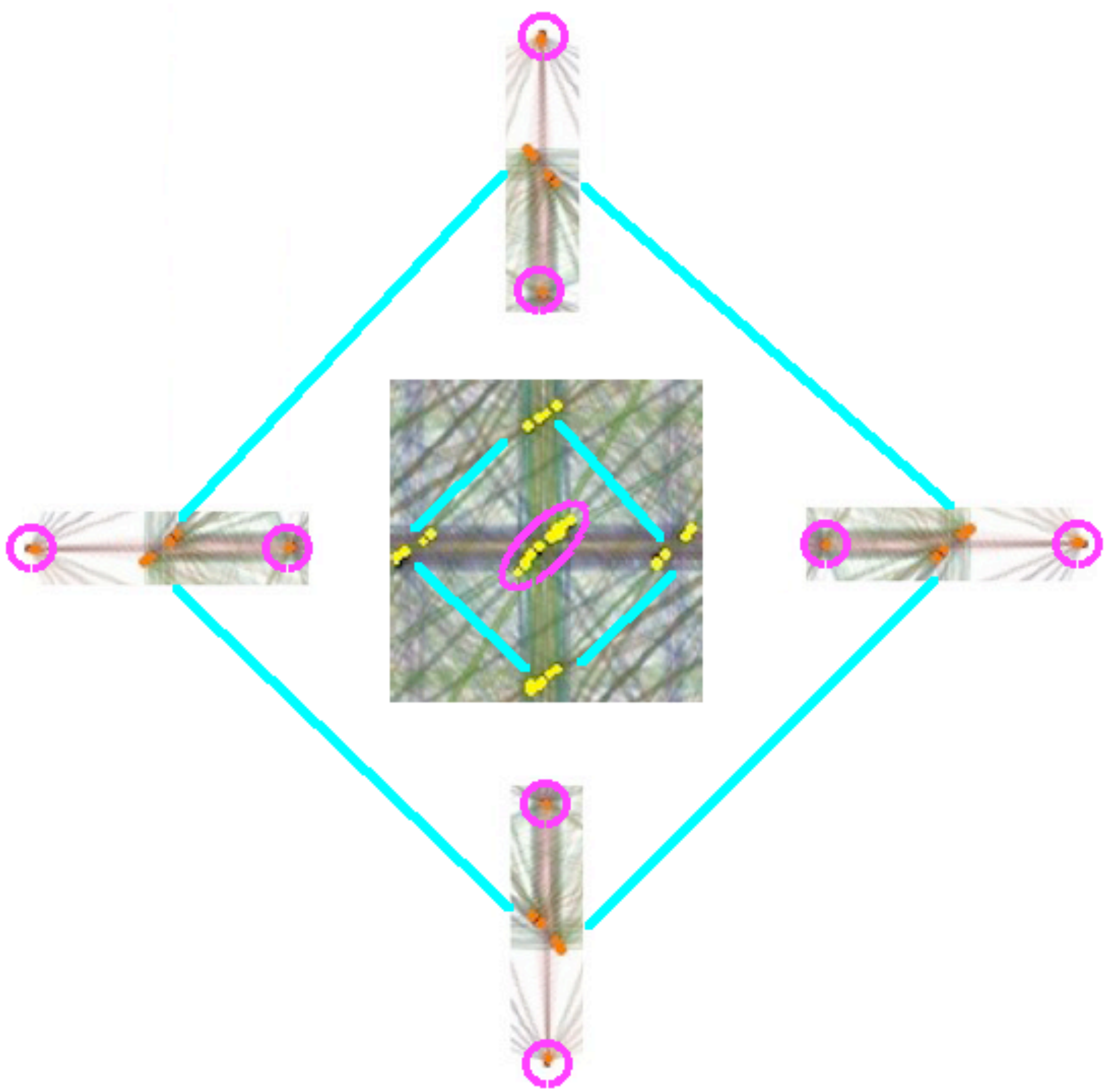


Standard Model SU(3)xSU(2)xU(1) generators are in
the 16 root vectors (magenta) plus 4 Cartan Subalgebra elements.

Conformal Spin(2,4) MacDowell-Mansouri Gravity generators are in
the 32 root vectors (cyan) plus 4 Cartan Subalgebra elements.

Those 16+32 = 48 Root Vectors can also be seen as
24 (yellow) + 24 (orange) of the two D4 subalgebras of E8.

```
(    ±i ±j       ±ie ±je     )/2
(       ±j ±k        ±je ±ke )/2
(    ±i    ±k    ±ie     ±ke )/2
```



16 Standard Model Root Vector Generators:

```
(    +i +j       +ie +je     )/2  W+ boson

(    +i +j       +ie -je     )/2  h92DualW+
(    +i +j       -ie +je     )/2  h92DualGlrg
(    +i -j       +ie +je     )/2  h92DualGlcm
(    -i +j       +ie +je     )/2  h92DualGlgm

(    +i +j       -ie -je     )/2  gluon_rg
(    -i -j       +ie +je     )/2  gluon_cm
(    +i -j       +ie -je     )/2  gluon_gb
(    -i +j       -ie +je     )/2  gluon_my
(    +i -j       -ie +je     )/2  gluon_br
(    -i +j       +ie -je     )/2  gluon_yc

(    -i -j       -ie +je     )/2  h92DualW-
(    -i -j       +ie -je     )/2  h92DualGlmy
(    -i +j       -ie -je     )/2  h92DualGlbr
(    +i -j       -ie -je     )/2  h92DualGlyc

(    -i -j       -ie -je     )/2  W- boson
```

4 Cartan = gamma and W0 and gluon_rgb and gluon_cmy
(note that gamma + W0 give photon + Z0)

The 8 (yellow) root vectors for W+ and W- and 6 gluons
are within the central (yellow) 24 of one D4 (D4SM) in E8.

The 8 (orange) root vectors for fermion connectors
are within the outer (orange) 24 of the other D4 (D4G) in E8.

The 16 (orange) root vectors for 4 Higgs and 12 Gravity bosons
are within the outer (orange) 24 of D4G in E8.

The 16 (yellow) root vectors for position/momentum connectors
are within the inner (yellow) 24 of D4SM in E8.

The 12 Standard Model generators live in the D4SM of E8
with 4 of the 8 Cartan Subalgebra elements of D8.
D4SM has an A3 = SU(4) subalgebra that contains color SU(3).

The 16 Conformal Gravity generators live in the D4G of E8
with 4 of the 8 Cartan Subalgebra elements of D8.
D4G has a Conformal A3=SU(2,2)=Spin(2,4) subalgebra.

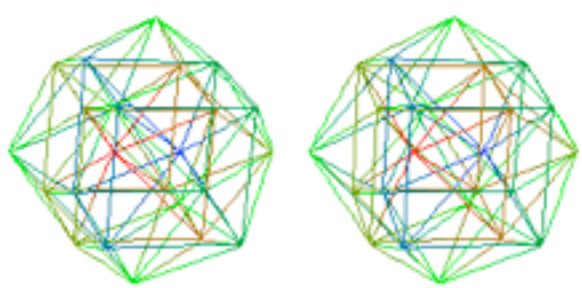
32 Conformal MacDowell-Mansouri Gravity Root Vector generators:

```
(       +j +k        +je +ke )/2  h92Dualgamma
(       +j +k        +je -ke )/2  h92DualC1
(       +j +k        -je +ke )/2  h92DualCi
(       +j -k        +je +ke )/2  h92DualCj
(       -j +k        +je -+e )/2  h92DualCk
(       +j +k        -je -ke )/2  conformal_rxy
(       -j -k        +je +ke )/2  conformal_rxz
(       +j -k        +je -ke )/2  conformal_1
(       -j +k        -je +ke )/2  conformal_i
(       +j -k        -je +ke )/2  conformal_j
(       -j +k        +je -ke )/2  conformal_k
(       -j -k        -je +ke )/2  h92DualCrxy
(       -j -k        +je -ke )/2  h92DualCrxz
(       -j +k        -je -ke )/2  h92DualCryz
(       +j -k        -je -ke )/2  h92DualCd
(       -j -k        -je -ke )/2  h92DualW0

(    +i    +k    +ie     +ke )/2  h92DualGlrgb
(    +i    +k    +ie     -ke )/2  h92DualCe
(    +i    +k    -ie     +ke )/2  h92DualCie
(    +i    -k    +ie     +ke )/2  h92DualCje
(    -i    +k    +ie     +ke )/2  h92DualCke
(    +i    +k    -ie     -ke )/2  conformal_btx
(    -i    -k    +ie     +ke )/2  conformal_bty
(    +i    -k    +ie     -ke )/2  conformal_e
(    -i    +k    -ie     +ke )/2  conformal_ie
(    +i    -k    -ie     +ke )/2  conformal_je
(    -i    +k    +ie     -ke )/2  conformal_ke
(    -i    -k    -ie     +ke )/2  h92DualCbtx
(    -i    -k    +ie     -ke )/2  h92DualCbty
(    -i    +k    -ie     -ke )/2  h92DualCbtz
(    +i    -k    -ie     -ke )/2  h92DualPrPh
(    -i    -k    -ie     -ke )/2  h92Dualcmy
```

4 Cartan = conformal_ryz and conformal_btz and conformal_d
and 1 Propagator Phase

Here are how the 48 Standard Model + Gravity Root Vectors appear with respect to decomposition into D4SM + D4G:

```
24 Standard Model Root Vector Generators of D4SM:

(    +i +j       +ie +je     )/2  W+ boson

(       +j +k        +je -ke )/2  h92DualC1
(       +j +k        -je +ke )/2  h92DualCi
(       +j -k        +je +ke )/2  h92DualCj
(       -j +k        +je -+e )/2  h92DualCk
(       -j -k        -je +ke )/2  h92DualCrxy
(       -j -k        +je -ke )/2  h92DualCrxz
(       -j +k        -je -ke )/2  h92DualCryz
(       +j -k        -je -ke )/2  h92DualCd

(    +i +j       -ie -je     )/2  gluon_rg
(    -i -j       +ie +je     )/2  gluon_cm
(    +i -j       +ie -je     )/2  gluon_gb
(    -i +j       -ie +je     )/2  gluon_my
(    +i -j       -ie +je     )/2  gluon_br
(    -i +j       +ie -je     )/2  gluon_yc

(    +i    +k    +ie     -ke )/2  h92DualCe
(    +i    +k    -ie     +ke )/2  h92DualCie
(    +i    -k    +ie     +ke )/2  h92DualCje
(    -i    +k    +ie     +ke )/2  h92DualCke
(    -i    -k    -ie     +ke )/2  h92DualCbtx
(    -i    -k    +ie     -ke )/2  h92DualCbty
(    -i    +k    -ie     -ke )/2  h92DualCbtz
(    +i    -k    -ie     -ke )/2  h92DualPrPh

(    -i -j       -ie -je     )/2  W- boson

4 Cartan = gamma and W0 and gluon_rgb and gluon_cmy
(note that gamma + W0 give photon + Z0)

D4SM Root Vectors form a 24-cell with 1+8+6+8+1 structure
(dual to D4G) of vertex + cube + octahedron + cube + vertex
```

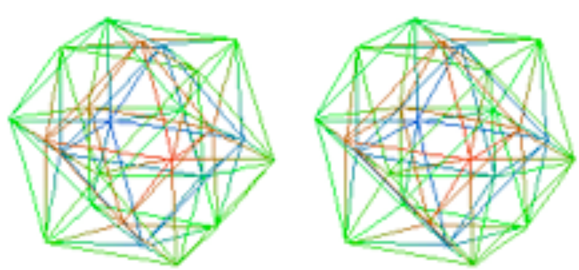


24 Conformal Gravity Root Vector generators of D4G:

```
(       +j +k        +je +ke )/2  h92Dualgamma
(    +i    +k    +ie     +ke )/2  h92DualGlrgb
(    +i +j       +ie -je     )/2  h92DualW+
(    +i +j       -ie +je     )/2  h92DualGlrg
(    +i -j       +ie +je     )/2  h92DualGlcm
(    -i +j       +ie +je     )/2  h92DualGlgm

(       +j -k        +je -ke )/2  conformal_1
(       -j +k        -je +ke )/2  conformal_i
(       +j -k        -je +ke )/2  conformal_j
(       -j +k        +je -ke )/2  conformal_k
(       +j +k        -je -ke )/2  conformal_rxy
(       -j -k        +je +ke )/2  conformal_rxz
(    +i    +k    -ie     -ke )/2  conformal_btx
(    -i    -k    +ie     +ke )/2  conformal_bty
(    +i    -k    +ie     -ke )/2  conformal_e
(    -i    +k    -ie     +ke )/2  conformal_ie
(    +i    -k    -ie     +ke )/2  conformal_je
(    -i    +k    +ie     -ke )/2  conformal_ke

(    -i -j       -ie +je     )/2  h92DualW-
(    -i -j       +ie -je     )/2  h92DualGlmy
(    -i +j       -ie -je     )/2  h92DualGlbr
(    +i -j       -ie -je     )/2  h92DualGlyc
(       -j -k        -je -ke )/2  h92DualW0
(    -i    -k    -ie     -ke )/2  h92DualGlcmy
```

4 Cartan = conformal_ryz and conformal_btz and conformal_d
and 1 propagator phase

D4G Root Vectors form a 24-cell with 6+12+6 structure
(dual to D4SM) of octahedron + cuboctahedron + octahedron



E8 Lattices

E8 Lattices are based on Octonions, which have 480 different multiplication products. E8 Lattices can be combined to form 24-dimensional Leech Lattices and 26-dimensional Bosonic String Theory, which describes E8 Physics when the strings are physically interpreted as World-Lines. A basic String Theory Cell has as its automorphism group the Monster Group whose order is
2^46 .3^20 .5^9 .7^6 .11^2 .13^3 .17.19.23.29.31.41.47.59.71 = about 8 x 10^53.

For more about the Leech Lattice and the Monster and E8 Physics,
see viXra 1210.0072 and 1108.0027 .

E8 Root systems and lattices are discussed by Robert A. Wilson in his 2009 paper "Octonions and the Leech lattice":
"... The (real) octonion algebra is an 8-dimensional (non-division) algebra with an orthonomal basis { 1=ioo , i0 , i1 , i2 , i3 , i4 , i5 , i6 } labeled
by the projective line PL(7) = { oo } u F7
...
The E8 root system embeds in this algebra ... take the 240 roots to be ...
112 octonions ... +/- it +/- iu for any distinct t,u
... and ...
128 octonions (1/2)(+/- 1 +/- i0 +/- ... +/- i6) ...[with]... an odd number of minus signs.
Denote by L the lattice spanned by these 240 octonions
...
Let s = (1/2)(- 1 + i0 + ... + i6) so s is in L ... write R for Lbar ...
...
(1/2) (1 + i0) L = (1/2) R (1 + i0) is closed under multiplication ... Denote this ...by A
... Writing **B = (1/2) (1 + i0) A (1 + i0)** ...from ... Moufang laws ... we have
L R = 2 B , and ... B L = L and R B = R ...[also]... 2 B = L sbar
...
the roots of B are
[**16 octonions**]... +/- it for t in PL(7)
... together with
[**112 octonions**]... (1/2) (+/- 1 +/- it +/- i(t+1) +/- i(t+3)) ...for t in F7
... and ...
[**112 octonions**]... (1/2) (+/- i(t+2) +/- i(t+4) +/- i(t+5) +/- i(t+6)) ...for t in F7
...
B is not closed under multiplication ... Kirmse's mistake
...[but]... as Coxeter ... pointed out ...
... **there are seven non-associative rings At** = (1/2) (1 + it) B (1 + it) ,
obtained from B by swapping 1 with it ... for t in F7
...
LR = 2B and BL = L ...[which]... appear[s] not to have been noticed before ... some work ... by Geoffrey Dixon ...".

Geoffrey Dixon says in his book "Division Algebras, Lattices, Physics, Windmill Tilting" using notation {e0,e1,e2,e3,e4,e5,e6,e7} for the Octonion basis elements that Robert A. Wilson denotes by {1=ioo,i0,i1,i2,i3,i4,i5 ,i6} and I sometimes denote by {1,i,j,k,e,ie,je,ke}: "...

$$\Xi_0 = \{\pm e_a\},$$
$$\Xi_2 = \{(\pm e_a \pm e_b \pm e_c \pm e_d)/2 : a,b,c,d \text{ distinct}, e_a(e_b(e_c e_d)) = \pm 1\},$$

$$\Xi^{even} = \Xi_0 \cup \Xi_2,$$
$$\mathcal{E}_8^{even} = \text{span}\{\Xi^{even}\},$$

$$\Xi_1 = \{(\pm e_a \pm e_b)/\sqrt{2} : a,b \text{ distinct}\},$$
$$\Xi_3 = \{(\textstyle\sum_{a=0}^{7} \pm e_a)/\sqrt{8} : \text{ even number of +'s}\},$$

$$\Xi^{odd} = \Xi_1 \cup \Xi_3,$$
$$\mathcal{E}_8^{odd} = \text{span}\{\Xi^{odd}\}$$

(spans over integers)
Ξeven has 16+224 = 240 elements ... Ξodd has 112+128 = 240 elements ...
E8even does not close with respect to our given octonion multiplication
...[but]...
the set Ξeven[0-a], derived from Ξeven by replacing each occurrence of e0 ... with ea, and vice versa, is multiplicatively closed. ...".

Geoffrey Dixon's Ξeven corresponds to Wilson's B which I denote as 1E8.

Geoffrey Dixon's Ξeven[0-a] correspond to Wilson's seven At
which I denote as iE8, jE8, kE8, eE8, ieE8, jeE8, keE8.

Geoffrey Dixon's Ξodd corresponds to Wilson's L.

My view is that **the E8 domains 1E8 = Ξeven = B is fundamental**
because
E8 domains iE8, jE8, kE8, eE8, ieE8, jeE8, keE = Ξeven[0-a] are derived from 1E8
and L and L s are also derived from 1E8 = Ξeven = B.

Using the notation {1,i,j,k,e,ie,je,ke} for Octonion basis
notice that in E8 Physics introduction of Quaternionic substructure
to produce (4+4)-dim M4 x CP2 Kaluza-Klein SpaceTime
requires breaking Octonionic light-cone elements
(+/- 1 +/- i +/- j +/- k +/- e +/- ie +/- je +/- ke) / 2
into Quaternionic 4-term forms like (+/- A +/- B +/- C +/- D) / 2.

To do that, consider that there are (8l4) = 70 ways to choose 4-term subsets
of the 8 Octonionic basis element terms. Using all of them produces
224 4-term subsets in each of the 7 Octonion Imaginary E8 lattices
iE8,jE8,kE8,eE8,ieE8,jeE8,keE8 each of which also has 16 1-term first-shell vertices.

56 of the 70 4-term subsets appear as 8 in each of the 7 Octonion Imaginary E8 lattices.

The other 70-56 = 14 4-term subsets occur in sets of 3 among 7x6 = 42 4-term subsets
as indicated in the following detailed list of the 7 Octonion Imaginary E8 lattices:

eE8:

```
112 of D8 Root Vectors
16 appear in all 7 of iE8,jE8,kE8,eE8,ieE8,jeE8,keE8
±1, ±i, ±j, ±k, ±e, ±ie, ±je, ±ke
96 appear in 3 of iE8,jE8,kE8,eE8,ieE8,jeE8,keE8
(±1  ±ke ±e  ±k )/2     (±i  ±j  ±ie ±je)/2     kE8  , eE8  , keE8
(±1  ±je ±j  ±e )/2     (±ie ±ke ±k  ±i )/2     jE8  , eE8  , jeE8
(±1  ±e  ±ie ±i )/2     (±ke ±k  ±je ±j )/2     iE8  , eE8  , ieE8

128 of D8 half-spinors appear only in eE8
(±1  ±ie ±je ±ke)/2     (±e  ±i  ±j  ±k )/2
(±1  ±k  ±i  ±je)/2     (±j  ±ie ±ke ±e )/2
(±1  ±i  ±ke ±j )/2     (±k  ±je ±e  ±ie)/2
(±1  ±j  ±k  ±ie)/2     (±je ±e  ±i  ±ke)/2
```

iE8:

```
112 of D8 Root Vectors
16 appear in all 7 of iE8,jE8,kE8,eE8,ieE8,jeE8,keE8
±1, ±i, ±j, ±k, ±e, ±ie, ±je, ±ke
96 appear in 3 of iE8,jE8,kE8,eE8,ieE8,jeE8,keE8
(±1  ±ie ±i  ±e )/2     (±j  ±k  ±je ±ke)/2     iE8  , eE8  , ieE8
(±1  ±ke ±je ±i )/2     (±j  ±k  ±e  ±ie)/2     iE8  , jeE8 , keE8
(±1  ±i  ±k  ±j )/2     (±e  ±ie ±je ±ke)/2     iE8  , jE8  , kE8

128 of D8 half-spinors appear only in iE8
(±1  ±k  ±ke ±ie)/2     (±i  ±j  ±e  ±je)/2
(±1  ±e  ±j  ±ke)/2     (±i  ±k  ±ie ±je)/2
(±1  ±j  ±ie ±je)/2     (±i  ±k  ±e  ±ke)/2
(±1  ±je ±e  ±k )/2     (±i  ±j  ±ie ±ke)/2
```

jE8:

```
112 of D8 Root Vectors
16 appear in all 7 of iE8,jE8,kE8,eE8,ieE8,jeE8,keE8
±1, ±i, ±j, ±k, ±e, ±ie, ±je, ±ke
96 appear in 3 of iE8,jE8,kE8,eE8,ieE8,jeE8,keE8
(±1  ±k  ±j  ±i )/2     (±e  ±ie ±je ±ke)/2     iE8  , jE8  , kE8
(±1  ±ie ±ke ±j )/2     (±i  ±k  ±e  ±je)/2     jE8  , ieE8 , keE8
(±1  ±j  ±e  ±je)/2     (±i  ±k  ±ie ±ke)/2     jE8  , eE8  , jeE8

128 of D8 half-spinors appear only in jE8
(±1  ±e  ±ie ±k )/2     (±i  ±j  ±je ±ke)/2
(±1  ±i  ±je ±ie)/2     (±j  ±k  ±e  ±ke)/2
(±1  ±je ±k  ±ke)/2     (±i  ±j  ±e  ±ie)/2
(±1  ±ke ±i  ±e )/2     (±j  ±k  ±ie ±je)/2
```

kE8:

```
112 of D8 Root Vectors
16 appear in all 7 of iE8,jE8,kE8,eE8,ieE8,jeE8,keE8
±1, ±i, ±j, ±k, ±e, ±ie, ±je, ±ke
96 appear in 3 of iE8,jE8,kE8,eE8,ieE8,jeE8,keE8
(±1  ±je ±k  ±ie)/2     (±i  ±j  ±e  ±ke)/2     kE8  , ieE8 , jeE8
(±1  ±j  ±i  ±k )/2     (±e  ±ie ±je ±ke)/2     iE8  , jE8  , kE8
(±1  ±k  ±ke ±e )/2     (±i  ±j  ±ie ±je)/2     kE8  , eE8  , keE8

128 of D8 half-spinors appear only in kE8
(±1  ±ke ±j  ±je)/2     (±i  ±k  ±e  ±ie)/2
(±1  ±ie ±e  ±j )/2     (±i  ±k  ±je ±ke)/2
(±1  ±e  ±je ±i )/2     (±j  ±k  ±ie ±ke)/2
(±1  ±i  ±ie ±ke)/2     (±j  ±k  ±e  ±je)/2
```

ieE8:

```
112 of D8 Root Vectors
16 appear in all 7 of iE8,jE8,kE8,eE8,ieE8,jeE8,keE8
±1, ±i, ±j, ±k, ±e, ±ie, ±je, ±ke
96 appear in 3 of iE8,jE8,kE8,eE8,ieE8,jeE8,keE8
(±1  ±j  ±ie ±ke)/2     (±i  ±k  ±e  ±je)/2     jE8  , ieE8 , keE8
(±1  ±i  ±e  ±ie)/2     (±j  ±k  ±je ±ke)/2     iE8  , eE8  , ieE8
(±1  ±ie ±je ±k )/2     (±i  ±j  ±e  ±ke)/2     kE8  , ieE8 , jeE8

128 of D8 half-spinors appear only in ieE8
(±1  ±je ±i  ±j )/2     (±k  ±e  ±ie ±ke)/2
(±1  ±ke ±k  ±i )/2     (±j  ±e  ±ie ±je)/2
(±1  ±k  ±j  ±e )/2     (±i  ±ie ±je ±ke)/2
(±1  ±e  ±ke ±je)/2     (±i  ±j  ±k  ±ie)/2
```

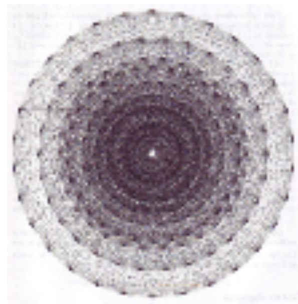
jeE8:

```
112 of D8 Root Vectors
16 appear in all 7 of iE8,jE8,kE8,eE8,ieE8,jeE8,keE8
±1, ±i, ±j, ±k, ±e, ±ie, ±je, ±ke
96 appear in 3 of iE8,jE8,kE8,eE8,ieE8,jeE8,keE8
(±1  ±e  ±je ±j )/2     (±i  ±k  ±ie ±ke)/2     jE8  , eE8  , jeE8
(±1  ±k  ±ie ±je)/2     (±i  ±j  ±e  ±ie)/2     kE8  , ieE8 , jeE8
(±1  ±je ±i  ±ke)/2     (±j  ±k  ±e  ±ie)/2     iE8  , jeE8 , keE8

128 of D8 half-spinors appear only in jeE8
(±1  ±i  ±k  ±e )/2     (±j  ±ie ±je ±ke)/2
(±1  ±j  ±ke ±k )/2     (±i  ±e  ±ie ±je)/2
(±1  ±ke ±e  ±ie)/2     (±i  ±j  ±k  ±je)/2
(±1  ±ie ±j  ±i )/2     (±k  ±e  ±je ±ke)/2
```

keE8:

```
112 of D8 Root Vectors
16 appear in all 7 of iE8,jE8,kE8,eE8,ieE8,jeE8,keE8
±1, ±i, ±j, ±k, ±e, ±ie, ±je, ±ke
96 appear in 3 of iE8,jE8,kE8,eE8,ieE8,jeE8,keE8
(±1  ±i  ±ke ±je)/2     (±j  ±k  ±e  ±ie)/2     iE8  , jeE8 , keE8
(±1  ±e  ±k  ±ke)/2     (±i  ±j  ±ie ±je)/2     kE8  , eE8  , keE8
(±1  ±ke ±j  ±ie)/2     (±i  ±k  ±e  ±je)/2     jE8  , ieE8 , keE8

128 of D8 half-spinors appear only in keE8
(±1  ±j  ±e  ±i )/2     (±k  ±ie ±je ±ke)/2
(±1  ±je ±ie ±e )/2     (±i  ±j  ±k  ±ke)/2
(±1  ±ie ±i  ±k )/2     (±j  ±e  ±je ±ke)/2
(±1  ±k  je  ±j )/2     (±i  ±e  ±ie ±ke)/2
```

Coxeter said in "Integral Cayley Numbers" (Duke Math. J. 13 (1946) 561-578 and in "Regular and Semi-Regular Polytopes III" (Math. Z. 200 (1988) 3-45):
"... the 240 integral Cayley numbers of norm1 ... are the vertices of 4_21



... ...

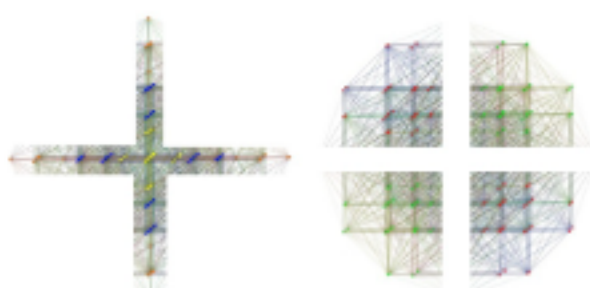
The polytope 4_21 ... has cells of two kinds ...
a seven-dimensional "cross polytope" (or octahedron-analogue) B_7
... there are ... 2160 B_7's ...
and ...
a seven-dimensional regular simplex A_7
... there are 17280 A_7's
...
the 2160 integral Cayley numbers of norm 2 are
the centers of the 2160 B_7's of a 4_21 of edge 2
...
the 17280 integral Cayley numbers of norm 4 (other than the doubles of those of norm 1) are the centers of the 17280 A_7's of a 4_21 of edge 8/3 ...

[Using notation of {a1,a2,a3,a4,a5,a6,a7,a8} for Octonion basis elements we have]

norm 1

112 like (+/- a1 +/- a2)
[which correspond to 112 = 16 + 96 = 16 + 6x16 in each of the 7 E8 lattices]

128 like (1/2) (- a1 + a2 + a3 + ... + a8) with an odd number of minus signs
[which correspond to 128 = 8x16 in each of the 7 E8 lattices]



112 **128**

norm 2

16 like +/- 2 a1
[which correspond to 16 fo the 112 in each of the 7 E8 lattices]

1120 like +/- a1 +/- a2 +/- a3 +/- a4
[which correspond to 70x16 = (56+14)x16 that appear in the 7 E8 lattices

with each of the 14 appearing in three of the 7 E8 lattices so that
the 14 account for (14/7)x3x16 = 6x16 = 96 in each of the 7 E8 lattices
and for 14x16 = **224 of the 1120**
and
with each of the 56 appearing in only one of the 7 E8 lattices so that
the 56 account for (56/7)x16 = 128 in each of the 7 E8 lattices
and for 56x16 = **896 = 7x128 of the 1120**]

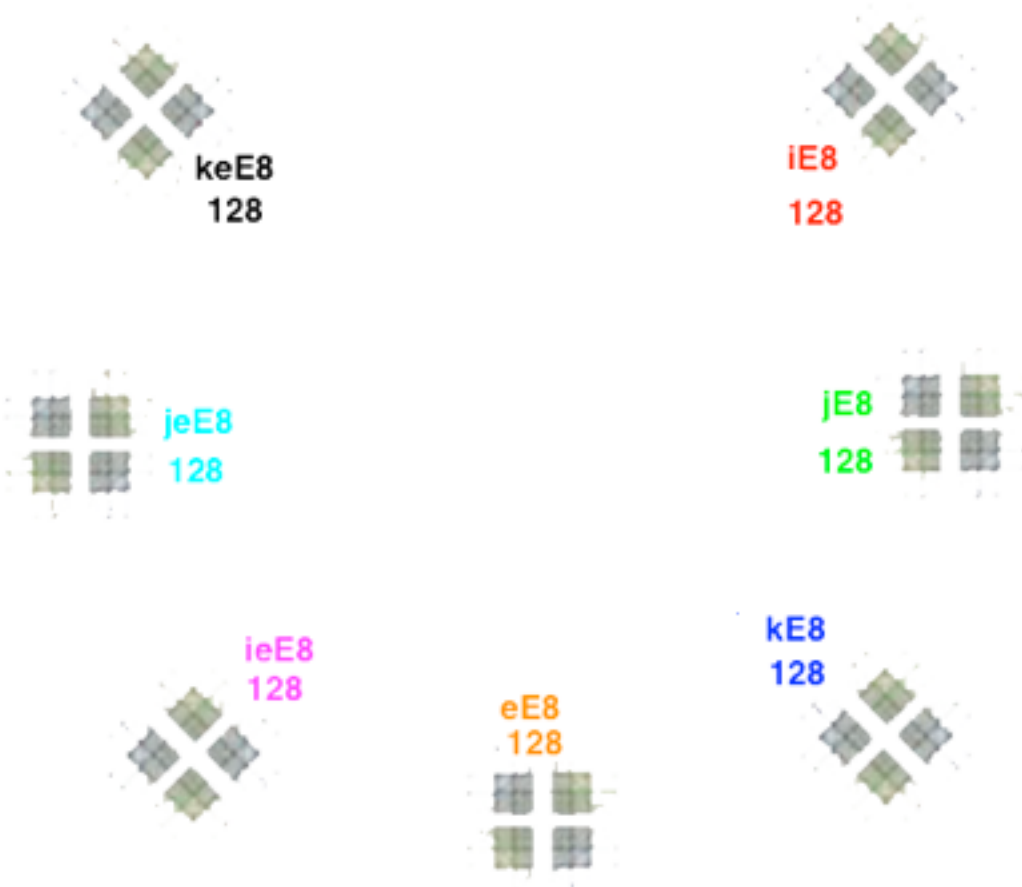
1024 like (1/2)(3a1 + 3a2 + a3 + a4 + ... + a8) with an even number of minus signs
[which correspond to **8x128** = 8 copies of the 128-dim Mirror D8 half-spinors that
are not used in the 7 E8 lattices. ...] ...".

One of the 128-dimensional Mirror D8 half-spinors from the 1024
combines with
the 128 from the 1120 corresponding to the one of the 7 E8 lattices that corresponds
to the central norm 1 240 = 112+128
and
the result is formation of a 128+128 = 256 corresponding to the Clifford Algebra Cl(8)
so that
the norm 2 second layer contains 7 copies of 256-dimensional Cl(8)

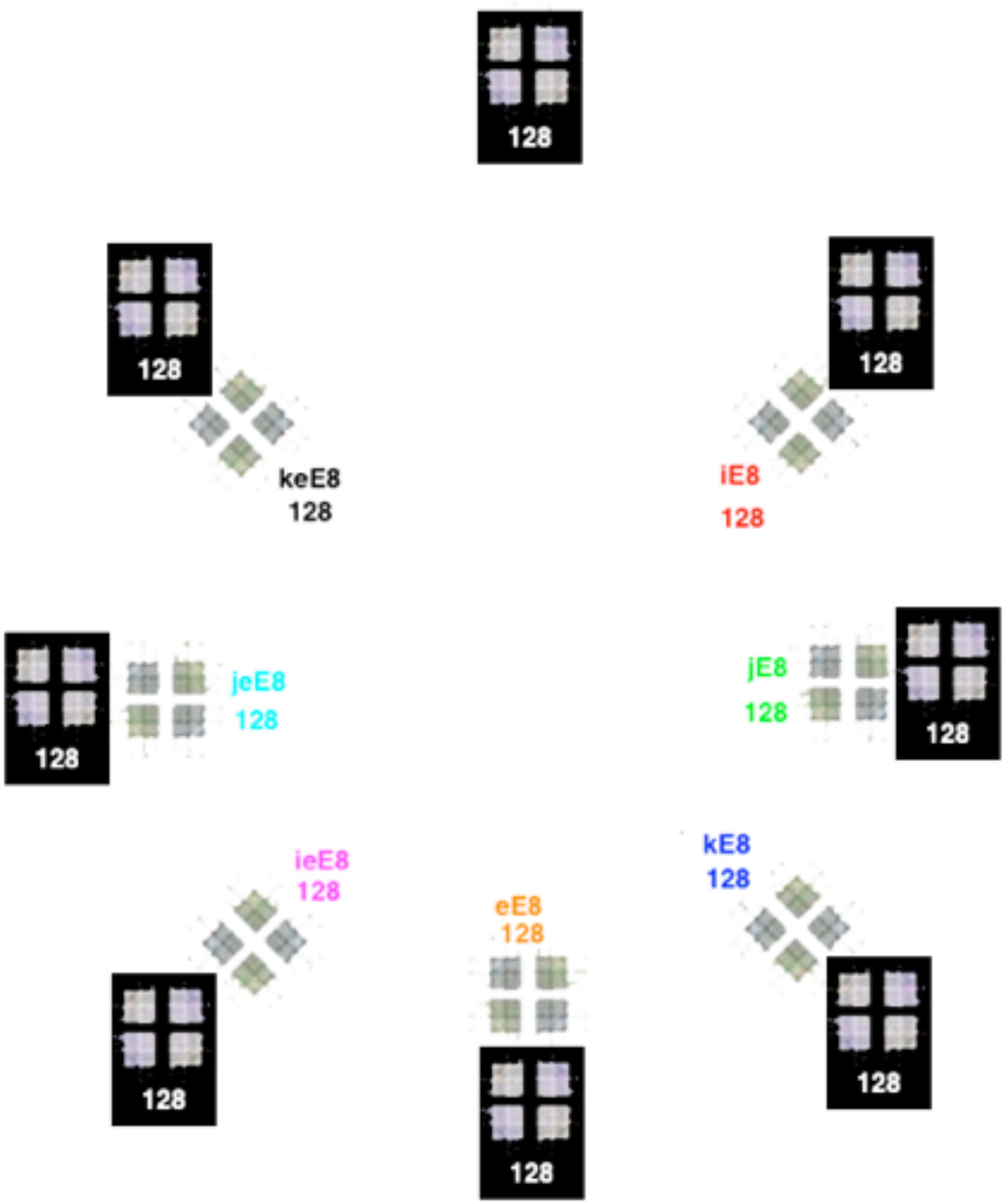
so the 2160 norm 2 vertices can be seen as

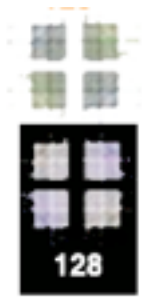
7(128+128) + 128 + 16 + 224 = 2160 vertices.

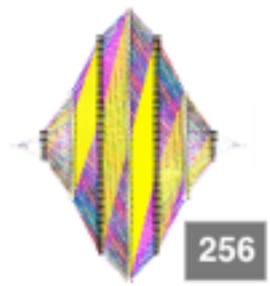
7x128 from the 1120 are the D8 half-spinor vertices of iE8, jE8, kE8, eE8, ieE8, jeE8, keE8



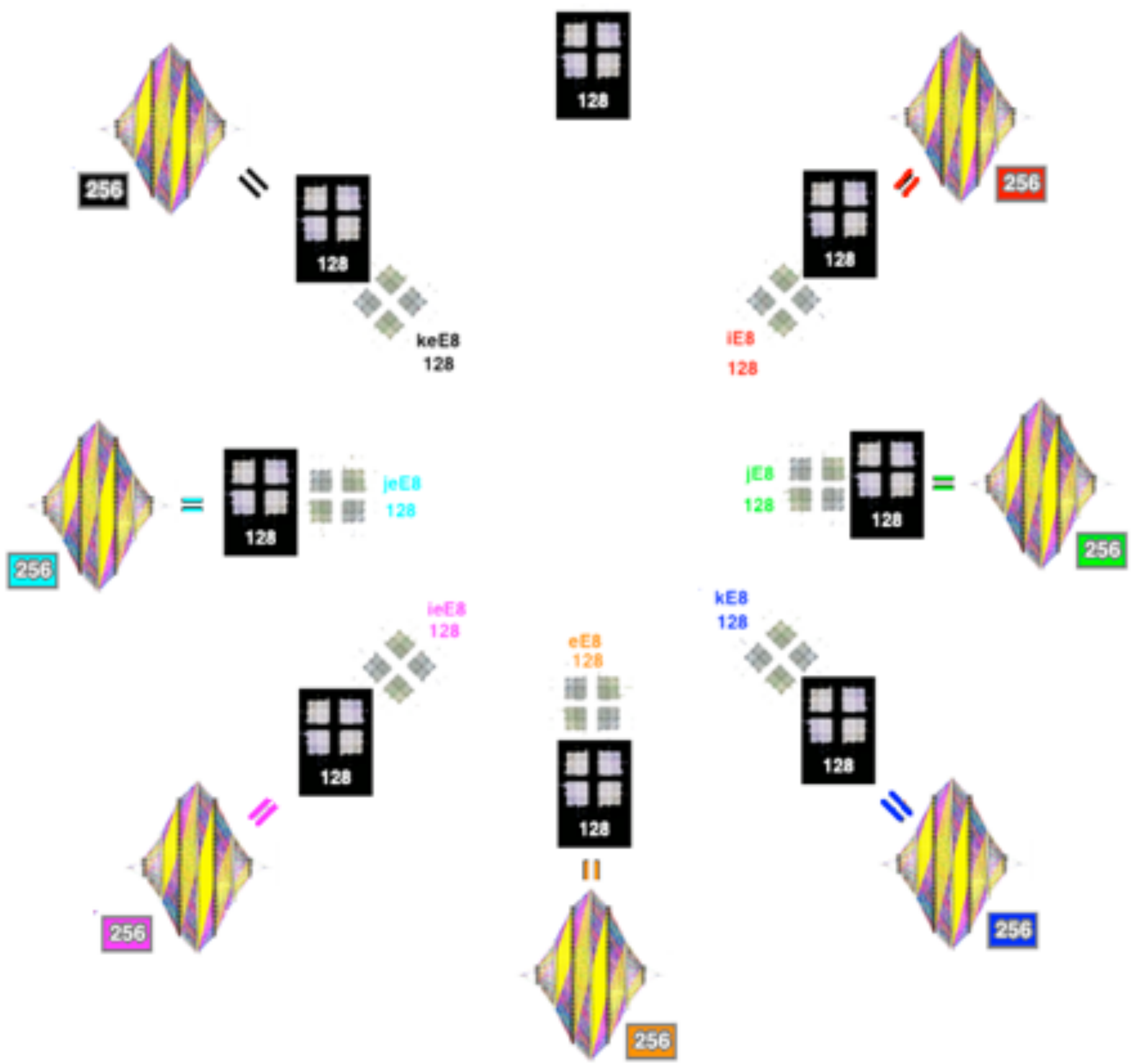
7x128 from the 1024 are Mirror D8 half-spinors that are not vertices of the 7 Imaginary E8 lattices iE8, jE8, kE8, eE8, ieE8, jeE8, keE8.
The 8th 128 is a Mirror D8 half-spinor, also not in the 7 Imaginary E8 lattices.



Each of the 7 pairs of 128 

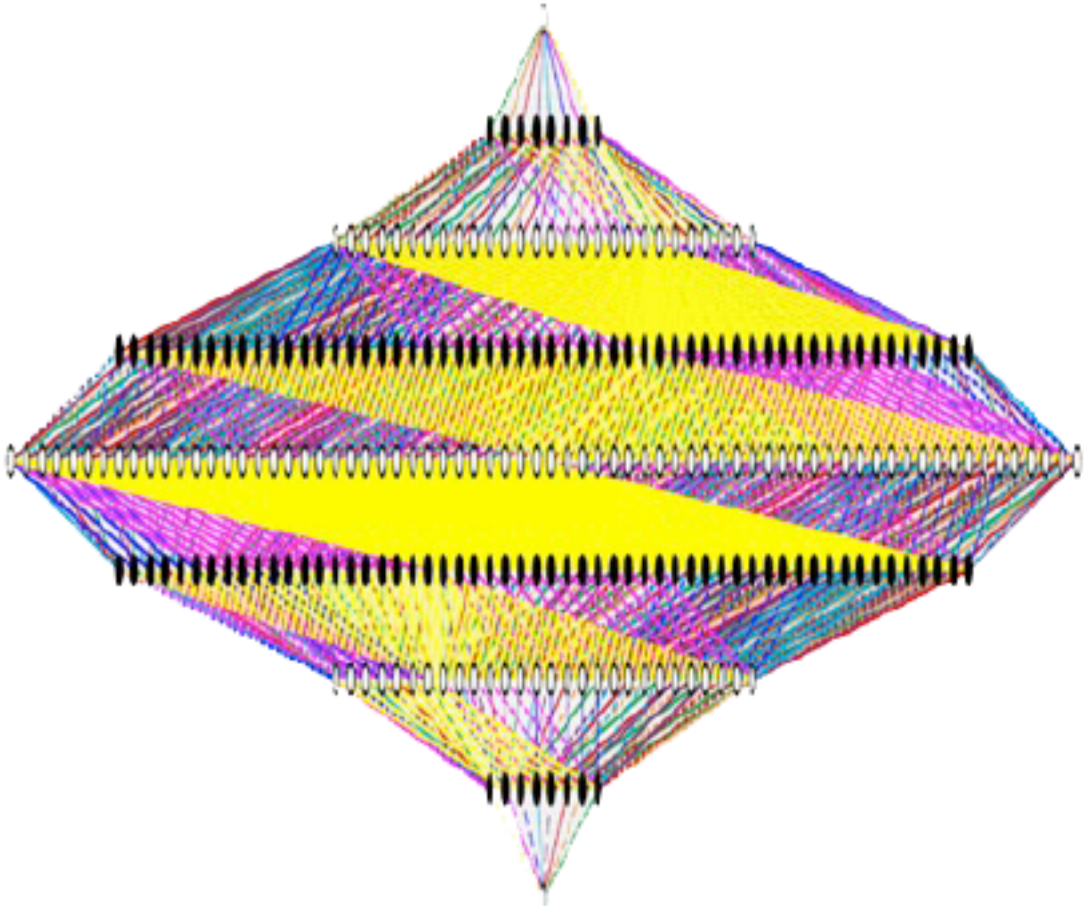
corresponds to 

a 256 Cl(8)



so that the 2160 second layer contains 7 sets of 256 vertices with each set corresponding to the Cl(8) Clifford Algebra and to the 256 vertices of an 8-dimensional light-cone (+/- 1 +/- i +/- j +/- k +/- e +/- ie +/- je +/- ke) / 2

The 256 vertices of each pair 128+128 form an 8-cube with 1024 edges, 1792 square faces, 1792 cubic cells, 1120 tesseract 4-faces, 448 5-cube 5-faces, 112 6-cube 6-faces, and 16 7-cube 7-faces. The image format of African Adinkra for 256 Odu of IFA



shows Cl(8) graded structure 1 + 8 + 28 + 56 + 70 + 56 + 28 + 8 + 1 of 8-cube vertices. Physically they represent **Operators in H92 x Sl(8)** Generalized Heisenberg Algebra that is the **Maximal Contraction of E8**:

Odd-Grade Parts of Cl(8) =
= **128 D8 half-spinors** of one of iE8, jE8, kE8, eE8, ieE8, jeE8, keE8
8+56 grades-1,3 = Fermion Particle 8-Component Creation (AntiParticle Annihilation)
56+8 grades-5,7 = Fermion AntiParticle 8-Component Creation (Particle Annihilation)

Even-Grade Subalgebra of Cl(8) = **128 Mirror D8 half-spinors =**
28 grade-2 = Gauge Boson Creation (16 for Gravity, 12 for Standard Model)
28 grade-6 = Gauge Boson Annihilation (16 for Gravity , 12 for Standard Model)
(each 28 = 24 Root Vectors + 4 of Cartan Subalgebra)
64 of grade-4 = 8-dim Position x Momentum
1+(3+3)+1 grades-0,4,8 = Primitive Idempotent:
(1+3) = Higgs Creation; (3+1) = Higgs Annihilation
= **112 D8 Root Vectors + 8 of E8 Cartan Subalgebra + 8 Higgs Operators**

8 of E8 Cartan Subalgebra + 8 Higgs Operators = 2 copies of 4-dim 16-cell

(images from Bathsheba)


The 16-cell has 24 edges, midpoints of which are the 24 vertices of a 24-cell.
The 24-cell has 96 edges, Golden Ratio points of which when added to its 24 vertices, form the 96+24 = 120 vertices of a 600-cell.

128 vertices of the D8 half-spinors + 112 vertices of D8 Root Vectors = 240 =
= 2 copies of 4-dim {3,3,5} 600-cell (images from Bathsheba)


Each 600-cell lives inside a 16-cell.

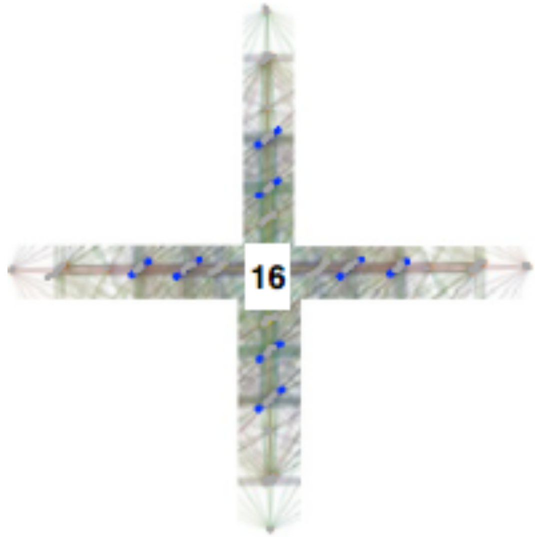
So,
the 256 vertices of **Cl(8)**
(which represents Creation/Annihilation Operators in the Generalized Heisenberg Algebra H92 x Sl(8) that is the Maximal Contraction of E8)
contain
dual 16-cell structure of E8 Cartan Subalgebra + Cl(8) Primitive Idempotent Higgs
as well as
the dual 600-cell structure of the 240 E8 Root Vector vertices

The 128 Mirror D8 half-spinors correspond to 16 + 112 of the 16 + 224.

The 16 + 224 corresponds to an 8th set of 240 Root Vector vertices for an 8th E8 lattice denoted 1E8.

It does not close under the Octonion Product used for the 7 Imaginary E8 lattices
(that is the basis for Kirmse's mistake)
but it does close under another of the 480 Octonion products.

16 live within the 112 D8 adjoint Root Vectors



in all of the 7 E8 lattices iE8, jE8, kE8, eE8, ieE8, jeE8, keE8.

224 = 7 sets of 32 with 3 sets of 32 = 96 within the 112 D8 adjoint Root Vectors

in the 7 E8 lattices iE8, jE8, kE8, eE8, ieE8, jeE8, keE8.

The 112 D8 Root Vector vertices in iE8, jE8, kE8, eE8, ieE8, jeE8, keE8
(+/- 1 , +/- 1 , 0 , 0 , 0 , 0 , 0 , 0)
for all 4 possible +/- signs times all (8l2) = 28 permutations of pairs of basis elements can be written in matrix form with each "4" representing possible signs and with the overall pattern of (1+2+3) + (4x4) + (3+2+1) representing the 28 permutations as

	1	i	j	k	e	ie	je	ke
1	-	4	4	4	4	4	4	4
i			4	4	4	4	4	4
j				4	4	4	4	4
k					4	4	4	4
e						4	4	4
ie							4	4
je								4
ke								-

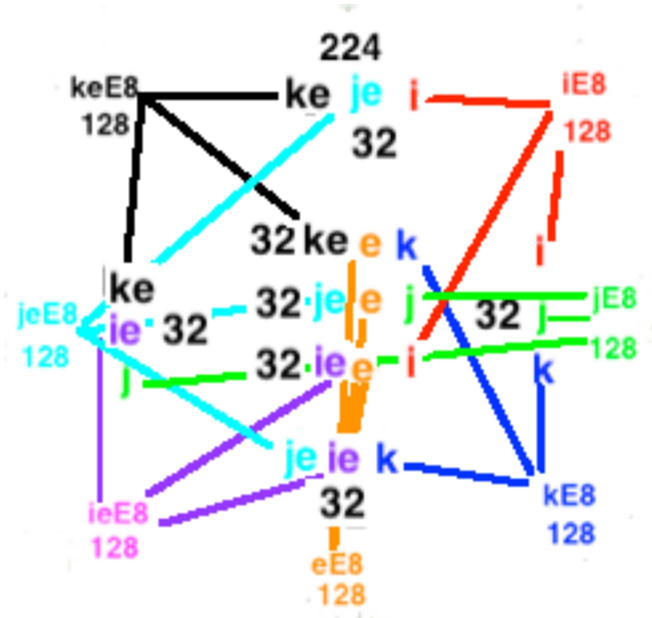
The 4 x 6 = 24 in the (1,i,j,k) x (1,i,j,k) block corresponding to M4 Physical Spacetime are the Root Vectors of a D4 in D8 in E8 with a U(2,2) subgroup that contains the SU(2,2) = Spin(2,4) Conformal Group of Gravity.

The 4 x 4x4 = 64 in the (1,i,j,k) x (e,ie,je,ke) block represents (4+4)-dim M4 x CP2 Kaluza-Klein Spacetime position and momentum.

The 4 x 6 = 24 in the (e,ie,je,ke) x (e,ie,je,ke) block corresponding to CP2 Internal Symmetry Space are the Root Vectors of another D4 in D8 in E8 with a U(4) subgroup that contains the SU(3) Color Force Group of the Standard Model.
The coset structure CP2 = SU(3) / U(1)xSU(2) gives the ElectroWeak U(1) and SU(2).

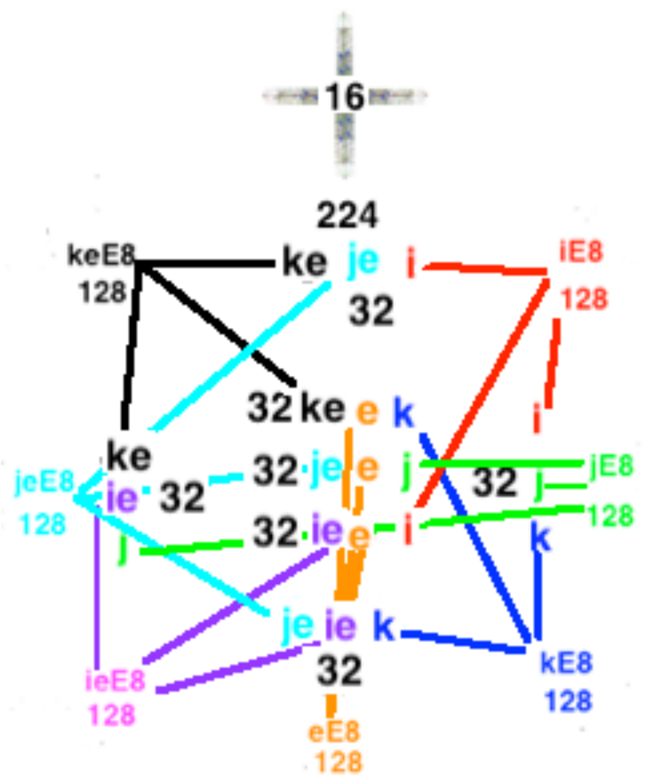
In each of the 7 E8 Root Vector sets for iE8, jE8, kE8, eE8, ieE8, jeE8, keE8
64 of the 128 D8 half-spinor vertices represent 8 components of 8 Fermion Particles
and
64 of the 128 D8 half-spinor vertices represent 8 components of 8 Fermion AntiParticles
where
the 8 fundamental Fermion Particle/AntiParticle types are:
neutrino, red down quark, green down quark, blue down quark;
blue up quark, green up quark, red up quark, electron.

The **224** are arranged as

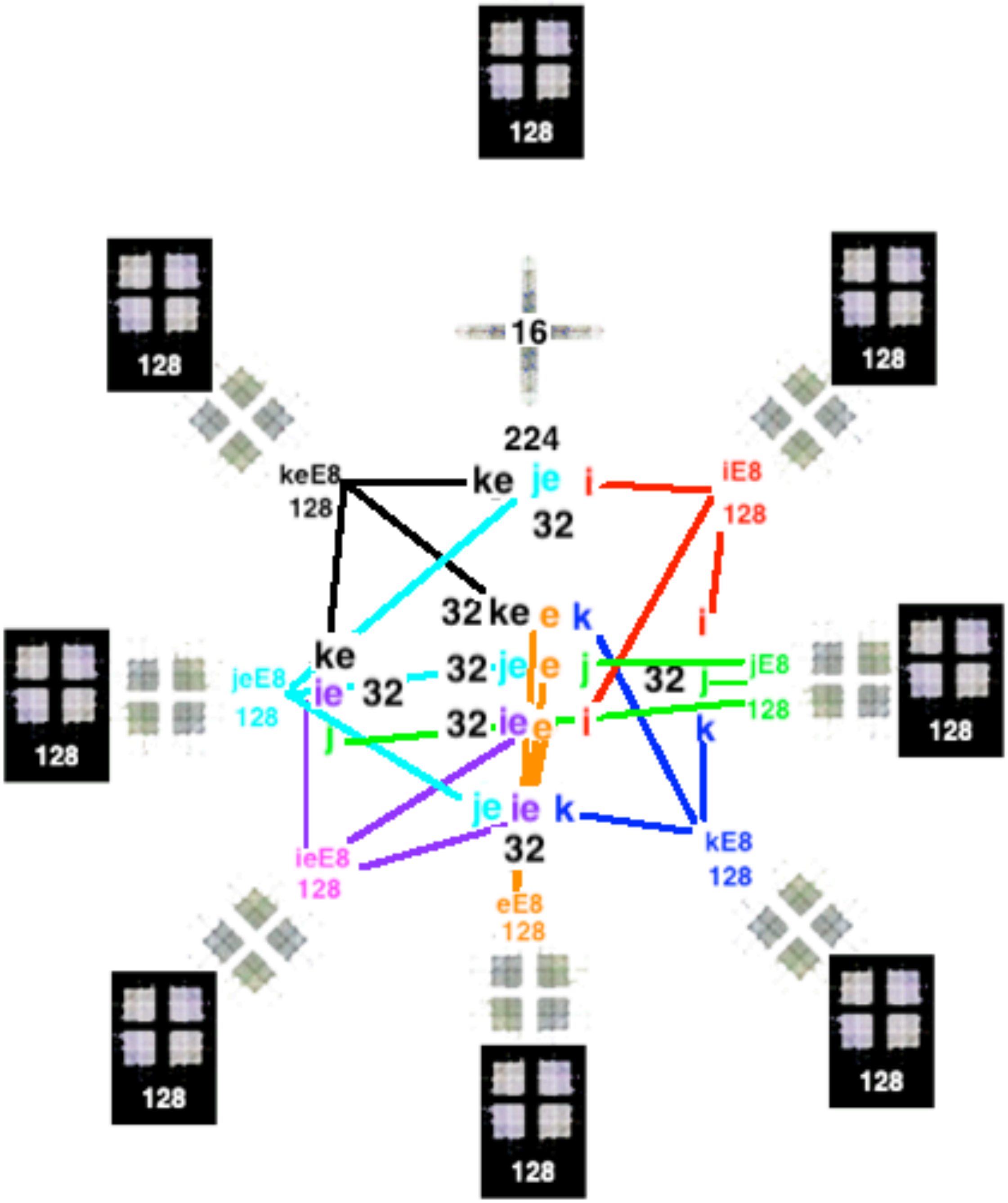


so that each of the sets of 32 connect with 3 of iE8, jE8, kE8, eE8, ieE8, jeE8, keE8 and each of iE8, jE8, kE8, eE8, ieE8, jeE8, keE8 connect with 3 of the sets of 32.

The **224** combined with the **16** give the **240 of 1E8**



The 7(128+128) + 128 + 16 + 224 structure
of all 2160 second layer E8 vertices
is



E8 Physics Model and 26D String Theory with Monster Group Symmetry

viXra 1210.0072 Frank Dodd (Tony) Smith, Jr. - 2012

A physically realistic Lattice Bosonic String Theory with Strings = World-Lines and Monster Group Symmetry containing gravity and the Standard Model can be constructed consistently with the E8 physics model
248-dim E8 = 120-dim adjoint D8 + 128-dim half-spinor D8
= (28 + 28 + 64) + (64 + 64)

Joseph Polchinski, in his books String Theory vols. I and II(Cambridge 1998), says:
"... the **closed ... unoriented ... bosonic string ... theory** has the maximal 26-dimensional Poincare invariance ... It is possible to have a consistent theory ... [with]... the **dilaton** ... the [**string-**]**graviton** ...[and]... the **tachyon** ...[whose]... negative mass-squared means that the no-string 'vacuum' is actually unstable ... ".
The **dilaton** of E8 Physics sets the Planck scale as the scale for the 16 dimensions that are orbifolded fermion particles and anti-particles and the 4 dimensions of the CP2 Internal Symmetry Space of M4xCP2 spacetime.
The remaining 26-16-4 = 6 dimensions are the Conformal Physical Spacetime with Spin(2,4) = SU(2,2) symmetry that produces M4 Physical Spacetime.
The **string-graviton** of E8 Physics is a spin-2 interaction among strings.
If Strings = World Lines and World Lines are past and future histories of particles, then string-graviton interactions determine a Cramer Transaction Quantum Theory discussed in quantum-ph/0408109. Roger Penrose in "Road to Reality" (Knopf 2004) says: "... **quantum** mechanics ... alternates between ... **unitary** evolution **U** ... and state reduction R ... quantum state **reduction** ... is ... **objective** ... **OR** ... it is always a gravitational phenomenon ... [A] conscious event ... would be ... orchestrated OR ... of ... large-scale quantum coherence ... of ... microtubules ...".
String-Gravity produces Sarfatti-Bohm Quantum Potential with Back-Reaction.
It is distinct from the MacDowell-Mansouri Gravity of stars and planets.
The **tachyon** produces the instability of a truly empty vacuum state with no strings.
It is natural, because if our Universe were ever to be in a state with no strings, then tachyons would create strings = World Lines thus filling our Universe with the particles and World-Lines = strings that we see. Something like this is necessary for particle creation in the Inflationary Era of non-unitary Octonionic processes.

Our construction of a 26D String Theory consistent with E8 Physics uses a structure that is not well-known, so I will mention it here before we start:

There are 7 independent E8 lattices, each corresponding to one of the 7 imaginary octionions denoted by iE8, jE8, kE8, EE8, IE8, JE8, and KE8 and related to both D8 adjoint and half-spinor parts of E8 and with 240 first-shell vertices. An 8th E8 lattice 1E8 with 240 first-shell vertices related to the D8 adjoint part of E8 is related to the 7 octonion imaginary lattices (viXra 1301.0150v2) .
It can act as an effectively independent lattice as part of the basis subsets {1E8,EE8} or {1E8,iE8,jE8,kE8}.
With that in mind, here is the construction:

Step 1:

Consider the 26 Dimensions of Bosonic String Theory as the 26-dimensional traceless part J3(O)o

a	O+	Ov
O+*	b	O-
Ov*	O-*	-a-b

(where Ov, O+, and O- are in Octonion space with basis {1,i,j,k,E,I,J,K} and a and b are real numbers with basis {1})

of the 27-dimensional Jordan algebra J3(O) of 3x3 Hermitian Octonion matrices.

Step 2:

Take a D3 brane to correspond to the Imaginary Quaternionic associative subspace spanned by {i,j,k} in the 8-dimenisonal Octonionic Ov space.

Step 3:

Compactify the 4-dimensional co-associative subspace spanned by {E,I,J,K} in the Octonionic Ov space as a CP2 = SU(3)/U(2), with its 4 world-brane scalars corresponding to the 4 covariant components of a Higgs scalar.

Add this subspace to D3, to get D7.

Step 4:

Orbifold the 1-dimensional Real subspace spanned by {1} in the Octonionic Ov space by the discrete multiplicative group Z2 = {-1,+1}, with its fixed points {-1,+1} corresponding to past and future time. This discretizes time steps and gets rid of the world-brane scalar corresponding to the subspace spanned by {1} in Ov. It also gives our brane a 2-level timelike structure, so that its past can connect to the future of a preceding brane and its future can connect to the past of a succeeding brane.

Add this subspace to D7, to get D8.

D8, our basic Brane, looks like two layers (past and future) of D7s.

Beyond D8 our String Theory has 26 - 8 = 18 dimensions, of which 25 - 8 have corresponding world-brane scalars:

- 8 world-brane scalars for Octonionic O+ space;
- 8 world-brane scalars for Octonionic O- space;
- 1 world-brane scalars for real a space; and
- 1 dimension, for real b space, in which the D8 branes containing spacelike D3s are stacked in timelike order.

Step 5:

To get rid of the world-brane scalars corresponding to the Octonionic O+ space, orbifold it by the 16-element discrete multiplicative group Oct16 = {+/-1,+/-i,+/-j,+/-k,+/-E,+/-I,+/-J,+/-K} to reduce O+ to 16 singular points {-1,-i,-j,-k,-E,-I,-J,-K,+1,+i,+j,+k,+E,+I,+J,+K}.

- Let the 8 O+ singular points {-1,-i,-j,-k,-E,-I,-J,-K} correspond to the fundamental fermion particles {neutrino, red up quark, green up quark, blue up quark, electron, red down quark, green down quark, blue down quark} located on the past D7 layer of D8.
- Let the 8 O+ singular points {+1,+i,+j,+k,+E,+I,+J,+K} correspond to the fundamental fermion particles {neutrino, red up quark, green up quark, blue up quark, electron, red down quark, green down quark, blue down quark} located on the future D7 layer of D8.

The 8 components of the 8 fundamental first-generation fermion particles = 8x8 = 64 correspond to the 64 of the 128-dim half-spinor D8 part of E8.

This gets rid of the 8 world-brane scalars corresponding to O+, and leaves:

- 8 world-brane scalars for Octonionic O- space;
- 1 world-brane scalars for real a space; and
- 1 dimension, for real b space, in which the D8 branes containing spacelike D3s are stacked in timelike order.

Step 6:

To get rid of the world-brane scalars corresponding to the Octonionic O- space, orbifold it by the 16-element discrete multiplicative group Oct16 = {+/-1,+/-i,+/-j,+/-k,+/-E,+/-I,+/-J,+/-K} to reduce O- to 16 singular points {-1,-i,-j,-k,-E,-I,-J,-K,+1,+i,+j,+k,+E,+I,+J,+K}.

- Let the 8 O- singular points {-1,-i,-j,-k,-E,-I,-J,-K} correspond to the fundamental fermion anti-particles {anti-neutrino, red up anti-quark, green up anti-quark, blue up anti-quark, positron, red down anti-quark, green down anti-quark, blue down anti-quark} located on the past D7 layer of D8.
- Let the 8 O- singular points {+1,+i,+j,+k,+E,+I,+J,+K} correspond to the fundamental fermion anti-particles {anti-neutrino, red up anti-quark, green up anti-quark, blue up anti-quark, positron, red down anti-quark, green down anti-quark, blue down anti-quark} located on the future D7 layer of D8.

The 8 components of the 8 fundamental first-generation fermion anti-particles = 8x8 = 64 correspond to the 64 of the 128-dim half-spinor D8 part of E8.

This gets rid of the 8 world-brane scalars corresponding to O-, and leaves:

- 1 world-brane scalars for real a space; and
- 1 dimension, for real b space, in which the D8 branes containing spacelike D3s are stacked in timelike order.

Step 7:

Let the 1 world-brane scalar for real a space correspond to a Bohm-type Quantum Potential acting on strings in the stack of D8 branes.

Interpret strings as world-lines in the Many-Worlds, short strings representing virtual particles and loops.

Step 8:

Fundamentally, physics is described on HyperDiamond Lattice structures.

There are 7 independent E8 lattices, each corresponding to one of the 7 imaginary octionions denoted by iE8, jE8, kE8, EE8, IE8, JE8, and KE8 and related to both D8 adjoint and half-spinor parts of E8 and with 240 first-shell vertices.

An 8th E8 lattice 1E8 with 240 first-shell vertices related to the D8 adjoint part of E8 is related to the 7 octonion imaginary lattices.

Give each D8 brane structure based on Planck-scale E8 lattices so that each D8 brane is a superposition/intersection/coincidence of the eight E8 lattices.
(see viXra 1301.0150v2)

Step 9:

Since Polchinski says "... If r D-branes coincide ... there are r^2 vectors, forming the adjoint of a U(r) gauge group ...", make the following assignments:

- a gauge boson emanating from D8 from its 1E8 and EE8 lattices is a U(2) ElectroWeak boson thus accounting for the photon and W+, W- and Z0 bosons.
- a gauge boson emanating from D8 from its IE8, JE8, and KE8 lattices is a U(3) Color Gluon boson thus accounting for the 8 Color Force Gluon bosons.

The 4+8 = 12 bosons of the Standard Model Electroweak and Color forces correspond to 12 of the 28 dimensions of 28-dim Spin(8) that corresponds to the 28 of the 120-dim adjoint D8 part of E8.

- a gauge boson emanating from D8 from its 1E8, iE8, jE8, and kE8 lattices is a U(2,2) boson for conformal U(2,2) = Spin(2,4)xU(1) MacDowell-Mansouri gravity plus conformal structures consistent with the Higgs mechanism and with observed Dark Energy, Dark Matter, and Ordinary matter.

The 16-dim U(2,2) is a subgroup of 28-dim Spin(2,6) that corresponds to the 28 of the 120-dim adjoint D8 part of E8.

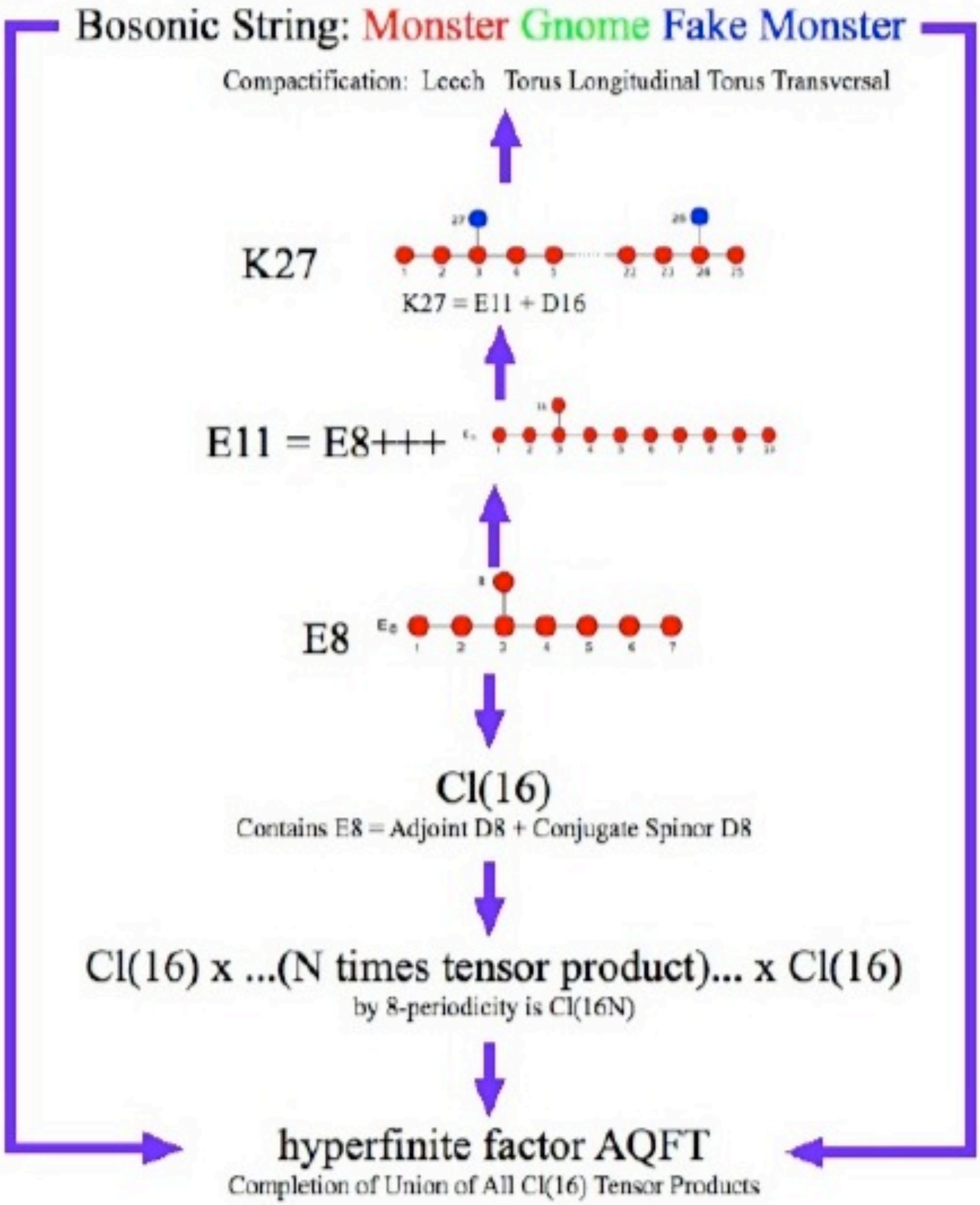
Step 10:

Since Polchinski says "... there will also be r^2 massless scalars from the components normal to the D-brane. ... the collectives coordinates ... X^u ... for the embedding of n D-branes in spacetime are now enlarged to nxn matrices. This 'noncummutative geometry' ...[may be]... an important hint about the nature of spacetime. ...", make the following assignment:

The 8x8 matrices for the collective coordinates linking a D8 brane to the next D8 brane in the stack are needed to connect the eight E8 lattices of the D8 brane to the eight E8 lattices of the next D8 brane in the stack.

The 8x8 = 64 correspond to the 64 of the 120 adjoint D8 part of E8.

We have now accounted for all the scalars
and
have shown that the model has the physics content of the realistic E8 Physics model with Lagrangian structure based on E8 = (28 + 28 + 64) + (64 + 64)
and AQFT structure based on Cl(16) with real Clifford Algebra periodicity and generalized Hyperfinite II1 von Neumann factor algebra.


Bosonic String: Monster Gnome Fake Monster
Compactification: Leech Torus Longitudinal Torus Transversal
K27
K27 = E11 + D16
E11 = E8+++
E8
Cl(16)
Contains E8 = Adjoint D8 + Conjugate Spinor D8
Cl(16) x ...(N times tensor product)... x Cl(16)
by 8-periodicity is Cl(16N)
hyperfinite factor AQFT
Completion of Union of All Cl(16) Tensor Products

A Single Cell of E8 26-dimensional Bosonic String Theory, in which Strings are physically interpreted as World-Lines, can be described by taking the quotient of its 24-dimensional O+, O-, Ov subspace modulo the 24-dimensional Leech lattice. Its automorphism group is the largest finite sporadic group, the Monster Group, whose order is

8080, 17424, 79451, 28758, 86459, 90496, 17107, 57005, 75436, 80000, 00000

=

2^46 .3^20 .5^9 .7^6 .11^2 .13^3 .17.19.23.29.31.41.47.59.71

or about 8 x 10^53.

A Leech lattice construction is described by Robert A. Wilson in his 2009 paper "Octonions and the Leech lattice":

"... The (real) octonion algebra is an 8-dimensional (non-division) algebra with an orthonomal basis { 1=ioo , i0 , i1 , i2 , i3 , i4 , i5 , i6 } labeled
by the projective line PL(7) = { oo } u F7

...

The E8 root system embeds in this algebra ... take the 240 roots to be ...
112 octonions ... +/- it +/- iu for any distinct t,u
... and ...
128 octonions (1/2)(+/- 1 +/- i0 +/- ... +/- i6) which have an odd number of minus signs.
Denote by L the lattice spanned by these 240 octonions

...

Let s = (1/2)(- 1 + i0 + ... + i6) so s is in L ... write R for Lbar ...

...

(1/2) (1 + i0) L = (1/2) R (1 + i0) is closed under multiplication ... Denote this ...by A
... Writing B = (1/2) (1 + i0) A (1 + i0) ...from ... Moufang laws ... we have
L R = 2 B , and ... B L = L and R B = R ...[also]... 2 B = L sbar

...

the roots of B are
[16 octonions]... +/- it for t in PL(7)
... together with
[112 octonions]... (1/2) (+/- 1 +/- it +/- i(t+1) +/- i(t+3)) ...for t in F7
... and ...
[112 octonions]... (1/2) (+/- i(t+2) +/- i(t+4) +/- i(t+5) +/- i(t+6)) ...for t in F7

...

the octonionic Leech lattice ... contains the following 196560 vectors of norm 4 ,
where M is a root of L and j,k are in J = { +/- it I t in PL(7) },
and all permutations of the three coordinates are allowed:

(2 M, 0 , 0)	**Number: 3x240 = 720**
(M sbar, +/- (M sbar) j , 0)	**Number: 3x240 x 16 = 11520**
((M s) j , +/- M k , +/- (M j) k)	**Number: 3x240 x 16 x 16 = 184320**

...

The key to the simple proofs above is the observation that LR = 2B and BL = L: these remarkable facts appear not to have been noticed before ... some work ... by Geoffrey Dixon ...". Geoffrey Dixon says in his book "Division Algebras, Lattices, Physics, Windmill Tilting" using notation {e0,e1,e2,e3,e4,e5,e6,e7} for the Octonion basis elements that Robert A. Wilson denotes by {1=ioo,i0,i1,i2,i3,i4,i5 ,i6} and I often denote by {1,i,j,k,E,I,J,K}: "...

$$\begin{aligned} \Xi_0 &= \{\pm e_a\}, \\ \Xi_2 &= \{(\pm e_a \pm e_b \pm e_c \pm e_d)/2 : a,b,c,d \text{ distinct}, \\ &\quad e_a(e_b(e_c e_d)) = \pm 1\}, \end{aligned}$$

$$\begin{aligned} \Xi^{even} &= \Xi_0 \cup \Xi_2, \\ \mathcal{E}_8^{even} &= \text{span}\{\Xi^{even}\}, \end{aligned}$$

$$\begin{aligned} \Xi_1 &= \{(\pm e_a \pm e_b)/\sqrt{2} : a,b \text{ distinct}\}, \\ \Xi_3 &= \{(\textstyle\sum_{a=0}^{7} \pm e_a)/\sqrt{8} : \text{ even number of +'s}\}, \end{aligned}$$

$$\begin{aligned} \Xi^{odd} &= \Xi_1 \cup \Xi_3, \\ \mathcal{E}_8^{odd} &= \text{span}\{\Xi^{odd}\} \end{aligned}$$

(spans over integers) ...

Ξeven has 16+224 = 240 elements ... Ξodd has 112+128 = 240 elements ...
E8even does not close with respect to our given octonion multiplication ...[but]...
the set Ξeven[0-a], derived from Ξeven by replacing each occurrence of e0 ... with ea, and vice versa, is multiplicatively closed. ...".
Geoffrey Dixon's Ξeven corresponds to B
Geoffrey Dixon's **Ξeven[0-a] corresponds to the seven At**
Geoffrey Dixon's Ξodd corresponds to L
Ignoring factors like 2 , j , k , and +/-1 the Leech lattice structure is:

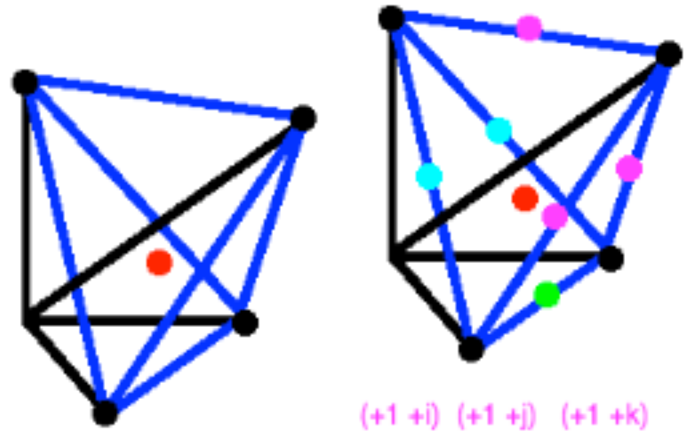
(L , 0 , 0)	Number: 3x240 = 720
(B , B , 0)	Number: 3x240 x 16 = 11520
(L s , L , L)	Number: 3x240 x 16 x 16 = 184320

(Ξodd , 0 , 0)	Number: 3x240 = 720
(Ξeven , Ξeven , 0)	Number: 3x240 x 16 = 11520
(Ξodd s , Ξodd , Ξodd)	Number: 3x240 x 16 x 16 = 184320

My view is that **the E8 domain B is fundamental**
and the E8 domains L and L s are derived from it.

That view is based on analogy with the 4-dimensional 24-cell
and its dual 24-cell. Using Quaternionic coordinates {1,i,j,k}
the 24-cell of 4-space has one Superposition Vertex for each 16-region of 4-space.

A Dual 24-cell gives a new Superposition Vertex at each edge of the region.



The Initial 24-cell Quantum Operators act with respect to 4-dim Physical Spacetime.
{1,i,j,k} represent time and 3 space coordinates.
(1/2)(+1+i+j+k) represents a fundamental first-generation Fermion particle/antiparticle (there is one for for each of the 16-regions).
The Dual 24-cell Quantum Operators act with respect to 4-dim CP2 Internal Symmetry Space. Since CP2 = SU(3)/SU(2)xU(1),
(+1 +i) (+1 +j) (+1 +k) are permuted by S3 to form the Weyl Group of Color Force SU(3),
(+i +j) (+i +k) are permuted by S2 to form the Weyl Group of Weak Force SU(2),
(+j +k) is permuted by S1 to form the Weyl Group of Electromagnetic Force U(1).
The B-type 24-cell is fundamental because it gives Fundamental Fermions.
The L-type dual 24-cell is derivative because it gives Standard Model Gauge Bosons.

Robert A.Wilson in "Octonions and the Leech lattice" also said
"... **B is not closed under multiplication** ... Kirmse's mistake
...[but]... as Coxeter ... pointed out ...
... there are seven non-associative rings At = (1/2) (1 + it) B (1 + it) , obtained from B by swapping 1 with it ... for t in F7 ...".
H. S. M. Coxeter in "Integral Cayley Numbers" (Duke Math. J. 13 (1946) 561-578) said
"... Kirmse ... defines ... an integral domain ... which he calls J1 [Wilson's B] ...[but]...
J1 itself is not closed under multiplication ... Bruck sent ... a revised description ...[of a]... domain J ... derived from J1 by transposing two of the i's [imaginary Octonions]...
It is closed under multiplication ... there are ... seven such domains, since the (7choose2) = 21 possible transpositions fall into 7 sets of 3, each set having the same effect. In each of the seven domains, one of the ... seven i's ... plays a special role, viz., that one which is not affected by any of the three transpositions. ...
J contains ... 240 units ... ". J is one of Wilson's seven At and, in Octonionic coordinates {1,i,j,k,e,ie,je,ke}, is shown below with physical interpretation color-coded as
8-dim Spacetime Coordinates x 8-dim Momentum Dirac Gammas
Gravity SU(2,2)=Spin(2,4) in a D4 + Standard Model SU(3)xU(2) in a D4
8 First-Generation Fermion Particles x 8 Coordinate Components
8 First-Generation Fermion AntiParticles x 8 Coordinate Components

112 = (16+48=64) + (24+24=48) Root Vectors corresponding to D8:

±1, ±i, ±j, ±k, ±e, ±ie, ±je, ±ke,

(	±1	±i			±e	±ie			)/2
(	±1		±j		±e		±je		)/2
(	±1			±k	±e			±ke	)/2

(			±j	±k			±je	±ke	)/2
(		±i		±k		±ie		±ke	)/2
(		±i	±j			±ie	±je		)/2

128 = 64 + 64 Root Vectors corresponding to half-spinor of D8:

(	±1					±ie	±je	±ke	)/2
(	±1		±j	±k		±ie			)/2
(	±1	±i		±k			±je		)/2
(	±1	±i	±j					±ke	)/2

(		±i	±j	±k	±e				)/2
(		±i			±e		±je	±ke	)/2
(			±j		±e	±ie		±ke	)/2
(				±k	±e	±ie	±je		)/2

The above Coxeter-Bruck J is, in the notation I usually use, denoted 7E8 .
It is one of Coxeter's seven domains (Wilson's seven {A0,A1,A2,A3,A4,A5,A6})
that I usually denote as { 1E8 , 2E8 , 3E8 , 4E8 , 5E8 , 6E8, 7E8 } .

Since the Leech lattice structure is

(L , 0 , 0)	**Number: 3x240 = 720**
(B , B , 0)	**Number: 3x240 x 16 = 11520**
(L s , L , L)	**Number: 3x240 x 16 x 16 = 184320**

if you replace the structural B with 7E8 and **the Leech lattice structure** becomes

(L , 0 , 0)	**Number: 3x240 = 720**
(7E8 , 7E8 , 0)	**Number: 3x240 x 16 = 11520**
(L s , L , L)	**Number: 3x240 x 16 x 16 = 184320**

and **the Leech lattice of E8 26-dim String Theory is the Superposition of 8 Leech lattices based on each of { B , 1E8 , 2E8 , 3E8 , 4E8 , 5E8 , 6E8, 7E8 }** just as the D8 branes of E8 26-dim String Theory are each the Superposition of the 8 domains { B , 1E8 , 2E8 , 3E8 , 4E8 , 5E8 , 6E8, 7E8 } .

What happens to a Fundamental Fermion Particle whose World-Line string intersects a Single Cell ?

The Fundamental Fermion Particle does not remain a single Planck-scale entity. **Tachyons create clouds of particles/antiparticles** as described by Bert Schroer in hep-th/9908021: "... any compactly localized operator applied to the vacuum generates clouds of pairs of particle/antiparticles ... More specifically it leads to the impossibility of having a local generation of pure one-particle vectors unless the system is interaction-free ...".

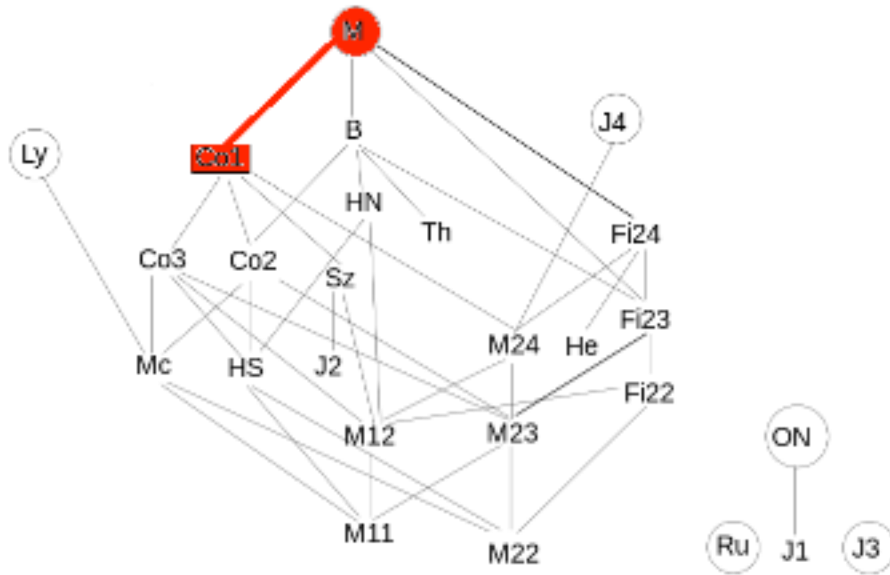
What is the structural form of the Fundamental Fermion Cloud ?

In "**Kerr-Newman [Black Hole]** solution as a Dirac particle", hep-th/0210103, H. I. Arcos and J. G. Pereira say: "... For $m^2 < a^2 + q^2$, with m, a, and q respectively the source mass, angular momentum per unit mass, and electric charge, the Kerr-Newman (KN) solution of Einstein's equation reduces to a naked singularity of circular shape, enclosing a disk across which the metric components fail to be smooth ... due to its topological structure, the extended KN spacetime does admit states with half-integral angular momentum. ... The state vector ... evolution is ... governed by the Dirac equation. ... for symmetry reasons, the electric dipole moment of the KN solution vanishes identically, a result that is within the limits of experimental data ... a and m are thought of as parameters of the KN solution, which only asymptotically correspond respectively to angular momentum per unit mass and mass. Near the singularity, a represents the radius of the singular ring ... With ... renormalization ... for the usual scattering energies, the resulting radius is below the experimental limit for the extendedness of the electron ...".

What is the size of the Fundamental Fermion Kerr-Newman Cloud ?

The FFKN Cloud is one Planck-scale Fundamental Fermion Valence Particle plus an effectively neutral cloud of particle/antiparticle pairs. The symmetry of the cloud is governed by the 24-dimensional Leech lattice by which the Single Cell was formed.

Here (adapted from Wikipedia) is a chart of the Monster M and its relation to other Sporadic Finite Groups and some basic facts and commentary:



The largest such subgroups of M are B, Fi24, and Co1.

B, the Baby Monster, is sort of like a downsized version of M,
as B contains Co2 and Fi23 while M contains Co1 and Fi24.

Fi24 (more conventionally denoted Fi24') is of order 1255205709190661721292800 = 1.2 x 10^24 It is the centralizer of an element of order 3 in the monster group M and is a triple cover of a 3-transposition group. It may be that Fi24' symmetry has its origin in the Triality of E8 26-dim String Theory.

The order of Co1 is 2^21.3^9.5^4.7^2.11.13.23 or about 4 x 10^18.
Aut(Leech Lattice) = double cover of Co1.
The order of the double cover 2.Co1 is 2^22.3^9.5^4.7^2.11.13.23 or about 0.8 x 10^19.
Taking into account the non-sporadic part of the Leech Lattice symmetry
according to the ATLAS at brauer.maths.qmul.ac.uk/Atlas/v3/spor/M/
the maximal subgroup of M involving Co1 is 2^(1+24).Co1 of order
139511839126336328171520000 = 1.4 x 10^26
As **2.Co1 is the Automorphism group of the Leech Lattice modulo to which the Single Cell was formed**, and as
the E8 26-dim String Theory Leech Lattice is a superposition of 8 Leech Lattices, 8 x 2^(1+24).Co1 describes the structure of the FFKN Cloud. Therefore,
the volume of the FFKN Cloud should be on the order of 10^27 x Planck scale, and
the FFKN Cloud should contain on the order of 10^27 particle/antiparticle pairs and **its size should be** somewhat larger than, but roughly similar to,
10^(27/3) x 1.6 x 10^(-33) cm = roughly **10^(-24) cm**.

The full 26-dimensional Lattice Bosonic String Theory can be regarded as an infinite-dimensional Affinization of the Theory of a Single Cell.

James Lepowsky said in math.QA/0706.4072:

"... **the Fischer-Griess Monster M ... was constructed by Griess as a symmetry group (of order about 10^54) of a remarkable new commutative but very, very highly nonassociative, seemingly ad-hoc, algebra B of dimension 196,883**. The "structure constants" of the Griess algebra B were "forced" by expected properties of the conjectured-to-exist Monster. It was proved by J. Tits that M is actually the full symmetry group of B. ...

There should exist a (natural) infinite-dimensional Z-graded module for M (i.e., representation of M)

V = DIRSUM(n=-1,0,1,2,3,...) V_n ...

such that

... the graded dimension of the graded vector space V ... = ... SUM(n=-1,0,1,2,3,...) (dim V_n) q^n

where

J(q) = q^(-1) + 0 + 196884q + higher-order terms,

the classical modular function with its constant term set to 0. J(q) is the suitably normalized generator of the field of SL(2, Z)-modular invariant functions on the upper half-plane, with q = exp(2 pi i tau) , tau in the upper half-plane ...

Conway and Norton conjectured ... for every g in M (not just g = 1), the the generating function

... the graded trace of the action of g on the graded space V ... = ... SUM(n=-1,0,1,2,3,...) (tr g | V_n) q^n

should be the analogous "Hauptmodul" for a suitable discrete subgroup of SL(2,R), a subgroup having a fundamental "genus-zero property," so that its associated field of modular-invariant functions has a single generator (a Hauptmodul) ... (... the graded dimension is of course the graded trace of the identity element g = 1.) The Conway-Norton conjecture subsumed a remarkable coincidence that had been noticed earlier

- that **the 15 primes giving rise to the genus-zero property ... are precisely the primes dividing the order of the ... Monster** ...

the McKay-Thompson conjecture ... that there should exist a natural ... infinite-dimensional Z-graded M-module V whose graded dimension is J(q) ... was (constructively) proved .... The graded traces of some, but not all, of the elements of the Monster - the elements of an important subgroup of M, namely, a certain involution centralizer involving the largest Conway sporadic group Co1 - were consequences of the construction, and these graded traces were indeed (suitably) modular functions ... We called this V **"the moonshine module V[flat]"** ...

The **construction ... needed ... a natural infinite-dimensional "affinization" of the Griess algebra B acting on V[flat**]

This "affinization," which was part of the new algebra of vertex operators, is analogous to, but more subtle than, the notion of affine Lie algebra .... More precisely, the vertex operators were needed for a "commutative affinization" of a certain natural 196884-dimensional enlargement B' of B, with an identity element (rather than a "zero" element) adjoined to B. This enlargement B' naturally incorporated the Virasoro algebra - the central extension of the Lie algebra of formal vector fields on the circle - acting on V[flat] ...

The vertex operators were also needed for a natural "lifting" of Griess's action of M from the finite-dimensional space B to the infinite-dimensional structure V[flat], including its algebra of vertex operators and its copy of the affinization of B'.

Thus the Monster was now realized as the symmetry group of a certain explicit "algebra of vertex operators" based on an infinite-dimensional Z-graded structure whose graded dimension is the modular function J(q).

Griess's construction of B and of M acting on B was a crucial guide for us, although we did not start by using his construction; rather, we recovered it, as **a finite-dimensional "slice" of a new infinite-dimensional construction** using vertex operator considerations. ...

The initally strange-seeming finite-dimensional Griess algebra was now embedded in a natural new infinite-dimensional space on which a certain algebra of vertex operators acts ... At the same time, the Monster, a finite group, took on a new appearance by now being understood in terms of a natural infinite-dimensional structure. ... the largest sporadic finite simple group, the Monster, was "really" infinite-dimensional ...

The very-highly-nonassociative Griess algebra, or rather, from our viewpoint, the natural modification of the Griess algebra, with an identity element adjoined, coming from a "forced" copy the Virasoro algebra, became simply the conformal-weight-two subspace of an algebra of vertex operators of a certain "shape." ...

the constant term of J(q) is zero, and this choice of constant term, which is not uniquely determined by number-theoretic principles, is not traditional in number theory. It turned out that the vanishing of the constant term ... was canonically "forced" by the requirement that the Monster should act naturally on V[flat] and on an associated algebra of vertex operators.

This vanishing of the degree-zero subspace of V[flat] is actually analogous in a certain strong sense to the absence of vectors in the Leech lattice of square-length two; the Leech lattice is a distinguished rank-24 even unimodular (self-dual) lattice with no vectors of square-length two.

In addition, this vanishing of the degree-zero subspace of V[flat] and the absence of square-length-two elements of the Leech lattice are in turn analogous to the absence of code-words of weight 4 in the Golay error-correcting code, a distinguished self-dual binary linear code on a 24-element set, with the lengths of all code-words divisible by 4. In fact, the Golay code was used in the original construction of the Leech lattice, and the Leech lattice was used in the construction of V[flat]

This was actually to be expected ... because it was well known that the automorphism groups of both the Golay code and the Leech lattice are (essentially) sporadic finite simple groups; the automorphism group of the Golay code is the Mathieu group M24 and the automorphism group of the Leech lattice is a double cover of the Conway group Co1 mentioned above, and both of these sporadic groups were well known to be involved in the Monster ... in a fundamental way....

The Golay code is actually **unique** subject to its distinguishing properties mentioned above ... and **the Leech lattice is unique** subject to its distinguishing properties mentioned above ... **Is V[flat] unique? If so, unique subject to what? ... this uniqueness is an unsolved problem** ...

V[flat] came to be viewed in retrospect by string theorists as an inherently string-theoretic structure: the "chiral algebra" underlying the Z2-orbifold conformal field theory based on the Leech lattice.

The string-theoretic geometry is this: One takes the torus that is the quotient of 24-dimensional Euclidean space modulo the Leech lattice, and then one takes the quotient of this manifold by the "negation" involution x -> -x, giving rise to an orbit space called an "orbifold"&emdash;a manifold with, in this case, a "conical" singularity. Then one takes the "conformal field theory" (presuming that it exists mathematically) based on this orbifold, and from this one forms a "string theory" in two-dimensional space-time by compactifying a 26-dimensional "bosonic string" on this 24-dimensional orbifold. The string vibrates in a 26-dimensional space, 24 dimensions of which are curled into this 24-dimensional orbifold ...

Borcherds used ... ideas, including his results on generalized Kac-Moody algebras, also called Borcherds algebras, together with certain ideas from string theory, including the "physical space" of a bosonic string along with the "no-ghost theorem" ... to prove the remaining Conway-Norton conjectures for the structure V[flat] ... What had remained to prove was ... that ... the conjugacy classes outside the involution centralizer - were indeed the desired Hauptmoduls ... He accomplished this by constructing a copy of his "Monster Lie algebra" from the "physical space" associated with V[flat] enlarged to a central-charge-26 vertex algebra closely related to the 26-dimensional bosonic-string structure mentioned above. He transported the known action of the Monster from V[flat] to this copy of the Monster Lie algebra, and ... he proved certain recursion formulas ... ... he succeeded in concluding that all the graded traces for V[flat] must coincide with the formal series for the Hauptmoduls ...

this vertex operator algebra V[flat] has the following three simply-stated properties ...

- (1) V[flat], which is an irreducible module for itself ... , is its only irreducible module, up to equivalence ... every module for the vertex operator algebra V [flat] is completely reducible and is in particular a direct sum of copies of itself. Thus the vertex operator algebra V[flat] has no more representation theory than does a field! (I mean a field in the sense of mathematics, not physics. Given a field, every one of its modules - called vector spaces, of course - is completely reducible and is a direct sum of copies of itself.)
- (2) dim V[flat]_0 = 0. This corresponds to the zero constant term of J(q); while the constant term of the classical modular function is essentially

arbitrary, and is chosen to have certain values for certain classical number-theoretic purposes, the constant term must be chosen to be zero for the purposes of moonshine and the moonshine module vertex operator algebra.

- (3) The central charge of the canonical Virasoro algebra in V[flat] is 24. "24" is the "same 24" so basic in number theory, modular function theory, etc. As mentioned above, this occurrence of 24 is also natural from the point of view of string theory.

These three properties are actually "smallness" properties in the sense of conformal field theory and string theory. These properties allow one to say that V[flat] essentially defines the smallest possible nontrivial string theory ... (These "smallness" properties essentially amount to: "no nontrivial representation theory," "no nontrivial gauge group," i.e., "no continuous symmetry," and "no nontrivial monodromy"; this last condition actually refers to both the first and third "smallness" properties.)

Conversely, conjecturally ... V[flat] is the unique vertex operator algebra with these three "smallness" properties (up to isomorphism). This conjecture ... remains unproved. It would be the conformal-field-theoretic analogue of the uniqueness of the Leech lattice in sphere-packing theory and of the uniqueness of the Golay code in error-correcting code theory ...

Proving this uniqueness conjecture can be thought of as the "zeroth step" in the program of classification of (reasonable classes of) conformal field theories. M. Tuite has related this conjecture to the genus-zero property in the formulation of monstrous moonshine.

Up to this conjecture, then, we have the following remarkable characterization of the largest sporadic finite simple group: **The Monster is the automorphism group of the smallest nontrival string theory** that nature allows ... **Bosonic 26-dimensional space-time ... "compactified" on 24 dimensions, using the orbifold construction V[flat]** ... or more precisely, the automorphism group of the vertex operator algebra with the canonical "smallness" properties. ...

This definition of the Monster in terms of "smallness" properties of a vertex operator algebra provides a remarkable motivation for the definition of the precise notion of vertex (operator) algebra. The discovery of string theory (as a mathematical, even if not necessarily physical) structure sooner or later must lead naturally to the question of whether this "smallest" possible nontrivial vertex operator algebra V . exists, and the question of what its symmetry group (which turns out to be the largest sproradic finite simple group) is.

And on the other hand, the classification of the the finite simple groups - a mathematical problem of the absolutely purest possible sort - leads naturally to the question of what natural structure the largest sporadic group is the symmetry group of; the answer entails the development of string theory and vertex operator algebra theory (and involves modular function theory and monstrous moonshine as well).

The Monster, a singularly exceptional structure - in the same spirit that the Lie algebra E8 is "exceptional," though M is far more "exceptional" than E8 - helped lead to, and helps shape, the very general theory of vertex operator algebras. (The exceptional nature of structures such as E8, the Golay code and the Leech lattice in fact played crucial roles in the construction of V[flat] ...

V[flat] is defined over the field of real numbers, and in fact over the field of rational numbers, in such a way that the Monster preserves the real and in fact rational structure, and that the Monster preserves a rational-valued positive-definite symmetric bilinear form on this rational structure. ...

the "orbifold" construction of V[flat] ...[has been]... interpreted in terms of algebraic quantum field theory, specifically, in terms of local conformal nets of von Neumann algebras on the circle ...

the notion of vertex operator algebra is actually the "one-complex-dimensional analogue" of the notion of Lie algebra. But at the same time that it is the "one-complex-dimensional analogue" of the notion of Lie algebra, the notion of vertex operator algebra is also the "one-complex- dimensional analogue" of the notion of commutative associative algebra (which again is the corresponding "one-real-dimensional" notion). ... This analogy with the notion of commutative associative algebra comes from the "commutativity" and "associativity" properties of the vertex operators ... in a vertex operator algebra ...

The remarkable and paradoxical-sounding fact that the notion of vertex operator algebra can be, and is, the "one-complex-dimensional analogue" of BOTH the notion of Lie algebra AND the notion of commutative associative algebra lies behind much of the richness of the whole theory, and of string theory and conformal field theory.

When mathematicians realized a long time ago that complex analysis was qualitatively entirely different from real analysis (because of the uniqueness of analytic continuation, etc., etc.), a whole new point of view became possible. In vertex operator algebra theory and string theory, there is again a fundamental passage from "real" to "complex," this time leading from the concepts of both Lie algebra and commutative associative algebra to the concept of vertex operator algebra and to its theory, and also leading from point particle theory to string theory. ...

While a string sweeps out a two-dimensional (or, as we've been mentioning, one-complex-dimensional) "worldsheet" in space-time, **a point particle of course sweeps out a one-real-dimensional "world-line"** in space-time, with time playing the role of the "one real dimension," and this "one real dimension" is related in spirit to the "one real dimension" of the classical operads that I've briefly referred to - the classical operads "mediating" the notion of associative algebra and also the notion of Lie algebra (and indeed, any "classical" algebraic notion), and in addition "mediating" the classical notion of braided tensor category. The "sequence of operations performed one after the other" is related (not perfectly, but at least in spirit) to the ordering ("time-ordering") of the real line.

But as we have emphasized, the "algebra" of vertex operator algebra theory and also of its representation theory (vertex tensor categories, etc.) is "mediated" by an (essentially) one-complex-dimensional (analytic partial) operad (or more precisely, as we have mentioned, the infinite-dimensional analytic structure built on this). When one needs to compose vertex operators, or more generally, intertwining operators, after the formal variables are specialized to complex variables, one must choose not merely a (time-)ordered sequencing of them, but instead, a suitable complex number, or more generally, an analytic local coordinate as well, for each of the vertex operators.

This process, very familiar in string theory and conformal field theory, is a reflection of how the one-complex-dimensional operadic structure "mediates" the algebraic operations in vertex operator algebra theory.

Correspondingly, "algebraic" operations in this theory are not instrinsically "time-ordered"; they are instead controlled intrinsically by the one-complex-dimensional operadic structure. The "algebra" becomes intrinsically geometric.

"Time," or more precisely, as we discussed above, the one-real-dimensional world-line, is being replaced by a one-complex-dimensional world-sheet.

This is the case, too, for the vertex tensor category structure on suitable module categories. In vertex operator algebra theory, "algebra" is more concerned with one-complex-dimensional geometry than with one-real-dimensional time. ...".

26D Strings, Bohmions, and Quantum Consciousness

James Lepowsky said in math.QA/0706.4072:
"... Bosonic 26-dimensional space-time ... string theory ...
[is]... the smallest nontrivial string theory that nature allows ...
[when] "compactified" on 24 dimensions ...[its]... automorphism group ...
is the largest sporadic finite simple group: The Monster ...".

In **26-dimensional Bosonic String Theory**, interpret **Strings as Particle World-Lines** and formulate quantum events based on interactions among entire World-Line histories along the lines proposed by Andrew Gray in quant-ph/9712037 (v2 August 2004).

Green, Schwartz, and Witten say in their book "Superstring Theory" vol. 1 (Cambridge 1986)
"... For the ... closed ... bosonic string .... The first excited level ... consists of ...
SO(24) ... little group of a ...[26-dim]... massless particle ...
massless ... spin two state ... and ...
a scalar ... '**dilaton**' ...
the ground state is ... a **tachyon** ...".

The **SO(24) little group** is related to the Monster automorphism group.

As to the **massless spin two state**, although Green, Schwartz, and Witten say "... we might try to identify ... the massless ... spin two state ... as the graviton ..."
here I will identify the massless spin two state with what I call the Bohmion: the carrier of the Bohm Force of the Bohm-Sarfatti Quantum Potential.

Peter R. Holland says in his book "The Quantum Theory of Motion" (Cambridge 1993) "... the total force ... from **the quantum potential ... does not necessarily fall off with distance** and indeed the forces between particles may become stronger ... This is because ...
the quantum potential ... depends on the form of ...[the quantum state]... rather than ... its ... magnitude ...".

Quantum Consciousness and related phenomena are based on Resonant Connections among Quantum State Forms.

Carver Mead says in his book "Collective Electrodynamics" (MIT 2000)
"... the energy shifts back and forth between ... two...coupled ... resonators ... despite an arbitrary separation between the resonators ...".

The **Quantum State Form of a Conscious Brain** is determined by the **configuration of a subset of its 10^18 Tubulin Dimers** with math description in terms of a large Real Clifford Algebra factorizable by 8-Periodicity into the tensor product of many copies of Cl(8).
(for details about Real Clifford Algebras see viXra 1304.0071)

As to the **dilaton,** Green, Schwartz, and Witten say: "...

$$S_{26} = -\frac{1}{2\kappa^2} \int d^{26}x\sqrt{g}e^{-2\Phi}\left(R - 4D_\mu\Phi D^\mu\Phi + \frac{1}{12}H_{\mu\nu\rho}H^{\mu\nu\rho}\right)$$

describes the long-wavelength limit of the massless modes of the bosonic closed string ... [the term] $\int d^{26}x\sqrt{g}e^{-2\Phi}R$ ... can be put in the form $\int d^{26}x\sqrt{g}R$ ... by absorbing a suitable power of $e^{-\Phi}$ in the definition of the spacetime metric $g_{\mu\nu}$ ...".

Deformation by $e^{-\Phi}$ causes the Einstein metric to differ from the string metric thus breaking scale invariance.

Joseph Polchinski says in his books "String Theory" (Vols. I and II Cambridge 1998): "... The massless dilaton appears in the tree-level spectrum of every string theory ... At the classical level, the massless dilaton ... can be understood as a Goldstone boson of ... spontaneously broken scale invariance ...".

Like the massless Higgs goes to mass for 3 Weak Bosons and Higgs Scalar, the massless dilaton goes from mediating a long-range scalar gravity-type force to the nonlocality of the Bohm-Sarfatti Quantum Potential whereby the Quantum Force carried by Bohmions does not necessarily fall off with distance.

As to the **tachyon** Joseph Polchinski says: "... the negative mass-squared means that the no-string 'vacuum' is actually unstable ...". Closed string tachyons localized at orbifolds may be physically equivalent to what Schroer describes in hep-th/9908021 as "... any compactly localized operator applied to the vacuum generates clouds of pairs of particle/antiparticles, unless the system is free ...". Since orbifolds are identified with fermion particles, their localized tachyons can be physically interpreted as describing the virtual particle-antiparticle clouds that dress the fermion particles. ...".

(for details about fermions as orbifolds see viXra 1210.0072)

Here is how all this works with Penrose-Hameroff Quantum Consciousness:

In Journal of Cosmology 14 (2011) (journalofcosmology.com/Consciousness160.html) Penrose and Hameroff said "... consciousness depends on biologically 'orchestrated' quantum computations in collections of microtubules within brain neurons ...

Microtubules are lattices of tubulin dimers ...[]... Discrete states of tubulins ... act as bits, switching between states, and interacting ... with neighbor tubulin states ...

orchestrated ... reduction of the quantum state ... (**Orch OR**) is taken to ... be a **quantum-gravity process** related to the fundamentals of spacetime geometry ... [and to] result in a moment of conscious awareness and/or choice ...

'OR' here refers to the ... viewpoint that ... the reduction R of the quantum state ('collapse of the wavefunction') ... is an actual physical phenomenon which is not part of the conventional unitary formalism U of quantum theory (or quantum field theory) ...

OR is taken to ... result from the mass displacement between the ... quantum-superposed alternative... quantum state[s] ... being sufficient, in gravitational terms, for the superposition to become unstable. ...
the superposition reduces to one of the alternatives in a time scale τ that can be estimated (for a superposition of two states each of which can be taken to be stationary on its own) according to the formula

$$\tau \approx \hbar/EG.$$

Here $\hbar$ (=h/2π) is Dirac's form of Planck's constant h and EG is the gravitational self-energy of the difference between the two mass distributions of the superposition.
... For a superposition for which each mass distribution is a rigid translation of the other, EG is the energy it would cost to displace one component of the superposition in the gravitational field of the other, in moving it from coincidence to the quantum-displaced location ...
The separation is ... a space-time separation, not just a spatial one. Thus the time of separation contributes as well as the spatial displacement. Roughly speaking, it is the product of the temporal separation T with the spatial separation S that measures the overall degree of separation, and OR takes place when this overall separation reaches a critical amount. ...
To estimate S, we compute (in the Newtonian limit of weak gravitational fields) the **gravitational self-energy EG of the difference between the mass distributions of the ... superposed states**. ... The quantity S is ... given by: S ≈ EG and T ≈ τ, whence

$$\tau \approx \hbar/EG \text{ , i.e. } EG \approx \hbar/\tau.$$

Thus ... OR occurs with the resolving out of one particular space-time geometry from the previous superposition when, on the average, $\tau \approx \hbar/EG$

... this is ... an element of proto-consciousness

... In the 1970s neurophysiologist Benjamin Libet performed experiments on patients having brain surgery while awake ... Libet determined that conscious perception of a stimulus required up to 500 msec of brain activity post-stimulus, but that conscious awareness occurred at 30 msec poststimulus, i.e. that subjective experience was referred 'backward in time'. .. The Orch OR scheme allows conscious experience to be temporally non-local to a degree, where this temporal non-locality would spread to the kind of time scale τ that would be involved in the relevant Orch OR process, which might indeed allow this temporal non-locality to spread to a time τ = 500ms ...".

Here is my view of the role of gravitational self-energy:

First consider Superposition of States involving one tubulin

with one electron of mass m and two different position states separated by a .
The Superposition Separation Energy Difference is the gravitational energy

$$E = G m^2 / a$$

For any single given tubulin a = 1 nanometer = 10^{-7} cm ...

Since the human brain is on the order of 10 cm, its volume is about 10^3 cm^3.
Since the human brain has about 10^{18} tubulin electrons,
the human brain has about $10^{18} / 10^3 = 10^{15}$ tubulin electrons/cm^3

Since for an electron Compton radius = 10^{-11} cm
and Schwarzschild radius = 10^{-55} cm
and since the speed of light c = 3 x 10^{10} cm/sec
and since $E_{electron} = G m^2 / a$
we have for a single Electron

$$T = h / (G m^2 / a) = (h / m c) (c^2 / G m) (a / c) =$$
$$=(\text{Compton} / \text{Schwarzschild}) (a / c)$$

where
$2 G m / c^2$ = Schwarzschild Radius of a classical black hole of mass m and
h / m c = Compton Radius of the Sidharth Kerr-Newman naked singularity model of an elementary particle of mass m
so that (ignoring for simplicity some factors like 2 and pi etc)

$$T = h / E = (\text{Compton} / \text{Schwarzschild}) (a / c) = 10^{26} \text{ sec} = 10^{19} \text{ years}$$

Now consider the case of N Tubulin Electrons in Coherent Superposition

Jack Sarfatti said "... Since all the Electrons are nonlocally connected into a coherent whole ... change m to M = Nm for a network of N connected ...[tubulin/electrons]... We are ... looking at ... the gravity self energy of the whole. Since it also has to be a metric fluctuation ... use Wheeler's "L" for the scale of ... metric quantum gravity fluctuation ... So how do we relate L to the microdisplacements of the pieces of the whole?
The obvious thing ... is $L^3 = N a^3$ [where] "a" is the displacement of each piece. ...".

Jack Sarfatti defines
the Superposition Energy E_N of N superposed Conformation Electrons in N Tubulins as

$$E_N = G M^2 / L$$

where L is the mesoscopic quantum phase coherence length
for the collective mode of N Conformation Electrons of total mass M in the N Tubulins, so that

$$E_N = N^2 G m^2 / a N^{(1/3)} =$$
$$= N^{(5/3)} G m^2 / a=$$
$$= N^{(5/3)} E_{electron}$$

To get the decoherence time for the system of N Tubulin Electrons recall that
(ignoring for simplicity some factors like 2 and pi etc)

T_electron = h / E_electron = (Compton / Schwarzschild) (a / c) = 10^26 sec

so that

T_N = h / E_N = h / N^(5/3) E_electron =
= N^(-5/3) T_electron =
= N^(-5/3) 10^26 sec

So we have the following rough approximate Table of Decoherence Times T_N
for various phenomena and structures involving various Numbers of Tubulin Dimers:

Time T_N	Number of Tubulins	Scale L = = N^(1/3) s
10^(-43) sec (Planck)	10^41	500 km
10^(-5) sec	10^18	1 cm
5 x 10^(-4) sec (2 kHz)	10^17	0.5 cm
25 x 10^(-3) sec (40 Hz)	10^16	0.2 cm
100 x 10^(-3) sec (EEG alpha)	4 x 10^15	0.16 cm
500 x 10^(-3) sec (Libet)	1.5 x 10^15	0.11 cm

Note that Quantum States involving 10^15 to 10^16 tubulins
(0.1 to 1 % of the 10^18 tubulins in a human brain)
give Decoherence Times on the order of human brain waves such as
Beta waves (14 to 30 Hz),
Alpha waves (8 to 13 Hz),
Theta waves (4 to 7 Hz), and
Delta waves (1 to 3 Hz).
and Schumann resonances such as
7.8, 14, 20, 26, 33, 39 and 45 Hz
and the Libet conscious perception time of up to 500 msec.

Penrose and Hameroff also said (journalofcosmology.com/Consciousness160.html)

"... **How could microtubule quantum states in one neuron extend to those in other neurons throughout the brain?** ...".

Penrose and Hameroff propose quantum tunneling through gap junctions
but I favor the use of

**Resonant Connections among Quantum States
mediated by Bohmions of the Bohm-Sarfatti Quantum Potential**

where the Bohmions are the massless spin 2 states of 26-dim Bosonic String Theory with Strings physically interpreted as World-Lines and fermions arising from orbifolding.
(for details about fermions as orbifolds see viXra 1210.0072)

Peter R. Holland says in his book "The Quantum Theory of Motion" (Cambridge 1993) "... the total force ... from the quantum potential ... does not necessarily fall off with distance and indeed the forces between particles may become stronger ... This is because ...
the quantum potential ... depends on the form of ...[the quantum state]...
rather than ... its ... magnitude ...".

so

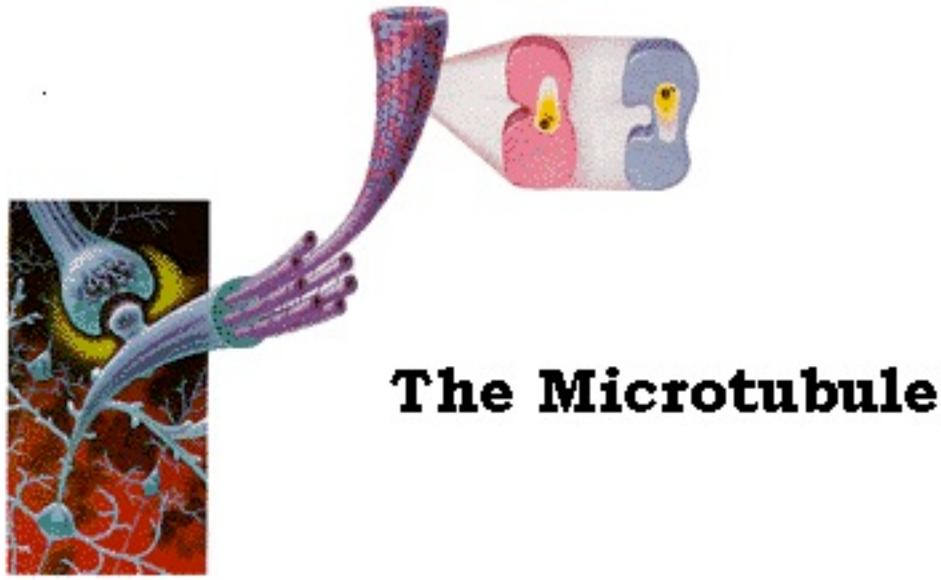
**Resonant Connections do not decline as inverse square of distance like Gravity
but if based on Bohmions of the Bohm-Sarfatti Quantum Potential
can be strong connections regardless of spacetime separation**.

The Bohm-Sarfatti Quantum Potential is so called because
since it is derived from the massless spin 2 states of 26-dim Bosonic String Theory
it inherits a Back-Reaction Property similar to that of General Relavity
advocated by Jack Sarfatti whose basic idea is
a reciprocal Back-Reaction of the particles of the Quantum State Form on the Quantum Potential just as General Relativlty has Matter/Enmergy back-reaction on Geometry. Jack Sarfatti has pointed out that conscious back-reaction could violate the assumption of equilibrium that ordinary quantum theory uses to obtain the Born approximation, noting that Antony Valentini in quant-ph/0203049 said "... pilot-wave theory indeed allows ... one to consider arbitrary 'nonequilibrium' initial distributions ...".

The Resonant Connection process is like that of Quantum Electrodynamics

described by Carver Mead in his book "Collective Electrodynamics" (MIT 2000)
There is a "... **first-order effect in which energy** flows from the high-amplitude resonator to the low-amplitude resonator ... the energy **shifts back and forth between ... two ... coupled ... resonators ... despite an arbitrary separation between the resonators** ... With the two resonators coupled, the energy shifts back and forth between the two resonators in such a way that the total energy is constant ...
The conservation of energy holds despite an arbitrary separation between the resonators ...".

Paola Zizzi drew **analogy between the Inflation Era of our Universe and the Quantum Consciousness process of human thought formation**.
The human brain contains about 10^18 tubulins in cylindrical microtubules.
Each tubulin contains a Dimer that can be in one of two binary states.



in the illustration (from a Rhett Savage web site), the red dimer has its electron in the down state and the blue dimer has its electron in the up state.
Each tubulin is about 8 x 4 x 4 nanometers in size
and contains about 450 molecules (amino acids) each with about 20 atoms.

If about 10% of the brain is involved in a given conscious thought,
it involves about 10% of 10^18 or about 10^17 tubulins.

Since 10^17 is about 2^56,
the mathematics of that thought is described by the Clifford algebra Cl(56)
which
is (by 8-periodicity) Cl(56) = Cl(7x8) =
= Cl(8) x ...(7 times tensor product)... x Cl(8) =
= 7 states of the basic Clifford algebra Cl(8)

That may account for

> "The Magical Number Seven, Plus or Minus Two:
> Some Limits on our Capacity for Processing Information"

by George Miller available on the web at psychclassics.yorku.ca/Miller/

Clifford Algebra Iterated Growth:

David Finkelstein's Cl(16) Fundamental Quantum Structure
of Nested Real Clifford Algebras:

Start with Empty Set = **0**

Real Dimension of each Clifford Algebra of Precursor Space:

0 = Cl(**0**) = {-1,+1} = Yin and Yang emerge from Tai Chi
\
1 = Cl(Cl(**0**)) = Real
\
1 + 1 = Cl(1) = Cl(Cl(Cl(**0**))) = Complex
\
1 + 2 + 1 = Cl(2) = Cl(Cl(1)) = Cl(Cl(Cl(Cl(**0**)))) = Quaternion
\
1 + 4 + 6 + 4 + 1 = Cl(4) = Cl(Cl(2)) = Cl(Cl(Cl(Cl(Cl(**0**)))))
\
1 + 16 + 120 + ... = Cl(16) = Cl(Cl(4)) = Cl(Cl(Cl(Cl(Cl(Cl(**0**))))))
\
1 + 65,536 + ... = Cl(65,536) = Cl(Cl(16)) = Cl(Cl(Cl(Cl(Cl(Cl(Cl(**0**)))))))
(by Real Clifford Algebra 8-Periodicity) = Cl(16) x...(16 times)...x Cl(16)

John von Neumann said ("Why John von Neumann did not Like the Hilbert Space Formalism of Quantum Mechanics (and What he Liked Instead)" by Miklos Redei in Studies in the History and Philosophy of Modern Physics 27 (1996) 493-510):

> "... if we wish to generalize the lattice of all linear closed subspaces
> from a Euclidean space to infinitely many dimensions,
> then one does not obtain Hilbert space ... our "case I_infinity" ...
> but that configuration, which Murray and I called "case II1" ...".

Completion of the Union of All Finite Tensor Products of Cl(16) with itself gives a generalized Hyperfinite II1 von Neumann Factor that in turn gives a realistic Algebraic Quantum Field Theory (AQFT).
Since Cl(16) is the Fundamental Building Block of a realistic AQFT with the structure of a generalized Hyperfinite II1 von Neumann Factor,
in order to understand how realistic AQFT works in detail,
we must understand the Geometric Structure of Cl(16).

Spinor Iterated Growth:

----

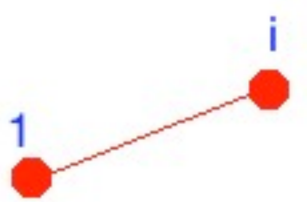
0 = Integers

----

1 = Real Numbers (basis = {1})

----

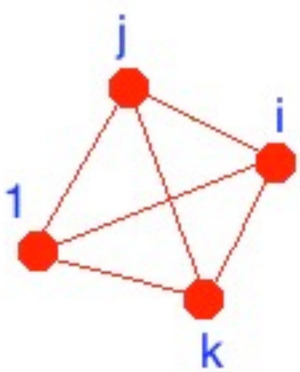
2 = Complex Numbers C (basis = {1,i}) = Cl(1) = half-spinors of Cl(4) Minkowski



These half-spinors are the basis of the conventional Fermionic Fock Space Hyperfinite II1 von Neumann Factor Algebraic Quantum Field Theory (AQFT)

----

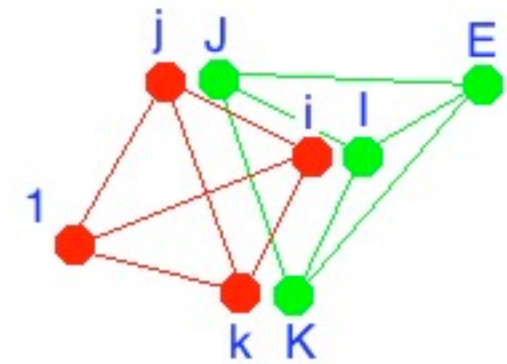
4 = Quaternions Q (basis = {1,i,j,k}) = Cl(2) = half-spinors of Cl(6) Conformal



WHICH CORRESPOND TO TETRAHEDRA

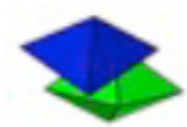
----

8 = Octonions O (basis = {1,i,j,k,I,J,K,E}) = half-spinors of Cl(8)

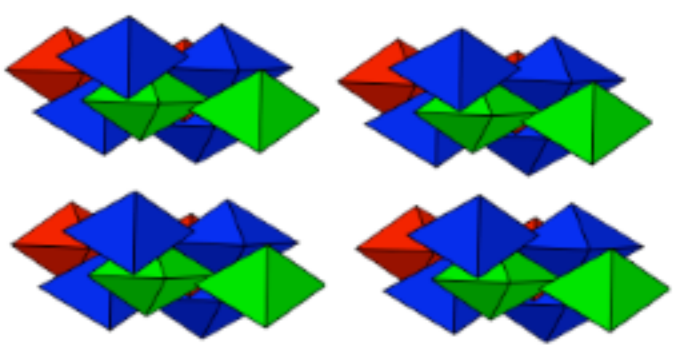


WHICH CORRESPOND TO
Chen-Engel-Glotzer (arXiv 1001.0586) DIMER PAIRS OF TETRAHEDRA

2^(8/2) = 2^4 = 16 = full spinors of Cl(8) = vectors of Cl(16)



2^(16/2) = 2^8 = 256 = 4 x 64 = full spinors of Cl(16)



These are the basis of the unconventional generalization of
the Hyperfinite II1 von Neumann Factor
that I use for Algebraic Quantum Field Theory (AQFT)

2^(256/2) = 2^128 = 3.4 x 10^38 = full spinors of Cl(256)
Such a large number as 2^128 is useful in describing
the inflationary expansion of our universe
and the production of the large number of particles that it contains.

Iterated Growth Physics

Clifford: n grows to 2^n
0
2^0 = 1 = Cl(0)
2^1 = 2 = Cl(1)
2^2 = 4 = Cl(2)
2^4 = 16 = Cl(4)
2^16 = 65,536 = Cl(16) = Cl(8)xCl(8)
65,536 + 1 = 65,537 = 2^2^4 + 1 is the largest known Fermat Prime
===

Spinor: n grows to 2^(n/2) (The Spinor Growth Pattern is due to David Finklestein.)

0 = Integers
2^(0/2) = 2^0 = 1 = Real Numbers
2^(1/2) = sqrt(2)
2^(sqrt(2)/2) = 2^0.707 = 1.63
2^(1.63/2) = 1.76
2^(1.76/2) = 1.84
... approaches 2 ...

2 = Complex Numbers C = Cl(1) = half-spinors of Cl(4) Minkowski
2^(2/2) = 2 fixed

4 = Quaternions Q = Cl(2) = half-spinors of Cl(6) Conformal
2^(4/2) = 2^2 = 4 fixed

Complex and Quaternion Qantum Processes are Unitary
and do not grow

6 = Conformal Physical Space
2^(6/2) = 2^3 = 8 Octonions O
2^(8/2) = 2^4 = 16 = full spinors of Cl(8)
Cl(8) triality: 8vector = 8+half-spinor = 8-half-spinor
F4 = 28 + 8+8+8
2^(16/2) = 2^8 = 256 = full spinors of Cl(16)
Cl(16) triality: 64vector = 64++half-half-spinor = 64--half-half-spinor
E8 = (28+28) + 64+64+64
2^(256/2) = 2^128 = 3.4 x 10^38 = full spinors of Cl(256)
2^127 = 1.7 x 10^38 = half-spinor of Cl(256) = (Mplanck / Mproton)^2
2^127 - 1 is a Mersenne Prime

Octonion Quantum Processes are NonUnitary
and can grow during Big Bang Inflation
until the Zizzi Inflation Decoherence Limit of sqrt(2^128) = 2^64 qubits is reached.

Each qubit at the Decoherence End of Inflation corresponds
to a Planck Mass Black Hole which transforms into
2^64 = 10^19 first-generation fermion particle-antiparticle pairs.
The resulting 2^64 x 2^64 = 2^128 = 10^19 x 10^19 = 10^38 fermion pairs
constitutes a Zizzi Quantum Register of order n_reh = 10^38 = 2^128.
Since, as Paola Zizzi says in gr-qc/0007006 :
"... the quantum register grows with time ... At time Tn = (n+1) Tplanck
the quantum gravity register will consist of (n+1)^2 qubits ...",
we have the number of qubits at Reheating:
Nreh = (n_reh)^2 = (2^128)^2 = 2^256 = 10^77
Since each qubit at Reheating should correspond to fermion particle-antiparticle pairs
we have the result that
the number of particles in our Universe at Reheating is about 10^77 nucleons.
==

64-dim Spinor Structure CxQxO

(The 64-dim spinor structure CxQxO is due to Geoffrey Dixon.)
Cl(16) Triality 64 = 2x4x8 = (CxQ)xO

CxQ represents:

1xQ = 4-dim Minkowski M4 Physical Spacetime
ixQ = 4-dim CP2 Internal Symmetry Space

O represents:

8vector = 8 gammas of 8-dim Octonionic Spacetime
8+half-spinor = 8 fermion particles (e, ur, ug, ub ; db, dg, db, nu)
8-half-spinor = 8 fermion antiparticles

CxQxO represents:

64vector = 8 components of each of 8 gammas
64++half-half-spinor = 8 components of each of 8 fermion particles
64--half-half-spinor = 8 components of each of 8 fermion antiparticles

----

Full spinors of Cl(256)
2^128 = 2^64 x 2^64 =
= all possible states/subsets of 8 components
of 8 fermion particles and 8 fermion antiparticles

==

The Fermat primes, of the form 2^2^k + 1, include:

2 + 1 = 2^1 + 1 = 2^2^0 + 1 = 2+1 = 3
2^2 + 1 = 2^2 + 1 = 2^2^1 + 1 = 4+1 = 5
2^2^2 + 1 = 2^4 + 1 = 2^2^2 + 1 = 16+1 = 17
2^2^2^2 + 1 = 2^16 + 1 = 2^2^4 + 1 = 65,536+1 = 65,537
2^2^3 + 1 = 2^8 + 1 = 257 is the only other known Fermat prime.

The Mersenne Primes, of the form 2^k - 1 for prime k, include:

2^2 -1 = 4-1 = 3
2^3 - 1 = 8-1 = 7
2^7 - 1 = 128-1 = 127
2^127 - 1 = approximately 1.7 x 10^38
2^(2^127 - 1) - 1 may or may not be prime. Its primality is not now known.
Some other Mersenne Primes are 2^k - 1
for k = 5, 13, 17, 19, 31, 61, 89, 107, 521, 607, and 1279.

Cl(16) has 2^16 = 65,536 elements with graded structure

1
16
120
560
1820
4368
8008
11440
12870
11440
8008
4368
1820
560
120
16
1

The 16-dim grade-1 Vectors of Cl(16) are D8 = Spin(16) Vectors that are acted upon by the 120-dim grade-2 Bivectors of Cl(16) which form the D8 = Spin(16) Lie algebra.

Cl(16) has, in addition to its 16-dim D8 Vector and 120-dim D8 Bivector bosonic commutator structure, a fermionic anticommutator structure related to its sqrt(65,536) = 256-dim spinors which reduce to 128-dim D8 +half-spinors plus 128-dim D8 -half-spinors.
Pierre Ramond in hep-th/0112261 said:

> "... the coset F4 / SO(9) ... is the sixteen-dimensional Cayley projective plane ... [represented by]... the SO(9) spinor operators [which] satisfy Bose-like commutation relations ... Curiously, if ...[the scalar and spinor 16 of F4 are both]... anticommuting, the F4 algebra is still satisfied ...".

The same reasoning applies to other exceptional groups that have octonionic structure and spinor component parts, including:

E6 = D5 + U(1) + 32-dim full spinor of D5
and
248-dim E8 = 120-dim D8 + 128-dim half-spinor of D8.

F4 and E8:
Commutators and AntiCommutators

Frank Dodd (Tony) Smith Jr. - 2013

Abstract:

Realistic Physics models must describe both commutator Bosons and anticommutator Fermions so that spin and statistics are consistent.
The usual commutator structure of Lie Algebras can only describe Bosons, so a common objection to Physics models that describe both Bosons and Fermions in terms of a single unifiying Lie Algebra (for example, Garrett Lisi's E8 TOE) is that they violate consistency of spin and statistics by using Lie Algebra commutators to describe Fermions.
However,
Pierre Ramond has shown in hep-th/0112261 as shown that the exceptional Lie Algebra F4 can be described using anticommutators as well as commutators.
This essay uses the periodicity property of Real Clifford Algebras to show that E8 can also be described using anticommutators as well as commutators so that it may be possible to construct a realistic Physics model that uses the exceptional Lie Algebra E8 to describe both Bosons and Fermions.

E8 also inherits from F4 Triality-based symmetries between Bosons and Fermions that can give the useful results of SuperSymmetry without requiring conventional SuperPartner particles that are unobserved by LHC.

Realistic Physics models must describe both
integer-spin Bosons whose statistics are described by commutators
(examples are Photons, W and Z bosons, Gluons, Gravitons, Higgs bosons)
and
half-integer-spin Fermions whose statistics are described by anticommutators.
(examples are 3 generations of Electrons, Neutrinos, Quarks and their antiparticles)

Lie Algebra elements are usually described by commutators of their elements
so
if a Physics model attempts to describe both Bosons and Fermions as elements of a single unifiying Lie Algebra (for example, Garrett Lisi's E8 TOE)
a common objection is:

since the Lie Algebra is described by commutators,
it can only describe Bosons and cannot describe Fermions
therefore
models (such as Garrett Lisi's) using E8 as a single unifying Lie Algebra
violate the consistency of spin and statistics and are wrong.

However, Pierre Ramond has shown in hep-th/0112261 as shown that the exceptional Lie Algebra F4 can be described using anticommutators as well as commutators.

The periodicity property of Real Clifford Algebras shows that E8 inherits from F4 a description using anticommutators as well as commutators so that
it may be possible to construct a realistic Physics model that uses
the exceptional Lie Algebra E8 to describe both Bosons and Fermions.

Here are relevant quotes from hep-th/0112261 by Pierre Ramond:
"... exceptional algebras relate tensor and spinor representations
of their orthogonal subgroups,
while Spin-Statistics requires them to be treated differently ...
all representations of the exceptional group F4 are generated by three sets
of oscillators transforming as 26. We label each copy of 26 oscillators as

$$A_0^{[\kappa]},\ A_i^{[\kappa]},\ i=1,\cdots,9,\ B_a^{[\kappa]},\ a=1,\cdots,16,$$

and their hermitian conjugates, and where k = 1, 2, 3.
Under SO(9), the A[k]_i transform as 9, B[k]_a transform as 16, and A[k]_0 is a scalar. They satisfy the commutation relations of ordinary harmonic oscillators

$$[A_i^{[\kappa]}, A_j^{[\kappa']\dagger}] = \delta_{ij}\,\delta^{[\kappa][\kappa']}, \qquad [A_0^{[\kappa]}, A_0^{[\kappa']\dagger}] = \delta^{[\kappa\kappa']}.$$

Note that the SO(9) spinor operators satisfy Bose-like commutation relations

$$[B_a^{[\kappa]}, B_b^{[\kappa']\dagger}] = \delta_{ab}\,\delta^{[\kappa][\kappa']}.$$

The generators T_ij and T_a

$$T_{ij} = -i\sum_{\kappa=1}^{4}\left\{\left(A_i^{[\kappa]\dagger}A_j^{[\kappa]} - A_j^{[\kappa]\dagger]}A_i^{[\kappa]}\right) + \frac{1}{2}B^{[\kappa]\dagger}\gamma_{ij}B^{[\kappa]}\right\},$$

$$T_a = -\frac{i}{2}\sum_{\kappa=1}^{4}\left\{(\gamma_i)^{ab}\left(A_i^{[\kappa]\dagger}B_b^{[\kappa]} - B_b^{[\kappa]\dagger}A_i^{[\kappa]}\right)' - \sqrt{3}\left(B_a^{[\kappa]\dagger}A_0^{[\kappa]} - A_0^{[\kappa]\dagger}B_a^{[\kappa]}\right)\right\},$$

satisfy the F4 algebra,

$$[T_{ij}, T_{kl}] = -i\,(\delta_{jk}T_{il} + \delta_{il}T_{jk} - \delta_{ik}T_{jl} - \delta_{jl}T_{ik}),$$

$$[T_{ij}, T_a] = \frac{i}{2}(\gamma_{ij})_{ab}T_b,$$

$$[T_a, T_b] = \frac{i}{2}(\gamma_{ij})_{ab}T_{ij},$$

so that the structure constants are given by

$$f_{ij\,ab} = f_{ab\,ij} = \frac{1}{2}(\gamma_{ij})_{ab}.$$

The last commutator requires the Fierz-derived identity

$$f_{ij\,ab} = f_{ab\,ij} = \frac{1}{2}(\gamma_{ij})_{ab}.$$

from which we deduce

$$3\,\delta^{ac}\delta^{db} + (\gamma^i)^{ac}(\gamma^i)^{db} - (a \leftrightarrow b) = \frac{1}{4}(\gamma^{ij})^{ab}(\gamma^{ij})^{cd}.$$

To satisfy these commutation relations, we have required both A_0 and B_a to obey Bose commutation relations **(Curiously, if both [A_0 and B_a] are anticommuting, the F4 algebra is still satisfied).** ...".

The 1 + 9 + 16 = 26 oscillators

$$A_0^{[\kappa]},\ A_i^{[\kappa]},\ i=1,\cdots,9,\ B_a^{[\kappa]},\ a=1,\cdots,16,$$

represent the 26-dim lowest-dimensional non-trivial represenstion of 52-dim F4 .

The 36 + 16 = 52 generators

$$T_{ij} = -i\sum_{\kappa=1}^{4}\left\{\left(A_i^{[\kappa]\dagger}A_j^{[\kappa]} - A_j^{[\kappa]\dagger]}A_i^{[\kappa]}\right) + \frac{1}{2}B^{[\kappa]\dagger}\gamma_{ij}B^{[\kappa]}\right\} ,$$

$$T_a = -\frac{i}{2}\sum_{\kappa=1}^{4}\left\{(\gamma_i)^{ab}\left(A_i^{[\kappa]\dagger}B_b^{[\kappa]} - B_b^{[\kappa]\dagger}A_i^{[\kappa]}\right) - \sqrt{3}\left(B_a^{[\kappa]\dagger}A_0^{[\kappa]} - A_0^{[\kappa]\dagger}B_a^{[\kappa]}\right)\right\} ,$$

represent the 52-dim adjoint representation of F4 written as commutators.

By the remark shown in bold in the quote above, Ramond states that
the 16 Spinor oscillators B[k]_a can be written as anticommutators
as well as commutators and that both cases produce the 52-dim F4 algebra.

Physically, this means that if you use the 52-dim F4 to build a physics model
with Fermions being represented by 16-dim F4 / SO(9) = OP2
then
you can use the anticommutator structure of the 16-dim B[k]_a
to satisfy the spin-statistics theorem
because
the B[k]_a represent a 16 of SO(9) which is also OP2 = F4 / SO(9) .

To see how this anticommutator structure extends to E8,
note that the lowest-dimensional non-trivial representation of E8 is 248-dim
and that the adjoint representation of E8 is also 248-dim.

As shown by T. Fulton in J. Phys. A: Math. Gen. 18 (1985) 2863-2891
(quotation slightly modified due to typographical considerations etc):

"... Schwinger ... has studied the generators and irreps of SU(2) in terms of two Bose oscillators (hereafter abbreviated SHO) ... the algebras of the classical groups ... A(n) ; B(n) ; D(n) ; C(n) have been realized in terms of Fermi oscillators. The spinor irreps of the orthogonal groups ... are the only ones which ... have been constructed ... using Fermi oscillators. These irreps involve various nunbers of Fermi oscillator creation operators a†_i
...
the elementary spinor irreps of the orthogonal groups can be written in terms of a single SHO creation operator acting on the vacuum state ... the 'vacuum state' does not have zero weight, but is an element of a spinor irrep.
...
For D(n), the 'vacuum state', together with all states formed by even powers of a† operating on this state, up to the maximum possible such power, constitute the elements of one of the elementary spinor irreps; all possible odd powers of a† , operating on the 'vacuum', constitute the set of all elements of the other elementary spinor irrep ...
...
F(4) ... we have elementary irrep f1 = 26-dim
so that D(13) = SO(26) contains F4
The other elementary irrep of F(4) is f2 = 52-dim
...
E(8) ... we have elementary irrep f1 = 248-dim
The simplest embedding of E(8) is to choose D(124) = SO(248) contains E(8) ...".

Physically, if you use the 248-dim E8 to build a physics model
with Fermions being represented by 128-dim E8 / D8 = (OxO)P2
and
if E8 inherits anticommutator structure from F4
then
you can use anticommutator structure of the 128-dim (OxO)P2
to satisfy the spin-statistics theorem.

To see how the anticommuting property of the 16 B_a elements of F4
can be inherited by some of the elements of E8,
consider that 52-dimensional F4 is made up of:

28-dimensional D4 Lie Algebra Spin(8) (in commutator part of F4)
8-dimensional D4 Vector Representation V8 (in commutator part of F4)
8-dimensional D4 +half-Spinor Representation S+8 (in anticommutator part of F4)
8-dimensional D4 -half-Spinor Representation S-8 (in anticommutator part of F4)

Since 28-dimensional D4 Spin(8) is the BiVector part BV28
of the Real Clifford Algebra Cl(8) with graded structure
Cl(8) = 1 + V8 + BV28 + 56 + 70 + 56 + 28 + 8 + 1
and with Spinor structure
Cl(8) = (S+8 + S-8) x (8 + 8)

F4 can be embedded in Cl(8) (blue commutator part, red anticommutator part):

F4 = V8 + BV28 + S+8 + S-8

Note that V8 and S+8 and S-8 are related by the Triality Automorphism.

Also consider the 8-periodicity of Real Clifford Algebras,
according to which for all N

Cl(8N) = Cl(8) x ...(N times tensor product)... Cl(8)

so that in particular Cl(16) = Cl(8) x Cl(8)
where Cl(16) graded structure is 1 + 16 + BV120 + 560 + ... + 16 + 1
and Cl(16) Spinor structure is ((S+64 + S-64) + (64 + 64)) x (128 + 128)
and Cl(16) contains 248-dimensional E8 as

E8 = BV120 + S+64 + S-64

where BV120 = 120-dimensional D8 Lie Algebra Spin(16)
and S+64 + S-64 = 128-dimensional D8 half-Spinor Representation

Consider two copies of F4 embedded into two copies of Cl(8).

For commutator structure:

The tensor product of the two copies of Cl(8) can be seen as

1 + V8 + BV28 + 56 + 70 + 56 + 28 + 8 + 1
x
1 + V8 + BV28 + 56 + 70 + 56 + 28 + 8 + 1

which produces the Real Clifford Algebra Cl(16) with graded structure

1 + 16 + BV120 + 560 + 1820 + ... + 16 + 1

where the Cl(16) BiVector BV120 is made up of 3 parts

BV120 = BV28x1 + 1xBV28 + V8xV8

that come from the V8 and BV28 commutator parts of the two copies of F4.

This gives the commutator part of E8 as BV120 inheriting commutator structure from the two copies of F4 embedded in two copies of Cl(8) whose tensor product produces Cl(16) containing E8.

For anticommutator structure:

The tensor product of the two copies of 256-dim Cl(8) can also be seen as

((S+8 + S-8) x (8 + 8))
x
((S+8 + S-8) x (8 + 8))

which produces the 2^16 = 65,536 = 256x256-dim Real Clifford Algebra Cl(16)

((S+8 + S-8) x (S+8 + S-8))
x
((8 + 8) x (8 + 8))

with 256-dimensional Spinor structure

((S+8 + S-8) x (S+8 + S-8)) =

= ((S+8 x S+8) + (S-8 x S-8)) + ((S+8 x S-8) + (S-8 x S+8))

that comes from the S+8 and S-8 anticommutator parts of the two copies of F4.

Since the (S+8 x S-8) and (S-8 x S+8) terms inherit mixed helicities from F4

only the (S+8 x S+8) and (S-8 x S-8) terms inherit consistent helicity from F4.

Therefore, define S+64 = (S+8 x S+8) and S-64 = (S-8 x S-8)
so that
(S+64 + S-64) = 128-dimensional D8 half-Spinor Representation

This gives the anticommutator part of E8 as S+64 + S-64 inheriting anticommutator structure from the two copies of F4 embedded in two copies of Cl(8) whose tensor product produces Cl(16) containing E8.

The result is that 248-dimensional E8 is made up of:

BV120 = 120-dimensional D8 Lie Algebra Spin(16) (commutator part of E8)

128-dimensional (S+64 + S-64) D8 half-Spinor (anticommutator part of E8)

Note that since the V8 and S+8 and S-8 components of F4 are related by Triality, and since
the E8 component BV120 contains 64-dimensional V8xV8
and
the 64-dimensional E8 component S+64 = S+8 x S+8
and
the 64-dimensional E8 component S-64 = S-8 x S-8

E8 inherits from the two copies of F4 a Triality relation

V8xV8 = S+64 = S-64

The commutator - anticommutator structure of E8 allows construction of realistic Physics models that not only unify both Bosons and Fermions within E8
but
also contain Triality-based symmetries between Bosons and Fermions
that can give the useful results of SuperSymmetry
without requiring conventional SuperPartner particles that are unobserved by LHC.

CONCLUSION:

Unified E8 Physics models can be constructed without violating spin-statistics, so that evaluation of such models as Garrett Lisi's E8 TOE and my E8 Physics model at http://vixra.org/abs/1108.0027 should be based on other criteria such as consistency with experimental observations.

Cl(16) contains E8

Cl(16) = 1 + 16 + **120** + 560 + 1820 + ... + 16 + 1=
= ((**64++ + 64--**) + (64+- + 64-+)) x 256

Cl(16) = Cl(8)$_1$ x Cl(8)$_2$ where Cl(8)$_1$ contains F4$_1$ and Cl(8)$_2$ contains F4$_2$

(8ofF4$_1$ x 8ofF4$_2$) + 28ofF4$_1$x1ofCl(8)$_2$ + 28ofF4$_2$x1ofCl(8)$_1$ = 64 + 28 + 28 = 120ofE8

(**8+**ofF4$_1$ + **8-**ofF4$_1$) x (**8+**ofF4$_2$ + **8-**ofF4$_2$) gives (**64++ + 64--**)ofE8

(because the terms (64+- + 64-+) are rejected as unphysical mixed helicity)

(The notation -> denotes a Maximal Contraction)

E8 = **120** + (**64++ + 64--**)

E8 -> A7 = SU(8) part of U(8) for 8 position x 8 momentum = 64 generators

- semidirect product with -

28 + 64 + 1 + 64 + 28 Heisenberg Algebra H92 for

28 gauge bosons for A2xA1xA0 = SU(3)xSU(2)xU(1) plus

D3 = Spin(2,4) = A3 = SU(2,2) Conformal Gravity

and for

64 = 8x8 of 8 components of 8 Fermions

E7.5 ->E7 - semidirect product with - 28+1+28 Heisenberg Algebra H28

28 = Quaternionic Jordan Algebra J4(Q) =(bijection)= D4

D4 = Spin(8) = (Standard Model + Gravity) Gauge Bosons

E7 -> E6 - semidirect product with - 27+1+27 Heisenberg Algebra H27

27 has 3 diagonal generators of Quaternion SU(2) + (8 + 8) Fermions) + 8 Spacetime vectors

27 = Octonionic Jordan Algebra J3(O) =(bijection)= J4(Q)o

E6 -> D5 = Spin(10) - semidirect product with - 16+1+16 Heisenberg Algebra H16

16 = (26-10)-dim Fermion part of J3(O)o

E6 = F4 + J3(O)o 26-dim traceless part of 27-dim Jordan Algebra J3(O)

F4 = **8** + **28** + (**8+** + **8-**)

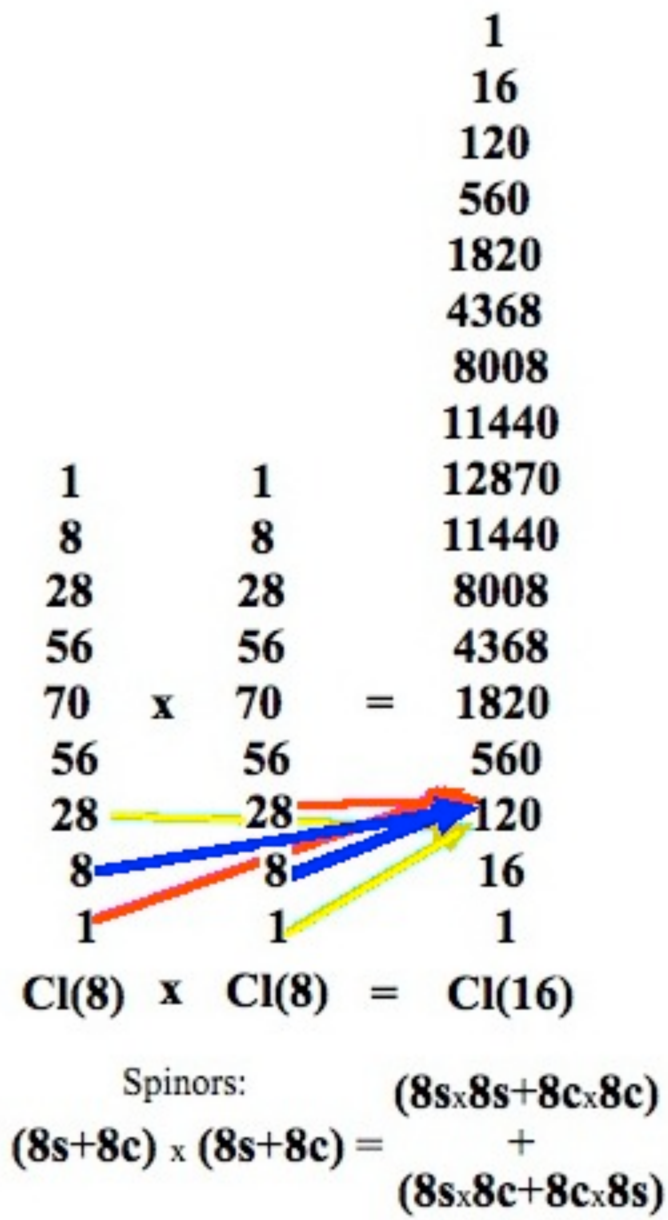
F4 -> B3 = (Spin(6) + S6) - semidirect product with - 15+1+15 Heisenberg Algebra H15

for 15 D3 = Spin(2,4) Conformal Gauge Bosons

6-dim Conformal Spacetime reduces to M4 Physical Spacetime

Cl(8) = 1 + **8** + **28** + 56 + 70 + 56 + 28 + 8 + 1 = (**8+** + **8-**) x 16

To study the E8 substructure of Cl(16),
note that the 120-dimensional bosonic Cl(16) bivector part of E8 decomposes, with respect to factoring Cl(16) into the tensor product Cl(8) x Cl(8) allowed by 8-periodicity, into 1x28 + 8x8 + 28x1



The 256-dim spinor of Cl(16) decomposes as the direct sum of
the two 128-dim half-spinor representations, i.e., as one generation and one anti-generation.

248-dim E8 contains the 128-dim D8 Cl(16) half-spinor representation of one generation of Fermion Particles and AntiParticles,
but
does not contain any of the anti-generation D8 Cl(16) half-spinor.

Note that if you tried to build a larger Lie Algebra than E8 within Cl(16)
by using the anti-generation D8 Cl(16) half-spinor, you would fail
because the construction would be mathematically inconsistent,
so
E8 is the Maximal Lie Algebra within Cl(16).

Decompose, with respect to factoring Cl(16) into Cl(8) x Cl(8),
the 128-dim fermion one-generation representation into two 64-dim fermion representations in terms of their 8 covariant components with respect to 8-dim spacetime as:

one 64 = 8x8 representing 8 fundamental left-handed fermion particles in terms of their 8 covariant components with respect to 8-dim spacetime
and
the other 64 = 8x8 representing 8 fundamental right-handed fermion antiparticles.

Cl(Cl(4)) = Cl(16) containing E8

Frank Dodd (Tony) Smith, Jr. - 2011

Cl(4):
1 grade-0: s
4 grade-1: x y z t - M4 physical spacetime
6 grade-2: a b c d e f - M4L Lorentz transformations
4 grade-3: x y z t - CP2 internal symmetry space
1 grade-4: s

Cl(Cl(4)) = Cl(16) for which Physical Interpretations
are based on Triality whereby
x y z t x y z t corresponds to
8-dim M4xCP2 Kaluza-Klein SpaceTime
8 elementary Fermion Particles
8 elementary Fermion AntiParticles.
The 8-dim M4xCP2 Kaluza-Klein interpretation is
used for Cl(16) grade-1 in which
x y z t x y z t occur as single elements
The 8 Fermion Particle - 8 Fermion AntiParticle
interpretation is used for the gauge forces of grade-2
in which x y z t x y z t occur as antisymmetric pairs.

1 grade-0:
s

16 grade-1:
s
x y z t - M4 physical spacetime
a b c d e f
x y z t - CP2 internal symmetry space
s

Further Physical Interpretations:
Even-Odd Clifford Dual to M4 physical spacetime:
s a b c
Even-Odd Clifford Dual to CP2 internal symmetry space:
d e f s

120 grade-2:

```
sx sy sz st
sa sb sc sd se sf
sx sy sz st                ss

xy xz xt
xa xb xc xd xe xf
xx xy xz xt                xs

yz yt
ya yb yc yx ye yf
yx yy yz yt                ys

zt
za zb zc zd ze zf
zx zy zz zt                zs

ta tb tc td te tf
tx ty tz tt                ts

ab ac ad ae af
ax ay az at                as

bc bd be bf
bx by bz bt                bs

cd ce cf
cx cy cz ct                cs

de df
dx dy dz dt                ds
ef
ex ey ez et                es
fx fy fz ft                fs
xy xz xt                   xs
yz yt                      ys
zt                         zs
                           ts
```

Physical Interpretations of the 120 grade-2 elements:

28-dim D4 Spin(8) for Standard Model Gauge Groups:

xy xz xt
yz yt
zt

xx xy xz xt |
yx yy yz yt | - This is U(4) that contains SU(3).
zx zy zz zt | U(2) = SU(2)xU(1) arises from
tx ty tz tt | CP2 = SU(3)/U(2) by Batakis.

xy xz xt
yz yt
zt

28-dim D4 Spin(8) for Conformal Gravity:

sa sb sc sd se sf

ss

ab ac ad ae af |
bc bd be bf | - This is Spin(2,4) Conformal Group
cd ce cf | that gives
de df | Gravity by MacDowell-Mansouri.
ef |

as
bs
cs
ds
es
fs

64-dim to describe 8-dim Kaluza-Klein SpaceTime:
Consider 8-dim K-K as Octonion Spacetime
with Octonion basis {1,i,j,k,E,I,J,K}.
For each of the 8 x y z t x y z t Position dimensions
there are 8 Momentum dimensions represented by
s a b c s d e f and basis elements {1,i,j,k,E,I,J,K}.
The a b c correspond to an SU(2) and so to {i,j,k}.
The d e f correspond to another SU(2) and to (I,J,K}.

8 s-terms for Real Part of Octonion SpaceTime:
sx sy sz st
sx sy sz st

8 s-terms for E-Imaginary Part of Octonion SpaceTime:
xs
ys
zs
ts
xs
ys
zs
ts

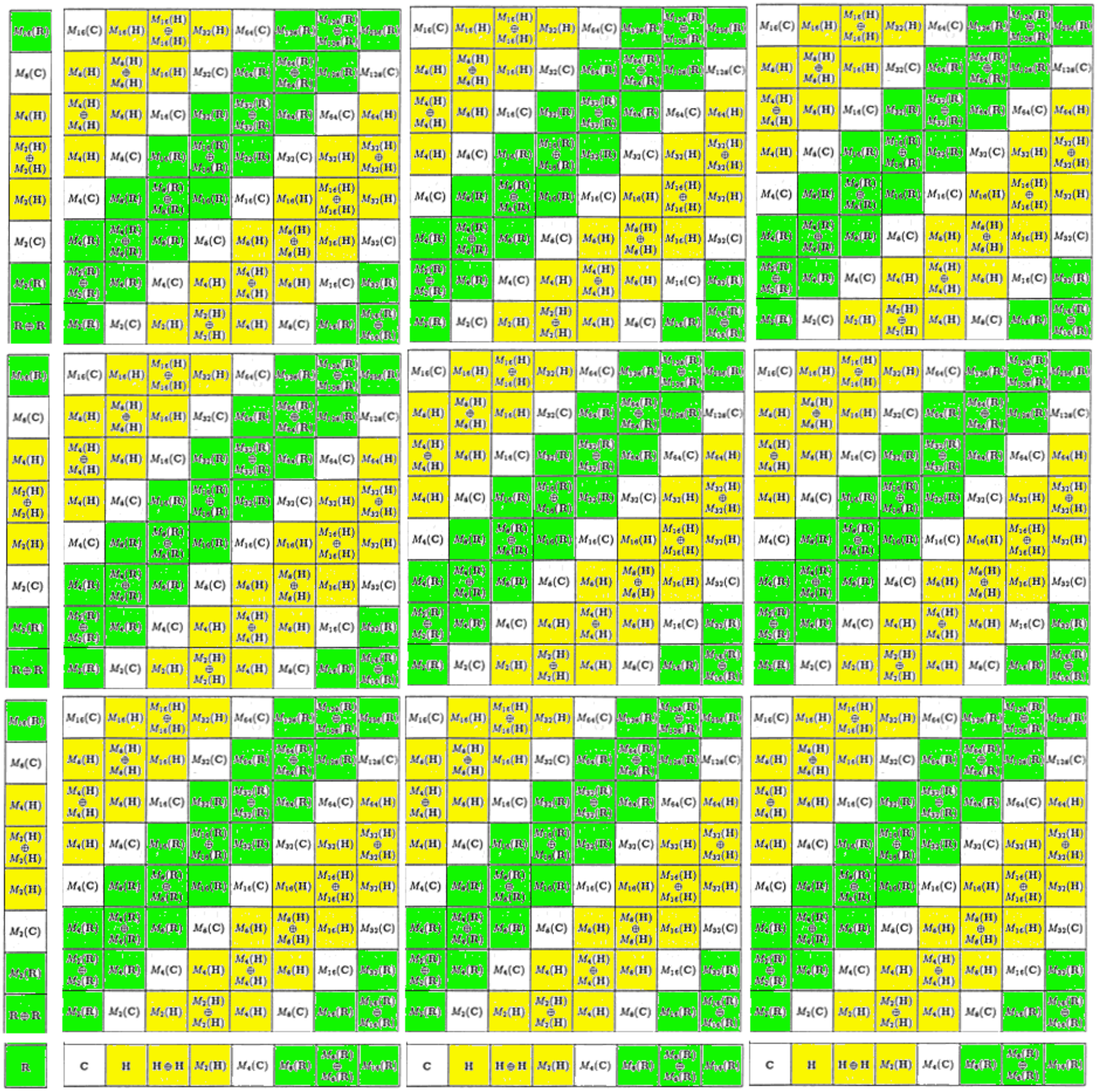
24 M4 ijkIJK components of Octonion SpaceTime:
xa xb xc xd xe xf
ya yb yc yx ye yf
za zb zc zd ze zf
ta tb tc td te tf

24 CP2 ijkIJK components of Octonion SpaceTime:
ax ay az at
bx by bz bt
cx cy cz ct
dx dy dz dt
ex ey ez et
fx fy fz ft

E8 is constructed from Cl(16) using grade-2 and half-Spinors so consider Spinors of Clifford Algebras:

$M_{16}(\mathbf{R})$	$M_{16}(\mathbf{C})$	$M_{16}(\mathbf{H})$	$M_{16}(\mathbf{H}) \oplus M_{16}(\mathbf{H})$	$M_{32}(\mathbf{H})$	$M_{64}(\mathbf{C})$	$M_{128}(\mathbf{R})$	$M_{128}(\mathbf{R}) \oplus M_{128}(\mathbf{R})$	$M_{256}(\mathbf{R})$
$M_{8}(\mathbf{C})$	$M_{8}(\mathbf{H})$	$M_{8}(\mathbf{H}) \oplus M_{8}(\mathbf{H})$	$M_{16}(\mathbf{H})$	$M_{32}(\mathbf{C})$	$M_{64}(\mathbf{R})$	$M_{64}(\mathbf{R}) \oplus M_{64}(\mathbf{R})$	$M_{128}(\mathbf{R})$	$M_{128}(\mathbf{C})$
$M_{4}(\mathbf{H})$	$M_{4}(\mathbf{H}) \oplus M_{4}(\mathbf{H})$	$M_{8}(\mathbf{H})$	$M_{16}(\mathbf{C})$	$M_{32}(\mathbf{R})$	$M_{32}(\mathbf{R}) \oplus M_{32}(\mathbf{R})$	$M_{64}(\mathbf{R})$	$M_{64}(\mathbf{C})$	$M_{64}(\mathbf{H})$
$M_{2}(\mathbf{H}) \oplus M_{2}(\mathbf{H})$	$M_{4}(\mathbf{H})$	$M_{8}(\mathbf{C})$	$M_{16}(\mathbf{R})$	$M_{16}(\mathbf{R}) \oplus M_{16}(\mathbf{R})$	$M_{32}(\mathbf{R})$	$M_{32}(\mathbf{C})$	$M_{32}(\mathbf{H})$	$M_{32}(\mathbf{H}) \oplus M_{32}(\mathbf{H})$
$M_{2}(\mathbf{H})$	$M_{4}(\mathbf{C})$	$M_{8}(\mathbf{R})$	$M_{8}(\mathbf{R}) \oplus M_{8}(\mathbf{R})$	$M_{16}(\mathbf{R})$	$M_{16}(\mathbf{C})$	$M_{16}(\mathbf{H})$	$M_{16}(\mathbf{H}) \oplus M_{16}(\mathbf{H})$	$M_{32}(\mathbf{H})$
$M_{2}(\mathbf{C})$	$M_{4}(\mathbf{R})$	$M_{4}(\mathbf{R}) \oplus M_{4}(\mathbf{R})$	$M_{8}(\mathbf{R})$	$M_{8}(\mathbf{C})$	$M_{8}(\mathbf{H})$	$M_{8}(\mathbf{H}) \oplus M_{8}(\mathbf{H})$	$M_{16}(\mathbf{H})$	$M_{32}(\mathbf{C})$
$M_{2}(\mathbf{R})$	$M_{2}(\mathbf{R}) \oplus M_{2}(\mathbf{R})$	$M_{4}(\mathbf{R})$	$M_{4}(\mathbf{C})$	$M_{4}(\mathbf{H})$	$M_{4}(\mathbf{H}) \oplus M_{4}(\mathbf{H})$	$M_{8}(\mathbf{H})$	$M_{16}(\mathbf{C})$	$M_{32}(\mathbf{R})$
$\mathbf{R} \oplus \mathbf{R}$	$M_{2}(\mathbf{R})$	$M_{2}(\mathbf{C})$	$M_{2}(\mathbf{H})$	$M_{2}(\mathbf{H}) \oplus M_{2}(\mathbf{H})$	$M_{4}(\mathbf{H})$	$M_{8}(\mathbf{C})$	$M_{16}(\mathbf{R})$	$M_{16}(\mathbf{R}) \oplus M_{16}(\mathbf{R})$
$\mathbf{R}$	$\mathbf{C}$	$\mathbf{H}$	$\mathbf{H} \oplus \mathbf{H}$	$M_{2}(\mathbf{H})$	$M_{4}(\mathbf{C})$	$M_{8}(\mathbf{R})$	$M_{8}(\mathbf{R}) \oplus M_{8}(\mathbf{R})$	$M_{16}(\mathbf{R})$

Extend the 8x8 part like a checkerboard
and extend the 1x7 and 7x1 strips for p =0 and q = 0
The extension (at the stage of extending by 3 steps) looks like this:



The real (green) and complex (white) and quaternion (yellow) types continue in a consistent pattern after extension.

Real Spinors (signatures (2,2) (3,1))
Cl(4) = M4(R) = 4x4 Real Matrix Algebra
Cl(8) = M16(R) (signature (0,8))
Cl(16) = M16(R) (x) M16(R) = M256(R) (signature (0,16))

Physically, the Real Structures describe
High-Energy (near Planck scale) Octonionic Physics.

Cl(4) Spinors:
4-dim x y z t space on which M4(R) matrices act.
With Spinors defined in terms
of Even Subalgebra of Clifford Algebra,
M4(R) reduces to M2(R) + M2(R)
and Cl(4) Spinors reduce to sum of half-Spinors as
2-dim x y space plus 2-dim z t space.

Cl(8) Spinors:
16-dim space on which M16(R) matrices act.
M16(R) reduces to M8(R) + M8(R)
and Cl(8) Spinors reduce to sum of half-Spinors as
8-dim x y z t x y z t +space plus
8-dim x y z t x y z t -space
where Triality has been used to represent half-Spinors
in terms of vectors x y z t x y z t that can be seen
as Cl(4) structures.

Cl(Cl(4)) = Cl(16) Spinors:
256-dim space on which M256(R) matrices act.
M256(R) reduces to M128(R) + M128(R)
and Cl(16) Spinors (8+ + 8-)x(8+ + 8-) =
= (64++ + 64--) + (64+- + 64-+) = 128pure + 128mixed
which reduces to sum of half-Spinors as
128-dim pure space plus 128-dim mixed space.
Only the pure half-Spinor 128-dim space is used to
construct E8 = 120-dim grade-2 + 128-dim half-Spinor.
The pure 128-dim half-Spinor 64++ + 64-- describes:
8 covariant components of 8 Fermion Particles by 64++
8 covariant components of 8 AntiParticles by 64-- .

Quaternion Spinors (signatures (0,4) (1,3) (4,0))
Cl(4) = M2(H) = 2x2 Quaternion Matrix Algebra

Cl(8) = M8(H) (signature (2,6))
Cl(16) = M8(H) (x) M8(H) = M128(H) (signature (4,12))
Physically, Quaternionic Structures describe
Low-Energy (with respect to Planck scale) Physics
which emerges after
Octonion Symmetry is broken
by "freezing out" a preferred Quaternion Substructure
at the End of Inflation
so
Quaternionic Structure is relevant for Low-Energy physics described by Cl(4) and observed directly by us now,
but not relevant for Cl(8) or Cl(16) which describe High-Energy physics such as that of the Inflationary Era.

Cl(4) Spinors:
8-dim space on which M2(H) matrices act.
With Spinors defined in terms
of Even Subalgebra of Clifford Algebra,
M2(H) reduces to H+H
and Cl(4) Spinors reduce to sum of half-Spinors as
4-dim space plus 4-dim space
which enables Cl(4) to describe Fermion Particles as
Lepton + RGB Quarks Particles by one H of H+H plus
Lepton + RGB Quarks AntiParticles by the other H of H+H
but Cl(4) is not large enough to distingush Neutrinos
from Electrons. To do that it should be expanded into
Cl(6) of the Conformal Group (signature (2,4))
with Cl(6) = M4(H) and Even Subalgebra M2(H) + M2(H)
giving a half-Spinor H+H for 8 Fermion Particles and
another half-Spinor H+H for 8 Fermion AntiParticles.
In a sense, this expands 4+4=8-dim Batakis Kaluza-Klein
to a 6+4=10-dim CNF6 x CP2 Kaluza-Klein,
with the M4 Minkowski M4 physical SpaceTime becoming a
conformal CNF6 physical SpaceTime
that is related to Segal Conformal Dark Energy.

Higgs as Primitive Idempotent:

Clifford Algebra Primitive Idempotents are described by Pertti Lounesto in his book Clifford Algebras and Spinors (Second Edition, LMS 286, Cambridge 2001) in whch he said at pages 226-227 and 29:
"... Primitive idempotents and minimal left ideals An orthonormal basis of R(p,q) induces a basis of Cl(p,q), called the standard basis.
Take a non-scalar element e_T, e_T^2 = 1, from the standard basis of Cl(p,q).
Set e = (1/2)(1 + e_T) and f = (1/2)(1 - e_T), then e + f = 1 and ef = fe = 0.
So Cl(p,q) decomposes into a sum of two left ideals
Cl(p,q) =Cl(p,q) e + Cl(p,q) f , where [for n = p + q]
dim Cl(p,q) e = dim Cl(p,q) f = [dim] (1/2) Cl(p,q) = 2^(n-1).
Furthermore,
if { e_T_1 , e_T_2 , ... , e_T_k } is a set of non-scalar basis elements
such that e_T_i^2 = 1 and e_T_i e_T_j = e_T_j e_T_i ,
then letting the signs vary independently in the product
(1/2)(1 +/- e_T _1) (1/2)(1 +/- e_T_2) ... (1/2)(1 +/- e_T_k) ,
one obtains 2^k idempotents which are mutually annihilating and sum up to 1.
The Clifford algebra Cl(p,q) is thus decomposed into a direct sum of 2^k left ideals, and by construction, each left ideal has dimension 2^(n - k) .
In this way one obtains a minimal left ideal by forming a maximal product of non-annilating and commuting idempotents.
The Radon-Hurwitz number r_i for i in Z is given by

i	0	1	2	3	4	5	6	7
r_i	0	1	2	2	3	3	3	3

and the recursion formula r_(i + 8) = r_ i + 4 .
For the negative values of i one may observe that r_(-1) = -1
and r_(-i) = 1 - i + r_(i + 2) for i > 1 .
r_-8 = 1 - 8 + r_10

Theorem. In the standard basis of Cl(p,q) there are always
k = q - r_(q - p) non-scalar elements e_T_i , e_T_i^2 = 1 ,
which commute, e_T_i e_T_j = e_T_j e_T_i ,
and generate a group of order 2^k .

The product of the corresponding mutually non-annihilating idempotents,
f = (1/2)(1 +/- e_T _1) (1/2)(1 +/- e_T_2) ... (1/2)(1 +/- e_T_k) ,

is primitive in Cl(p,q).

Thus, the left ideal S = Cl(p,q) f is minimal in Cl(p,q).
Example ... In the case of R(0,7) we have k = 7 - r_7 = 4. Therefore the idempotent
f = (1/2)(1 + e_124) (1/2)(1 + e_235) (1/2)(1 + e_346) (1/2)(1 + e_457)
is primitive to Cl(0,7) = 2^Mat(8,R). ...".

Further example of R(0,8) is discussed by Pertti Lounesto in his book
"Spinor Valued Regular Functions in Hypercomplex Analysis"
(Report-HTKKMAT-A154 (1979) Helsinki University of Technology) said
[in the quote below I have changed his notation for a Clifford algebra
from R_(p,q) to Cl(p,q)] at pages 40-42:
"... To fix a minimal left ideal V of Cl(p,q)
we can choose a primitive idempotent f of Cl(p,q) so that V = Cl(p,q) f .
By means of an orthonormal basis { e_1 , e_2 , ... , e_n }
for [the grade-1 vector part of Cl(p,q)] Cl^1(p,q) we can construct
a primitive idempotent f as follows:
Recall that the 2^n elements
e_A = e_a_1 e_a_2 ... e_a_k ,
1 < a_1 < a_2 < ... < s_k < n
constitute a basis for Cl(p,q). ...
dim_R V = 2^X , where X = h or X = h + 1 according as
p - q = 0, 1, 2 mod 8 or p - q = 3, 4, 5, 6, 7 mod 8 and h = [n / 2] .
Select n - X elements e_A, e_A^2 = 1 , so they are pairwise commuting
and generate a group of order 2^(n - X) .
Then the idempotent ...
f = (1/2)(1 + e_A_1) (1/2)(1 + e_A_2) ... (1/2)(1 + e_A_(n - X))
is primitive ...
To prove this note that the dimension of (1/2)(1 + e_A) Cl(p,q) is (2^n) / 2
and so the dimension of Cl(p,q) f is (2^n) / (2^(n - X)) = 2^X .
Hence,
if there exists such an idempotent f , then f is primitive.
To prove that such an idempotent f exists in every Clifford algebra Cl(p,q)
we may first check the lower dimensional cases and then proceed by making use
of the isomorphism Cl(p,q) x Cl(0,8) = Cl(p, q + 8)
and the fact that Cl(0,8) has a primitive idempotent
f = (1/2)(1 + e_1248) (1/2)(1 + e_2358) (1/2)(1 + e_3468) (1/2)(1 + e_4578)
= (1/16)(1 + e_1248 + e_2358 + e_3468 + e_4578 + e_5618 + e_6728 + e_7138
- e_3567 - e_4671 - e_5712 - e_6123 - e_7234 - e_1345 - e_2456 + e_J)
with four factors [and where J = 12345678] ...
The division ring F = f Cl(p,q) f = { PSI in V | PSI f = f PSI }
is isomorphic to R, C, or H

according as $p - q = 0, 1, 2, \bmod 8$, $p - q = 3 \bmod 4$, or $p - q = 4, 5, 6 \bmod 8$. ...".
In "Idempotent Structure of Clifford Alghebras" (Acta Applicandae Mathematicae 9 (1987) 165-173) Pertti Lounesto and G. P. Wene said:
"... An idempotent e is primitive if it is not a sum of two nonzero annihilating idempotents and minimal if it is a minimal element in the set of all nonzero idempotents with order relation $f \leq e$ if and only if $ef = f = fe$.
These last two properties of an idempotent e are equivalent. An idempotent e is primitive if e is the only nonzero idempotent of the subring eAe.
A subring S of A is a left ideal if ax is in S for all a in A and x in S.
A left ideal is minimal if it does not contain properly any nonzero left ideals.
... if S is a minimal left ideal of A,
then either Ss = 0 or S = Ae for some idempotent e.
Spinor spaces are minimal left ideals of a Clifford algebra.
Any minimal left ideal S of a Clifford algebra A = Rp,q is of the form S = Ae for some primitive idempotent e of Rp,q.
... if e is a primitive idempotent of Rp,q then

$$\begin{matrix} e & 0 \\ 0 & 0 \end{matrix}$$

is a primitive idempotent of Rp,q(2) = Rp+1,q+1
... The maximum number of mutually annihilating primitive idempotents in the Clifford algebra Rp,q is 2^k where k = q - r_q-p .
...[where]... r_i ...[is the]... Radon-Hurwitz number ...
These mutually annihilating primitive idempotents sum up to 1.
If mutually annihilating primitive idempotents sum up to 1,
then in a simple ring, such a sum has always the same number of summands.
... Lattices Generated by Idempotents
A lattice is a partially ordered set where each subset of two elements has a least upper bound and a greatest lower bound. Any set of idempotents of a ring A is partially ordered under the ordering defined by $e \leq f$ if and only if $ef = e = fe$.
If e and f are commuting idempotents, then ef and e + f - ef are, respectively, a greatest lower bound and a least upper bound relative to the partial ordering defined. Hence, any set of commuting idempotents generate a lattice.
This lattice is complemented and distributive.
...
Let e1 , e2 , ... , es in Rp,q be a set of mutually annihilating primitive idempotents summing up to 1. Then the set e1 , e2 , ... , es generates a complemented and distributive lattice of order 2^s , where s = 2^k , k = q - r_q-p
...
EXAMPLE [I have changed the example from R3,1 to R0,8 and paraphrased]
In the Clifford algebra R0,8 = R(16) we have k = 8 - r_8 = 8 - 4 = 4

and so primitive idempotents can have 4 commuting factors of type (1/2)(1 + eT) . Furthermore s = 2^k = 16 and so R0,8 can be represented by 16x16 matrices R(16), and there are 2^s = 2^16 = 65,536 commuting idempotents in the lattice generated by the 16 mutually annihilating primitive idempotents ...
this lattice looks like ... a 16-dimensional analogy of the cube ...”.

The Clifford algebra R0,8 = Cl(0,8) is 2^8 = 16x16 = 256-dimensional with graded structure such that it
is represented by the geometric structure of a simplex.

The Spinors of R0,8 = Cl(0,8) are sqrt(256) = 16-dimensional with no simplex-type graded structure so that it
is represented by the geometric structure of a cube.

248-dim E8 = 120-dim Cl(16) bivectors + 128-dim Cl(16) half-spinors and Cl(16) = Cl(8) x Cl(8)
so the structure of the 128-dim Cl(16) half-spinors is important for E8 Physics.

The Clifford algebra Cl(16) (also denoted R0,16) is the real 256x256 matrix algebra R(256) for which we have k = 16 - r_16 = 16 - 8 = 8
and so primitive idempotents can have 8 commuting factors of type (1/2)(1 + eT) . Furthermore s = 2^k = 256 and so R0,16 can be represented by 256x256 matrices R(256), and there are 2^s = 2^256 = 1.158 x 10^77 commuting idempotents in the lattice generated by the 256 mutually annihilating primitive idempotents.

E8 lives in Cl(16) as
248-dim E8 = 120-dim bivectors of Cl(16) + 128-dim half-spinor of Cl(16).
Since Cl(16) bivectors are all in one grade of Cl(16)
and Cl(16) half-spinors have no simplex-type graded structure
E8 does not get detailed graded structure from Cl(16) gradings,
but only the Even-Odd grading obtained by
splitting 128-dim half-spinor into two mirror image 64-dim parts:
E8 = 64 + 120 + 64

E8 has only a Cl(16) half-spinor so there are in E8 Physics 2^(s/2) = 2^128 commuting idempotents in the lattice generated by the 128 mutually annihilating primitive idempotents. 2^128 = about 3.4 x 10^38 the square root of which is about the ratio (Hadron mass / Planck mass)^2 of the Effective Mass Factor for Gravity strength.

The typical Hadron mass can be thought of in terms of superposition of Pions:

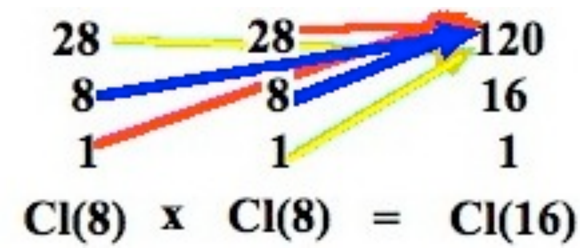
In E8 Physics, at a single spacetime vertex, a Planck-mass black hole is the Many-Worlds quantum sum of all possible virtual first-generation particle-antiparticle fermion pairs permitted by the Pauli exclusion principle to live on that vertex.Once a Planck-mass black hole is formed, it is stable in in E8 Physics. Less mass would not be gravitationally bound at the vertex. More mass at the vertex would decay by Hawking radiation.Since Dirac fermions in 4-dimensional spacetime can be massive (and are massive at low enough energies for the Higgs mechanism to act), the Planck mass in 4-dimensional spacetime is the sum of masses of all possible virtual first-generation particle-antiparticle fermion pairs permitted by the Pauli exclusion principle. A typical combination should have several quarks, several antiquarks, a few colorless quark-antiquark pairs that would be equivalent to pions, and some leptons and antileptons. Due to the Pauli exclusion principle, no fermion lepton or quark could be present at the vertex more than twice unless they are in the form of boson pions, colorless first-generation quark-antiquark pairs not subject to the Pauli exclusion principle. Of the 64 particle-antiparticle pairs, 12 are pions. A typical combination should have about 6 pions.
If all the pions are independent, the typical combination should have a mass of 0.14x6 GeV = 0.84 GeV. However, just as the pion mass of 0.14 GeV is less than the sum of the masses of a quark and an antiquark, pairs of oppositely charged pions may form a bound state of less mass than the sum of two pion masses. If such a bound state of oppositely charged pions has a mass as small as 0.1 GeV, and if the typical combination has one such pair and 4 other pions, then the typical combination should have a mass in the range of 0.66 GeV so that

sqrt(3.4 x 10^38) = 1.84 x 10^19
while Planck Mass = 1.22 x 10^19 GeV = 1.30 x 10^19 Proton Mass =
= 1.85 x 10^19 Hadron Mass

In terms of the Graded Structure of Cl(16)
the 256 Cl(16) Primitive Idempotents can be understood
in terms of graded structures of the Cl(8) and E8 substructures of Cl(16):

The detailed E8 graded structure 8 + 28 + 56 + 64 + 56 + 28 + 8
comes from the grades of the Cl(8) factors of Cl(16) = Cl(8)xCl(8).

The Even 120 of E8 breaks down in terms of Cl(8) factors as



120 = 1x28 + 8x8 + 28x1 = 28 + 64 + 28

The Odd 128 = 64 + 64 breaks down as

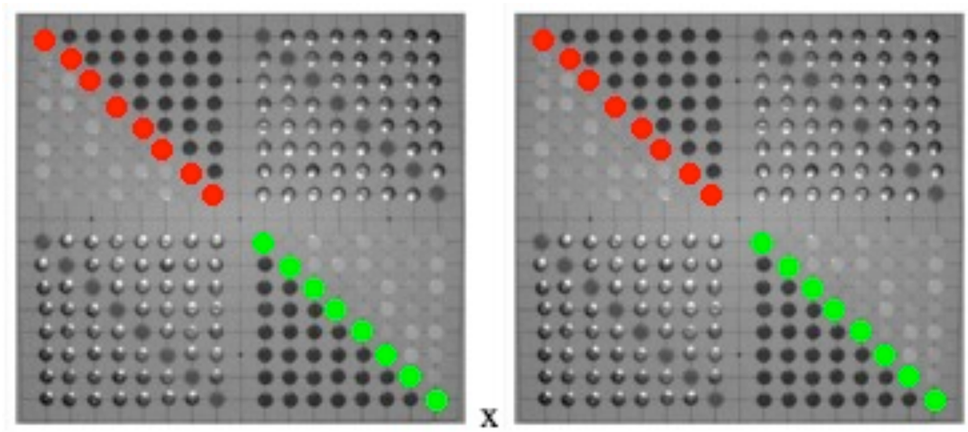
Spinors:

$$(8s+8c) \times (8s+8c) = \begin{matrix} (8s \times 8s + 8c \times 8c) \\ + \\ (8s \times 8c + 8c \times 8s) \end{matrix}$$

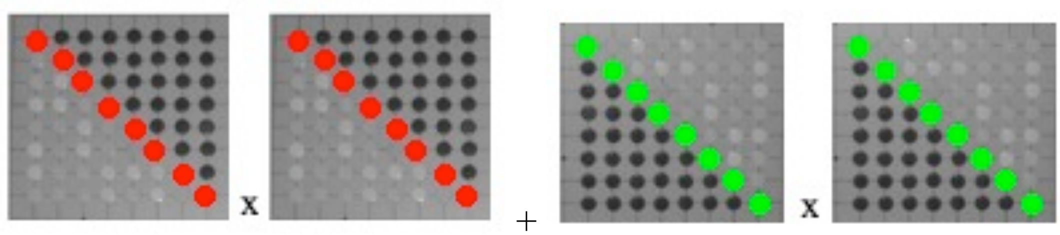
to become
64 + 64 = 8 + 56 + 56 + 8

Here are some details about the half-spinors of E8:

The +half-spinors (red) and -half-spinors (green) of Cl(8) are the 8+8 = 16 diagonal entries of the 16x16 real matrix algebra that is Cl(8), so that Cl(16) = Cl(8) x Cl(8) can be represented as:

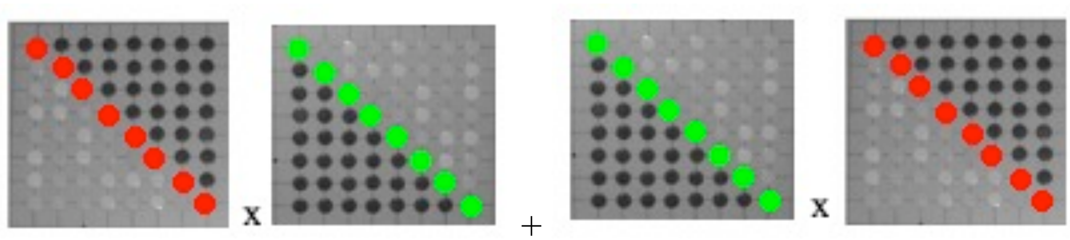


and
the 16x16 = 256 spinors of Cl(16) (the diagonal entries of R(256)) can be represented as the sum of the diagonal product terms

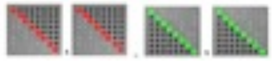


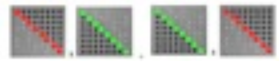
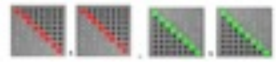
64+64 = 128
(these two (pure red and pure green) are the Cl(16) +half-spinor which decomposes physically into particles (red) and antiparticles (green))

+

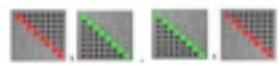
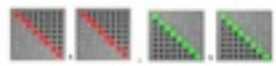


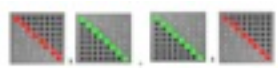
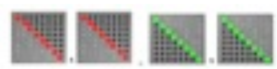
64+64 = 128
(these two (mixed red and green) are the Cl(16) -half-spinor which do not decompose readily into particles (red) and antiparticles (green))

grade-0: 1 PurePI 

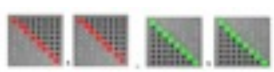
grade-1: 16 NotPI
grade-2: 120 NotPI
grade-3: 560 NotPI
grade-4: 1820 = 1792 + 14 MixedPI 
+ 14 PurePI 

grade-5: 4368 NotPI
grade-6: 8008 NotPI
grade-7: 11440 NotPI

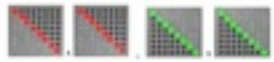
grade-8: 12870 = 12672 + 100 MixedPI 
+ 98 PurePI 

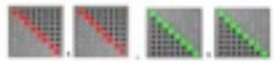
grade-9: 11440 NotPI
grade-10: 8008 NotPI
grade-11: 4368 NotPI
grade-12: 1820 = 1792 + 14 MixedPI 
+ 14 PurePI 

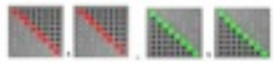
grade-13: 560 NotPI
grade-14: 120 NotPI
grade-15: 16 NotPI

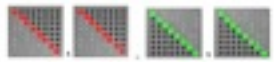
grade-16: 1 PurePI 

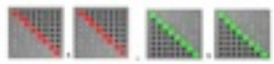
Only the PurePI Cl(16) +half-spinor has scalar grade-0 and pseudoscalar grade-16

grade-0:   1 PurePI 

grade-4: 14 PurePI 

grade-8: 98 PurePI 

grade-12: 14 PurePI 

grade-16: 1 PurePI 

so it is the only half-spinor that can physically represent a Higgs scalar
and is the only half-spinor in the E8 of E8 Physics.

Further, for E8 to describe a consistent E8 Physics model, it must be that

E8 = Cl(16) bivectors + Cl(16) +half-spinor
with physical distinction between particles and antiparticles
and that
E8 does not contain the Cl(16) -half-spinor made up of particle/antiparticle mixtures.

In the context of physics models,
the Cl(16) -half-spinors correspond to fermion antigenerations that are not realistic
and their omission from E8 allows E8 Physics to be chiral and realistic.

E8 with graded structure 8 + 28 + 56 + 64 + 56 + 28 + 8 lives in Cl(16)
as
248-dim E8 = 120-dim bivectors of Cl(16) + 128-dim half-spinor of Cl(16).

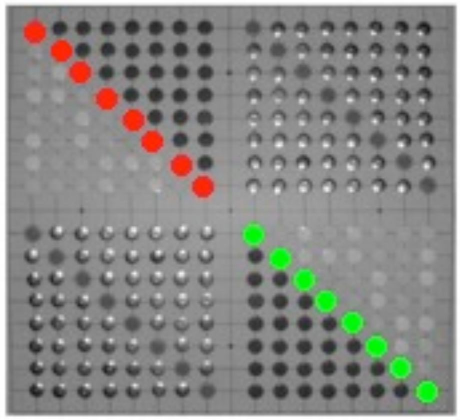
The two half-spinors of Cl(16) are Left Ideals of a Cl(16) Primitive Idempotent.

Due to 8-periodicity of Real Clifford Algebras Cl(16) = Cl(8) x Cl(8)
where x is tensor product. Let Primitive Idempotent be denoted by PI
and J = 12345678 :

Cl(16)PI = Cl(8)PI x Cl(8)PI

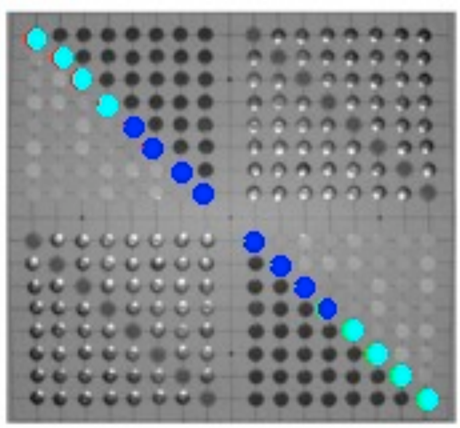
Cl(8)PI = (1/16) (1 + e_1248) (1 + e_2358) (1 + e_3468) (1 + e_4578) =

= (1/16)(1
+ e_1248 + e_2358 + e_3468 + e_4578 + e_5618 + e_6728 + e_7138



- e_3567 - e_4671 - e_5712 - e_6123 - e_7234 - e_1345 - e_2456
+ e_J) =

= (1/16)(
1 +
+ e_1248 + e_2358 + e_3468
- e_3567 - e_4671 - e_5712
+ e_J



+ e_4578 + e_5618 + e_6728 + e_7138
- e_6123 - e_7234 - e_1345 - e_2456
)

256-dim Cl(8) has graded structure 1 + 8 + 28 + 56 + 70 + 56 + 28 + 8 + 1
16-dim Cl(8)PI has graded structure 1 + 14 + 1 = 1 + (8+6) + 1
16-dim Cl(8)PI = 8-dim Cl(8)PIE8 + 8-dim Cl(8)PInotE8
where
8-dim Cl(8)PIE8 has graded structure of only 8 in the middle grade
plus
8-dim Cl(8)PInotE8 has graded structure 1 + 6 + 1
8-dim Cl(8)PIE8 is contained in the middle 64 of E8 graded structure
8 + 28 + 56 + 64 + 56 + 28 + 8
so that
since the physical interpretation of the middle 64 is
8 momentum components of 8-dim position spacetime
the 8-dim Cl(8)PIE8 corresponds to a one-component field over 8-dim spacetime
and
therefore Cl(8)PIE8 describes a scalar field over 8-dim spacetime
and so a Higgs field in E8 Physics spacetime.

8-dim Cl(8)PInotE8 with graded structure 1 + 6 + 1
corresponds to the part of Cl(8)PI that is in Cl(8) but not in E8
so that

Cl(8) with graded structure 1 + 8 + 28 + 56 + 70 + 56 + 28 + 8 + 1
=
Cl(8)PInotE8 with graded structure 1 + 6 + 1
+
E8 with graded structure 8 + 28 + 56 + 64 + 56 + 28 + 8

and
therefore Cl(8)PInotE8 describes the Clifford algebra structure beyond E8
(1 scalar and 6 middle-grade and 1 pseudoscalar)
that produces the half-spinors that belong to E8
and
therefore describes the coupling between the Higgs field and half-spinor Fermions.

The Higgs-Fermion coupling, below the freezing out of a preferred Quaternionic substructure of 8-dim Octonionic E8 Physics spacetime, produces
the Mayer Mechanism Higgs field of 8-dim Batakis Kaluza-Klein spacetime.

The Higgs-Fermion coupling, below ElectroWeak Symmetry Breaking Energy, gives mass to Fermions.

Since the 128-dim half-spinor part of E8 comes from
Cl(16)PI = Cl(8)PI x Cl(8)PI
the E8 Higgs-Fermion is based on
two copies (one from each Cl(8)PI factor) of a scalar Higgs field over spacetime

so that

two copies of Cl(8)PIE8 show that the E8 Physics Higgs field is a scalar doublet.

As Cottingham and Greenwood said in their book “An Introduction to the Standard Model of Particle Physics” (2nd ed, Cambridge 2007):
“... Higgs ... mechanism ...[uses]... a complex scalar field ... [i]n place of [which]... we [can] have two coupled real scalar fields ...”.

As Steven Weinberg said in his book “The Quantum Theory of Fields, v. II” (Cambridge 1996 at pages 317-318 and 356):
“... With only a single type of scalar doublet, there is just one ... term that satisfies SU(2) and Lorentz invariance ... At energies below the electroweak breaking scale, this yields an effective interaction ... this gives lepton number non-conserving neutrino masses at most of order (300 GeV)^2 / M ... For instance,
in the so-called see-saw mecanism, a neutrino mass of this order
would be produced by exchange of a heavy neutral lepton of mass M ...
M is expected to be of order 10^15 - 10^18 GeV,
so we would expect neutrino masses in the range 10^(-4) - 10^(-1) ...
A similar analysis shows that there are interactions of dimensionality six
that violate both baryon and lepton number conservation, involving
three quark fields and one lepton field. Such interactions would have
coupling constants of order M^(-2), and would lead to processes like
proton decay, with rates proportional to M^(-4). ...”.

and

the part of the Cl(16) Primitive Idempotent that is not in the E8 in Cl(16)
is the product Cl(8)PInotE8 x Cl(8)PInotE8 of two copies of Cl(8)PInotE8
each copy having graded structure 1 + 6 + 1 (grades 0 and 4 and 8)
so that
the part of the Cl(16) Primitive Idempotent that is not in the E8 in Cl(16)
has graded structure 1 + 12 + 38 + 12 + 1 (grades 0 and 4 and 8 and 12 and 16).
The total dimension of those Cl(16) grades are:
1 and 1820 and 128870 and 1820 and 1.

Cl(8) **256 = 1 + 8 + 28 + 56 + 70 + 56 + 28 + 8 + 1**

Primitive **16 = 1 + 6 + 1**
Idempotent **+ 8**

E8 Root Vectors 240 = 8 + 28 + 56 + 56 + 56 + 28 + 8

Greg Trayling and W. E. Baylis in Chapter 34 of “Clifford Algebras - Applications to Mathematics, Physics, and Engineering”, 2004, Proceedings of 2002 Cookeville Conference on Clifford Algebras, ed. by Rafal Ablamowicz
(see also hep-th/0103137) said:
“... the exact gauge symmetries U(1)Y x SU(2)L x SU(3)C of the minimal standard model arise ...[from]... symmetries of ... a ... space with ... four extra spacelike dimensions ...
[compare the Batakis M4xCP2 4+4=8-dimensional Kaluza-Klein model]...
Rather than embed the gauge broups into some master group, we infix the Dirac algebra into the ... Clifford algebra Cl(7) ...[in which]... the unit vectors
e1 ,e2 , ... , e7 are chosen to represent ... spacelike directions ...
We further choose e1 , e2 , e3 to represent ... physical space and ...
e4 , e5 , e6 , e7 to ... represent ... four ...dimensions ... orthogonal to physical space ... [compare the Cl(8) of E8 Physics which is represented by 16x16 matrices with two 8-dimensional half-spinor spaces and in which the 8 unit vectors
e0 , e1 ,e2 , ... , e7 represent Batakis 8-dimensional spacetime M4xCP2 where
e0 , e1 ,e2 , e3 represents M4 and e4 , e5 ,e6 , e7 represents CP2]...
To describe one generation of the standard model, we use the algebraic spinor PSI in Cl(7) ... there are eight independent primitive idempotents that can each be used to reduce PSI to a spinor representing a fermion doublet ...
Each of the eight ... primitive idempotents ... projects PSI onto one of eight minimal left ideals of Cl(7) ...
[compare the 8+8 = 16 primitive idempotents of Cl(8) which correspond to 8 first-generation fermion particles and their 8 antiparticles] ...
we previously disregarded the higher-dimensional vector components ... This ... vector space ... then ... affords a natural inclusion of the minimal Higgs field ...
The Higgs field ... arises here simply as a coupling to the higher-dimensional vector components ...”.

[compare the E8 Physics model relationship between the Higgs and the Cl(8) primitive idempotents which live in grades 0 and 4 and 8 of Cl(8)]

Klaus Dietz in arXiv quant-ph/0601013 said:
"... **m-Qubit states are embedded in Cl(2m) Clifford algebra**. ...
This ... allows us to arrange the 2^(2m) - 1 real coordinates of a m-Qubit state in multidimensional arrays which are shown to 'transforn\m' as O(2m) tensors ...
A hermitian 2^m x 2^m matrix requires 2^(2m) real numbers for a complete parameterization. Thus m-qubit states can be expanded in terms of I and the products introduced. Clifford numbers are the starting point for the construction of a basis in R-linear space of hermitian matrices:
this basis is construed as a Clifford algebra Cl(2m) ...".

Stephanie Wehner in arXiv 0806.3483 said:
"... A Clifford algebra of n generators is isomorphic to a ... algebra of matrices of size 2^(n/2) x 2^(n/2) for n even ...
we can view the operators G1 , ... , G2n as 2n orthogonal vectors forming a basis for a 2n-dimensional real vector space R2n ...
each operator Gi has exactly two eigenvalues +/-1 ...
we can express each Gi as Gi = G0i - G1i
where G0i and G1i are projectors onto the positive and negative eigenspace of Gi
... for all i,j with i =/= j   Tr(GiGj) = (1/2)Tr(GiGj + GjGi) = 0
that is all such operators are orthogonal with respect to the Hilbert-Schmidt inner product ... the collection of operators
1
Gj   (1 ≤ j ≤ 2n)
Gjk := iGjGk   (1 ≤ j < k ≤ 2n)
Gjkl := GjGkGl   (1 ≤ j < k < l ≤ 2n)
...
G12...(2n) := iG1G2 ... G2n =: G0
forms an orthogonal basis for ... the d x d matrices ... with d = 2^n ...
We saw ... how to construct such **a basis ... based on mutually unbiased bases ... the well-known Pauli basis, given by the 2^(2n) elements of the form**
Bj = B1j x ...[tensor product]... x Bnj with Bij in { I , sx , sy , sz } ...
we obtain a whole range of ... statements as we can find different sets of 2n anti-commuting matrices within the entire set of 2^(2n) basis elements ...
the subspace spanned by the elements G1 , ... , G2n plays a special role ...
when considering the state minimizing our uncertainty relation,
only the 1-vector coefficients play any role. The other coefficients do not contribute at all to the minimization problem. ...
Anti-commuting Clifford observables obey the strongest possible uncertainty relation for the von Neumann entropy: if we have no uncertainty for one of the measurements, we have maximum uncertainty for all others. ...".

Monique Combescure in quant-ph/060509, arXiv 0710.5642 and 0710.5643 said:
"... two basic unitary d x d matrices U , V ... constructed by Schwinger ... q := exp (2 i pi / d) ... are of the following form:

$$U := \mathrm{Diag}(1, q, q^2, ..., q^{d-1})$$

$$V := \begin{pmatrix} 0 & 1 & 0 & . & . & 0 \\ 0 & 0 & 1 & . & . & 0 \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ 0 & 0 & 0 & . & . & 1 \\ 1 & 0 & 0 & . & . & 0 \end{pmatrix}$$

... the matrices U and V are called
"generalized Pauli matrices on d-state quantum systems" ...
U, V generate the discrete Weyl-Heisenberg group ... U, V allows to find MUB's ...
in dimension d there is at most d+1 MUB, and exactly d + 1 for d a prime number
...
A d x d matrix C is called circulant ... if all its rows and columns are successive circular permutations of the first ... the theory of circulant matrices allows to recover the result that there exists p + 1 Mutually Unbiased Bases in dimension p, p being a... prime number ... Then the MUB problem reduces to exhibit a circulant matrix C which is a unitary Hadamard matrix, such that its powers are also circulant unitary Hadamard matrices. Then using Discrete Fourier Transform Fd which diagonalizes all circulant matrices, we have shown that a MUB in that case is just provided by the set of column vectors of the set of matrices
{ Fd, 1, C, C2, ... , C(d-1) }
...
the theory of block-circulant matrices with circulant blocks allows to show ...
that if d = p^n (p a prime number, n any integer)
there exists d + 1 mutually Unbiased Bases in Cd ...".

Stephen Brierley, Stefan Weigert, and Ingemar Bengtsson in arXiv 0907.4097 said:
“... All complex Hadamard matrices in dimensions two to five are known ...
In dimension three there is ... only one dephased complex Hadamard matrix up to equivalence. It is given by the (3 x 3) discrete Fourier matrix

$$F_3 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}$$

defining w = exp(2 pi i / 3)
...
In dimension d = 4 , all 4 x 4 complex Hadamard matrices are equivalent to a member of the ... one-parameter family of complex Hadamard matrices ...

$$F_4(x) = \frac{1}{2} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & ie^{ix} & -ie^{ix} \\ 1 & -1 & -ie^{ix} & ie^{ix} \end{pmatrix}, \quad x \in [0, \pi]$$

... There is one three-parameter family of triples ...
Only one set of four MU bases exists ...
there is a unique way to a construct five MU bases which is easily seen to be equivalent to the standard construction of a complete set of MU bases ... d = 4 ...

d	2	3	4	5	6
pairs	1	1	∞^1	1	$\geq \infty^3$
triples	1	1	∞^3	2	$\geq \infty^2$
quadruples	-	1	1	1	?
quintuples	-	-	1	1	?
sextuples	-	-	-	1	?

... The notion of equivalence used in this paper ... is mathematical in nature ...
Motivated by experiments, there is a finer equivalence of complete sets of MU bases based on the entanglement structure of the states contained in each basis ...
For dimensions that are a power of two, a complete set of MU bases can be realized using Pauli operators acting on each two-dimensional subsystem.
Two sets of MU bases are then called equivalent when they can be factored into the same number of subsystems. For d = 2, 4 this notion of equivalence also leads to a unique set of (d + 1) MU bases. However, for d = 8, 16, . . . complete sets of MU bases can have different entanglement structures even though they are equivalent up to an overall unitary transformation ...”.

P. Dita in arXiv 1002.4933 said:
"... Mutually unbiased bases (MUBs) constitute a basic concept of quantum information ... Its origin is in the Schwinger paper ... "Unitary operator bases", Proc.Nat. Acad. Sci.USA, 46 570-579 (1960) ...
Two orthonormal bases in Cd, A = (a1, . . . , ad) and B = (b1, . . . , bd), are called MUBs if ... the product A B* of the two complex Hadamard matrices generated by A and B is again a Hadamard matrix, where * denotes the Hermitian conjugate ...
The technique for getting MUBs for p prime was given by Schwinger ... who made use of the properties of the Heisenberg-Weyl group
...
[in this paper] An analytical method for getting new complex Hadamard matrices by using mutually unbiased bases and a nonlinear doubling formula is provided. The method is illustrated with the n = 4 case that leads to a rich family of eight-dimensional Hadamard matrices that depend on five arbitrary phases ... The ... matrices are new ... the only [prior] known result parametrized by five phases is the [n = 8] complex Hadamard matrix stemming from the Fourier matrix F8
...
real Sylvester-Hadamard matrices ...[have a]... solution for n = 8 ...

$$S = \begin{bmatrix} a & b & c & d & l & m & n & p \\ b & -a & -d & c & m & -l & p & -n \\ c & d & -a & -b & n & -p & -l & m \\ d & -c & b & -a & p & n & -m & -l \\ l & -m & -n & -p & -a & b & c & d \\ m & l & p & -n & -b & -a & d & -c \\ n & -p & l & m & -c & -d & -a & b \\ p & n & -m & l & -d & c & -b & -a \end{bmatrix}$$

... for real Hadamard matrices with dimension d = 2, 4, 8, 12 there is only one matrix under the usual equivalence ... there is an other type of matrix equivalence ... two matrices ... are equivalent if and only if they have the same spectrum ... However a simple spectral computation of the h1, h2, h3, h4 matrices shows that only the matrices h1 and h3 are equivalent, and h1 is not equivalent to h2 and h4, nor h2 is equivalent to h4 ...[so that]... we do not suggest the use of the new equivalence ... for real Hadamard matrices ... because it will cause dramatic changes in the field ...".

Standard Model Higgs compared to E8 Physics Higgs

The conventional Standard Model has structure:
spacetime is a base manifold;
particles are representations of gauge groups
- gauge bosons are in the adjoint representation
- fermions are in other representations (analagous to spinor)
- Higgs boson is in scalar representation.

E8 Physics (see vixra 1108.0027 and tony5m17h.net) has structure
(from 248-dim E8 = 120-dim adjoint D8 + 128-dim half-spinor D8):
spacetime is in the adjoint D8 part of E8 (64 of 120 D8 adjoints)
gauge bosons are in the adjoint D8 part of E8 (56 of the 120 D8 adjoints)
fermions are in the half-spinor D8 part of E8 (64+64 of the 128 D8 half-spinors.

There is no room for a fundamental Higgs in the E8 of E8 Physics.
However,
for E8 Physics to include the observed results of the Standard Model
it must have something that acts like the Standard Model Higgs
even though it will NOT be a fundamental particle.

To see how the E8 Physics Higgs works,
embed E8 into the 256-dimensional real Clifford algebra Cl(8):

Cl(8)	256 =	1	+	8	+	28	+	56	+	70	+	56	+	28	+	8	+	1
Primitive	16 =	1							+	6							+	1
Idempotent									+	8								
E8 Root Vectors	240 =			8	+	28	+	56	+	56	+	56	+	28	+	8		

The Cl(8) Primitive Idempotent is 16-dimensional and can be decomposed
into two 8-dimensional half-spinor parts each of which is related by Triality
to 8-dimensional spacetime and has Octonionic structure. In that decomposition:
the 1+6+1 = (1+3)+(3+1) is related to two copies of
a 4-dimensional Associative Quaternionic subspace of the Octonionic structure
and
the 8 = 4+4 is related to two copies of
a 4-dimensional Co-Associative subspace of the Octonionic structure
(see the book “Spinors and Calibrations” by F. Reese Harvey)

The 8 = 4+4 Co-Associative part of the Cl(8) Primitive Idempotent
when combined with the 240 E8 Root Vectors
forms the full 248-dimensional E8.
It represents a Cartan subalgebra of the E8 Lie algebra.

The (1+3)+(3+1) Associative part of the Cl(8) Primitive Idempotent is the Higgs of E8 Physics.

The half-spinors generated by the E8 Higgs part of the Cl(8) Primitive Idempotent represent:

neutrino; red, green, blue down quarks; red, green, blue up quarks; electron

so
the E8 Higgs effectively creates/annihilates the fundamental fermions and
the E8 Higgs is effectively a condensate of fundamental fermions.

In E8 Physics the high-energy 8-dimensional Octonionic spacetime reduces, by freezing out a preferred 4-dim Associative Quaternionic subspace, to a 4+4 -dimensional Batakis Kaluza-Klein of the form M4 x CP2 with 4-dim M4 physical spacetime.

Since the (1+3)+(3+1) part of the Cl(8) Primitive Idempotent
includes the Cl(8) grade-0 scalar 1
and 3+3 = 6 of the Cl(8) grade-4 which act as pseudoscalars for 4-dim spacetime
and the Cl(8) grade-8 pseudoscalar 1
the E8 Higgs transforms with respect to 4-dim spacetime as a scalar (or pseudoscalar) and in that respect is similar to Standard Model Higgs.

Not only does the E8 Higgs fermion condensate transform with respect to 4-dim physical spacetime like the Standard Model Higgs but
the geometry of the reduction from 8-dim Octonionic spacetime to 4+4 -dimensional Batakis Kaluza-Klein, by the Mayer mechanism, gives E8 Higgs the ElectroWeak Symmetry-Breaking Ginzburg-Landau structure.

Since the second and third fermion generations emerge dynamically from the reduction from 8-dim to 4+4 -dim Kaluza-Klein, they are also created/annihilated by the Primitive Idempotent E8 Higgs and are present in the fermion condensate.
Since the Truth Quark is so much more massive that the other fermions,
the E8 Higgs is effectively a Truth Quark condensate.
When Triviality and Vacuum Stability are taken into account,
the E8 Higgs and Truth Quark system has 3 mass states.

Since it creates/annihilates Fermions,
the (1+3)+(3+1) Associative part of the Cl(8) Primitive Idempotent
is a Fermionic Condensate Higgs structure.
The creation/annihilation operators have graded structure similar to part of a Heisenberg algebra

$$64 + 0 + 64$$

Since it creates/annihilates the 8-dimensional SpaceTime
represented by the Cartan Subalgebra of the E8 Lie Algebra,
the 8 = 4+4 Co-Associative part of the Cl(8) Primitive Idempotent
is a Bosonic Condensate Spacetime structure.
The creation/annihilation operators correspond to position-momentum related by Fourier Transform and to an 8x8 = 64-dimensional U(8)

E8 has two D4 Lie subalgebras D4 and D4* related by Fourier Transform:

28-dimensional D4 acting on M4 4-dim Physical SpaceTime and containing a Spin(2,4) subalgebra for Conformal MacDowell-Mansouri Gravity;
and
28-dimensional D4* acting on CP2 Internal Symmetry Space and containing a U(4) subalgebra for the Batakis Standard Model gauge groups.

Taken together, the D4 and U(8) and D4* have graded structure

$$28 + 64 + 28$$

that breaks down into a semi-simple 63-dimensional SU(8)

$$63$$

and a Heisenberg Algebra

$$28 + 1 + 28$$

When the Fermionic 64 + 0 + 64 is added, the Heisenberg Algebra becomes

$$92 + 1 + 92$$

and the total 92 + U(8) + 92 is seen to be the contraction of E8 into the semidirect product of semisimple SU(8) and Heisenberg Algebra 92 + U(1) + 92

Robert Hermann in “Lie Groups for Physicists” (Benjamin 1966) said:
“... Let G be a Lie group ... imbed G into the associative algebra U(G) ... the universal ... enveloping algebra ...
the “polynomials” of the .. basis [elements] of G ... form a basis for U(G) ...
the center of U(G) ...[is]... the Casimir operators of G ...[whose]... number ...[is]... equal to ... the dimension of its Cartan subalgebras ...
every polynomial ... invariant under AdG ... arise[s] ... from a Casimir operator ...
when G is semisimple, Ad G acting on G admits an invariant polynomial of degree 2 ... the Killing form ... This is the simplest such Casimir operator
...
there is a group-theoretical construction which in certain situations reduces to the Fourier transform. To describe it, we need ... a Lie group G, two subgroups L and H of G, and linear representations ... of L and H ... on a vector space U, which determines vector bundles E and E’ over G/L and G/H. ...
A cross section PSI of ... E’ over G/H is an eigenvector of each Casimir operator of U(G) .... its transform PSI*, considered as a function on G/K, is also an eigenfunction of each Casimir operator of U(G). ...”.

Rutwig Campoamor-Stursberg in “Contractions of Exceptional Lie Algebras and SemiDirect Products” (Acta Physica Polonica B 41 (2010) 53-77) said:
“... it is of interest to analyze whether ... semidirect products ... of semisimple and Heisenberg Lie algebras ... appear as contractions of semisimple Lie algebras ...
Let s be a ... semisimple Lie algebra. For the indecomposable semidirect product g = s + Hn the number of Casimir operators is given by N(g) = rank(s) + 1
... In some sense, the Levi subalgebra s determines these Casimir invariants, to which the central charge (the generator of the centre of the Heisenberg algebra) is added. ... the quadratic Casimir operator will always contract onto the square of the centre generator of the Heisenberg algebra ...
... We have classified all contractions of complex simple exceptional Lie algebras onto semidirect products ... s + h_N ... of semisimple and Heisenberg algebras.
An analogous procedure holds for the real forms of the exceptional algebras ...
Contractions of E8 ... E8 contains D8 contains A7 ...[and for E8]... N = 92
... This reduction gives rise to the contraction ...[E8 to A7 + H92]...
E8 ... has primitive Casimir operators ... of degrees ...[2,8,12,14,18,20,24,30]...
D8 ... has primitive Casimir operators ... of degrees ...[2,4,6,8,10,12,14,8]...
A7 ... has primitive Casimir operators ... of degrees ...[2,3,4,5,6,7,8]...”.

The E8 primitive Casimirs 2 ,8, 12, 14, 18, 20, 24, 30 contract as follows:

2 to the center U(1) of H92.
8, 12, 14 to the 8, 12, 14 of D8 and to the 4=8/2, 6=12/2, 7=14/2 of A7

18, 20, 24, 30 to the 4=18-14, 6=20-14, 10=24-14, 8=(1/2)(30-14) of D8
and to the 2=4/2, 3=6/2, 5=10/2, 8 of A7

The 2, 8, 12, 14 of E8 are dual to the 30, 24, 20, 18 of E8 such that
2+30 = 8+24 = 12+20 = 14+18 = 32.

The E8 primitive Casimirs correspond to the Cartan subalgebras of E8 and of D8
and also to 8-dim Spacetime and 4+4-dim Batakis Kaluza-Klein M4 x CP2

The 2, 8, 12, 14 Casimirs of E8 correspond to
the (1+3)-dim M4 Batakis Physical Spacetime

The 18, 20, 24, 30 Casimirs of E8 correspond to
the 4-dim CP2 Batakis Internal Symmetry Space

Weyl Symmetric Polynomial Degrees and Topological Types:

E8:
degrees - 2, 8, 12, 14, 18, 20, 24, 30
note that 1, 7, 11, 13, 17, 19, 23, and 29 are all relatively prime to 30
type - 3, 15, 23, 27, 35, 39, 47, 59; center = Z1 = 1 = trivial

D8 Spin(16):
degrees - 2, 4, 6, 8, 10, 12, 14, 8
type - 3, 7, 11, 15, 19, 23, 27, 15; center = Z2 + Z2

A7 SU(8):
degrees - 2, 3, 4, 5, 6, 7, 8
type - 3, 5, 7, 9, 11, 13, 15; center = Z8

Luis J. Boya has written a beautiful paper “Problems in Lie Group Theory” math-ph/0212067 and here are a few of the interesting things he says:

"... Given a Lie group in a series G(n) ... how is the group G(n+1) constructed?

For the **orthogonal series (Bn and Dn)** ... given O(n) acting on itself, that is, the adjoint (adj) representation, and the vector representation, n, ...
Adj O(n) + Vect O(n) -> Adj O(n+1) ...

For the unitary series SU(n) ... Adj SU(n) + Id + n + n* = Adj SU(n+1) ...

For the symplectic series
Sp(n) = Cn ... Adj Sp(n) + Adj Sp(1) + 2(n + n*) = Adj Sp(n+1) ...

For **G2** ... **Adj SU(3) + n + n* -> G2** ...[in addition, I conjecture the existence of an alternate construction: **Adj O(4) + Vect O(4) + Spin O(4) = G2** ,
where Spin O(4) is its Spin representation, a notation that I will continue to use in the rest of this quotation instead of the notation Spin(4) that Boya uses, because I want to reserve the notation Spin(4) for the covering group of SO(4). Note that Spin O(n) for even n is reducible to two copies of mirror image half-spinor representations half-Spin O(n)]...

For the **exceptional groups, the F4 & E series** ...

- **Adj SO(9) + Spin O(9) -> Adj F4 (36+16=52)**
- **Adj SO(10) + Spin O(10) + Id -> Adj E6 (45+32+1=78)**
- **Adj SO(12) + Spin O(12) + Sp(1) -> Adj E7 (66+64+3=133)**
- **Adj SO(16) + [half-]Spin O(16) -> Adj E8 ([120+128=248])**

Notice that 8+1 , 8+2 , 8+4 , and 8+8 appear. In this sense the octonions appear as a "second coming " of the reals, completed with the spin, not the vector irrep. ...
This confirms that the F4 E6-7-8 corresponds to
the octo, octo-complex, octo-quater and octo-octo birings,
as the Freudenthal Magic Square confirms. ...
Another ... question ... is the geometry associated to the exceptional groups ...
Are we happy with G2 as the automorphism group of the octonions, F4 as the isometry of the [octonion] projective plane, E6 (in a noncompact form) as the collineations of the same, and E7 resp. E8 as examples of symplectic resp. metasymplectic geometries? ... one would like to understand the exceptional groups ... as automorphism groups of some natural geometric objects. ...

The gross topology of Lie groups is well-known. The non-compact case reduces to compact times an euclidean space (Malcev-Iwasawa). The compact case is reduced to a finite factor, a Torus, and a semisimple compact Lie group.
H. Hopf determined in 1941 that the real homology of simple compact Lie groups is that of a product of odd spheres ...
The exponents of a Lie group are the numbers i such
that S(2i+1) is an allowed sphere ...
neither the U-series nor the Sp-series have torsion.
The exponents ... for U(n) ... are 0, 1, ... , n-1 ... and jump by two in Sp(n).

But for the orthogonal series one has to consider some Stiefel manifolds instead of spheres, which have the same real homology ...
It ... introduces (preciesely) 2-torsion:
in fact, Spin(n), n≥7 and SO(n), n≥3, have 2-torsion.
The low cases Spin(3,4,5,6) coincide
with Sp(1), Sp(1)xSp(1), Sp(2) and SU(4) , and have no torsion.

For ... G2 ... SU(2) -> G2 -> M11 ... where M11 is again a Steifel manifold,
with real homology like S11, but with 2-torsion ...

For F4 we do not get the sphere structure from any irrep,
and in fact F4 has 2- and 3-torsion. ...

2- and 3-torsion appears in ... E6 and E7 ...

E8 has 2-, 3- and 5-torsion ...
The Coxeter number of (dim - rank) of E8 is 30 = 2 x 3 x 5 ,
in fact a mnemonic for the exponents of E8 is:
they are the coprimes up to 30, namely (1,7,11,13,17,19,23,29) ...
The first perfect numbers are 6, 28, and 492,
associated to the primes 2, 3 and 5 (... Mersenne numbers ...) ...
496 = dim O(32) = dim E(8) x E(8) . Why the square?

It also happens in O(4) , dim = 6 (prime 2), as O(4) ...[is like]... O(3) x O(3) ;
even O(8) [dim = 28] (prime 3) is like S7 x S7 x G2 ...

The sphere structure of compact simple Lie groups has a curious "capicua" ...
Catalan word (cap i cua 0 = head and tail) ... form:
the exponents are symmetric from each end; for example ...

exponents of E6: 1,4,5,7,8,11. Differences: 3,1,2,1,3

exponents of E7: 1,5,7,9,11,13,17. Differences: 4,2,2,2,2,4 ...

exponents of E8 ... 1,7,11,13,17,19,23,29 ...[Differences 6,4,2,4,2,4,6]...

The real homology algebra of a simple Lie group is a Grassmann algebra,
as it is generated by odd (i.e., anticommutative) elements.
However, from them we can get, in the enveloping algebra,
multilinear symmetric forms, one for each generator; ...
in physics they are called Casimir invariants,
in mathematics the invariants of the Weyl group ...".

Martin Cederwall and Jakob Palmkvist, in "The octic E8 invariant"
hep-th/0702024, say:

"... The largest of the finite-dimensional exceptional Lie groups, E8, with Lie algebra e8, is an interesting object ... its root lattice is the unique even self-dual lattice in eight dimensions (in euclidean space, even self-dual lattices only exist in dimension 8n). ... Because of self-duality, there is only one conjugacy class of representations, the weight lattice equals the root lattice, and there is no "fundamental" representation smaller than the adjoint. ...
Anything resembling a tensor formalism is completely lacking. A basic ingredient in a tensor calculus is a set of invariant tensors, or "Clebsch-Gordan coefficients". The only invariant tensors that are known explicitly for E8 are the Killing metric and the structure constants ...

The goal of this paper is to take a first step towards a tensor formalism for E8 by explicitly constructing an invariant tensor with eight symmetric adjoint indices. ...

On the mathematical side, the disturbing absence of a concrete expression for this tensor is unique among the finite-dimensional Lie groups. Even for the smaller exceptional algebras g2, f4, e6 and e7, all invariant tensors are accessible in explicit forms, due to the existence of "fundamental" representations smaller than the adjoint and to the connections with octonions and Jordan algebras. ...

The orders of Casimir invariants are known for all finite-dimensional semi-simple Lie algebras. They are polynomials in U(g), the universal enveloping algebra of g, of the form t_(A1...Ak) T^(A1 . . . TAk), where t is a symmetric invariant tensor and T are generators of the algebra, and they generate the center U(g)^(g) of U(g).

The Harish-Chandra homomorphism is the restriction of an element in U(g)^(g) to a polynomial in the Cartan subalgebra h, which will be invariant under the Weyl group W(g) of g.

Due to the fact that the Harish-Chandra homomorphism is an isomorphism
from U(g)^(g) to U(h) W(g) one may equivalently consider
finding a basis of generators for the latter, a much easier problem.
The orders of the invariants follow more or less directly
from a diagonalisation of the Coxeter element,
the product of the simple Weyl reflections ...

In the case of e8, the center U(e8)^(e8) of the universal enveloping subalgebra is generated by elements of orders 2, 8, 12, 14, 18, 20, 24 and 30.
The quadratic and octic invariants correspond to primitive invariant tensors in terms of which the higher ones should be expressible. ...
the explicit form of the octic invariant is previously not known ...

E8 has a number of maximal subgroups, but one of them, Spin(16)/Z2, is natural for several reasons.
Considering calculational complexity,
this is the subgroup that leads to the smallest number of terms in the Ansatz.

Considering the connection to the Harish-Chandra homomorphism,
K = Spin(16)/Z2 is the maximal compact subgroup of the split form G = E8(8).

The Weyl group is a discrete subgroup of K,
and the Cartan subalgebra h lies entirely in the coset directions g/k ...

We thus consider the decomposition of the adjoint representation of E8 into representations of the maximal subgroup Spin(16)/Z2.

The adjoint decomposes into the adjoint 120 and a chiral spinor 128. ...

Our convention for chirality is GAMMA_(a1...a16) PHI = + e_(a1...a16) PHI .

The e8 algebra becomes (2.1)

[T^(ab) , T^(cd)] = 2 delta^([a)_([c) T^(b])_(d]) ,
[T^(ab) , PHI^(alpha)] = (1/4) (GAMMA^(ab) PHI)^(alpha) ,
[PHI^(alpha) , PHI^(alpha)] = (1/8) (GAMMA_(ab))^(alpha beta) T^(ab) ,

... The coefficients in the first and second commutators are related
by the so(16) algebra. The normalisation of the last commutator is free,
but is fixed by the choice for the quadratic invariant, which for the case above is

X2 = (1/2) T_(ab) T^(ab) + PHI_(alpha) PHI^(alpha) .

Spinor and vector indices are raised and lowered with delta .
Equation (2.1) describes the compact real form, E8(-248) .

By letting PHI -> i PHI one gets E8(8),
where the spinor generators are non-compact,
which is the real form relevant as duality symmetry in three dimensions
(other real forms contain a non-compact Spin(16)/Z2 subgroup).

The Jacobi identities are satisfied thanks to the Fierz identity

(GAMMA_(ab)_[(alpha beta) (GAMMA_(ab)_(alpha beta)] = 0 ,

which is satisfied for so(8) with chiral spinors, so(9), and so(16) with chiral spinors
(in the former cases the algebras are so(9), due to triality, and f4).

The Harish-Chandra homomorphism tells us that the "heart" of the invariant
lies in an octic Weyl-invariant of the Cartan subalgebra.
A first step may be to lift it to a unique Spin(16)/Z2-invariant in the spinor,
corresponding to applying the isomorphism fÅ|1 above.
It is gratifying to verify ... that there is indeed an octic invariant
(other than (PHI PHI)^4), and that no such invariant exists at lower order. ...

Forming an element of an irreducible representation containing a number of
spinors involves symmetrisations and subtraction of traces, which can be rather
complicated. This becomes even more pronounced when we are dealing with
transformation ... under the spinor generators, which will transform as spinors.

Then irreducibility also involves gamma-trace conditions. ... The transformation ... under the action of the spinorial generator is an so(16) spinor. The vanishing of this spinor is equivalent to e8 invariance. The spinorial generator acts similarly to a supersymmetry generator on a superfield ... The final result for the octic invariant is, up to an overall multiplicative constant:

$$
\begin{aligned}
X_8 = {} & \tfrac{1}{3072}\varepsilon^{a_1\cdots a_{16}}T_{a_1a_2}\ldots T_{a_{15}a_{16}} \\
& \quad - 30\mathrm{tr}T^8 + 14\mathrm{tr}T^6\mathrm{tr}T^2 + \tfrac{35}{4}(\mathrm{tr}T^4)^2 - \tfrac{35}{8}\mathrm{tr}T^4(\mathrm{tr}T^2)^2 + \tfrac{15}{64}(\mathrm{tr}T^2)^4 \\
& + \big[2\mathrm{tr}T^6 - \mathrm{tr}T^4\mathrm{tr}T^2 + \tfrac{1}{8}(\mathrm{tr}T^2)^3\big](\phi\phi) \\
& \quad + \big[(\tfrac{5}{4}\mathrm{tr}T^4 - \tfrac{1}{2}(\mathrm{tr}T^2)^2)T^{ab}T^{cd} + \tfrac{27}{4}\mathrm{tr}T^2T^{ab}(T^3)^{cd} \\
& \qquad - 15T^{ab}(T^5)^{cd} - 15(T^3)^{ab}(T^3)^{cd}\big](\phi\Gamma_{abcd}\phi) \\
& \quad + \big[\tfrac{1}{16}\mathrm{tr}T^2T^{ab}T^{cd}T^{ef}T^{gh} - \tfrac{5}{8}T^{ab}T^{cd}T^{ef}(T^3)^{gh}\big](\phi\Gamma_{abcdefgh}\phi) \\
& \quad - \tfrac{1}{192}T^{ab}T^{cd}T^{ef}T^{gh}T^{ij}T^{kl}(\phi\Gamma_{abcdefghijkl}\phi) \\
& + \big[7\mathrm{tr}T^4 - \tfrac{31}{8}(\mathrm{tr}T^2)^2\big](\phi\phi)^2 \\
& \quad - \tfrac{3}{64}T^{ab}T^{cd}T^{ef}T^{gh}(\phi\phi)(\phi\Gamma_{abcdefgh}\phi) \\
& \quad + \big[\tfrac{5}{64}T^{ab}T^{cd}T^{ef}T^{gh} - \tfrac{15}{16}T^{ab}T^{ce}T^{df}T^{gh} \\
& \qquad + \tfrac{5}{8}T^{ae}T^{bf}T^{cg}T^{dh}\big](\phi\Gamma_{abcd}\phi)(\phi\Gamma_{efgh}\phi) \\
& \quad + \big[\tfrac{3}{2}(T^3)^{ab}T^{cd} - \tfrac{1}{8}\mathrm{tr}T^2T^{ab}T^{cd}\big](\phi\phi)(\phi\Gamma_{abcd}\phi) \\
& \quad + \big[\tfrac{15}{16}(T^3)^{ab}T^{cd} - \tfrac{3}{16}\mathrm{tr}T^2T^{ab}T^{cd} + \tfrac{5}{4}(T^2)^{ac}(T^2)^{bd}\big](\phi\Gamma_{ab}{}^{ij}\phi)(\phi\Gamma_{cdij}\phi) \\
& \quad + \tfrac{15}{8}T^{ab}T^{cd}(T^2)^{ef}(\phi\Gamma_{abe}{}^{i}\phi)(\phi\Gamma_{cdfi}\phi) \\
& + \tfrac{1}{2}\mathrm{tr}T^2(\phi\phi)^3 + \tfrac{55}{32}T^{ab}T^{cd}(\phi\phi)^2(\phi\Gamma_{abcd}\phi) \\
& \quad + \tfrac{1}{8}T^{ab}T^{cd}(\phi\phi)(\phi\Gamma_{ab}{}^{ij}\phi)(\phi\Gamma_{cdij}\phi) \\
& \quad + \big[-\tfrac{1}{384}T^{ab}T^{cd} + \tfrac{7}{192}T^{ac}T^{bd}\big](\phi\Gamma_{ab}{}^{ij}\phi)(\phi\Gamma_{cd}{}^{kl}\phi)(\phi\Gamma_{ijkl}\phi) \\
& - \tfrac{57}{32}(\phi\phi)^4 + \tfrac{1}{12288}(\phi\Gamma_{ab}{}^{cd}\phi)(\phi\Gamma_{cd}{}^{ef}\phi)(\phi\Gamma_{ef}{}^{gh}\phi)(\phi\Gamma_{gh}{}^{ab}\phi) \\
& + \beta[-\tfrac{1}{2}\mathrm{tr}T^2 + (\phi\phi)]^4 \,.
\end{aligned}
\tag{2.3}
$$

Here, β is an arbitrary constant multiplying the fourth power of the quadratic invariant. The trace vanishes for $\beta = \frac{9}{127}$ (that such a value exists at all is non-trivial and provides a further check on the coefficients). The occurrence of the prime 127 is not incidental; taking the trace of $\delta^{(AB}\delta^{CD}\delta^{EF}\delta^{GH)}$ gives $\delta_{GH}\delta^{(AB}\delta^{CD}\delta^{EF}\delta^{GH)} = (\frac{1}{7}\cdot 248 + \frac{6}{7})\delta^{(AB}\delta^{CD}\delta^{EF)} = \frac{2\cdot 127}{7}\delta^{(AB}\delta^{CD}\delta^{EF)}$. The actual technique we use for calculating the trace is not to extract the eight-index tensor, but to act on the invariant with $\frac{1}{2}\frac{\partial}{\partial T_{ab}}\frac{\partial}{\partial T^{ab}} + \frac{\partial}{\partial \phi_a}\frac{\partial}{\partial \phi^a}$. We remind that eq. (2.3) gives the octic invariant for the compact form $E_{8(-248)}$. The corresponding expression for the split form $E_{8(8)}$ is obtained by a sign change of the terms containing ϕ^{4k+2}.

...".

E8 Maximal Contraction = Heisenberg Building Block of AQFT

Our Universe can be described by an **Algebraic Quantum Field Theory (AQFT)** that **is the Completion of the Union of all Tensor Products of a Building Block**. This Fundamental Building Block can be embedded in the Real Clifford Algebra Cl(8,8). By the 8-Periodicity Property of Real Clifford Algebras, all Tensor Products of it are themselves Real Clifford Algebras, and the Completion of the Union of all Tensor Products is well-behaved and constitutes a generalized hyperfinite II1 von Neumann factor AQFT. The purpose of this paper is to describe the structure of this Fundamental Building Block and its physical interpretation.

The Lie Algebra E8 is contained in Cl(8,8) as
248-dim E8 = 120-dim D8 + 128-dim-half-spinor D8
where D8 = Spin(8,8) is the Lie Algebra of the bivectors of Cl(8,8)
and half-spinor D8 is an irreducible half-spinor of Cl(8,8).

The Fundamental Building Block is the Maximal Contraction of E8
which, since it leads to an AQFT, is called the E8 Quantum Contraction (E8QC). Maximal Contractions of Exceptional Lie Algebras (includng E8) are describe by Rutwig Campoamor-Stursberg in "Contractions of Exceptional Lie Algebras and SemiDirect Products" (Acta Physica Polonica B 41 (2010) 53-77).

Here, **E8QC** is written as the 5-Graded Lie Algebra with structure

28 + 64 + (SL(8,R) + 1) + 64 + 28

Central Even Grade 0 = SL(8,R) + 1 where the 1 is an anticommuting scalar and SL(8,R) has bosonic commutators. As Polar Coordinates, SL(8,R) represents a local 8-dim spacetime as SL(8,R) = Spin(8) + Traceless Symmetric 8x8 Matrices.

Odd Grades -1 and +1 each = 64 = 8x8 = Creation/Annihilation Operators for 8 components of 8 fundamental fermions with fermionic anticommutators.

Even Grades -2 and +2 each = 28-dim Spin(4,4) = Creation/Annihilation Operators for 28 Gauge Bosons with bosonic commutators.

The use of bosonic commutators for the Even Grades {-2,0,+2} of E8QC
and fermionic anticommutators for the Odd Grades {-1,+1} of E8QC,
and the consequent physically realistic spin/statistics relationships,
is justified by Pierre Ramond's remark in hep-th/0112261
"... "... the coset F4 / SO(9) ... is the sixteen-dimensional Cayley projective plane ... [represented by]... the SO(9) spinor operators [which] satisfy Bose-like commutation relations ... Curiously, if ...[the scalar and spinor 16 of F4 are both]... anticommuting, the F4 algebra is still satisfied ..."
which is based on 52-dim F4 = 36-dim Spin(9) + 16-dim-spinor Spin(9)
and applying it to
248-dim E8 = 120-dim Spin(8,8) + 128-dim-half-spinor Spin(8,8).

The structure SL(8,R) = Spin(8) + Traceless Symmetric 8x8 Matrices
is described by V. V. Gorbatsevich, A. L. Onishchik and E. B. Vinberg
in their book "Lie Groups and Lie Algebras III,
Structure of Lie Groups and Lie Algebras, Springer-Verlag (1994).

The **AQFT** is the Completion of the Union of all Tensor Products

E8QC x ...(N times)... x E8QC

Since each factor E8QC is embedded in a copy of Cl(8,8)
and since Cl(8,8) x ...(N times)... x Cl(8,8) = Cl(8N,8N) by 8-Periodicity,
the Union of all E8QC x ...(N times)... x E8QC is well-behaved
as is its Completion, the AQFT.

To see some structural properties of the AQFT,
consider that

E8QC x ...(N times)... x E8QC

is equal to the N times Tensor Product of

28 + 64 + (SL(8,R) + 1) + 64 + 28

with itself and
look at some interesting substructures of the Tensor Product:

AntiCommuting Parts of E8QC:

Look at the Tensor Products of the anticommuting parts of E8QC
of Grades {-1,0,+1}.

64 + 1 + 64

Fermionic Fock Space:

64 + 1 + 64

They are the Real Clifford Algebra analogues of the Complex Clifford Algebra Spinors derived from the 2x2 Complex Clifford Algebra whose Tensor Products are governed by 2-Periodicity. As John Baez said in his Week 175:

"... the algebra of 2^n x 2^n [Complex] matrices is a [Complex] Clifford algebra ... take the union of all these algebras ... and ... complete this and get a von Neumann algebra ... this ... is called "the hyperfinite II1 factor" ...
the hyperfinite II1 factor is a kind of infinite-dimensional Clifford algebra ...
[It is]... just another name for the algebra generated by creation and annihilation operators on the fermionic Fock space over C^(2n). ...".

Similarly, using 8-Peridocity of Real Clifford Algebras,
the Completion of the Union of the Tensor Products of the 64 + 1 + 64
which is structurally a Heisenberg-type Algebra of Creation/Annihilation Operators
for anticommuting 8 fundamental fermions, each with 8 components,
produces their physically realistic fermionic Fock space.

Note - At this stage there is only one generation of fermions.
The second and third generations appear only as a consequence
of breaking 8-dim spacetime into 4+4 dim Kaluza-Klein.

Commuting Parts of E8QC:

Look at the Tensor Products of the commuting parts of E8QC
of Even Grades {-2,0,+2}.

28 + SL(8,R) + 28

Consider the 28 + 28 and the SL(8,R) separately:

Bosonic Fock Space:

28 + 28

The 28 + 28 represent Creation/Annihilation Operators for 28 Gauge Bosons, and the Completion of the Union of their Tensor Products produces their physically realistic bosonic Fock space.

As to physical interpretations of the 28 Gauge Bosons:

16 of them give you conformal gravity because:
28-dimensional ALT8(R) can be seen as so(4,4)
28-dim so(4,4) has a 16-dim subgroup u(2,2)
u(2,2) contains su(2,2) = so(2,4) = Conformal Group Lie Algebra
which gives gravity (Einstein-Hilbert LaGrangian with cosmological constant)
by MacDowell-Mansouri mechanism.
(see section 14.6 of the book by Rabindra Mohapatra
Unification and Supersymmetry, 2nd edition, Springer-Verlag 1992)

The other 12 give you the SU(3)xSU(2)xU(1) Standard Model Gauge Bosons but they appear only as a consequence of breaking 8-dim spacetime into 4+4 dim Kaluza-Klein with 4-dim physical spacetime and 4-dim CP2 internal symmetry space. The CP2 gives the 12 Standard Model Gauge Bosons, because CP2 = SU(3) / SU(2)xU(1) as a symmetric space.
(see Class. Quantum Grav. 3 (1986) L99-L105 by N. A. Batakis)

The breaking of 8-dim spacetime into 4+4 dim Kaluza-Klein produces the Higgs.
(see Hadronic Journal 4 (1981) 108-152 by Meinhard Mayer
and articles by M. E. Mayer and A. Trautman in the proceedings book
New Developments in Mathematical Physics, 20th Universitatswochen fur Kernphysik in Schladming, Springer-Verlag 1981)

The Higgs as a T-quark condensate in 8-dim spacetime is described by Hashimoto, Tanabashi, and Yamawaki at hep-ph/0311165

8-dimensional Spacetime:

SL(8,R)

SL(8,R) represents local Polar Coordinates for 8-dim Spacetime due to the structure SL(8,R) = Spin(8) + Traceless Symmetric 8x8 Matrices (see the book by V. V. Gorbatsevich, A. L. Onishchik and E. B. Vinberg Lie Groups and Lie Algebras III, Structure of Lie Groups and Lie Algebras, Springer-Verlag 1994)

Tensor Products of the SL(8,R) structure represent a Global Spacetime in which each factor describes 8-dim Polar Coordinates at a Point.

With the Completion of the Union of all Tensor Products,
the Global Spacetime becomes an 8-dimensional Condensate
of Mutually Consistent 8-dim SL(8,R) Polar Coordinate systems.

At high (Planck-scale) Energies, throughout the Inflationary Era,
the 8-dim Condensate Spacetime has Octonionic structure
whose Non-Unitarity allows creation of the matter of our Universe.
(see,
for Octonionic Non-Unitarity, pages 50-52 and 561 of the book
Quaternionic Quantum Mechanics and Quantum Fields, Oxford 1995,
by Stephen L. Adler
and
for Clifford Algebra structure of the Inflationary Era,
papers by Paula Zizzi including but not limited to gr-qc/0007006)

Upon cooling at the end of Inflation,
a preferred Quaternionic substructure freezes out of the 8-dim Spacetime
producing a 4+4 Kaluza-Klein Spacetime M4 + CP2 where
M4 is 4-dim Minkowski physical spacetime
and CP2 = SU(3)/SU(2)xU(1) is 4-dim internal symmetry space.
After Octonionic structure is replaced by Quaternionic structure,
Quantum Processes become Unitary.

h92Duals and Quantum Heisenberg Algebra

E8 Physics consists of two levels:

The first level is **Lagrangian** Classical Action Structure made up of:

Integration over 8-dim Spacetime - 64 E8 Root Vectors

```
                ±1, ±i, ±j, ±k, ±e, ±ie, ±je, ±ke
                 (±1 ±i       ±e ±ie         )/2
                 (±1    ±j    ±e     ±je     )/2
                 (±1       ±k ±e         ±ke )/2
```

of Dirac Fermion term - 128 E8 Root Vectors

```
 (-1             ±ie ±je ±ke )/2    electron         8 components
 (-1    ±j ±k    ±ie         )/2    red up quark     8 components
 (-1 ±i    ±k        ±je     )/2    green up quark   8 components
 (-1 ±i ±j               ±ke )/2    blue up quark    8 components
 (   ±i ±j ±k -e             )/2    neutrino         8 components
 (   ±i       -e     ±je ±ke )/2    red down quark   8 components
 (      ±j    -e ±ie     ±ke )/2    green down quark 8 components
 (         ±k -e ±ie ±je     )/2    blue down quark  8 components
(-1             ±ie ±je ±ke )/2 positron             8 components
(-1    ±j ±k    ±ie         )/2 red up antiquark     8 components
(-1 ±i    ±k        ±je     )/2 green up antiquark   8 components
(-1 ±i ±j               ±ke )/2 blue up antiquark    8 components
(   ±i ±j ±k -e             )/2 antineutrino         8 components
(   ±i       -e     ±je ±ke )/2 red down antiquark   8 components
(      ±j    -e ±ie     ±ke )/2 green down antiquark 8 components
(         ±k -e ±ie ±je     )/2 blue down antiquark  8 components
```

and
of Standard Model Gauge Boson term -
8 Root Vectors + 4 Cartan Subalgebra elements

```
           (    +i +j      +ie +je     )/2  W+ boson
           (    +i +j      -ie -je     )/2  gluon_rg
           (    -i -j      +ie +je     )/2  gluon_cm
           (    +i -j      +ie -je     )/2  gluon_gb
           (    -i +j      -ie +je     )/2  gluon_my
           (    +i -j      -ie +je     )/2  gluon_br
           (    -i +j      +ie -je     )/2  gluon_yc
           (    -i -j      -ie -je     )/2  W- boson
```

and
of Conformal MacDowell-Mansouri Gravity term -

```
12 Root Vectors + 4 Cartan Subalgebra elements
         (       +j -k        +je -ke )/2    conformal_1
         (       -j +k        -je +ke )/2    conformal_i
         (       +j -k        -je +ke )/2    conformal_j
         (       -j +k        +je -ke )/2    conformal_k
         (       +j +k        -je -ke )/2  conformal_rxy
         (       -j -k        +je +ke )/2  conformal_rxz
         (    +i    +k    -ie     -ke )/2  conformal_btx
         (    -i    -k    +ie     +ke )/2  conformal_bty
         (    +i    -k    +ie     -ke )/2    conformal_e
         (    -i    +k    -ie     +ke )/2   conformal_ie
         (    +i    -k    -ie     +ke )/2   conformal_je
         (    -i    +k    +ie     -ke )/2   conformal_ke
```

The Lagrangian construction uses
64+128+8+4+12+4 = 220 generators of E8
(212 Root Vectors + 8 Cartan Subalgebra elements)

Although the Lagrangian gives nice Standard Model + Gravity physics results that can be compared with experiments (and so seen to be realistic) it is fundamentally a Classical structure (General Relativity of an Einstein-Hilbert Action plus Standard Model Gauge Theory) with Quantum phenomena by ad hoc Sum-Over-Histories Path Integrals.

Fundamental Quantum structure should appear as a natural Algebraic Quantum Field Theory which can be derived from real Clifford Algebra periodicity and embedding of E8 in the real Cl(16) Clifford Algebra to produce a generalized Hyperfinite II1 von Neumann factor AQFT that has the structure of a Quantum Heisenberg-type algebra that arises from the maximal contraction of E8:

E8 -> SL(8) + h_92

where SL(8) is 63-dimensional
and h_92 is 92+1+92 = 185-dimensional.
The 92 sets of creation/annihilation operators act on the 64 components (in 8-dim spacetime) of 8 fermions plus 12 Standard Model bosons plus 16 Conformal Gravity generators.

This second level **Heisenberg Algebra** Quantum structure is made up of

Position/Momentum Operators -

16 Root Vectors

```
(       +j +k        +je -ke )/2  h92Dual  C1
(       +j +k        -je +ke )/2  h92Dual  Ci
(       +j -k        +je +ke )/2  h92Dual  Cj
(       -j +k        +je -+e )/2  h92Dual  Ck
(       -j -k        -je +ke )/2  h92DualCrxy
(       -j -k        +je -ke )/2  h92DualCrxz
(       -j +k        -je -ke )/2  h92DualCryz
(       +j -k        -je -ke )/2  h92Dual  Cd
(    +i    +k    +ie     -ke )/2  h92Dual  Ce
(    +i    +k    -ie     +ke )/2  h92Dual Cie
(    +i    -k    +ie     +ke )/2  h92Dual Cje
(    -i    +k    +ie     +ke )/2  h92Dual Cke
(    -i    -k    -ie     +ke )/2  h92DualCbtx
(    -i    -k    +ie     -ke )/2  h92DualCbty
(    -i    +k    -ie     -ke )/2  h92DualCbtz
(    +i    -k    -ie     -ke )/2  h92DualPrPh
```

and

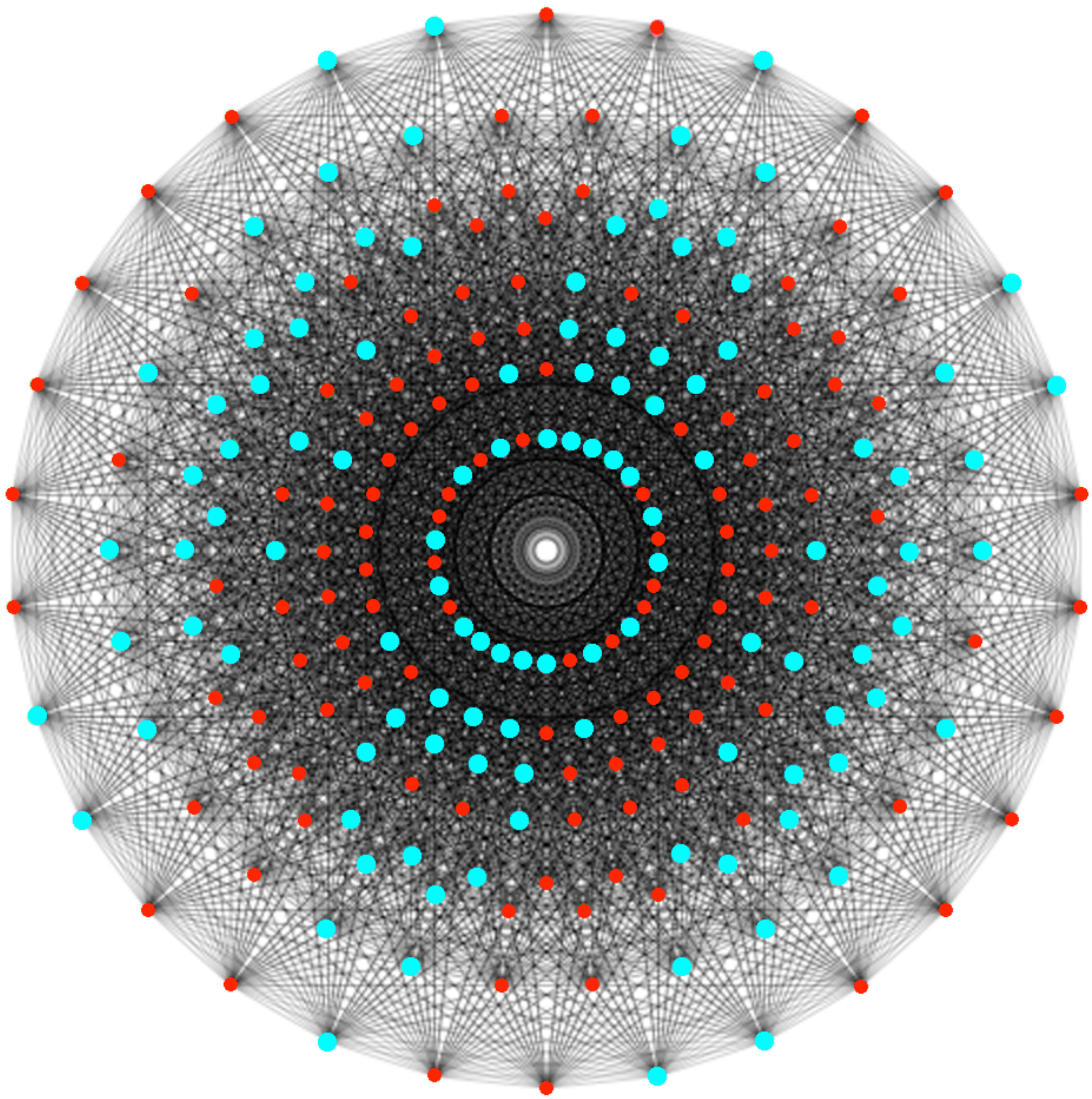
Creation Operators -

12 Root Vectors

```
(       +j +k        +je +ke )/2  h92Dualgamma
(    +i    +k    +ie     +ke )/2  h92DualGlrgb
(    +i +j       +ie -je     )/2  h92Dual   W+
(    +i +j       -ie +je     )/2  h92Dual Glrg
(    +i -j       +ie +je     )/2  h92Dual Glcm
(    -i +j       +ie +je     )/2  h92Dual Glgm
(    -i -j       -ie +je     )/2  h92Dual   W-
(    -i -j       +ie -je     )/2  h92Dual Glmy
(    -i +j       -ie -je     )/2  h92Dual Glbr
(    +i -j       -ie -je     )/2  h92Dual Glyc
(       -j -k        -je -ke )/2  h92Dual   W0
(    -i    -k    -ie     -ke )/2  h92DualGlcmy
```

The Heisenberg construction uses all 248 E8 generators including the 16+12 = 28 not used in Lagrangian construction.

The cube/square-type projection used above is not the only useful projection of the 240 E8 Root Vectors. Another is the projection to 8 circles each with 30 Root Vectors:



The image above adapted from the web site of David Madore at www.madore.org/~david/ shows in cyan the 112 root vectors of the D8 subalgebra of E8 that represent Spacetime, the Standard Model, and Gravity/Higgs
and in red the 128 root vectors of the D8 half-spinor in E8 that represent first-generation fermion particles and antiparticles.

David Madore uses xhtml to show the E8 Root Vectors in a coordinate system which the 240 Root Vectors are "... at the (112) points having coordinates (±1,±1,0,0,0,0,0,0) (where both signs can be chosen independently and the two non-zero coordinates can be anywhere) together with those having coordinates (±1/2,±1/2,±1/2,±1/2,±1/2,±1/2,±1/2,±1/2) (where all signs can be chosen independently except that there must be an even number of minuses) ...".

The relationship between David Madore's coordinates and the coordinates used in this paper is indicated by H. S. M. Coxeter in "Integral Cayley Numbers" (Duke Mathematical Journal, Vol. 13, No. 4, December 1946) reprinted in his book "The Beauty of Geometry: Twelve Essays" (1968, Dover edition 1999):

"... An alternative notation. In terms of the combinations

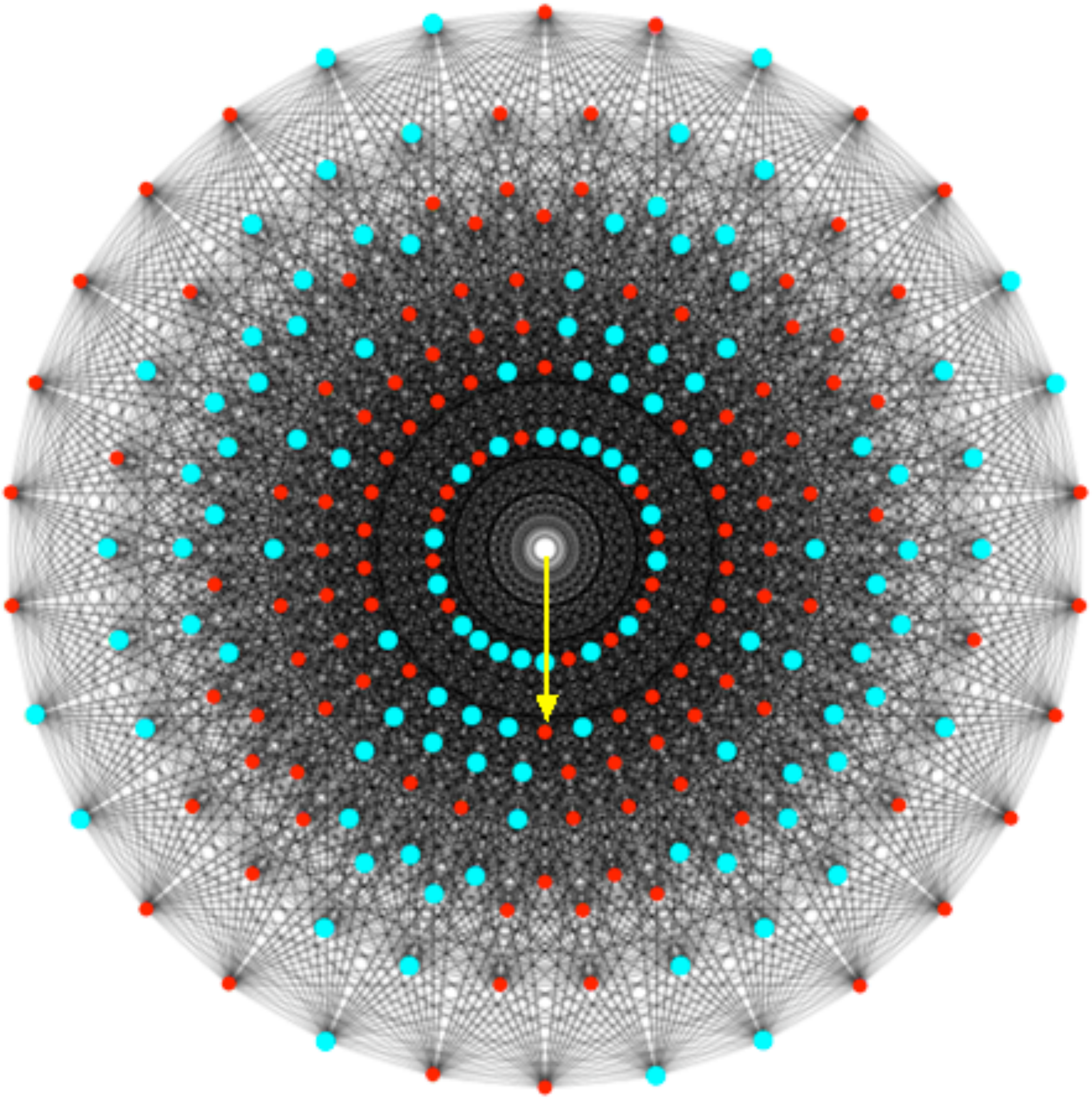
```
L1 = (1/2)( 1 + e )
L2 = (1/2)( i + ie)
L3 = (1/2)( j + je)
L4 = (1/2)( k + ke)
L5 = (1/2)( 1 - e )
L6 = (1/2)( i - ie)
L7 = (1/2)( j - je)
L8 = (1/2)( k - ke)
```

... all expressions of the form ± Lr ± Ls
.. and also (1/2)(± L1 ± L2 ± L3 ± L4 ± L5 ± L6 ± L7 ± L8)
with any odd number of minus signs ...
with r =/= s ... are the 112 + 128 units ...".

Note that
Coxeter chose the odd number of minus signs for the 128
while
David Madore the even number of minus signs for the 128.

A nice feature of David Madore's e8w.xhtml.html web page is that you can see by pointing the cursor at each point a lot of data including the coordinates of that point.

For example as shown in the following image, pointing the cursor over the point indicated by the yellow arrow shows the data set out below the Root Vector diagram:



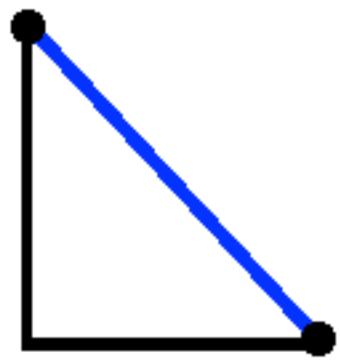
Position number 44,
coordinates: ½(-1,-1,1,1,1,1,-1,-1),
as FR: □-1,-1,-1,-1,-1,-1,-1,-1□,
projected: □0.00000,0.38547□.
Occupied by root 44,
coordinates: ½(-1,-1,1,1,1,1,-1,-1),
as FR: □-1,-1,-1,-1,-1,-1,-1,-1□,
projected: □0.00000,0.38547□.
Position↔occupied angle: 0.

You can calculate from the coordinates that
the indicated Root Vector represents
one of the 8 components of the red down antiquark.

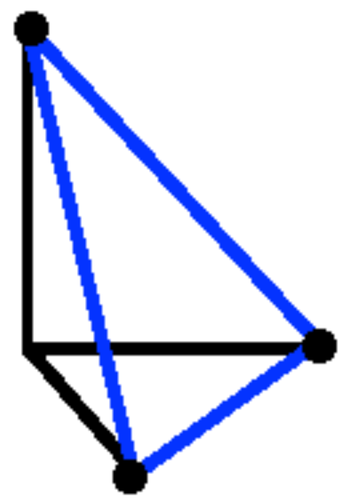
Simplex Superpositions

Frank Dodd (Tony) Smith, Jr. - 2012

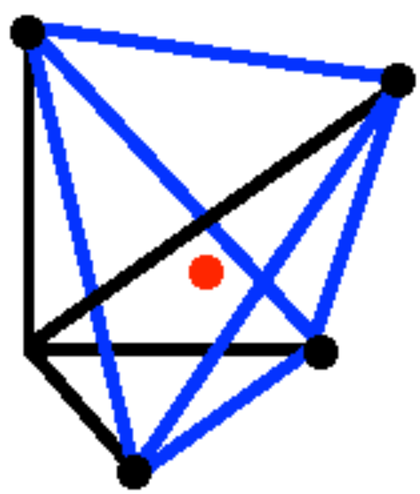
In E8 Physics Quantum Creation and Annihilation Operators come from the Maximal Contraction of E8 (semi-direct product of Sl(8) and H92 where H92 is a Heisenberg Algebra with graded structure 28+64+1+64+28). Superpositions of Quantum Operators can be described: With square/cubic tilings of 2-space and 3-space, there is no Superposition Vertex that corresponds to Superposition of any of the Basis Vertex States.



+1, +i



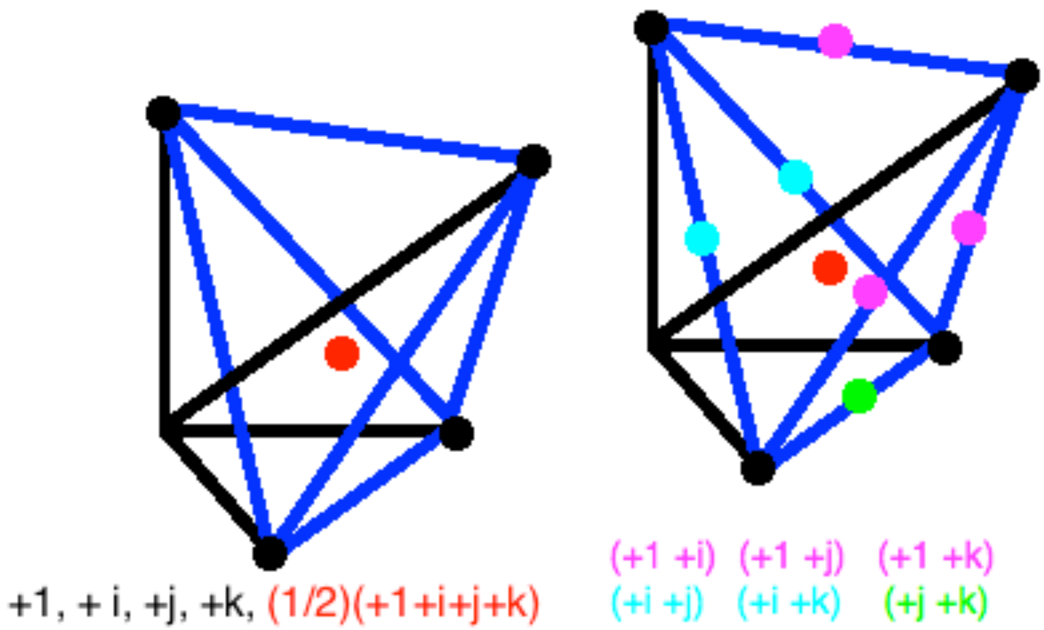
+1, +i, +j



+1, + i, +j, +k, (1/2)(+1+i+j+k)

Superposition Vertices begin at Quaternions and the 24-cell D4 tiling of 4-space.

A Dual 24-cell gives a new Superposition Vertex at each edge of the Simplex/Tetrahedron.



The Initial 24-cell Quantum Operators act with respect to 4-dim Physical Spacetime.
For example,
(1/2)(+1+i+j+k) represents Creation of the 4-dimensional space of the SU(2,2) = Spin(2,4) Conformal Group of Gravity of 4-dimensional Physical Spacetime
with {1,i,j,k} representing time and 3 space coordinates.

The Dual 24-cell Quantum Operators act with respect to 4-dim CP2 Internal Symmetry Space.
For example, bearing in mind that CP2 = SU(3)/SU(2)xU(1),
(+1 +i) (+1 +j) (+1 +k) are permuted by S3 to form the Weyl Group of the Color Force SU(3),
(+i +j) (+i +k) are permuted by S2 to form the Weyl Group of the Weak Force SU(2),
(+j +k) is permuted by S1 to form the Weyl Group of the Electromagnetic Force U(1).

The 4+4 dimensional Kaluza-Klein structure of the Initial 24-cell plus the Dual 24-cell of 4-dim Physical Spacetime plus 4-dim CP2 Internal Symmetry Space
is inherited from the Octonionic 8-dimensional structure of E8 lattices.

An Octonionic E8 lattice structure has 8 representative 8-vertex Simplex Basis Vertices

```
+1,  +i,  +j,  +k,  +e,  +ie,  +je,  +ke
```

plus 14 Superposition Vertices.

6 of the Superposition Vertices

```
(+1 +ke +e  +k)/2          (+i  +j  +ie  +je)/2
(+1 +je +j  +e)/2          (+ie +ke +k   +i)/2
(+1 +e  +ie +i)/2          (+ke +k  +je  +j)/2
```

project to (+1 +i) (+1 +j) (+1 +k) (+i +j) (+i +k) (+j +k) of CP2 Internal Symmetry Space.

8 of the Superposition Vertices

```
(+1 +ie +je +ke)/2         (+e  +i  +j  +k)/2
(+1 +k  +i  +je)/2         (+j  +ie +ke +e)/2
(+1 +i  +ke +j)/2          (+k  +je +e  +ie)/2
(+1 +j  +k  +ie)/2         (+je +e  +i  +ke)/2
```

project to (1/2)(+1+i+j+k) of 4-dim Physical Spacetime.

When you consider all 7 of the E8 lattices, you get 8 additional Superposition Vertices

```
(+1 +i +j  +k)/2           (+e +ie +je +ke)/2
(+1 +i +je +ke)/2          (+j +k  +e  +ie)/2
(+1 +j +ie +ke)/2          (+i +k  +e  +je)/2
(+1 +k +ie +je)/2          (+i +j  +e  +ke)/2
```

that also project to (1/2)(+1+i+j+k) of 4-dim Physical Spacetime,
and
the 8+8 = 16 E8-type vertices represent the 16 generators of U(2,2)
which contains the Conformal Group SU(2,2) = Spin(2,4).

As to the 8-vertex Simplex Basis Vertices

```
+1,  +i,  +j,  +k,  +e,  +ie,  +je,  +ke
```

they represent Quantum Creation Operators for the 8 fundamental fermion particles
neutrino; red down quark, green down quark, blue down quark;
electron; red up quark, green up quark, blue up quark

or, equivalently by Triality,
for the corresponding 8 fundamental fermion antiparticles
or
for the 8 dimensions of 8-dim spacetime.

Therefore, the 4-dim Simplex Basis Vertices to which they project can represent
4 dimensions of 4-dim Physical Spacetime or 4 dimensions of CP2 Internal Symmetry Space
or a lepton plus 3 quark subset of fermion particles or antiparticles.

Heisenberg Hamiltonian Quantum Physics
and
E8 Lagrangian Classical Physics
contained in
Cl(8)xCl(8) = Cl(16)

Heisenberg Hamiltonian Quantum Physics

Since by 8-periodicity Cl(8) is the basic factor of all real Clifford algebras,
start with the Cl(8) Clifford algebra with graded structure
1 + 8 + 28 + 56 + 70 + 56 + 28 + 8 + 1.
The vector 8 corresponds to Octonionic 8-dimensional spacetime.
Its dual pseudovector 8 corresponds to 8-dimensional momentum space.
The tensor product of the vector 8 and the pseudovector 8
produces 8x8 = 64-dimensional U(8).
U(8) is the symmetry group of the Hamiltonian
of the 8-dimensional isotropic harmonic oscillator.

The semidirect product of U(8) and the Heisenberg group H8 forms
the Heisenberg motion group G8 with graded structure 8 + 64 + 8.

Generalize G8 beyond the simple 8-dimensional harmonic oscillator
to a fully realistic physics model by forming the semidirect product
of U(8) and the Heisenberg group H92 to
get G248 with dimension 64 + 92+1+92 -1 = 248
(the -1 being due to the merging of 1 of the 64 of U(8) with the 1 of H92)
and graded structure 28 + 64 + 64 + 64 + 28 (grades -2,-1,0,1,2).

The central grade 0 represents the 64 dimensions of the semidirect product
of U(8) and the central 1-dimensional element of H92.

The odd grades -1 and 1 represent 64 creation and 64 annihilation operators
of the 8 components (with respect to 8-dim spacetime) of 8 fermion particles.

The even grades -2 and 2 represent two sets of 28 Spin(8) gauge bosons.

Break Octonionic spacetime symmetry by a Quaternionic structure creating
4+4 dim Kaluza-Klein spacetime and morphing Spin(8) into Spin*(8) = Spin(2,6).

One of the sets of 28 becomes Spin(2,6) with 16-dim U(2,2) subgroup that includes the Spin(2,4) Conformal Group of MacDowell-Mansouri Gravity. Spin(2,4) / Spin(0,2)xSpin(1,3) corresponds to 4-complex-dim bounded domain whose Shilov boundary 4-real-dim RP1xS3
corresponds to Minkowski physical spacetime of 4+4 Kaluza-Klein.
What is in 28 outside the 16 U(2,2) = Spin(0,2)xSpin(2,4) Gravity generators:
Spin(2,6) / Spin(0,2)xSpin(2,4) has real dimension 12
and is the G248 graded dual of the 12-dim Standard Model.

The other set of 28 becomes Spin*(8) with U(4) subgroup that gives
the Standard Model SU(3) and SU(2)xU(1) by the Batakis mechanism by which
SU(3) and its isotropy group for CP2 = SU(3) / SU(2)xU(1)
gives 12 Standard Model group generators.
CP2 corresponds to 4-dim Internal Symmetry Space of 8-dim Kaluza-Klein.
What is in the other 28 outside the 12 Standard Model generators:
First, look at the U(4):
U(4) = U(1) x SU(4)
SU(4) / SU(3)xU(1) = CP3 so SU(4) = SU(3) plus U(1) plus CP3
CP3 = C plus CP2
so U(4) = SU(3) plus CP2 plus U(1) plus U(1) plus C =
= SU(3) plus CP2 plus 4-dim T2C
and 4-dim T2C is in U(4) outside the 12 Standard Model generators
given by SU(3) plus the isotropy group for CP2.
Second, look at Spin*(8) / U(4):
Spin*(8) / SU(4)xU(1) has real dimension 28 - 16 = 12
So:
Spin*(8) / U(4) plus T2C has real dimension 16
and is the G248 graded dual of the 16-dim Conformal Gravity U(2,2).

Second and Third Generation Fermions emerge from
breaking Octonionic Symmetry of 8-dim SpaceTime
to Quaternionic Symmetry of 4+4 dim Kaluza-Klein.

The Higgs also emerges from that breaking to 4+4 dim Kaluza-Klein
and is represented by the Cl(8) Primitive Idempotent Structure
with grading 1 + 6 + 1 (grades 0,4,8) in the Cl(8) grading
1 + 8 + 28 + 56 + 70 + 56 + 28 + 8 + 1
in which the 8 + 28 + 56 + 64 + 56 + 28 + 8 correspond
to the G248 grading 28 + 64 + 64 + 64 + 28

E8 Lagrangian Classical Physics

To go from G248 Heisenberg Hamiltonian Quantum Physics
by an analog of the Legendre Transform to Classical Lagrangian Physics
expand G248 with 5-graded structure 28 + 64 + 64 + 64 + 28 (grades -2,-1,0,1,2)
to the E8 Lie Algebra with graded structure
8 + 28 + 56 + 64 + 56 + 28 + 8 (grades 1,2,3,4,5,6,7)

The central grade 4 represents the 64 = 8x8 dimensions of
Octonionic spacetime 8-dim position x 8-dim momentum
thus giving the base manifold over which the Lagrangian density is integrated.

The odd grades 1,3 represent 8+56= 64 sets of the 8 components (with respect to 8-dim spacetime) of 8 first-generation fermion particles.
The odd grades 5,7 represent 8+56= 64 sets of the 8 components (with respect to 8-dim spacetime) of 8 first-generation fermion antiparticles.
Together the grades 1,3,5,7 give the Dirac fermion term of the Lagrangian density.

The even grades 2 and 6 represent two sets of 28 Spin(8) gauge bosons.

The grade 2 Spin(8) becomes Spin(2,6) with 16-dim U(2,2) subgroup that
includes the Spin(2,4) Conformal Group of MacDowell-Mansouri Gravity to
produce a Gravity term of the Lagrangian density.

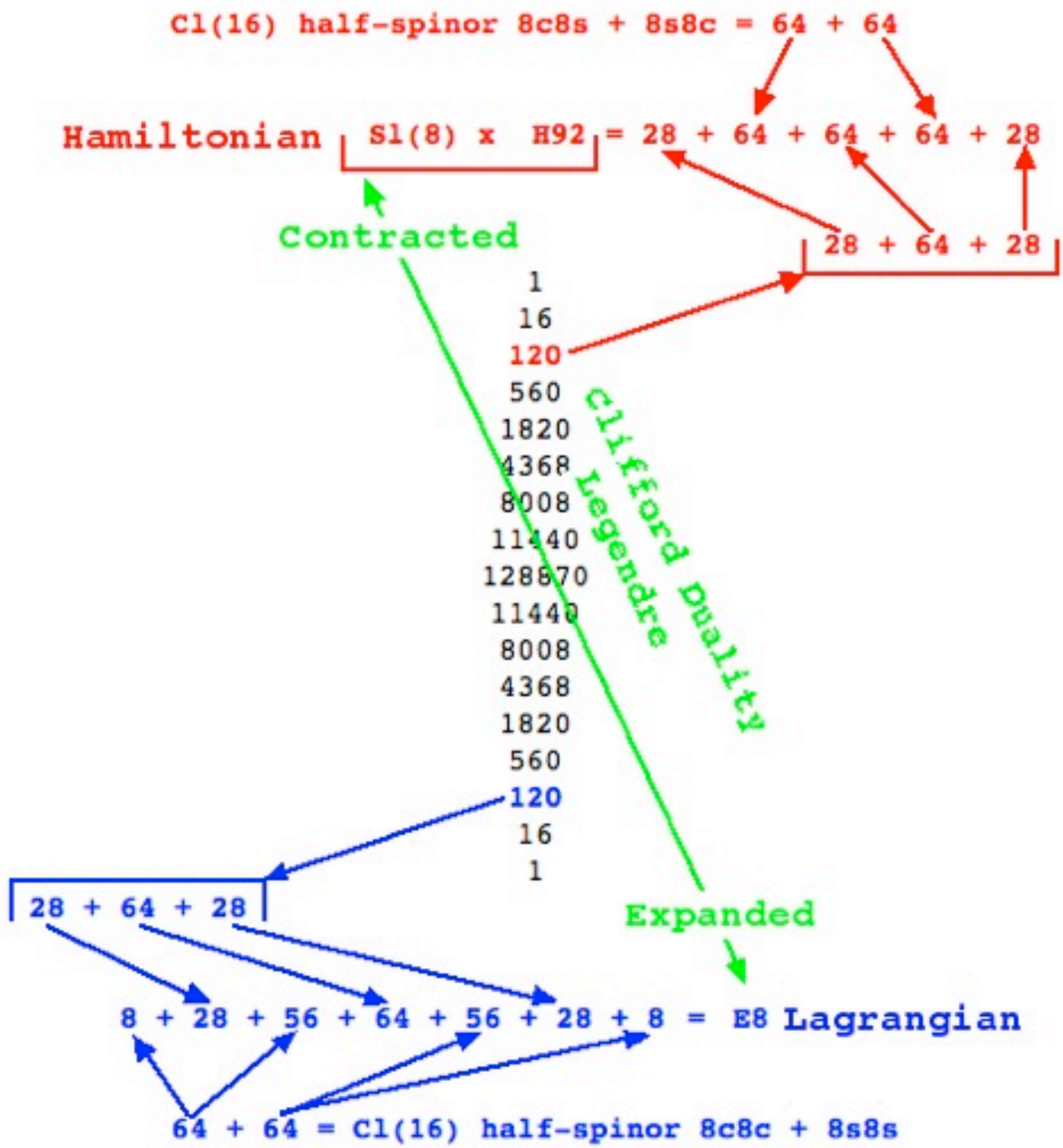
The grade 6 Spin(8) becomes Spin*(8) with U(4) subgroup that gives
the Standard Model SU(3) and SU(2)xU(1) by the Batakis mechanism by which
SU(3) and its isotropy group for CP2 = SU(3) / SU(2)xU(1) gives the Standard
Model SU(3)xSU(2)xU(1) gauge group term of the Lagrangian density.

Breaking Octonionic Symmetry of 8-dim SpaceTime to Quaternionic Symmetry of
4+4 dim Kaluza-Klein gives the resulting Lagrangian second and third generation
fermions and Mayer mechanism Higgs so that at our experimental energies the
resulting Lagrangian gives realistic Gravity plus Standard Model with Higgs.

Both Heisenberg Hamiltonian Quantum G248 and Classical Lagrangian E8
live inside Cl(16) the completion of the union of all tensor products of which,
by real Clifford periodicity, produce a realistic Algebraic Quantum Field Theory
as a generalized Hyperfinite II1 von Neumann factor.

G248 and E8 are related by Cl(16) duality as indicated in the following chart:

Cl(16) = Cl(8)xCl(8) = 256x256 = 65,536
Cl(8) spinor = 8s half-spinor + 8c half-spinor
Cl(16) spinor = 8s8s + 8s8c + 8c8s + 8c8c = 256



Schwinger - Hua - Wyler

Frank Dodd Tony Smith Jr - 2013 - vixra 1311.0088

In their book "Climbing the Mountain: The Scientific Biography of Julian Schwinger"
Jagdish Mehra and Kimball Milton said:

"... Schwinger ... always felt that the mathematics should emerge from the physics,
not the other way around ...

[Julian Schwinger said in conversations and interviews with Jagdish Mehra,
in Bel Air, California, March 1988]:

"... in 1966 ... Schwinger ... realized that he could base the whole machinery of particle physics on the abstraction of particle-creation and annihilation acts.
One can define a free action, say for a photon, in terms of propagation of virtual photons between photon sources, conserved in order to remove the scalar degree of freedom.
But a virtual photon can in turn act as a pair of electron-positron sources, through a `primitive interaction' between electrons and photons, essentially embodied in the conserved Dirac current.
So this multiparticle exchange gives rise to quantum corrections to the photon propagator, to vacuum polarization, and so on.
All this without any reference to renormalization or `high-energy speculations'. ...
The problem with conventional field theory is that it makes an implicit hypothesis that the physics is known down to zero distance ...
Source theory was ... that the physical quantities that you are interested in
were not the fields but the correlations between fields
and ... that the correlations between fields are really Green's functions
... which ... take into account not only how the particles behave
but how they are created.
The sources are the way of cataloging the various Green's functions.
The final point at which the theory asks to be compared with experiment ... involves just pure numbers, Green's functions and sources, not operator fields. ...
The whole point was to develop the space-time structure of a Green's function in general so it will be applicable both to stable particles and unstable particles.
...
Green's functions [were] universally recognized as carrying the information of physical interest ... one had differential equations for these Green's functions and then came the necessity of picking out of the vast infinity of solutions the physical ones of interest ...
This was enforced by appropriate boundary conditions, that the wave propagate outwards, that is, the idea of causality ... if you rotated the time axis into a complex space, then the boundary conditions ... would select just the physically acceptable states of the Green's function ... all representations of physical interest can be obtained from the ... Euclidean group ... attached [to]... the Lorentz group ... (the "unitary trick" of Weyl) ... a correspondence between the quantum theory of fields with its underlying Lorentz space, and a mathematical image in a Euclidean space ...".

The Schwinger Sources are finite regions in a Complex Domain spacetime corresponding to Green's functions of particle creation / annihilation.

What Complex Domains have Symmetries of Particle Physics ?

E8 8-dim Octonionic Spacetime (effective at high Planck-scale energies)
is by Triality isomorphic with the natural representation space of fundamental
First-Generation Fermion Particles (and AntiParticles)
so
Fermion Particles (and AntiParticles) are represented by Schwinger Sources
with Bounded Complex Domain structure of a Cartan domain.

David B. Lowdenslager in Annals of Mathematics 67 (1958) 467-484 said: "... For an irreducible Cartan domain ... there is only one linearly independent Riemannian metric ... the Bergman metric ... corresponding to ... Δ ... the Laplace-Beltrami operator ... solutions of ... Δ f = 0 ... are determined by their values ... on the ... Bergman-Shilov ... boundary B ... Let D be a classical Cartan domain, Δ an invariant Laplacian, and K a Poisson kernel for D. Then K as a function of D satisfies ΔK = 0 , for all b in B ...".

Steven G. Krantz in his book "Geometric Analysis of the Bergman Kernel and Metric" said:

"... the Bergman kernel ... K ... for Ω is related to the Green's function ... θ ... for the boundary value problem

$$\begin{aligned} \frac{\partial}{\partial \bar{z}_j} \triangle \beta &= 0 \qquad \text{on } \Omega, \quad j = 1, \ldots, k \\ \sum_{j-1}^{k} \frac{\partial \beta}{\partial \bar{z}_j} \cdot \overline{\alpha}_j &= 0 \qquad \text{on } \partial\Omega. \end{aligned}$$

... in this way

$$K_\Omega(z,t) = \triangle_z \theta(z,t)\,.$$

...".

Armand Wyler, in his 1972 IAS Princeton preprint "The Complex Light Cone Symmetric Space of the Conformal Group", said: "... the bounded realization Dn of SO(n,2) / SO(n)xSO(2) ... allows to define ... the Bergman metric, the invariant differential operators and their elementary solutions (Green functions) ...[and]... the Shilov boundary Qn ...[as]... the quotient space C(Mn) / P(Mn) of the conformal group by the Poincare group ... and give ... eigenvalues of Casimir operators in the Lie algebra of C(Mn) ...".
In Wyler's approach, the elementary solutions of the invariant differential operators in the Bounded Complex Schwinger Source Domains are Schwinger Green's functions.

Using Schwinger-type Euclidean Spin(10) version of the Spin(8,2) Conformal Group, the Fermion Schwinger Sources correspond to the Symmetric space
the Lie Sphere Spin(10) / Spin(8)xU(1)
which has local symmetry of the Spin(8) gauge group with respect to which the first generation spinor fermions are seen as +half-spinor and -half-spinor spaces,
so the Fermion Schwinger Source Bounded Complex Domain D8 is of type IV8
which has Shilov Boundary Q8 = RP1 x S7 .

The Complex Domain of type IV8 is described by L. K. Hua in his book "Harmonic Analysis of Functions of Several Complex Variables in the Classical Domains" as

(4) The domain $\mathfrak{R}_{IV}$ of n-dimensional ($n>2$) vectors

$$z=(z_1, z_2, \cdots, z_n) \qquad BDI\,(q=2)$$

(z_i are complex numbers) satisfying the conditions[2]

$$|zz'|^2+1-2\bar{z}\bar{z}'>0, \qquad |zz'|<1.$$

with Characteristic Manifold = Shilov Boundary = RP1 x S7

4°. The characteristic manifold of the domain $\mathfrak{R}_{IV}$ consists of vectors of the form $e^{i\theta}x$, where $0\leq\theta\leq\pi$, and $x=(x_1, \cdots, x_n)$ is a real vector which satisfies the condition $xx'=1$.

$$H(z,\ \theta,\ x)=\frac{1}{V(\mathfrak{C}_{IV})\,[(x-e^{-i\theta}z)\,(x-e^{-i\theta}z)']^{n/2}}, \qquad (4.7.11)$$

It is easy to calculate the magnitude of the volume $V(\mathfrak{C}_{IV})$:

$$V(\mathfrak{C}_{IV})=\frac{2\pi^{\frac{n}{2}+1}}{\Gamma\left(\frac{n}{2}\right)}.$$

The Poisson kernel of a type IV Complex Domain is

(4) For $\mathfrak{R}_{IV}$

$$P(z,\ \xi)=\frac{1}{V(\mathfrak{C}_{IV})}\cdot\frac{(1+|zz'|^2-2\bar{z}\bar{z}')^{\frac{n}{2}}}{|(z-\xi)(z-\xi)'|^n}, \qquad (4.8.9)$$

where $\xi\in\mathfrak{C}_{IV}$.

and the Bergman kernel of a type IV Complex Domain is

THEOREM 4.4.1. *The Bergman kernel of the domain* $\mathfrak{R}_{IV}$ *is*

$$\frac{1}{V(\mathfrak{R}_{IV})}(1+|zz'|^2-2z\bar{z}')^{-n},$$

where, by (2.5.7),

$$V(\mathfrak{R}_{IV})=\frac{\pi^n}{2^{n-1}\cdot n!}.$$

How big are the Schwinger Sources ?

Schwinger Sources as described above are continuous manifold structures
of Bounded Complex Domains and their Shilov Boundaries
but
E8 Physics at the Planck Scale has spacetime condensing out of Clifford structures
forming a Leech lattice underlying 26-dim String Theory of World-Lines
represents 8 + 8 + 8 = 24-dim of fermion particles and antiparticles and of spacetime.

The automorphism group of a single 26-dim String Theory cell modulo the Leech lattice
is the Monster Group of order about 8 x 10^53.

When a fermion particle/antiparticle appears in E8 spacetime it does not remain a single
Planck-scale entity becauseTachyons create a cloud of particles/antiparticles.
The cloud is one Planck-scale Fundamental Fermion Valence Particle plus an effectively
neutral cloud of particle/antiparticle pairs forming a Kerr-Newman black hole.

That cloud constitutes the Schwinger Source.
Its structure comes from the 24-dim Leech lattice part of the Monster Group which is
2^(1+24) times the double cover of Co1, for a total order of about 10^26.

(Since a Leech lattice is based on copies of an E8 lattice and since there are 7 distinct E8 integral domain
lattices there are 7 (or 8 if you include a non-integral domain E8 lattice)mdistinct Leech lattices.
The physical Leech lattice is a superposition of them, effectively adding a factor of 8 to the order.)

The volume of the Kerr-Newman Cloud is on the order of 10^27 x Planck scale,
so the Kerr-Newman Cloud should contain about 10^27 particle/antiparticle pairs
and its size should be about 10^(27/3) x 1.6 x 10^(-33) cm =
= roughly 10^(-24) cm.

How do the Schwinger Sources fit into the E8 Lagrangian Structure ?

The fundamental high-energy E8 Lagrangian for Octonionic 8-dim SpaceTime is

$$\int_{ST} GRb + StMb + Spf \quad .$$

an integral over SpaceTime ST of a Gravity boson term GRb,
a Standard Model boson term StMb, and a Spinor fermion term Spf.

Consider the Spinor fermion term Spf based on Schwinger Source Fermions.

In the conventional picture, the spinor fermion term is of the form m S S* where m is the fermion mass and S and S* represent the given fermion.
Although the mass m is derived from the Higgs mechanism, the Higgs coupling constants are, in the conventional picture, ad hoc parameters, so that effectively the mass term is, in the conventional picuture, an ad hoc inclusion.

E8 Physics does not put in the mass m in an ad hoc way,
but
constructs the Lagrangian integral such that
the mass m emerges naturally from the geometry of the spinor fermions by setting the spinor fermion mass term as the volume of the Schwinger Source Fermions.

Effectively the integral over the Schwinger Source spacetime region
of its Kerr-Newman cloud of virtual particle/antiparticle pairs plus the valence fermion gives the volume of the Schwinger Source fermion and defines its mass,
which, since it is dressed with the particle/antiparticle pair cloud,
gives quark mass as constituent mass.

Note that in the process of breaking Octonionic 8-dim SpaceTime
down to Quaternionic (4+4)-dim M4 x CP2 Kaluza-Klein
all Fermions are treated similarly so that ratios of their masses remain the same.

What about Gauge Bosons ?

The fundamental high-energy E8 Lagrangian for Octonionic 8-dim SpaceTime is

$$\int_{ST} GRb + StMb + Spf \quad .$$

an integral over SpaceTime ST of a Standard Model boson term StMb,
a Gravity boson term GRb, and a Spinor fermion term Spf.

What are the Schwinger Sources for the gauge boson terms StMb and GRb ?

The GRb bosons live in one of the two D4 Lie SubAlgebras of the E8 Lie Algebra.

The StMb bosons live in the other of the two D4 Lie SubAlgebras of the E8 Lie Algebra.

The process of breaking Octonionic 8-dim SpaceTime
down to Quaternionic (4+4)-dim M4 x CP2 Kaluza-Klein
creates differences in the way gauge bosons "see" M4 Physical SpaceTime.

Joseph Wolf (Journal of Mathematics and Mechanics 14 (1965) 1033) showed that
there are only 4 equivalence classes of 4-dimensional Riemannian Symmetric Spaces
with Quaternionic structures, with the following repesentatives:

S4 = 4-sphere = Spin(5) / Spin(4) where Spin(5) is the Schwinger-Euclidean version of
the Anti-DeSitter Group that gives MacDowell-Mansouri Gravity

CP2 = complex projective 2-space = SU(3) / U(2) with the SU(3) of the Color Force

S2 x S2 = SU(2)/U(1) x SU(2)/U(1) with two copies of the SU(2) of the Weak Force

S1 x S1 x S1 x S1 = U(1) x U(1) x U(1) x U(1) = 4 copies of the U(1) of the EM Photon
(1 copy for each of the 4 covariant components of the Photon)

The GRb bosons (Schwinger-Euclidean versions) live in
a Spin(5) subalgebra of the Spin(6) Conformal subalgebra of D4 = Spin(8).
They "see" M4 Physical spacetime as the 4-sphere S4
so that their part of the Physical Lagrangian is

$$\int_{S4} GRb \quad .$$

an integral over SpaceTime S4.
The Schwinger Sources for GRb bosons are the Complex Bounded Domains and Shilov Boundaries for Spin(5) MacDowell-Mansouri Gravity bosons.
However, due to Stabilization of Condensate SpaceTime
by virtual Planck Mass Gravitational Black Holes,
for Gravity, the effective force strength that we see in our experiments
is not just composed of the S4 volume and the Spin(5) Schwinger Source volume,
but is suppressed by the square of the Planck Mass.
The unsuppressed Gravity force strength is the Geometric Part of the force strength.

The Standard Model SU(3) Color Force bosons live in
a SU(3) subalgebra of the SU(4) subalgebra of D4 = Spin(8).
They "see" M4 Physical spacetime as the complex projective plane CP2
so that their part of the Physical Lagrangian is

$$\int_{CP2} (SU(3) \text{ part of StM})b \quad .$$

an integral over SpaceTime CP2.
The Schwinger Sources for SU(3) bosons are the Complex Bounded Domains and Shilov Boundaries for SU(3) Color Force bosons.
The Color Force Strength is given by
the SpaceTime CP2 volume and the SU(3) Schwinger Source volume.
Note that since the Schwinger Source volume is dressed with the particle/antiparticle pair cloud, the calculated force strength is
for the characteristic energy level of the Color Force (about 245 MeV).

The Standard Model SU(2) Weak Force bosons live in
a SU(2) subalgebra of the U(2) local group of CP2 = SU(3) / U(2)
They "see" M4 Physical spacetime as two 2-spheres S2 x S2
so that their part of the Physical Lagrangian is

$$\int_{S2xS2} \text{(SU(2) part of StM)b} \quad .$$

an integral over SpaceTime S2xS2.
The Schwinger Sources for SU(2) bosons are the Complex Bounded Domains and Shilov Boundaries for SU(2) Weak Force bosons.
However, due to the action of the Higgs mechanism,
for the Weak Force, the effective force strength that we see in our experiments
is not just composed of the S2xS2 volume and the SU(2) Schwinger Source volume,
but is suppressed by the square of the Weak Boson masses.
The unsuppressed Weak Force strength is the Geometric Part of the force strength.

The Standard Model U(1) Electromagnetic Force bosons (photons) live in
a U(1) subalgebra of the U(2) local group of CP2 = SU(3) / U(2)
They "see" M4 Physical spacetime as four 1-sphere circles S1xS1xS1xS1 = T4
(T4 = 4-torus) so that their part of the Physical Lagrangian is

$$\int_{T4} \text{(U(1) part of StM)b} \quad .$$

an integral over SpaceTime T4.
The Schwinger Sources for U(1) photons
are the Complex Bounded Domains and Shilov Boundaries for U(1) photons.
The Electromagnetic Force Strength is given by
the SpaceTime T4 volume and the U(1) Schwinger Source volume.

What are the results of Wyler-type calculations for Schwinger Sources ?

The Schwinger Source calculations using the Wyler approach give the following results, details of which can be found at http://vixra.org/abs/1310.0182 and my web sites.
Since calculations are for ratios of particle masses and force strengths,
the Higgs mass and the Geometric Part of the Gravity force strength are set so that the ratios agree with conventional observation data.

Particle/Force	Tree-Level	Higher-Order
e-neutrino	0	0 for nu_1
mu-neutrino	0	9 x 10^(-3) eV for nu_2
tau-neutrino	0	5.4 x 10^(-2) eV for nu_3
electron	0.5110 MeV	
down quark	312.8 MeV	charged pion = 139 MeV
up quark	312.8 MeV	proton = 938.25 MeV
		neutron - proton = 1.1 MeV
muon	104.8 MeV	106.2 MeV
strange quark	625 MeV	
charm quark	2090 MeV	
tauon	1.88 GeV	
beauty quark	5.63 GeV	
truth quark (low state)	130 GeV	(middle state) 174 GeV
		(high state) 218 GeV
W+	80.326 GeV	
W-	80.326 GeV	
W0	98.379 GeV	Z0 = 91.862 GeV
Mplanck=1.217x10^19 GeV		
Higgs VEV (assumed)	252.5 GeV	
Higgs (low state)	126 GeV	(middle state) 182 GeV
		(high state) 239 GeV
Gravity Gg (assumed)	1	
(Gg)(Mproton^2 / Mplanck^2)		5 x 10^(-39)
EM fine structure	1/137.03608	
Weak Gw	0.2535	
Gw(Mproton^2 / (Mw+^2 + Mw-^2 + Mz0^2))		1.05 x 10^(-5)
Color Force at 0.245 GeV	0.6286	0.106 at 91 GeV

The Third Grothendieck Universe: Clifford Algebra Cl(16) E8 AQFT

Realistic Physics/Math can be described using Three Grothendieck universes:

1 - Empty Set - the seed from which everything grows.

2 - Hereditarily Finite Sets - computer programs, discrete lattices, discrete Clifford algebras, cellular automata, Feynman Checkerboards.

3 - Completion of Union of all tensor products of Cl(16) real Clifford algebra - a generalized hyperfinite II1 von Neumann factor algebra that, through its Cl(16) structure, contains such useful Physics/Math objects as:

- Spinor Spaces
- Vector Spaces
- BiVector Lie Algebras and Lie Groups
  - Symmetric Spaces
  - Complex Domains, their Shilov boundaries, and Harmonic Analysis
  - E8 Lie Algebra
  - Sl(8)xH92 Algebra (Contraction of E8)
- Classical Physics Lagrangian structures
  - Base Manifold
  - Spinor Fermion term
  - Standard Model Gauge Boson term
  - MacDowell-Mansouri Gravity term
- Quantum Physics Hamiltonian/Heisenberg Algebra
  - Position/Momentum Spaces
  - Gravity + SM boson Creation/Annihilation Operators
  - Fermion Creation/Annihilation Operators

Daniel Murfet (Foundations for Category Theory 5 October 2006) said:
"... The most popular form of axiomatic set theory is Zermelo-Frankel (ZF) together with the Axiom of Choice (ZFC) ... this is not enough, because we need to talk about structures like the "category of all sets" which have no place in ZFC ...[more useful foundations include]
...
(a) An alternative version of set theory called NBG (due to von Neumann, Robinson, Bernays and Godel) which introduces classes to play the role of sets which are "too big" to exist in ZF
...
(b) Extend ZFC by adding a new axiom describing Grothendieck universes. Intuitively speaking, you fix a Grothendieck universe U and call elements of U sets, while calling subsets of U classes. ... This ... seems to be the only serious foundation available for modern research involving categories
...
(c) The first two options [(a) and (b)] are conversative, in that they seek to extend set theory by as little as possible to make things work. More exotically, we can introduce categories as foundational objects. This approach focuses on topoi as the fundamental logical objects (as well as the connection with the more familiar world of naive set theory). While such a foundation shows promise, it is not without its own problems ... and is probably not ready for "daily use".
...
Before we study Grothendieck universes, let us first agree on what we mean by ZFC. The first order theory ZFC has two predicate letters A,B but no function letter, or individual constants. Traditionally the variables are given by uppercase letters X1,X2, . . . (As usual, we shall use X, Y,Z to represent arbitrary variables). We shall abbreviate A(X, Y) by X in Y and B(X, Y) by X = Y .
Intuitively e is thought of as the membership relation and the values of the variables are to be thought of as sets (in ZFC we have no concept of "class"). The proper axioms are as follows (there are an infinite number of axioms since an axiom scheme is used):

Axiom of Extensionality Two sets are the same if and only if they have the same elements ...
Axiom of Empty Set There is a set with no elements. By the previous axiom, it must be unique ...
Axiom of Pairing If x, y are sets, then there exists a set containing x, y as its only elements, which we denote {x, y}. Therefore given any set x there is a set {x} = {x, x} containing just the set x ...

Axiom of Union For any set x, there is a set y such that the elements of y are precisely the elements of the elements of x ...
Axiom of Infinity There exists a set x such that the empty set is in x and whenever y is in x, so is y union {y} ...
Axiom of Power Set Every set has a power set. That is, for any set x there exists a set y, such that the elements of y are precisely the subsets of x ...
Axiom of Comprehension Given any set and any ... well formed formula ...
wf B(x) with x free, there is a subset of the original set containing precisely those elements x for which B(x) holds (this is an axiom schema) ... Here we make the technical assumption that the variables A,B,C do not occur in B ...
Axiom of Replacement Given any set and any mapping, formally defined as a wf B(x, y) with x, y free such that B(x, y1) and B(x, y2) implies y1 = y2, there is a set containing precisely the images of the original set's elements (this is an axiom schema) ...
Axiom of Foundation A foundation member of a set x is y in x such that y intersect x is empty. Every nonempty set has a foundation member ...
Axiom of Choice Given any set of mutually disjoint nonempty sets, there exists at least one set that contains exactly one element in common with each of the nonempty sets.
...
Looking at the axioms, only the Axiom of Replacement can produce a set outside our universe (beginning with sets inside the universe), although one could argue that the Axiom of Infinity also "produces" the set N, which may not belong to U . To get around the latter difficulty, we add the following axiom to ZFC ...
UA. Every set is contained in some universe ...
UA is equivalent to the existence of inaccessible cardinals,
and is therefore logically independent of ZFC
...
[This gives]... The first order theory ZFCU ...

Grothendieck Universes

Whatever foundation we use for category theory, it must somehow provide us with a notion of "big sets". In Grothendieck's approach, one fixes a particular set U (called the universe) and thinks of elements of U as "normal sets", subsets of U as "classes", and all other sets as "unimaginably massive".

...

Definition 3. A Grothendieck universe (or just a universe) is a nonempty set U with the following properties:

U1. If x in U and y in x then y in U (that is, if x in U then x subset U).

U2. If x, y in U then {x, y} in U .

U3. If x in U , then ... power set ... P(x) in U .

U4. If I in U and {xi}_(i in I) is a family of elements of U ,
then the union over i in I of the xi belongs to U .

...

Therefore
any finite union, product and disjoint union of elements of U belongs to U .
In particular every finite subset of U belongs to U ...
by our convention U contains N, and therefore also Z,Q,R,C and
all structures built from these using the theory of sets ...".

The Wikipedia article on Grothendieck universe said:
"... The idea of universes is due to Alexander Grothendieck,
who used them as a way of avoiding proper classes ...
There are two simple examples of Grothendieck universes:

The empty set,
and
The set of all hereditarily finite sets ... ".

The Third Grothendieck universe describes a realistic E8 AQFT.
It is the completion of the union of all tensor products of Cl(16)
which I will denote as
UCl16

The UCl16 universe gives category techniques useful in math and physics.

The real Clifford algebra Cl(16) = Cl(Cl(4)) = Cl(Cl(Cl(Cl(Cl(Cl(**θ**))))))
so UCl16 can be constructed by iterating Clifford Algebra construction
from empty **θ** to 0-dim Cl(**θ**) = {-1,+1} to 1-dim Cl(0) = R and so on.
Since Cl(16) = Cl(8)xCl(8) and real Clifford algebras have 8-periodicity,
UCl16 includes all arbitrarily large real Clifford algebras.

UCl16 is a hyperfinite von Neumann factor algebra, being
a real generalization of the usual complex hyperfinite II1 von Neumann factor.

Further, UCl16 inherits from its Cl(16) factors some structures that are useful
in areas including, but not limited to, physics model building
which structures can be seen in Category Theoretical terms:

- Vectors
- BiVector Lie Algebras and Lie Groups
  - Symmetric Spaces
  - Complex Domains, their Shilov boundaries, and Harmonic Analysis
- Spinors with Fermion properties
- E8 Lie Algebra
- Sl(8)xH92 Algebra (Contraction of E8)

Some other Categories useful with respect to physics model building are:

- Classical Physics Lagrangian
  - Lagrangian Spinor Fermion term
  - Lagrangian Base Manifold
  - Lagrangian MacDowell-Mansouri Gravity term
  - Lagrangian Standard Model Gauge Boson term

- Quantum Physics Hamiltonian/Heisenberg Algebra
  - Position/Momentum
  - Gravity + SM boson Creation/Annihilation
  - Fermion Creation/Annihilation

With respect to those Categories, there exist Functors

Cl(16) -> E8 -> Classical Physics Lagrangian
and
Cl(16) -> Sl(8)xH92 -> Quantum Physics Hamiltonian/Heisenberg Algebra

defined by

Cl(16) -> E8 and
E8 128 Spinors -> Lagrangian Spinor Fermion term
E8 64 Position/Momentum -> Lagrangian Base Manifold
E8 28 D4 Gravity -> Lagrangian M-M Gravity term
E8 28 D4 Standard Model -> Lagrangian SM Gauge Boson term

and

Cl(16) -> Sl(8)xH92 = Sl(8)xH(28+64) and
Sl(8) -> Position/Momentum
H28 -> Gravity + SM boson Creation/Annihilation
H64 -> Fermion Creation/Annihilation

Therefore **Path Integral quantization of Classical Physics Lagrangian**
has a Category Theoretical relationship with
Quantum Physics Hamiltonian/Heisenberg Algebraic Quantum Field Theory
that may show a Categorification of Lagrangian Path Integral
that is more directly related to the Standard Model + Gravity
than the
Chern-Simons theory whose Path Integral Quantization to a Topological Quantum Field Theory is described by Daniel Freed in Bull. AMS 46 (2009) 221-254.

Details of the physics structures mentioned above can be found in my paper Introduction to E8 Physics that is on the web at these URLs:
http://vixra.org/abs/1108.0027
http://www.valdostamuseum.org/hamsmith/E8physics2011.pdf
http://www.tony5m17h.net/E8physics2011.pdf

The wikipedia timeline of category theory says:
1958 - Grothendieck formulates topos theory based on algebraic geometry
1958 - Godement generalizes to monads
1963 - Grothendieck topos - categories = universes for doing all math
1963 - MacLane does n-categories (ribbons, braids, etc)
1964 - Lawvere does Elementary Theory of the Category of Sets (ETCS)
1972 - Grothendieck Universes for math
2006 - Lurie Higher Topos Theory

Kromer in his book Tool and Object says: "... the foundational debate
...
For Grothendieck, set theory is a foundation;
he assumes "more" than ZF ...[such as]... universes
...
Lawvere, however, assumes "less" ...".

Lawvere Approach

It seems to me that the Lawvere approach to AQFT leads to n-categorical higher topos stuff which seems to me to be so abstract
that it loses touch with concrete things needed to build physics models.
For example, the timeline also says:
1964 - Haag-Kastler-Segal Algebraic Quantum Field Theory (AQFT)
1988 - Witten Topological Quantum Field Theory (TQFT)

with the Lawvere path of AQFT leading to TQFT (and ribbons, braids, etc) which do not have enough detailed structure for construction of realistic physics models.

For example, Colin McLarty says in his 2009 paper
"What does it take to prove Fermat's Last Theorem?
Grothendieck and the logic of number theory" that it is
"... not entirely known ...[whether it]... go[es] beyond ... ZFC ...
[or]...
merely use[s] Peano Arithmetic (PA) or some weaker fragment of ... ZFC ..."
so
it seems to me that from the Lawvere approach it is not clear
that FLT has been proven.

Grothendieck Universe Approach

Colin McLarty in his 2009 paper "What does it take to prove Fermat's Last Theorem? Grothendieck and the logic of number theory" goes on to say:

"... Grothendieck ... universe is an uncountable transitive set U such
that {U,in} ... contains the powerset of each of its elements, and
for any function from an element of U to U the range is also an element of U ...
ZFC + U consists of ZFC plus the assumption of a universe ...
ZFC + U certainly implies more statements of arithmetic than ZFC alone
...
Grothendieck universes ... organize a context for ... explicit arithmetic
calculations proving FLT ... The great proofs in cohomological number theory,
such as Wiles[1995] or Deligne[1974], or Faltings[1983] ... in fact ...
use universes ...".

Therefore I prefer the Grothendieck universe approach to AQFT
1964 - Haag-Kastler-Segal Algebraic Quantum Field Theory (AQFT)
1972 - Grothendieck Universes for math

which I think does have sufficient detailed structure:

Streicher says in Universes in Toposes (2004): "... Grothendieck ...
introduced .. Grothendieck universe ... ZFC together with the requirement
that every set A be contained in some Grothendieck universe guaranteeing
at least an infinite sequence ... of Grothendieck universes ...
U_0 in U_1 in ... U_(n-1) in U_n in U_(n+1) in ...".

You can take U_0 as the empty set
and
U_1 as hereditarily finite sets
(which can be constructed from the power set
and which give you computer programs, discrete lattices,
discrete Clifford algebras, cellular automata, Feynman checkerboards, etc).

I would like to construct U_2 by noticing that the power set structure of U_1
is inherent in the basic construction of real Clifford algebras
which have the concrete structure of 8-periodicity
which allows you (since Cl(16) = Cl(8)xCl(8)) to construct
the completion of the union of all tensor products of Cl(16)
which seems to have algebraic structure that is
similar to the hyperfinite II1 von Neumann factor
and therefore to be a nice candidate for a realistic AQFT.

Of course, you can go beyond U_2 as far as you want to go,
but if you can build a realistic AQFT World from U_2
then my guess is that going beyond U_2 describes the Many-Worlds
of Many-Worlds Quantum Theory which gets you to
evolution of the Many-Worlds Multiverse by Quantum Game Theory
which in turn can be described by Clifford Algebra as in
http://arxiv.org/abs/1008.4689 by Chappell, Iqbal, and Abbott.

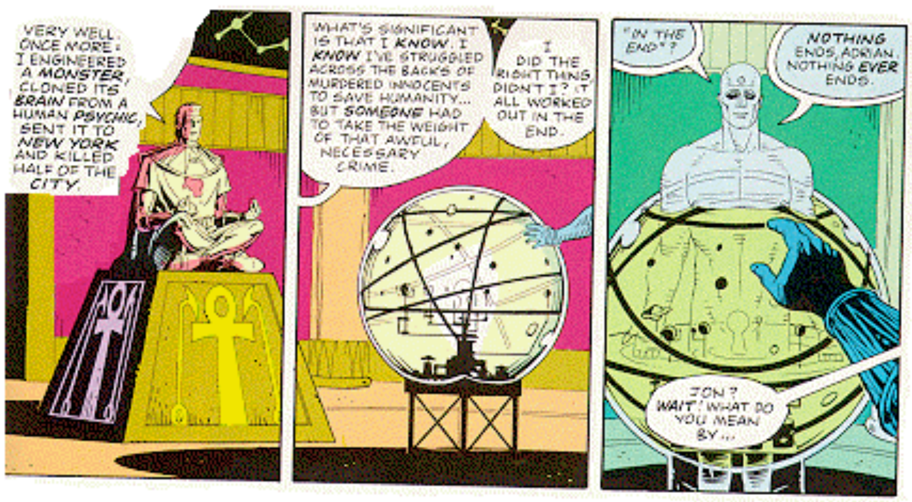
That, in turn, leads to the AQFT geometry of EPR phenomena
as described by Joy Christian at http://arxiv.org/abs/1201.0775

Marcus Chown, in the article Taming the Multiverse in New Scientist (14 July 2001, pages 27-30), says: "... David Deutsch ... thinks ... the multiverse ... could make real choice possible. ... In the multiverse ... there are alternatives ... Free will might have a sensible definition, Deutsch ... says...

"By making good choices ... we thicken the stack of universes in which versions of us live reasonable lives ...".

Each and every thing we do is a move in a vast never-ending Quantum Game .

As Jon (Dr. Manhattan) said in Watchmen (by Alan Moore and Dave Gibbons, DC Comics 1986, 1987):



"... Nothing EVER ends. ...". Each and every thing we do is a move in a vast never-ending Quantum Game.

A simple example: your World is post-World War II Humanity on Earth:
one Fate is a Dark Age - an alternative Fate is a Bright Age.

From time to time Human Choices lead to a Fork in the Path of Fates, one of three Fates:

to a Dark Age Basin of Attraction;

to Delay, Sit on the Fence, and stay on the Boundary Between Basins;

to a Bright Age Basin of Attraction.

BASIN OF ATTRACTION

DARK AGE

BASIN OF ATTRACTION

BRIGHT AGE

SCIENCE

Superstring Theory
Disconnected from Experiment
Consensus Conformity Enforced

Clifford Algebra AQFT
Consistent with Experiment
Realistic Creativity Rewarded

TECHNOLOGY

Electronic Communication Controlled
Chemical/Nuclear Energy Only
Isolation and Stagnation

Electronic Communication Open
Dark Energy Controlled
Exploration of Universe

SOCIETY

Financial Sector Dictatorship
People and Goods Controlled
Ideas Controlled and Limited

Free Movement of People and Goods
Free Exchange of Ideas
Financial Sector in Support Role

If Our World falls into a Bright Age Basin, we can enjoy a preview of Heaven.

If Our World falls into a Dark Age Basin, we are stuck in Hell.

If Our World is on the Boundary between Basins,
we still have Choices to make and a Mission to carry out.

Therefore, if your perception is of the World that demands most of your attention,
your perception is most likely to be that you live in a World on the Boundary Between Basins.

Let * represent a given state of the ManyWorlds, and let o represent various possible future states:

```
o     o       o     o
 \   /         \   /
  \ /           \ /
   o     o o     o
    \   /   \   /
     \ /     \ /
      o       o
       \     /
        \   /
         \ /
          *
```

The given state * might be a human mind, or a rock, or a glass of water, or anything else.

If there is no Resonant Connection between the given state * and the possible future states o, then the future of * will be spread at random among the possible future states o, each of which will become an actual future state * in the Worlds of the ManyWorlds:

```
*     *       *     *
 \   /         \   /
  \ /           \ /
   *     * *     *
    \   /   \   /
     \ /     \ /
      *       *
       \     /
        \   /
         \ /
          *
```

If there is a Resonant Connection between the given state * and one of the possible future states o:

```
o     o       o     o
 \   /         \   /
  \ /           \ /
   o     o o     o
    \   /   \   /
     \ /     \ /
      o       o
       \     /
        \   /
         \ /
          *
```

then the future of * will be concentrated at the possible future states related to the Resonant Connection o

and

the Non-Resonant possible future states o will be reduced or eliminated (depending on the strength of the Resonant Connection) from the set of each of which will become an actual future states * in the Worlds of the ManyWorlds:

```
o     o       *     *
 \   /         \   /
  \ /           \ /
   o     o o     *
    \   /   \   /
     \ /     \ /
      o       *
       \     /
        \   /
         \ /
          *
```

From this point of view, the set of all Worlds of the ManyWorlds looks like an environment in which ManyWorlds Abstract Beings live and interact by Resonant Connections

```
*     *       *     *
 \   /         \   /
  \ /           \ /
   *     * o     *
    \   /   \   /
     \ /     \ /
      *       *
       \     /
        \   /
         \ /
          *
```

How do different Basins of Attraction (such as * and *) in the ManyWorlds compete ?

By **Resonant Connections and the Quantum Zeno Effect interacting by Quantum Game Theory**

Direct perception of such Basins and Interactions beyond the physical universe of only one World can be achieved
by some cultivated humans, and, perhaps, by Dolphins and/or Conscious Quantum Computers.

Aden Ahmed in “On Quaternions, Octonions, and
the Quantization of Games”
and at http://arxiv.org/abs/0808.1391
uses the 3-sphere and the 7-sphere used by Joy Christian
in his paper at http://arxiv.org/abs/1201.0775
where he uses them to explain EPR phenomena.

Chappell, Iqbal, and Abbott at http://arxiv.org/abs/1008.4689
deal with quantum games and EPR using Clifford algbras.
Since the Quaternions are the $2^2 = 4$-dim Cl(2) Clifford algebra
could you use the $2^3 = 8$-dim Cl(3) Clifford algebra
instead of the non-associative Octonions in Quantum Games ?

In other words, are there two paths to study Quantum Games ?

Real Clifford Algebras:
Cl(1) = Complex
Cl(2) = Quaternion
Cl(3)
Cl(4)
Cl(5)
Cl(6)
Cl(7)
Cl(8)

Here, due to 8-periodicity,
there are no more really new
real Clifford algebras
because for any k, Cl(8k) =
= Cl(8) x (k times tensor product) x Cl(8)
In particular, at and beyond Cl(8) you seem
to get spinors that are not pure spinors
(see for example Penrose and Rindler,
Spinors and Space-Time, Vol 2, around page 453).

Cayley-Dickson Algebras:
2-dim Complex
4-dim Quaternion
8-dim Octonion

Here, due to
zero-divisors,
there are no more
division algebras
If you want to go to
bigger quantum games,
you have to deal with
zero-divisor
structure that first
appears in 16-dim
Sedenions.

Are the two paths (Clifford and Cayley-Dickson) equivalent ?

Historical Background

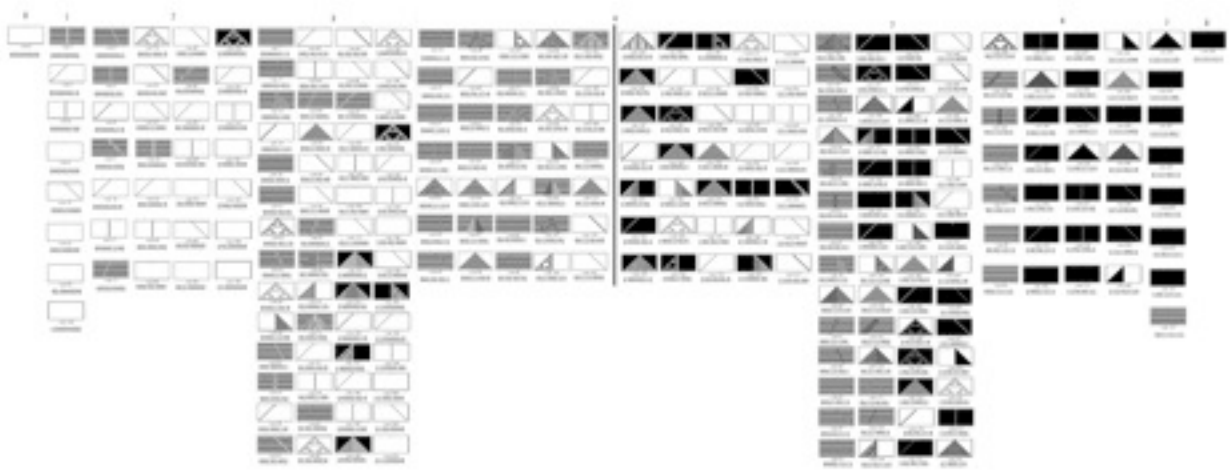
A little less than 15 billion years ago, our Universe emerged from the Void.

4 billion years ago, our Earth and Moon were orbiting our Sun.

2 billion years ago, bacteria built a nuclear fission reactor in Africa.

100,000 years ago, Humans were expanding from the African home-land to Eurasia and beyond.

12,000 years ago, Africans knew that the knowledge-patterns of 8 binary choices giving 2^8 = 256 = 16x16 possibilities could act as an Oracle. Did they realize then that those 256 possibilites corresponded to the



256 Fundamental Cellular Automata, some of which act as Universal Computers?

From Africa, the 16x16 Oracle-patterns spread, so that by the 13th century parts of them were found in:

Judaism as the 248 positive Commandments plus the 365 negative Commandments given to Moses during the 50 days from Egypt to Sinai;

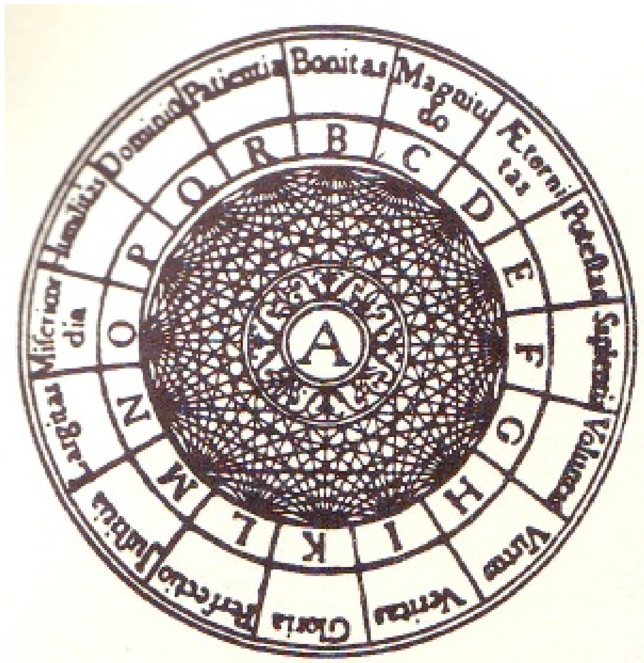
India as the 240 parts of the first sukt of the Rig Veda;

Japan as the 128 possibilities of Shinto Futomani Divination;

China as the 64 possibilities of the I Ching;

Mediterranean Africa as the 16 possibilities of the Ilm al Raml.

Near the end of the 13th century, Ramon Llull of Mallorca studied the 16 possibilities of the Ilm al Raml and realized that the 16x16 African Oracle-patterns had a Fundamental Organizational Principle that he summarized in a Wheel Diagram



with 16 vertices connected to each other by 120 lines, like the 120 bivectors of the Cl(16) Clifford Algebra that correspond to the D8 Lie Algebra that lives inside E8. He used such structures to show the underlying unity of all human religions. However, the establishments of the various religions refused to accept Ramon Llull's revelations, and his ideas were relegated to a few obscure publications, plus an effort to preserve some aspects of the 16x16 Oracle-patterns in the form of the 78 Tarot cards and the subset of 52 cards that remains popular into the 21st century.

Since Llull was Roman Catholic, the Islamic and Judaic bureaucracies could (and did) ignore his work as that of an irrelevant outsider. As to the Christians, in the 14th century, Dominican Inquisitors had Ramon Llull condemned as a heretic, his works were suppressed, and his ideas were relegated to a few obscure publications, plus an effort to preserve some aspects of the 16x16 Oracle-patterns in the form of the 78 Tarot cards and the subset of 52 cards that remains popular into the 21st century.

In the 17th century the Roman Inquisition burned Giordano Bruno at the stake and sentenced Galileo to house arrest for the rest of his life, all for the sake of the Roman Inquisition's enforcement of conformity to its Consensus.

Rediscovery of the full significance of Ramon Llull's Oracle-patterns did not happen until:

> after 20th century science experiments progressed beyond Gravity, Electromagnetism, and early Quantum Mechanics, and
>
> after Lise Meitner discovered the Uranium Fission Chain Reaction Process that led to the Fission Bombs that ended the Japanese part of World War II.

The Japanese defeat liberated Saul-Paul Sirag, a child of Dutch-American Baptist missionaries, from a Japanese concentration camp in Java.

During the 1950s and 1960s, David Finkelstein described Black Holes and worked on Quaternionic Physics, Hua Luogeng 华罗庚 returned to China where he wrote his book "Harmonic Analysis of Functions of Several Complex Variables in the Classical Domains", Jack Sarfatti studied physics (BA from Cornell and PhD from U. C. Riverside), and I learned about Lie Groups and Lie Algebras (AB in math from Princeton).

During the 1970s, Saul-Paul Sirag learned math and physics working with Arthur Young and the physics community developed the Standard Model showing how everything other than Gravity could be described, consistent with experimental results, by 3 forces of a Standard Model:

> Electromagnetism, with the symmetry of a circle, denoted by S1 = U(1)
>
> Weak Force with Higgs, with the symmetry of a 3-dimensional sphere, denoted by S3 = SU(2)
>
> Color Force, with symmetry related to a Star of David, denoted by SU(3)

From the 1980s on, I learned about Clifford Algebras from David Finkelstein at Georgia Tech; about Weyl Groups and Root Vectors from the work of Saul-Paul Sirag; about Quantum Consciousness, Space-Time and Higgs as Condensates, and Bohmian Back-Reaction from the work of Jack Sarfatti; and about Compton Radius Vortices from the work of B. G. Sidharth.

In contrast to the advances in experimental results and construction of the Standard Model of physics, the social structure of the Physics Scientific Community evolved during the 20th century into a rigid Physics Consensus Community much like the Inquisitorial Consensus Community of a few hundred years ago.

For example, in the USA physics community around the middle of the 20th century, J. Robert Oppenheimer enforced his dislike of the ideas of David Bohm by declaring, as head of the Princeton Institute for Advanced Study:

“... if we cannot disprove Bohm, then we must agree to ignore him ...”

As the 20th century ended and the 21st century began, the Physics Consensus Community continued to enforce conformity to Consensus so strongly that Stanford physicist Burton Richter said:

“... scientists are imprisoned by golden bars of consensus ...”

The rigidly enforced Physics Consensus Community was so void of independent thought that the 20th century ended without anyone seeing how Ramon Llull’s Oracle-patterns explained both Gravity and the Standard Model in a unified way,

but

in January 2008 the cover of the magazine of Science & Vie declared:

“Theorie du tout Enfin!

Un physicien ... chercheur hors norme ... aurait trouve la piece manquante”

The missing piece was a 248-dimensional Lie Algebra known as E8.

The beyond-the-norm physics researcher was a California-Hawaii Surfer Dude, Garrett Lisi, who realized that the structure of E8 could unify Gravity and the Standard Model in a way that satisfied Einstein's Criterion for a structure

"... based ... upon a faith in the simplicity ... of nature: there are no arbitrary constants ... only rationally completely determined constants ... whose ... value could ... not ... be changed without destroying the theory …".

Motivated by Garrett Lisi's E8 work, I constucted from E8 a Lagrangian that realistically describes physics in a Local Region. Since E8 lives inside the Clifford Algebra Cl(16) = Cl(8)xCl(8), if you let a copy of Cl(16) represent a Local Lagrangian Region, you can construct a Global Structure by taking the tensor products of the copies of Cl(16). Due to Real Clifford Algebra 8-periodicity, any Real Clifford Algebra, no matter how large, can be embedded in a tensor product of factors of Cl(8), and therefore of Cl(8)xCl(8) = Cl(16). My E8 model was constructed somewhat differently from Garrett Lisi's so as to avoid its problems that had been pointed out by Jaques Distler, Skip Garibaldi, et al.

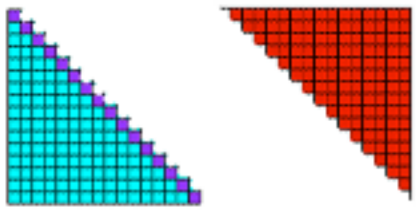
Just as the completion of the union of all tensor products of 2x2 complex Clifford algebra matrices produces the usual Hyperfinite II1 von Neumann factor that describes creation and annihilation operators on fermionic Fock space over C^(2n) (see John Baez's Week 175), we can take the completion of the union of all tensorhat products of Cl(16) = Cl(8)xCl(8) to produce a generalized Hyperfinite II1 von Neumann factor that gives a natural Algebraic Quantum Field Theory structure for E8 Physics, and corresponds to the El Aleph of Jorge Luis Borges.

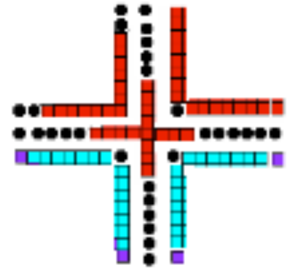
In some sense, the 240 Root Vectors of E8 are a seed from which El Aleph grows.

The following pages give some details of the historical development.

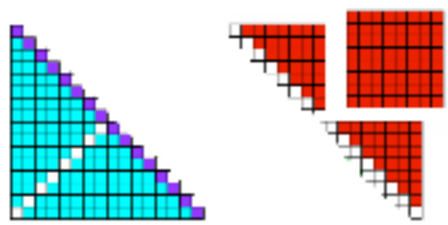
Historical Development of Physics:

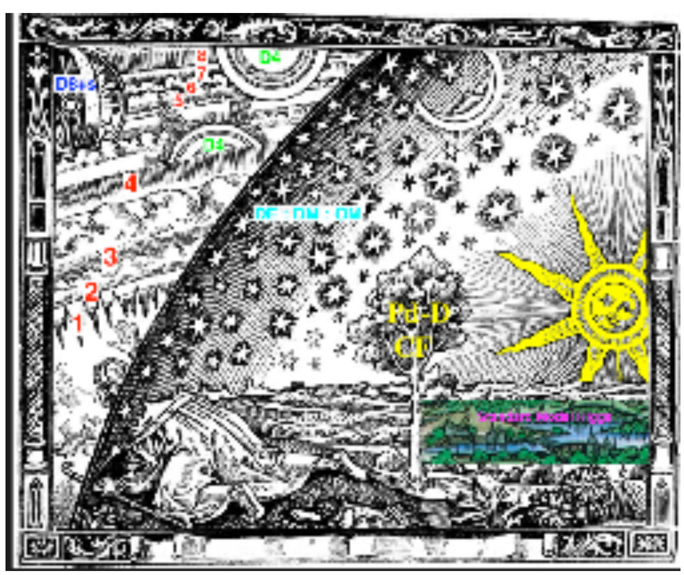
African IFA to RigVeda-Pachisi





to Tarot to Llull to E8 Physics





Flammarion's Naive Missionary Explorer sees

the intersection of Terrestrial Physics and AstroPhysics as a window
to
the Realm of Terrestrial-AstroPhysics Unification through E8 Physics.

Vedic Meru and Nile Pyramid:
African IFA to RigVeda-Pachisi to Tarot
to Llull to Cartan-Dirac-Riesz-E8Physics

Abstract

Ancient Africa developed IFA divination with
256 Odu analagous to the Real Clifford Algebra Cl(8)
and 16 Orishas analagous to the 8+8 Spinors of Cl(8)
but in Africa IFA was transmitted by oral tradition and not written down.
50,000 years ago humans emerging from Africa via the Arabian Sea settled in India
where ideas of IFA were written in the Sanskrit Rig Veda and encoded in Pachisi.
40,000 years ago humans migrating down the Nile settled in Giza
where ideas of IFA were encoded in the Great Pyramid.
By about 500 B.C. Indian Vedic Pachisi evolved into Tarot.
By about 1300 A.D. Tarot and some ideas of IFA were known in Mallorca
where Ramon Llull used them to develop structures of the D4 and D8 Lie Algebras
that lived in the IFA Clifford Algebra Cl(8) and its tensor square Cl(8)xCl(8) = Cl(16).
Although Llull's work was preserved in writing in Mallorca, his ideas were rejected
by the Intellectual Establishment of Paris and remained dormant for centuries.

Around 1890 Killing and Cartan rediscovered Llull's Lie Algebras.
Only in the 1900s did the work of Cartan
as further developed mathematicaly by Jovet, Sauter, and Riesz
and as applied by Dirac to physics
show how a D8 half-Spinor could be added to D8 itself to get E8 Physics
and to see how E8 Physics lives inside IFA Cl(8).

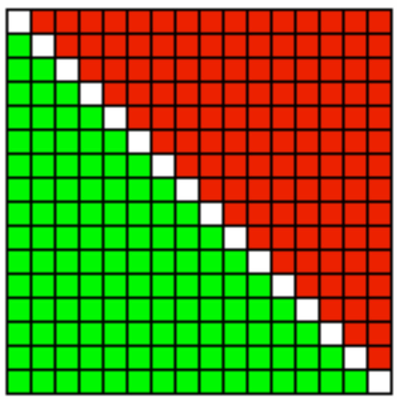
It has taken until now (the 2000s) for the formal Written Human Culture
to catch up with the informal Oral Ancient African Culture
in understanding a realistic Unified Theory of the Laws of Nature.

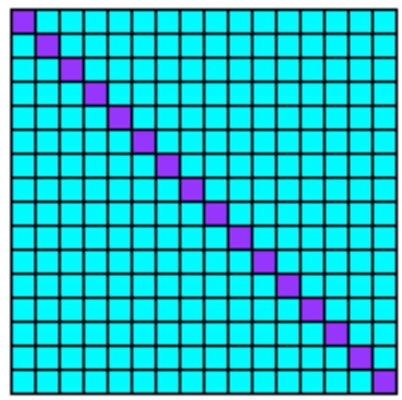
Introduction

The oldest and most comprehensive Information System of humans on Earth is African IFA Divination based on 16x16 = 256 elements (tony5m17h.net/VoudouFA.html)
It corresponds mathematically to the Cl(8) Real Clifford Algebra
with graded structure 256 = 1 + 8 + 28 + 56 + 70 + 56 + 28 + 8 + 1
and algebraic structure of M(16,R) = 16x16 Matrices of Real Numbers.

Here is how E8 Physics of Gravity and the Standard Model is encoded in IFA:

256-dim 16x16 =
= 120-dim Antisymmetric16x16 + 136-dim Symmetric 16x16





For Antisymmetric 16x16 each red entry above the diagonal is the negative of the corresponding green entry below the diagonal and the 16 diagonal entries are zero so the number of Antisymmetric entries is 120 corresponding to the D8 Lie Algebra.

For Symmetric 16x16 each cyan entry above the diagonal is equal to the corresponding cyan entry below the diagonal and the 16 diagonal entries are non-zero so the number of Symmetric entries is 120+16 = 136.
8 of the 136 Symmetric entries of the IFA Cl(8) 16x16 Matrix do not correspond to E8
but
the other 136-8 = 128 = 64 + 64 correspond to 128-dim half-spinor of D8.

Since 248-dim E8 = 120-dim D8 + 128-dim half-spinor of D8
by E8 / D8 rank 8 Type EVIII space (OxO)P2 the Octo-Octonionic Projective Plane

256-dim IFA Cl(8) contains 120+128 = 248-dim E8
and
encodes the structure of E8 Physics of Gravity and the Standard Model.

Some details of the IFA encoding of E8 Physics is set out Appendix2.

African IFA to RigVeda-Pachisi

From its home in Africa the IFA Information System spread, like humanity itself,

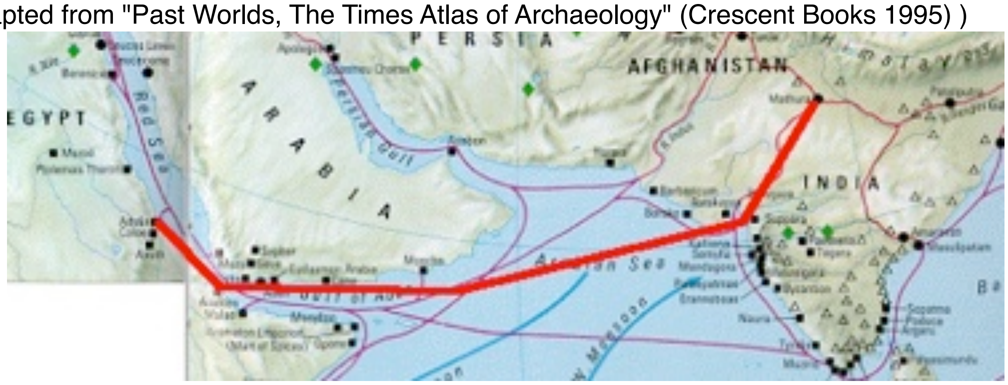
throughout the Earth. Some of its descendant systems, such as

128-element Shinto Divination
64-element I Ching
16-element Ilm Al Raml

are straightforward subsets of 256-element IFA
but
the Rig Veda and its related game Pachisi has a more intricate relationship to IFA.

Within its African home IFA was never written down but was oral tradition
but
when humans left Africa they had less of the direct contact with their Ancestral Home that is useful for preservation of oral tradition.

India was settled from Africa via the Arabian Sea in very early times.
(map adapted from "Past Worlds, The Times Atlas of Archaeology" (Crescent Books 1995))



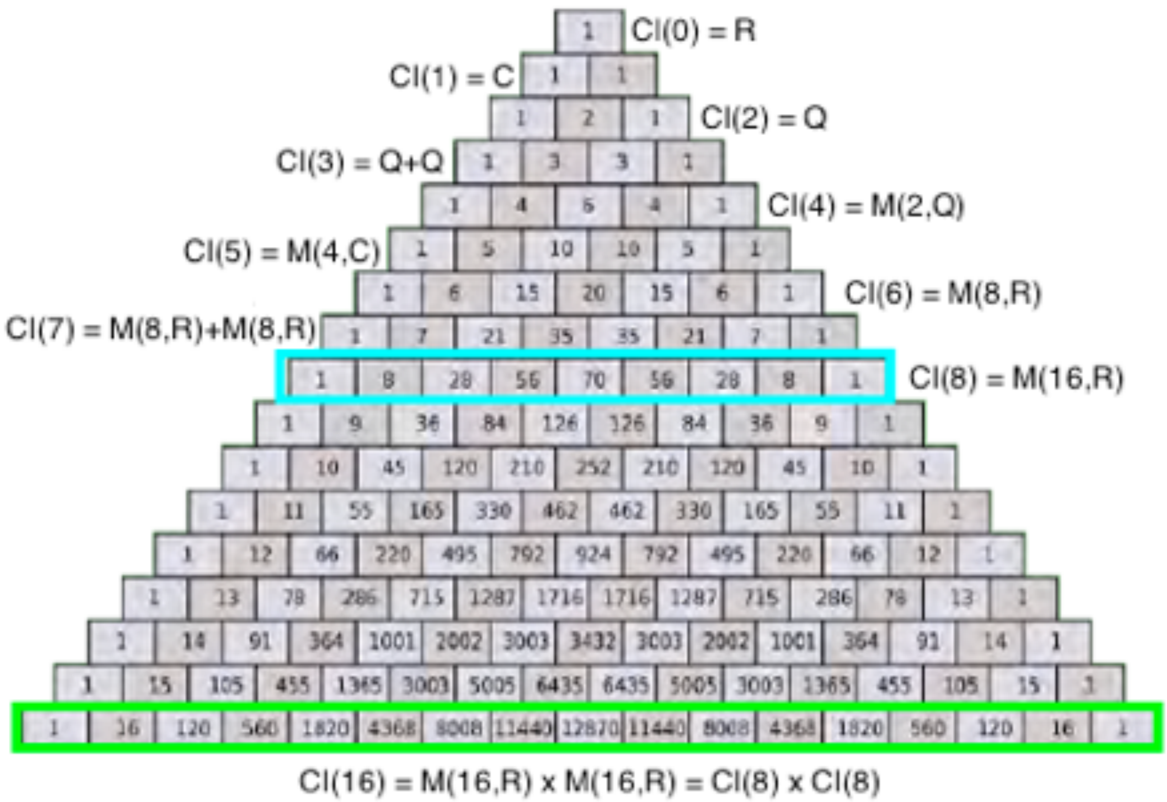
Indian priests of IFA chose to put the IFA Information System into writing,
so they developed Sanskrit from the African Geez language.

In a 16 October 2010 post to his blog at bafsudralam.blogspot.com Clyde Winters said:
"... The Naga were Semitic speaking people from Ethiopia …
The major gift of the Naga to India was the writing system: Deva-Nagari.

Nagari is the name for the Sanskrit script ... the ancient Ethiopic and Sanskrit writing are one and the same ... the name Nagari for Sanskrit betrays the Ethiopia origin of this form of writing. In Geez, the term nagar means 'speech, to speak'. ...".

Feuerstein, Kak, and Frawley, in their book In Search of the Cradle of Civilization (Quest 1995), say "... The principal and, taken in its totality, the oldest of the four Vedic hymnbodies is the Rig-Veda. ... The Sanskrit word ric, which for euphonic reasons is changed to rig, means literally "praise". ... The Sanskrit word veda means literally "knowledge" or "wisdom". ... **The Rig-Veda is** the oldest book in the Sanskrit language ... More than that, if we are correct, it is **the oldest book in the world** ...".

The Indian National Science Academy web site insaindia.org says
"... The Vedic Civilization ... evolved around ... the Vedas ... Vedic meters ... permutations and combinations of long and short sounds ... led ... to discover[y of] the Meru Prastara ...



now known as Pascal's Triangle ...".

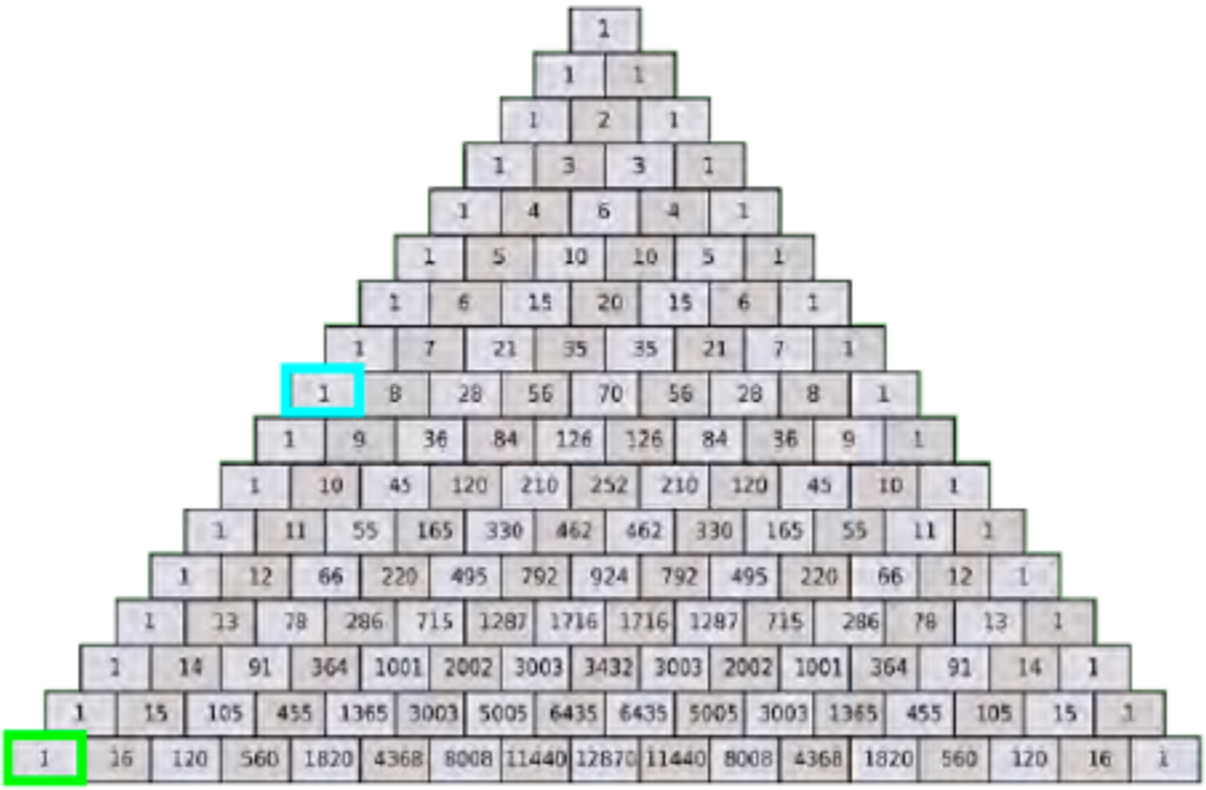
The row I have outlined in cyan contains the 1+8+28+56+70+56+28+8+1 = 256 = 16x16 elements of the Cl(8) Real Clifford Algebra of African IFA divination.
The other rows contain the 2^N elements of Cl(N) where N is the second number from the left in each row, so that the Meru Prastara describes all Real Clifford Algebras Cl(N), with the figure above showing Cl(0) through Cl(16) which I have outlined in green.

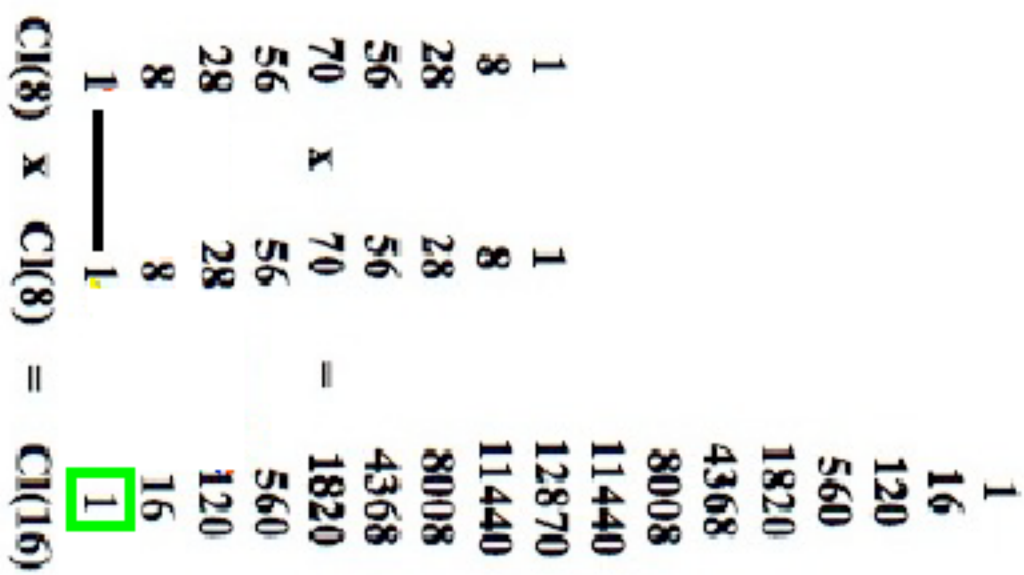
8-Periodicity of Real Clifford Algebras is shown by the Division Algebra sequence
R for Cl(0)
C for Cl(1) Q for Cl(2) Q+Q for Cl(3) Q for Cl(4)
C for Cl(5) R for Cl(6) R+R for Cl(7)   R for Cl(8) = M(16,R)
so that
Cl(16) = tensor product Cl(8) x Cl(8) is the case N = 2
of Cl(8N) = Cl(8) x ...(N times tensor product)... x Cl(8).
The case Cl(16) = Cl(8) x Cl(8) graded structure is

				1
				16
				120
				560
				1820
				4368
				8008
				11440
1		1		12870
8		8		11440
28		28		8008
56		56		4368
70	x	70	=	1820
56		56		560
28		28		120
8		8		16
1		1		1
Cl(8)	x	Cl(8)	=	Cl(16)

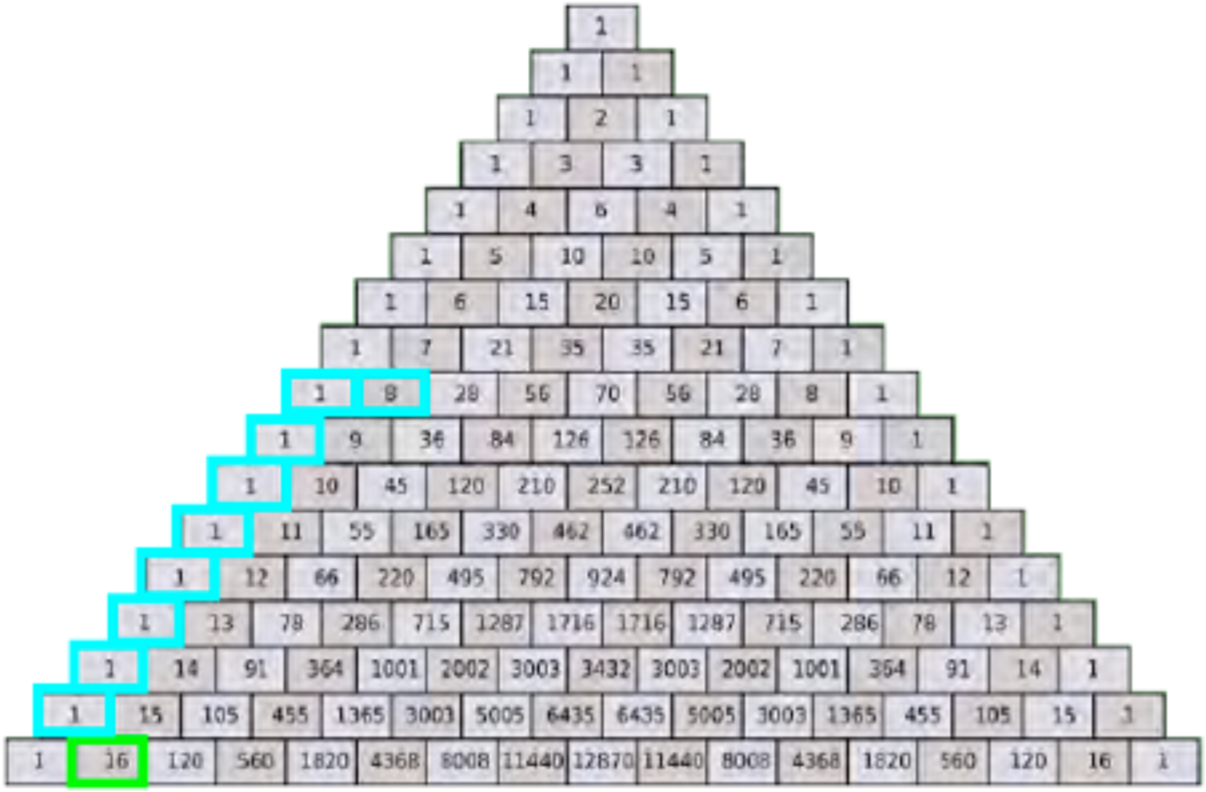
with grade 0 at the bottom, grade 16 of the top of Cl(16),
and grade 8 at the middle of Cl16) and the top of the Cl(8).
Here are more details for some illustrative examples:

Cl(16) Grade 0 has dimension 1 = 1x1

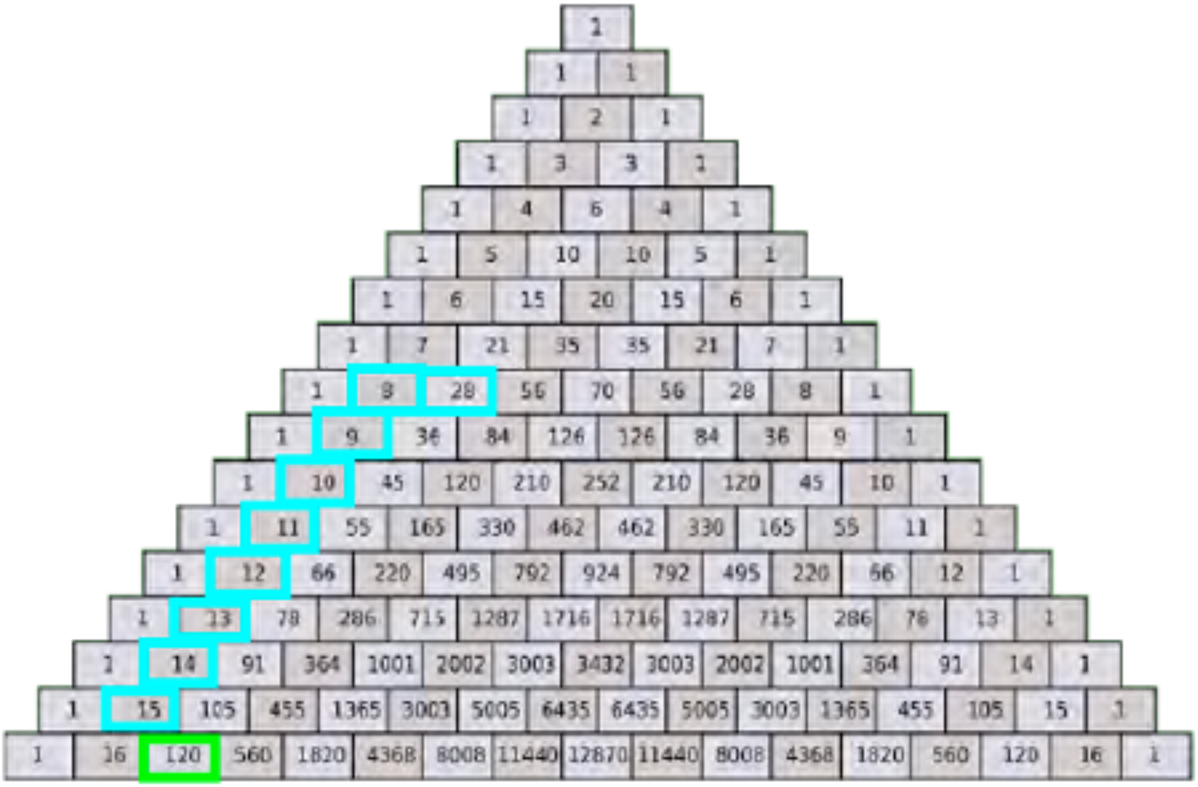




Cl(16) Grade 1 has dimension 16 = 8x1 + 1x8



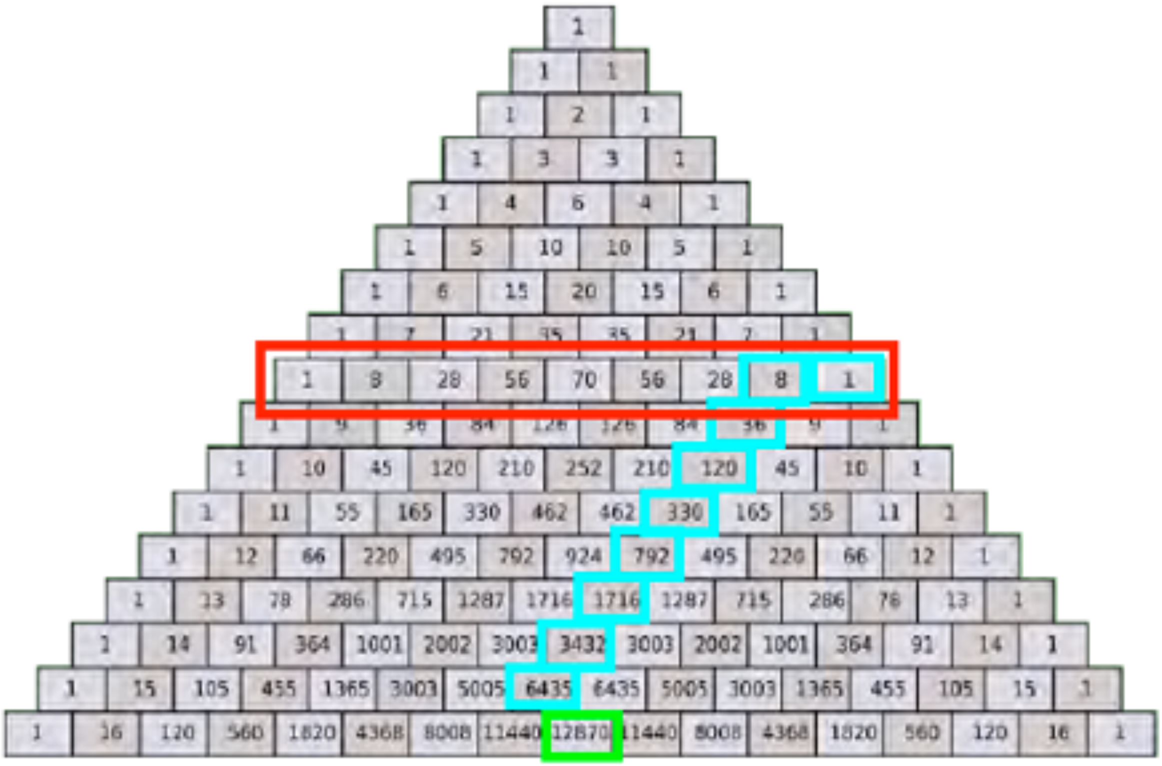
Cl(16) Grade 2 has dimension 120 = 8x8 + 1x28 + 28x1

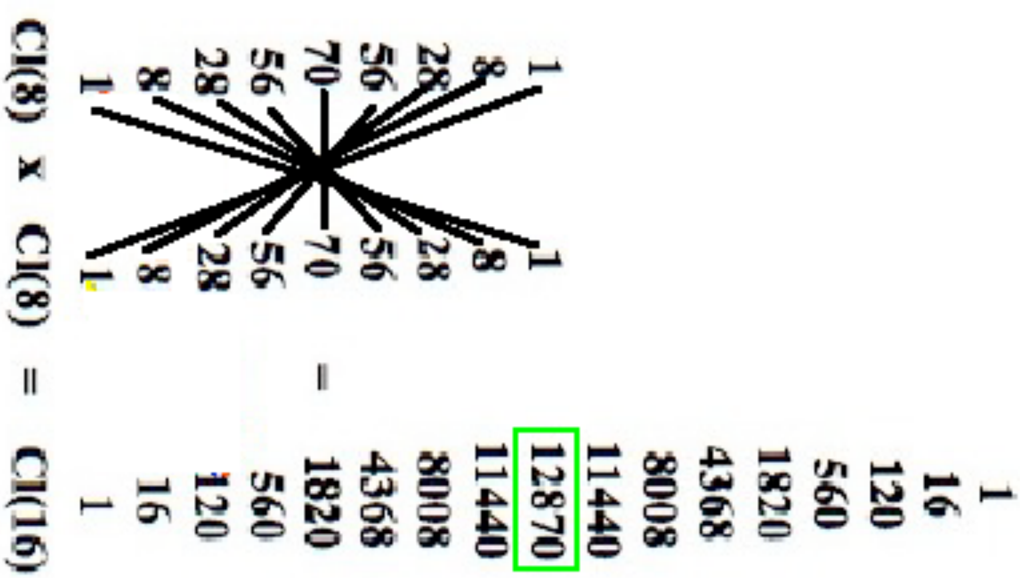


Cl(16) Grade 8 (its middle grade) has dimension

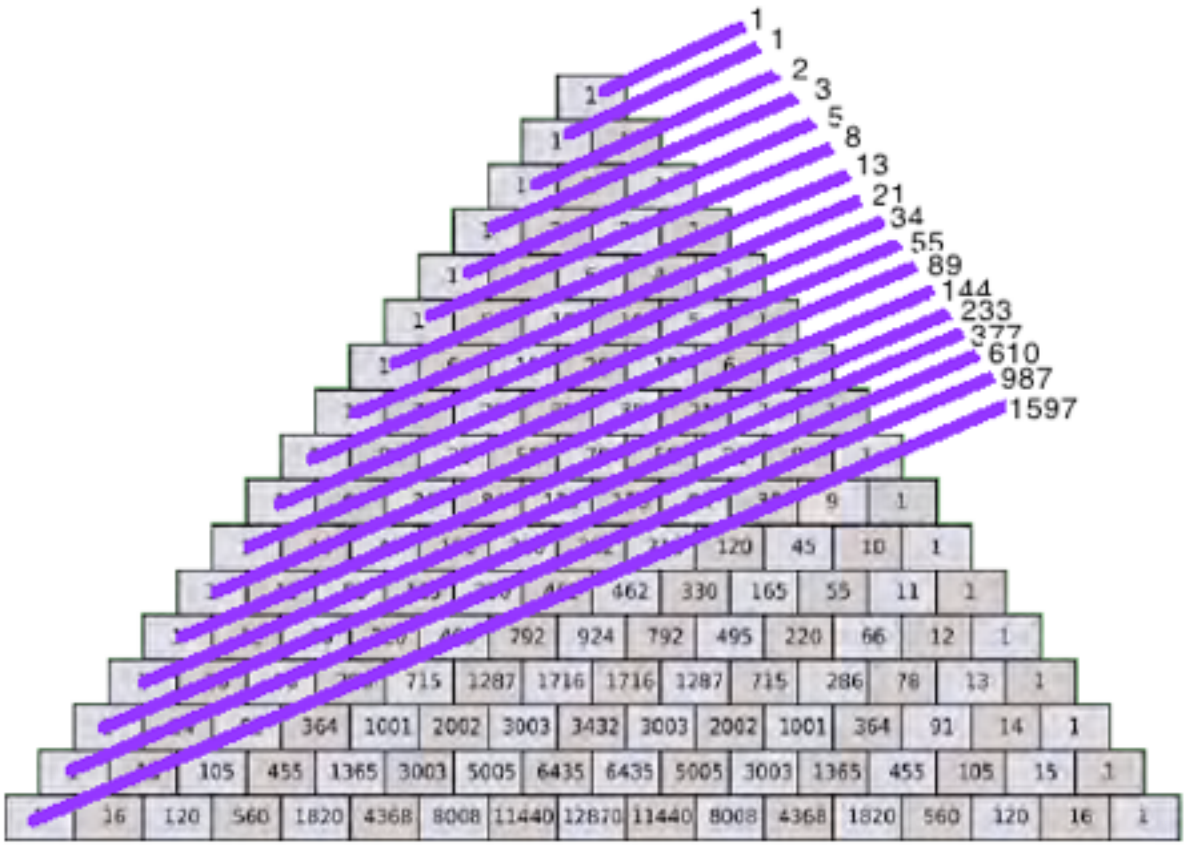
12,870 = 1x1 + 8x8 + 28x28 + 56x56 + 70x70 + 56x56 + 28x28 + 8x8 + 1x1

which is the sum of the squares of the grades of Cl(8)





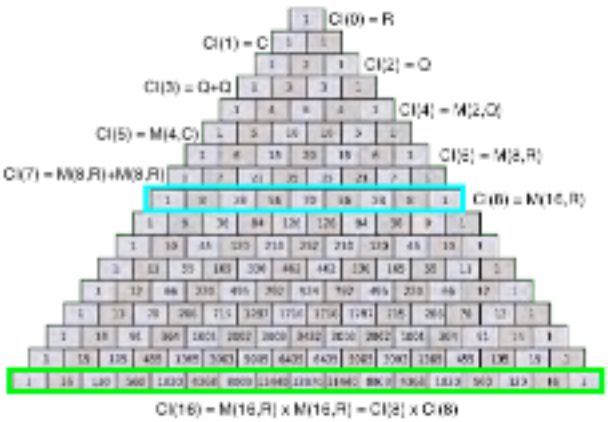
The Meru Prastara also encodes Fibonacci numbers and therefore related processes:



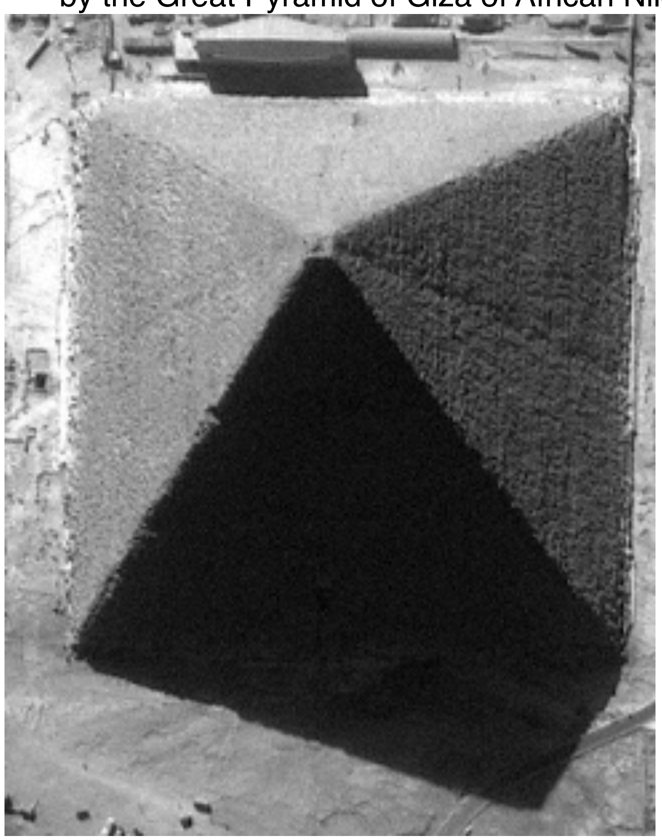
According to Wikipedia:
Prime Fibonacci numbers shown above are 1, 2, 3, 5, 13, 89, 233, and 1597.
Starting with 5, every second Fibonacci number is the length of the hypotenuse of a right triangle with integer sides, or in other words, the largest number in a Pythagorean triple. The length of the longer leg of this triangle is equal to the sum of the three sides of the preceding triangle in this series of triangles, and the shorter leg is equal to the difference between the preceding bypassed Fibonacci number and the shorter leg of the preceding triangle. The first triangle in this series has sides of length 5, 4, and 3. Skipping 8, the next triangle has sides of length 13, 12 (5 + 4 + 3), and 5 (8 – 3). Skipping 21, the next triangle has sides of length 34, 30 (13 + 12 + 5), and 16 (21 – 5).
The Fibonacci numbers occur as the ratio of successive convergents of the continued fraction for the Golden Ratio ϕ

$$\varphi = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \ddots}}}$$

The Golden Ratio structure and pyramidal shape show that the representation of Ancient African IFA by the Meru Prastara of African Migrants to India 50,000 years ago (about 4 Vedic Semi-Precession periods of 4800 + 3600 + 2400 + 1200 = 12,000 years with the 4th 1200 year Dark Iron Kali Yuga ending about 2012 / 2013 to be followed by a Bright Golden 4800 year Satya Yuga of reconciliation of technology and spirituality)



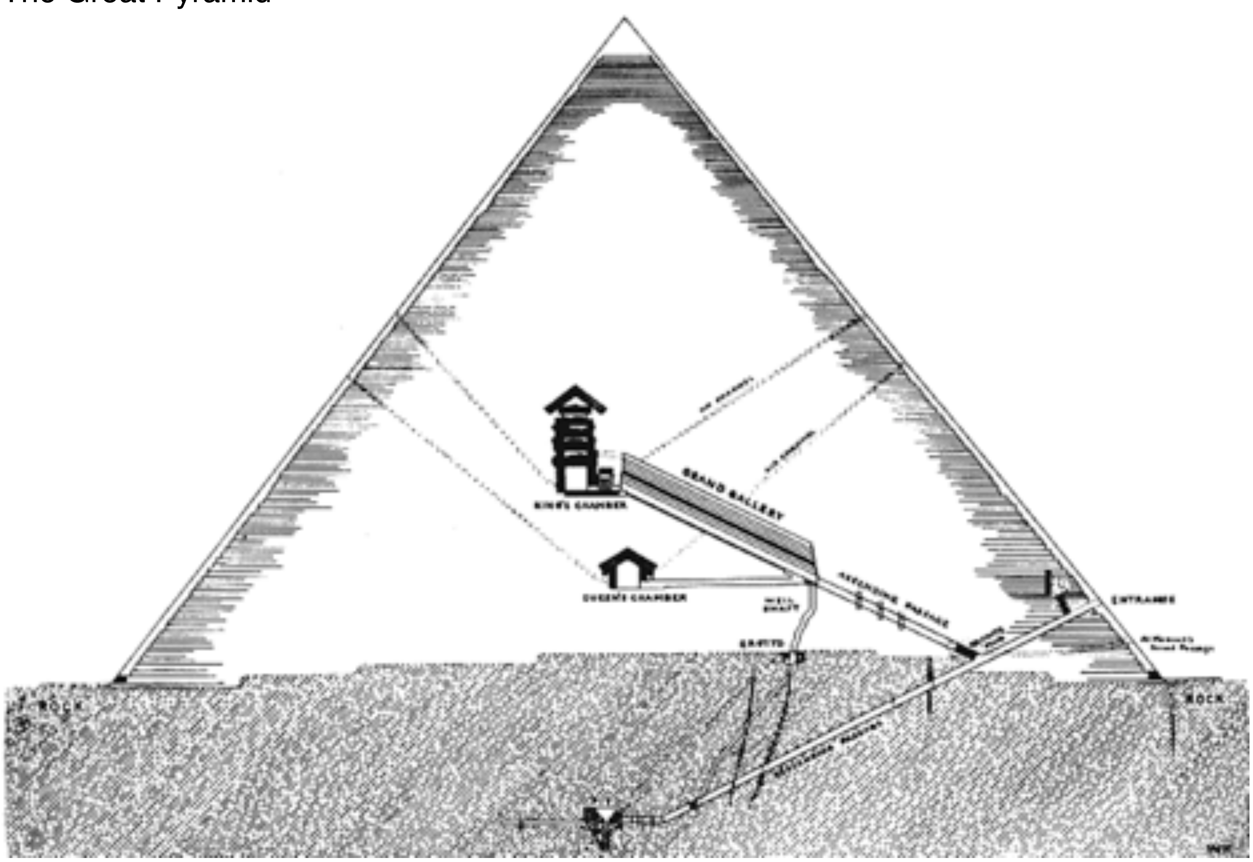
corresponds to its representation
by the Great Pyramid of Giza of African Nile Migrants of 40,000 years ago



The migration from Africa to the Mouth of the Nile about 40,000 years ago can be seen in terms of the chronology of the Egyptian historian Manetho who lived about 2,000 years ago in which the African Nile migration would be seen as occurring about 36,525 years ago, when the Geminga SuperNova Shock Wave hit Earth and when began an Ice Age Civilization as the Cro-Magnons from Africa entered Europe and displacd the Neanderthals.

As to the next 24,925 years (approximately Earth precession period of 26,000 years):
13,900 years, Manetho's Rule of Gods on Earth when the Great Pyramid and Sphinx might have been built, would have lasted until about 22,625 years ago at the Last Glacial Maximum on Earth.
The following 11,025 years, Manetho's Rule by Demigods and Spirits of the Dead, would have lasted until about 11,600 years ago at the Younger Dryas Cold Snap when the Vela X SuperNova was seen on Earth, the Taurid/Encke Comet fragmented,
and
a very sudden (50 years or so) Warming Event ended the Ice Age
and
began the Holocene Age warm climate with glacial retreat
and Manetho's Rule of Mortal Humans with increasing Technology but less Spirituality.

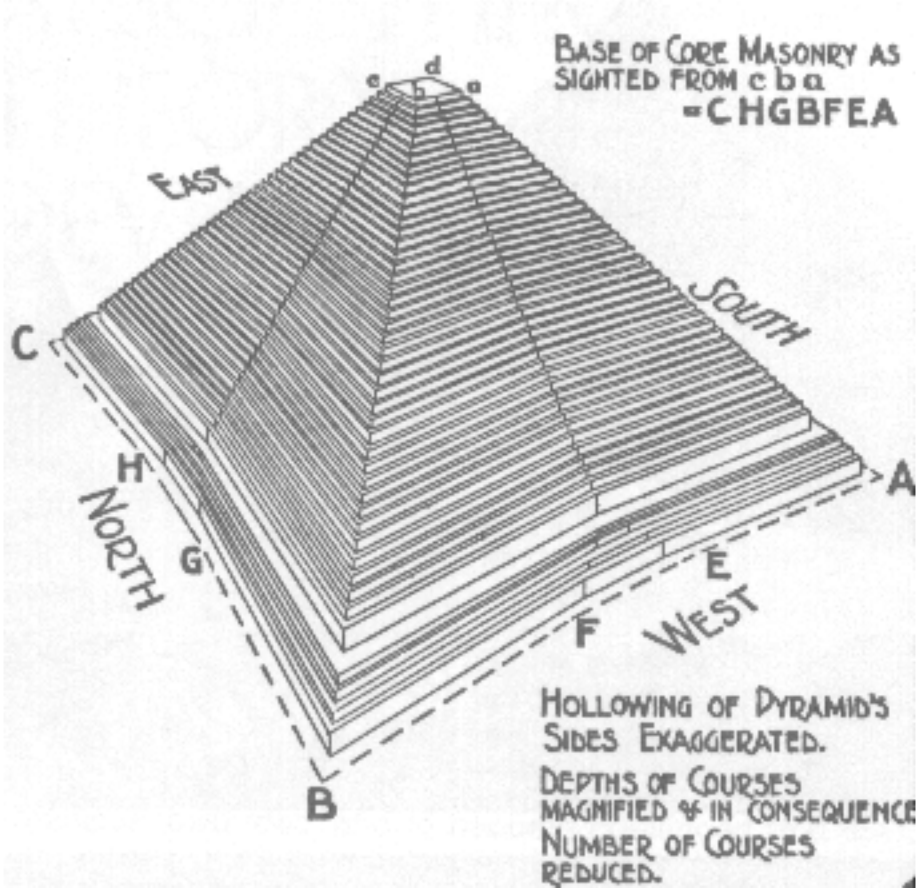
The Great Pyramid



(image due to David Davidson)

is built of 203 layers (courses) plus a now-missing capstone
for a total number of layers = 204 = 64 + 49 + 36 + 25 + 16 + 9 + 4 + 1 = SPN(8)
the Square Pyramidal Number of order 8.

Peter Tompkins in his book "Secrets of the Great Pyramid" said:
Sir Flinders Petrie noted a ... hollowing of the core masonry at the central portion of each face of the pyramid ... [in courses 1 through 168]

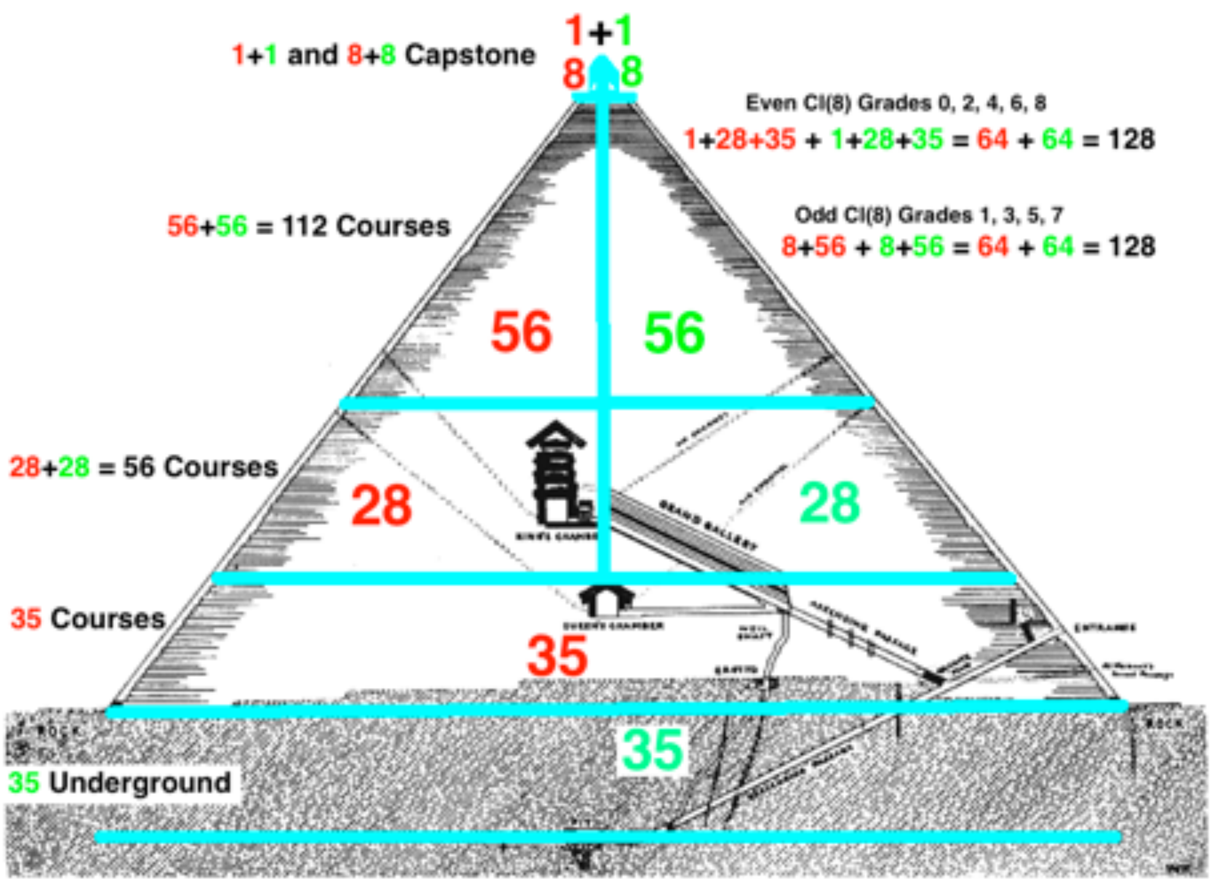


... Petrie found no evidence of hollowing along the lower-level casing stones ... [in courses 169 through 203]...".
From the top of the pyramid (course 0 in my notation) down through course 168 the mid-line of the "hollowing" splits the 4 faces in to 4+4 = 8 faces.
The mid-line corresponds to the duality splitting of the Meru Clifford Algebras whereby the middle-grade parts of Cl(N) for even N are split into two dual halves.
According to Flinders Petrie (www.ronaldbirdsall.com/gizeh/) some courses in the mid-line region down through course 168 are distinguished by thickness.
Some of them have interesting mathematical correspondences:

0 - Missing Capstone
1 - Real Numbers
2 - Complex Numbers
4 - Quaternions
8 - Octonions
24 - Leech Lattice and SPN(24) = 4900 = 70^2 is the only Square Pyramidal Number that is itself a square. In 24 dimensions it is analagous to the 2-dimensional 3-4-5 triangle whose sides square to 9+16 = 25 that gives the slope of the Second Pyramid.
168 - PSL(2,7) = SL(3,2) of Octonionic Fano Projective Plane

The top course 169 of the 35 courses 169 through 203 sitting on the ground level corresponds to the top of the Queen's Chamber

The Subterranean Pit is as deep below ground level as Queen's Chamber is above it so the Subterranean Pit depth equivalent to 35 courses is dual to the Queen's Chamber height of 35 courses just as the 70 mid-grade grade 4 elements of the Cl(8) Clifford Algebra are (35+35) 35 elements plus 35 elements, dual to each other.
(3+3) of (35+35) are the middle components of the 1+3+3+1 Higgs Primitive Idempotent whose 1+1 scalar+pseudoscalar compontents are the 1+1 of the Capstone.



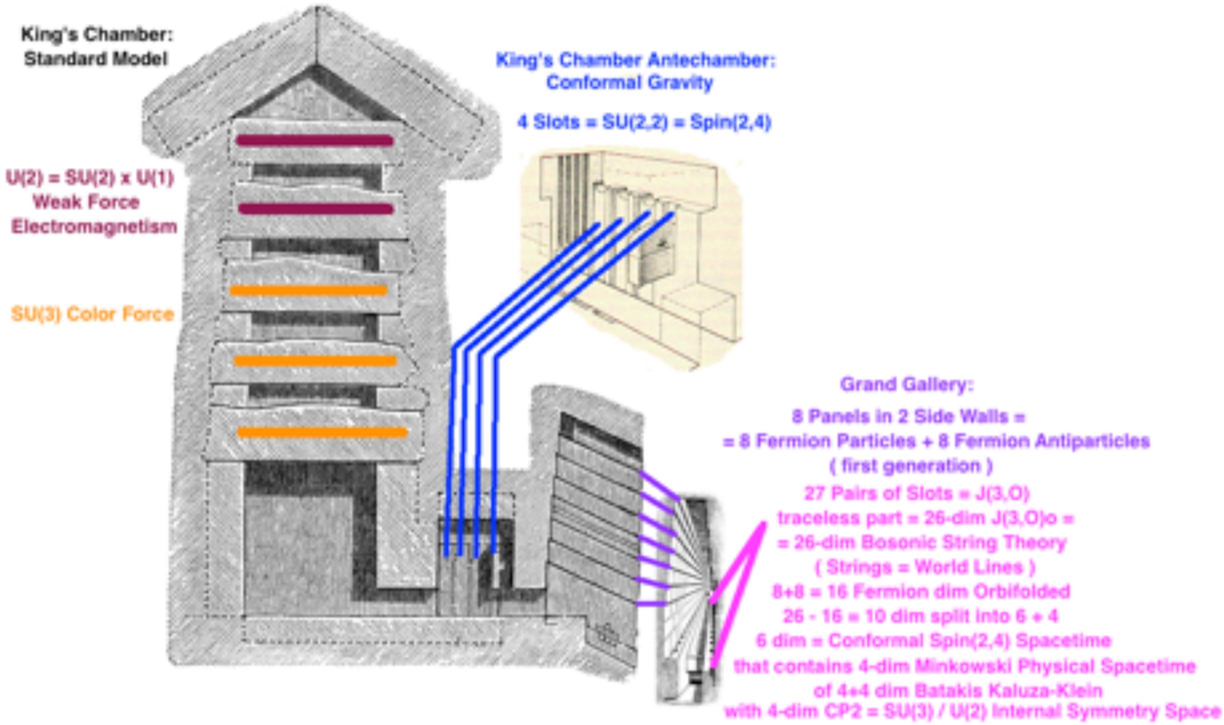
1 + 8 + 28 + 56 + (35 + 35) + 56 + 28 + 8 + 1

(image adapted from David Davidson image -
for a larger version of this image go to tony5m17h.net/GreatPyrCl8.png
or valdostamuseum.com/hamsmith/GreatPyrCl8.png)

The South-facing Shaft from the King's Chamber is at an angle of about 45 degrees looking at the Plane of our Milky Way Galaxy (Standard Model matter).
The North-facing Shaft from the King's Chamber is at an angle of about 32 degrees looking at the North Pole (Earth Rotation Gravity).

8+8 of the Capstone represent the 8 Fermion Particles and 8 Fermion Antiparticles of the first generation, with respect to the Time Component of 8-dim Spacetime. 56+56 of the top 112 Courses represent the 7 Spatial Components of the Fermions.

28+28 of the next 56 Courses represent the Standard Model and Conformal Gravity

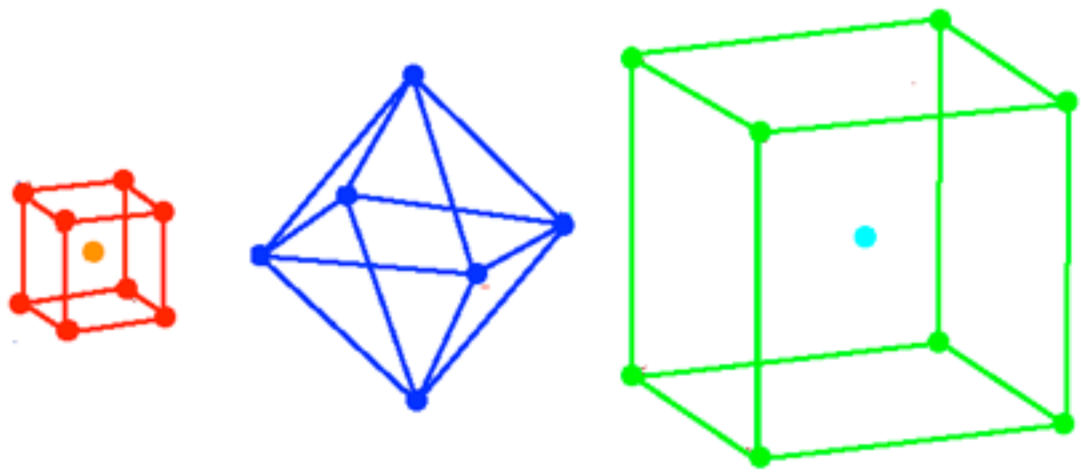


(image adapted from etc.usf.edu and Graham Hancock, Fingerprints of the Gods (Crown 1995) and gatesofegypt.blogspot - for a larger version of this image go to tony5m17h.net/GPyrStdMConfG.png or valdostamuseum.com/hamsmith/GPyrStdMConfG.png)

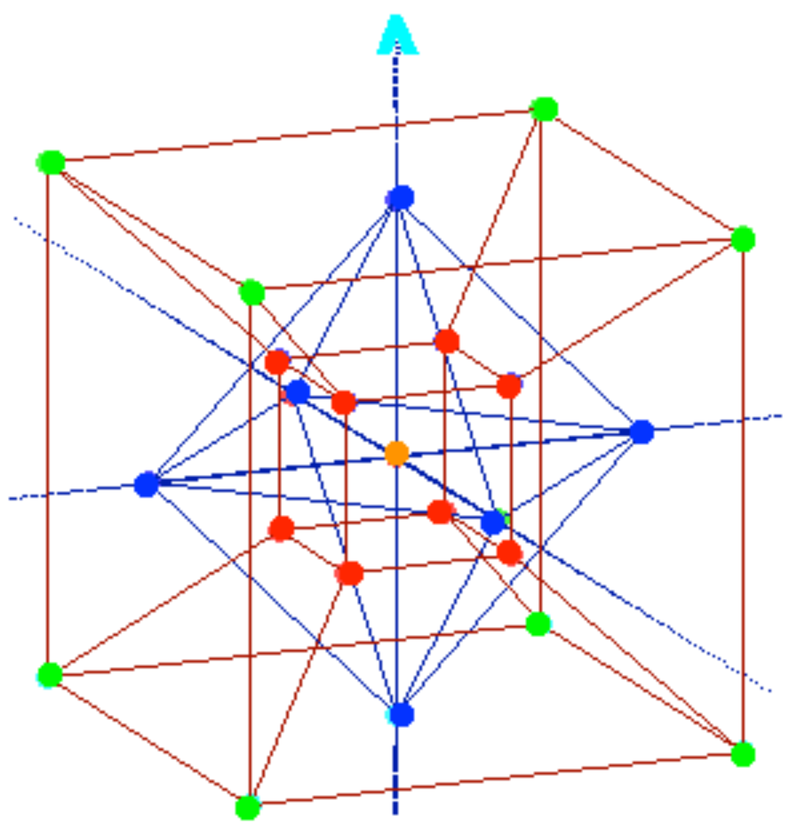
The South-facing and North-facing Shafts from the Queen's Chamber are angled at about 39 degrees, roughly the right-angle-complements of the Golden Angles of the faces of the Great Pryamid with the ground. They do not extend to the outer surface of the Great Pyramid, but only connect the Queen's Chamber Higgs with the Kings's Chamber and Antechamber of the Standard Model plus Conformal Gravity.

The Grand Gallery 26-dim Bosonic String Theory (Strings = World Lines) produces an effective Bohm-type Quantum Potential that provides the Superposition Separation Effect of Penrose-Hameroff Quantum Consciousness. Effectively, the Grand Gallery is the Loom the Weaves World-Line Histories (past, present, and future) into our Tapestry of Reality.

The Capstone 8 + 8 and 1 + 1
combined with
the 5 vertices of the Great Pyramid
plus a 6th underground vertex antipodal to the Capstone

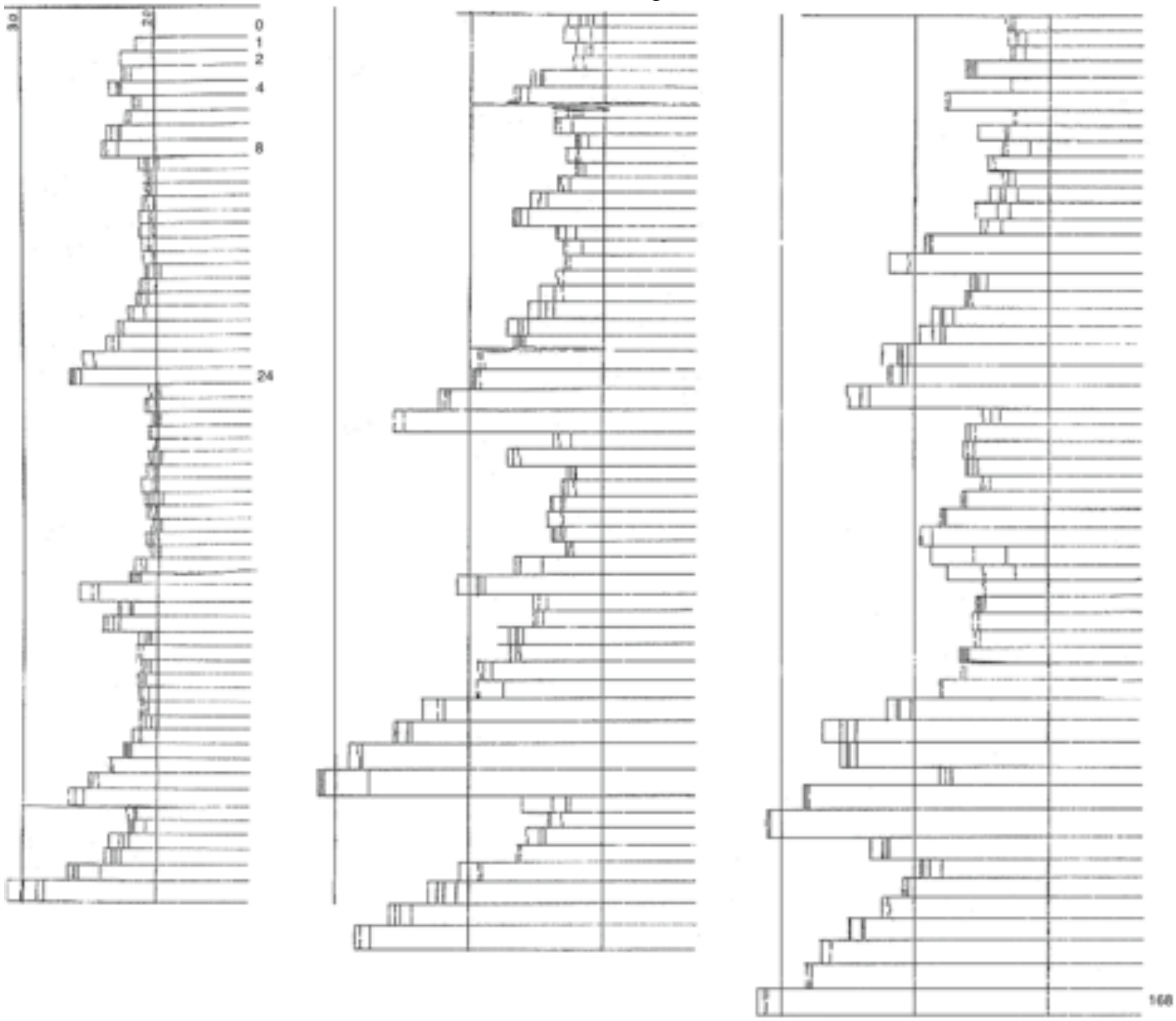


form a 24-cell (image with one vertex at infinity)

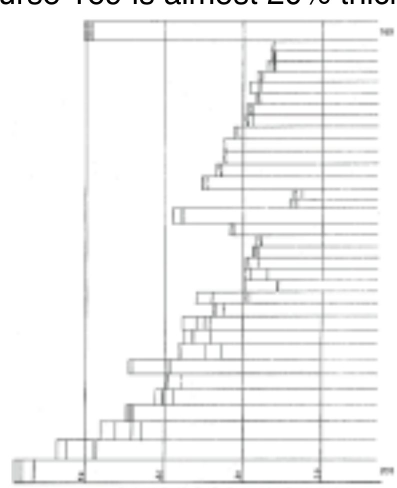


(image adapted from Frans Marcelis members.home.nl/fg.marcelis/)

Here is Flinders Petrie's chart of courses 1 through 168:



Here is Flinders Petrie's chart of the base courses 169 through 203:
(smaller scale - course 169 is almost 20% thicker than course 168)



The Vedic Civilization not only preserved ideas in Meru Prastara triangle structures and Sanskrit writing and prosody but also through games whose structure effectively outline basic ideas.
Perhaps the most important such Vedic game is Pachisi.

In "The Indian Games of Pachisi, Chaupar,and Chausar" (Expedition Spring 1964, 32-35) W. Norman Brown said: "... The Rig-Veda ... has references to the use of dice ... cowries, which are used in pachisi, are ... as old in India as the Harappa civilization ... At Mohenjo-daro in the Indus Valley, a portion of a triple-rowed gaming-diagram on brick was recovered, dating perhaps from the last part of the third millennium B.C. ...".

In "Shells as Evidence of the Migrations of Early Culture" (Manchester University Press 1917) J. Wilfrid Jackson said: "... In India the money-cowry seem to have been regarded with special favour for amuletic and currency purposes from very early times ... a cowry game ... is ... related to the Hindu game of Pachisi, also played with cowries. The shells are thrown as dice ... Games like Pachisi, in which cowries are used as dice, are known in the Maldive Islands ... Among the Nagas ... a warrior, having slain an enemy, had the privilege of wearing a kilt decorated with cowry-shells ... A similar custom ... is to be found in East Central Africa, where the Djibba tribe wear ... cowries ...".

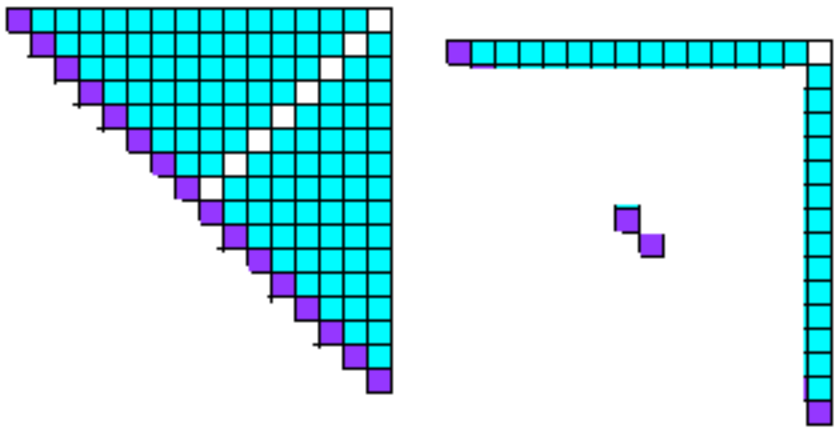
Cowries provide evidence not only for an early and strong Africa-India connection, but also for the world-wide reach of trade and ideas in ancient times. For example, Jackson also said: "... The money-cowry ... is, and has been for centuries, a sacred object among the Ojibwa and Menomini Indians of North America, and is employed in initiation ceremonies of the Grand Medicine Society. The use of this particular cowry by these Indians is of peculiar interest; in the first place, owing to it being alien to the American continent, and in the second place, in view of its intimate association with so many remarkable ... beliefs and practices in different parts of the Old World. ...". Since Lake Superior is the primary source on Earth of native copper, the Indians there had world-wide trade in ancient times even preceding the Bronze Age.

To preserve their heritage of the African IFA Information System,
priests of India not only wrote down the Sanskrit Rig Veda
(tony5m17h.net/RgVeda.html)
but
they also developed the game Pachisi
to keep the dynamics of IFA in popular culture.

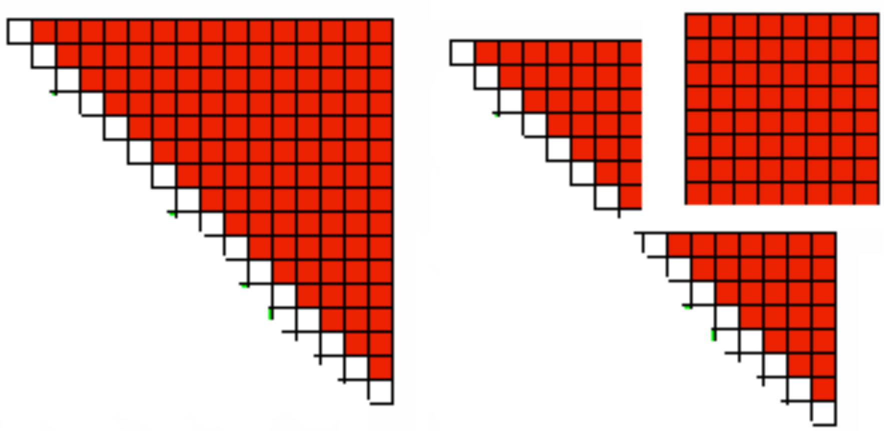
Here is how the structure of the IFA Information System has been simplified for transmission to Pachisi:

First, due to the diagonal-reflection symmetry of Antisymmetric and Symmetric matrices, only the upper triangular parts of the matrices need to be preserved:

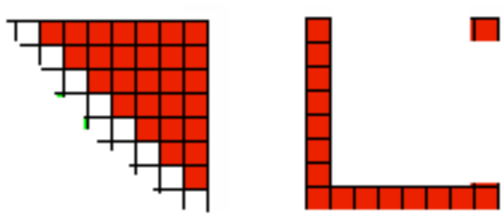
The Symmetric part was simplified by ignoring the part of Cl(8) not in E8 and then using only 32 entries (from outer shell and diagonal) of those 128 entries



The Antisymmetric part was first cut into 3 sections

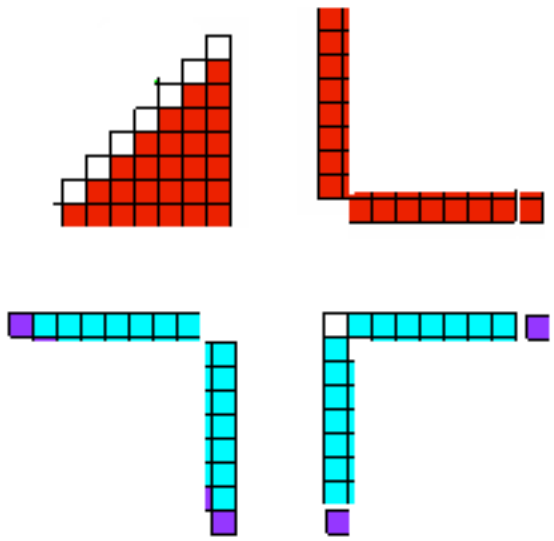


two similar triangular each with 28 entries and one square with 64 entries.
Using only one of the two similar 28-entry triangles plus 16 from the square (outer shell)

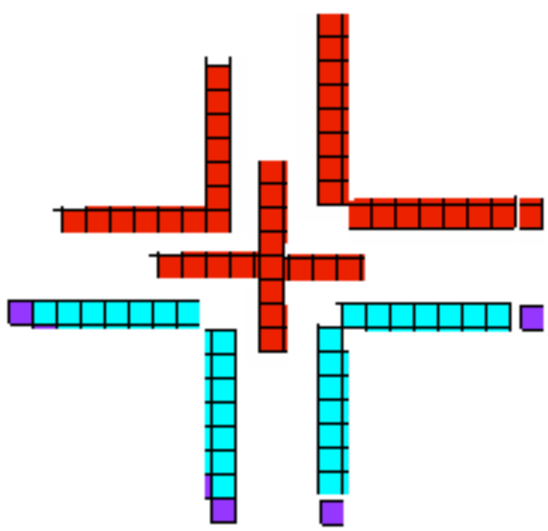


reduced the 120 Antisymmetric entries to 28 + (64-49) = 28+16 = 44 entries

thus reducing 248-dim E8 to 28 + 16 + 32 = 76 entries.
Since most of the IFA E8 structures are outer boundaries of square regions
it is natural to construct Pachisi as a boundary-progession board game
so the 30-entry Symmetric outer shell is broken into two parts which,
when added to the 28-entry and 15-entry Antisymmetric parts,
naturally fit together in this configuration

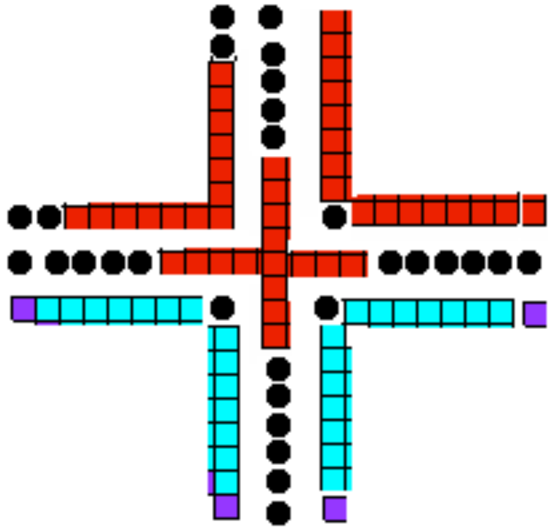


with 3 quadrants that look like a boundary-progression board game
but with one triangular quadrant that looks out of place. To make the board look more nearly consistent, move the interior 15 elements of the triangle to the interior of the board to get



with 8+8+8+8+8+8+7+6 = 61 outer plus 15 inner = 76 entries

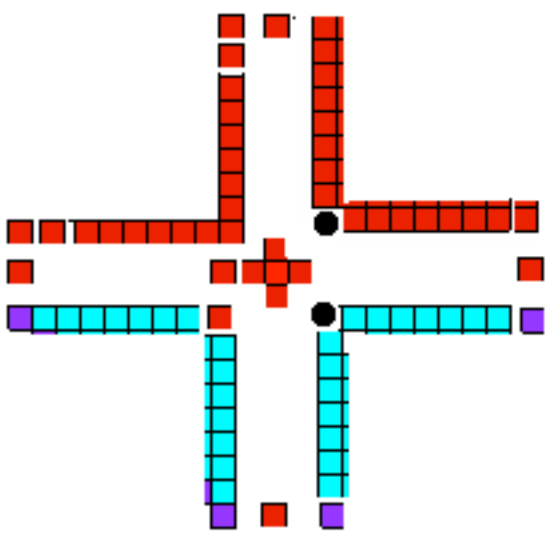
In order to fill out the Pachisi board 29 more entries are needed as filler



to get the total of 105 entries on the full game board for Pachisi.

RigVeda-Pachisi to Tarot

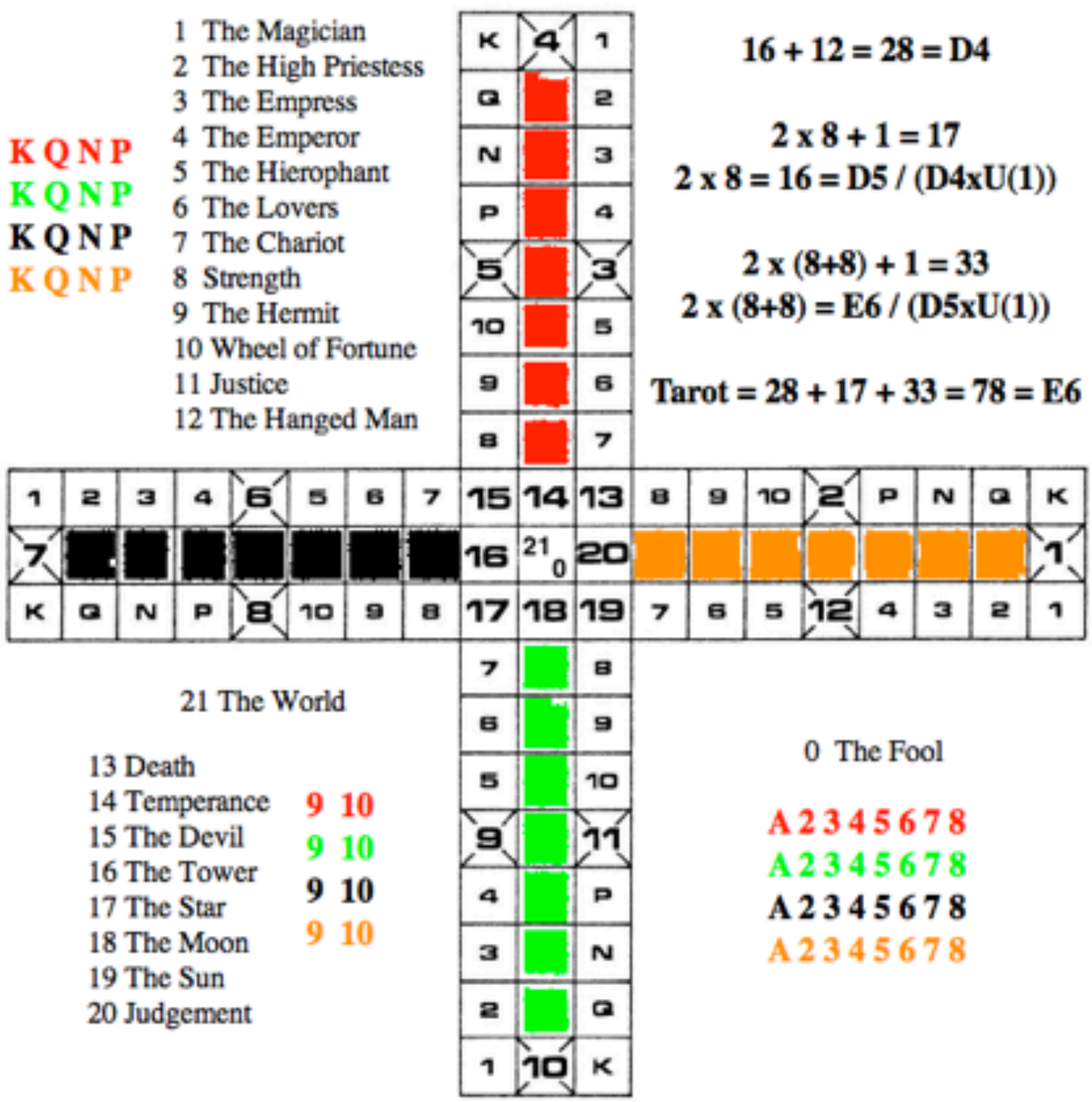
Tarot was developed from Pachisi by getting rid of 26 filler entries
and using the 15 inner entries: 4 to complete the exterior arms of D4;
4 to bridge between arms; 1 as an inner corner; and 6 in central configuration,
with the remaining 2 filler entries used for the U(1) of D5 / D4xU(1) and E6 / D5xU(1)
The red entry at the corner of the left-side blue pair of arms corresponds to
the Antisymmetric U(1) in the U(2,2) = U(1)xSpin(2,4) subalgebra of the D4 which
physically represents the propagator phase of Fermions from the Symmetric sector.



Tarot has 105 - 27 = 78 cards, corresponding to the 78-dim E6 Lie Algebra.

Stephen E. Franklin in "Origins of the Tarot Deck" (www.lordbalto.com) said:
"... Games are at their most basic level symbol systems not unlike simplified languages ... Pachisi may ... be thought of as inscribed in a 19x19 square ... the outer rim of which has been compressed around the central cross ...
The Tarot must ... have been created between 654 and 403 B.C. and the probability is high that it first appeared sometime very near the year 540 B.C. ...
the Tarot was not invented to play card games but survived ... in the same twilight zone ... as Latin, Old Church Slavonic and biblical Hebrew ...
The similarity of ... Tarot ... court cards to ... four-handed proto-chess ... and the resemblance between the four suits and the four varnas or classes of Hindu society, which appear at least as early as the Rigveda, all point to an Indian origin ... transported to the West ... by the Arabs or the Gypsies ...".

(image modified version of figure from Franklin's article)



Tarot E6 Lie Algebra structure is used in Realistic Physics Model construction.

(tony5m17h.net/stringbraneStdModel.html)

(in this paper I am oversimplifying many things, such as by ignoring signature)

The 16 KQNP correspond to a U(4) Conformal Lie Algebra for Gravity

K Q N P
K Q N P
K Q N P
K Q N P

These 12 cards correspond to a symmetric space Spin(8) / U(4) for the Standard Model Gauge Groups

1 The Magician
2 The High Priestess
3 The Empress
4 The Emperor
5 The Hierophant
6 The Lovers
7 The Chariot
8 Strength
9 The Hermit
10 Wheel of Fortune
11 Justice
12 The Hanged Man

which can be thought of as the completion of Spin(8) from a foundation of U(4)
so that
taken together those 16+12 = 28 cards correspond to the D4 Lie Algebra Spin(8)
which
is the grade-2 bivector part of the IFA Cl(8) Clifford Algebra with graded structure

256 = (8+8)x(8+8) = 1 + 8 + **28** + 56 + 70 + 56 + 28 + 8 + 1

These 1 + 8 + 8 = 17 cards correspond to two copies of 8-dim spacetime

21 The World

13 Death	
14 Temperance	9 10
15 The Devil	9 10
16 The Tower	9 10
17 The Star	9 10
18 The Moon	
19 The Sun	
20 Judgement	

plus
one card for the Complex U(1) needed to glue them together into a Complex spacetime
that is 8-Complex-dimensional (16-real-dimensional)
corresponding to a symmetric space Spin(10) / Spin(8) x U(1)
which can be thought of as the completion of Spin(10) from a foundation of Spin(8)
so that
taken together all 28 + 17 = 45 cards correspond to the D5 Lie Algebra Spin(10)
which
appears in the IFA Cl(8) Clifford Algebra with graded structure

256 = (8+8)x(8+8) = **1** + **8** + **28** + 56 + 70 + 56 + 28 + **8** + 1

These 1 + (8+8) + (8+8) = 33 correspond to two copies of (8+8)-dim spinors

(representing 8 fermion particles + 8 fermion antiparticles)

0 The Fool

A 2 3 4 5 6 7 8
A 2 3 4 5 6 7 8
A 2 3 4 5 6 7 8
A 2 3 4 5 6 7 8

plus
one card for the Complex U(1) needed to glue them together into Complex spinors corresponding to a symmetric space E6 / Spin(10) x U(1) which can be thought of as the completion of E6 from a foundation of Spin(10)

so that

all 28 + 17 + 33 = 78 Tarot cards correspond to the E6 Lie Algebra

which appears in the IFA Cl(8) Clifford Algebra

as spinor structure plus a U(1)

256 = (**8+8**)x(**8+8**) = 1 + 8 + **28** + 56 + 70 + 56 + 28 + 8 + **1**

Tarot to Llull

Ramon Llull (1232-1316) of Mallorca lived in a time and place of
a unique confluence of Islamic, Christian, and Jewish mystical ideas
on a Mediterranean island between Iberia and Africa
so
he was exposed to ideas including
Islamic 16-element Ilm al Raml derived from African 256-element IFA,
Christian-Crusader Troubadour 78-element Tarot,
Jewish Urim v'Thummim system revealed to Moses for decoding the 72 letters
on the 12 stones of the Breastplate of Judgment,
and
he was able to travel easily to Africa, the home of 256-element IFA.

According to Anthony Bonner's book Doctor Illuminatus (Princeton 1993):
"... In the history of Western mysticism,
there is nothing quite like ...[Llull's Quaternary Phase (1274-89)]...
with its curious blend of Troubadour, Franciscan, and Islamic influences,
mixed with Llull's own special outlook based on the Art ...".

"Llull's own special outlook" may have been to see that

72 letters of the Urim v'Thummim Breastplate are contained in the 78-element Tarot

the 78-element Tarot fits inside the 256-element IFA D4 Real Clifford Algebra as

$$1 + 8 + 28 + ... + 8 + 1$$

fits inside

$$256 = 1 + 8 + 28 + 56 + 70 + 56 + 28 + 8 + 1$$

and as

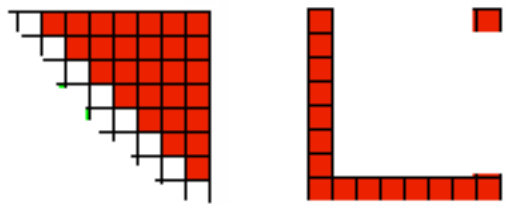
$$(8+8) + (8+8)$$

fits inside

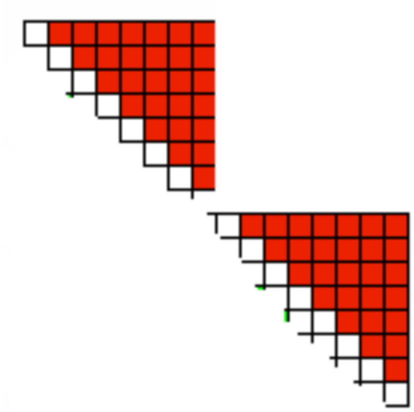
$$256 = (8+8)x(8+8)$$

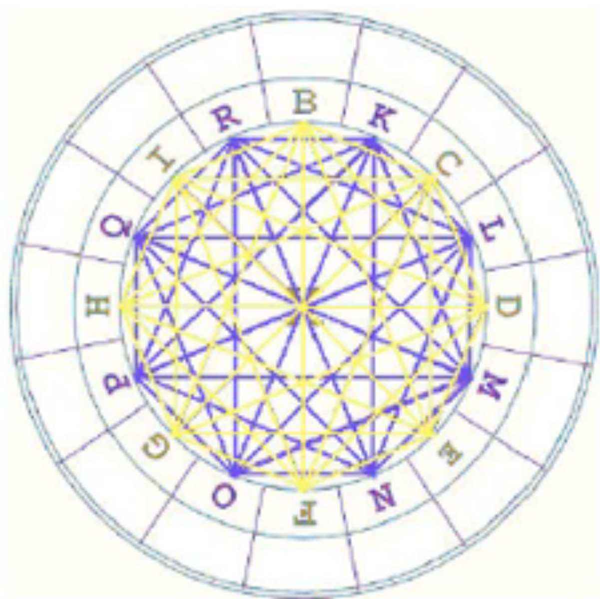
Troubadours propagated songs, poetry, and games such as Tarot.
Ramon Llull (1232-1316) of Mallorca studied the Islamic 16-element Ilm al Raml,
the Troubadour 78-element Tarot,
and the 256-element IFA
and
found a structure that he summarized in Wheel Diagrams with 16 vertices
connected to each other by lines such as (some images adapted from lullianarts.net web site)

Of **the 120-dim Antisymmetric Part of 16x16 Real matrices**
the 78-dim Tarot contains only a 16-dim partial boundary of its 64-dim U(8) square
and one 28-dim D4 triangle plus a single 1-dim U(1) from D5 / D4xU(1)



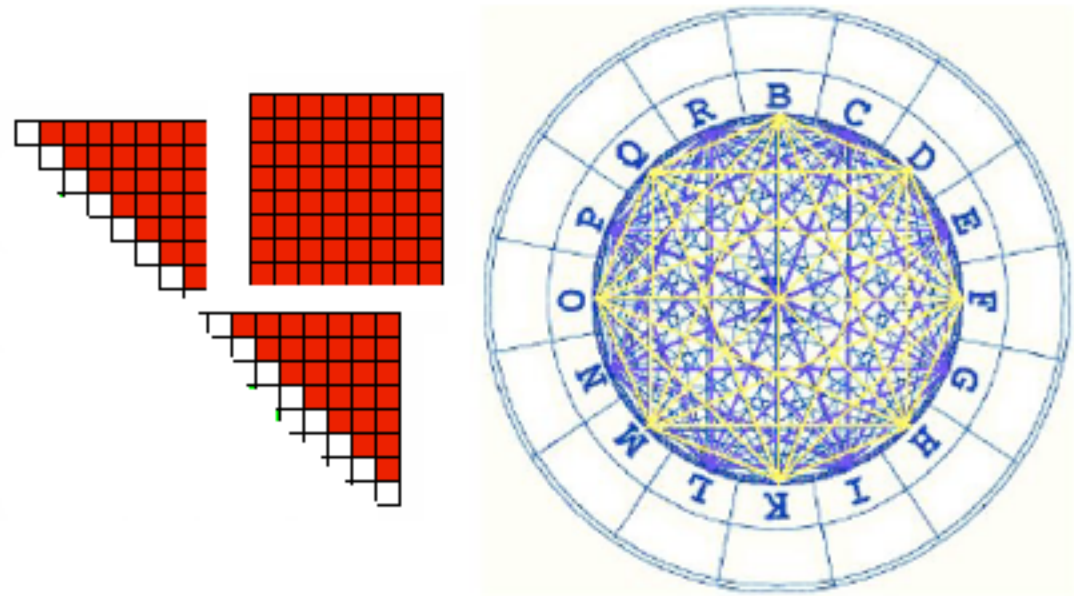
Llull expanded that 45-dim part of the Tarot to include the second D4 triangle
and to represent them in his Wheel Diagram X as two sets of 8 vertices,
for a total of 16 vertices, around the X-Wheel within which each set of 8 vertices
was connected with the other 7 of that set by 28 lines,
each line representing one generator of each of the two copies of 28-dim





The 28 gold lines represent the D4 containing U(2,2) that gives Conformal Gravity
and
the 28 purple lines represent second D4 containing the SU(3)
that when combined with Kaluza-Klein Internal Symmetry Space CP2 = SU(3) / U(2)
gives by the Batakis mechanism the Standard Model Gauge Groups SU(3)xSU(2)xU(1).

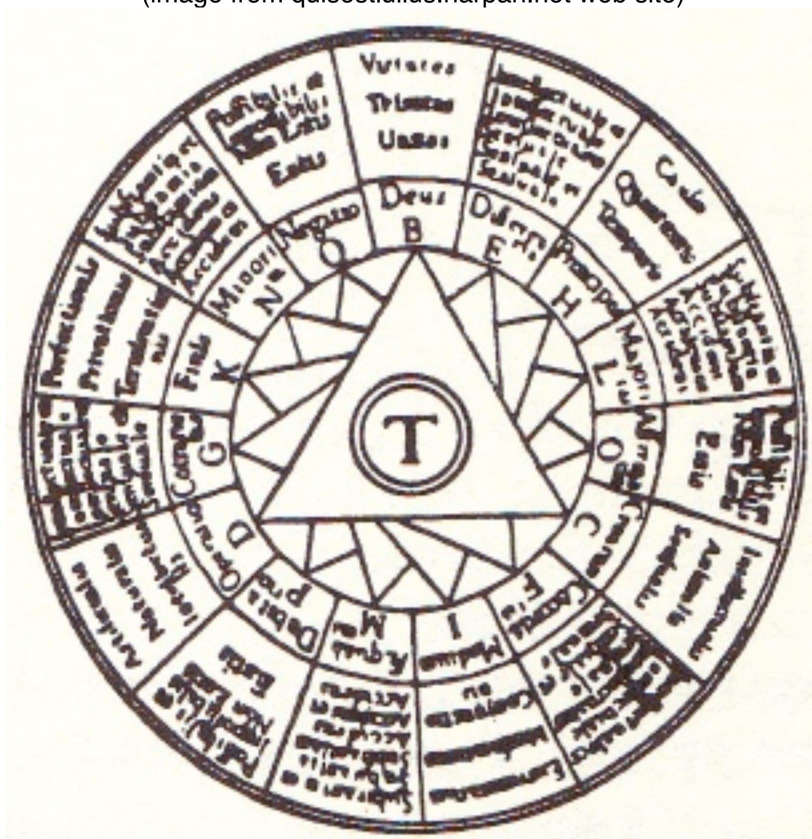
Llull further expanded the 16-dim square-partial-boundary to full square 64-dim size
by adding the 64 blue lines that connect a vertex of one D4 with a vertex of the other D4



Adding in 64 blue lines gives 28+28+64 = 120 lines of the Llullian A-'wheel
that represents the Spin(16) bivector Lie Algebra D8
of the Clifford Algebra Cl(16) = Cl(8) x Cl(8)

As to the details of the U(2,2) Conformal Gravity of the 28 Gold Lines,
Llull constructed a T-Wheel

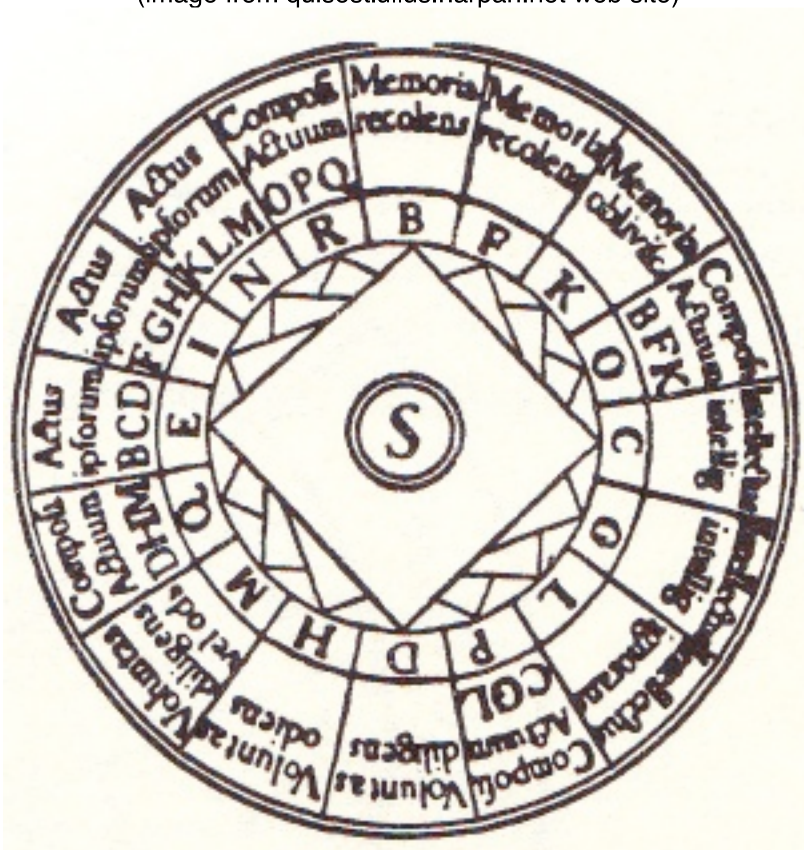
(image from quisestlullus.narpan.net web site)



The 3 vertices of the T-triangle in Llull's T-wheel correspond
to a 3-dimensional Cartan subalgebra of
the 15-dimensional Conformal Group SU(2,2) = Spin(2,4)
that lives in U(2,2) as U(2,2) = U(1) x U(2,2).
The other 4 triangles in Llull's T-wheel correspond
to the 12 vertices of the Cuboctahedron Root Vector Polytope
of the Conformal Group SU(2,2) = Spin(2,4).

As to the details of the Standard Model of the 28 Purple Lines,
Llull constructed an S-wheel

(image from quisestlullus.narpan.net web site)

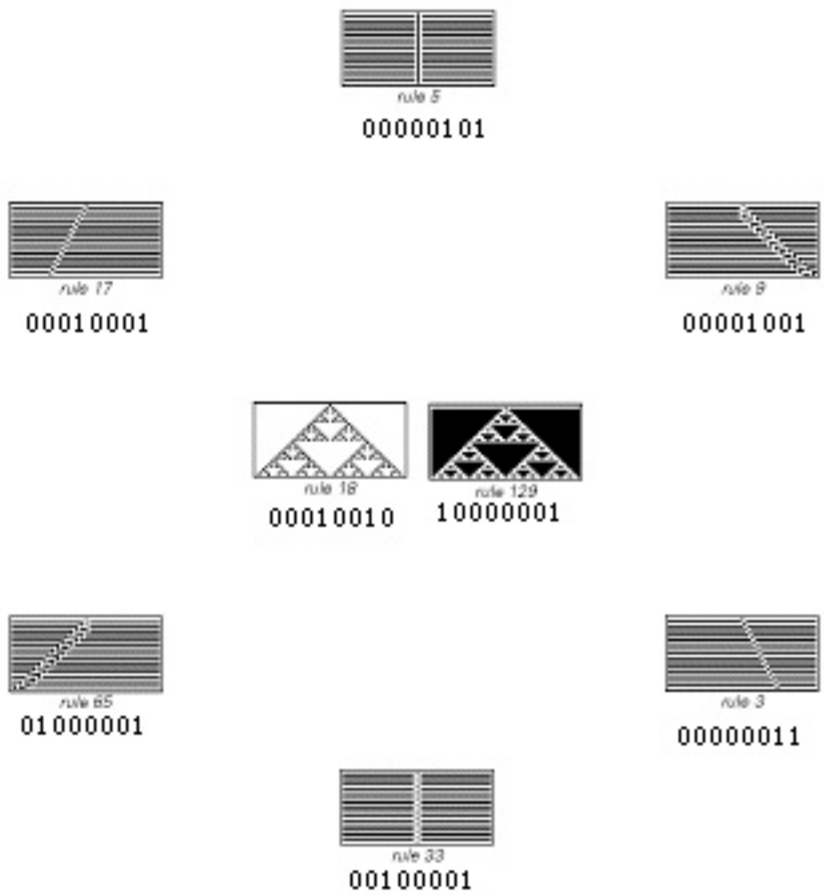


The 4 vertices of the S-square correspond to the 4-dimensional Quaternionic subspaces that emerge below the Planck energy to break Octonionic 8-dim Spacetime into (4+4)-dim Kaluza-Klein spacetime with 4-dim Minkowski Physical Spacetime plus 4-dim Internal Symmetry Space CP2 = SU(3) / SU(2)xU(1)

Two of the remaining 3 squares of Llull's S-wheel form the vertices of a cube

```
x-----x
|\    |\
| y-----y
| |   | |
x-|---x |
 \|    \|
  y-----y
```

Looking at the cube along a diagonal axis and projecting all 8 vertices onto a perpendicular plane



you see the Root Vector Diagram of SU(3) and its 8 gluons. (here I have identified the vertices with their corresponding Cellular Automata using a correspondence between the 256 Elementary Cellular Automata and the 256 Odu of IFA - for details see vixra.org/pdf/0907.0040v3.pdf).

Since each gluon links 4-dim Physical Spacetime to color Internal Symmetry Space, the gauge group SU(3) acts globally on CP2 Internal Symmetry Space, as can be seen by the fibration CP2 = SU(3) / U(2)

The third of the remaining squares, that is the final square, corresponds to the 3 SU(2) weak bosons and the U(1) electromagnetic photon.
Since SU(2)xU(1) = U(2), and since CP2 = SU(3) / U(2),
they act locally on CP2 Internal Symmetry Space.

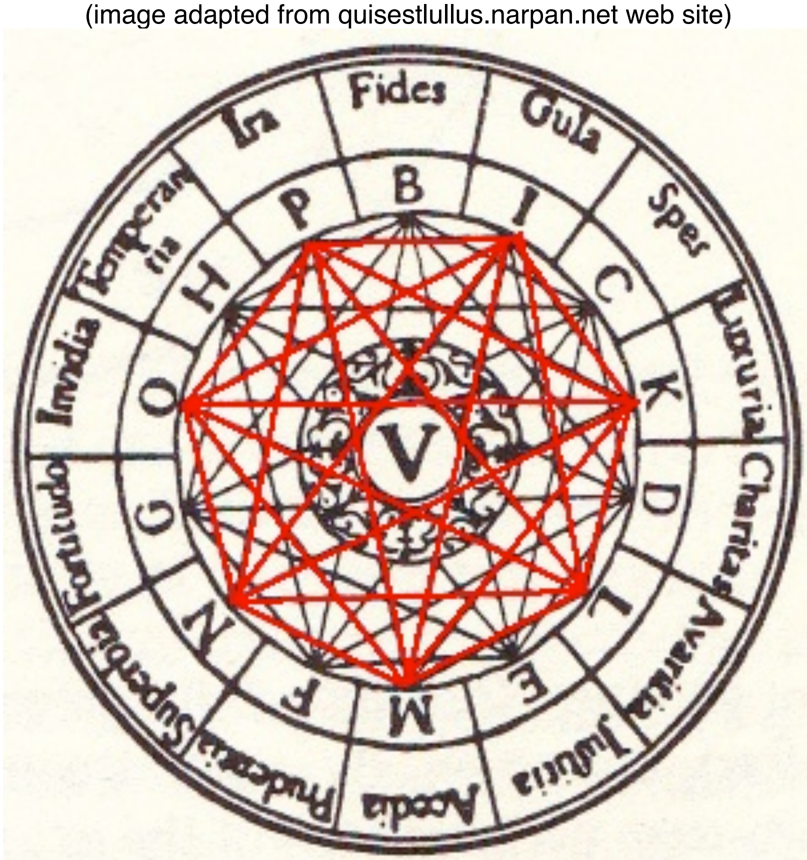
Each D4 Lie Algebra generates the 28 rotations of the 7-sphere S7 that lives
in 8-dim Euclidean space.
Although the 1-sphere S1 is the Lie Group U(1) based on Complex Numbers
and
the 3-sphere S3 is the Lie Group Sp(1) = SU(2) = Spin(3) based on Quarternions,

the Non-Associativity of Octonions prevents the 7-sphere S7 from forming a Lie Group.

If you try to make a Lie Algebra out of the 7 generators of S7
you find that the products do NOT form a closed 7-dim Lie Algebra
but
that you generate two more things:
a 14-dim G2 Lie Algebra that generates the Automorphisms of the Octonions and
a second 7-sphere S7.
If you put all those things together the S7 combines with the new 21-dim G2 x S7
to form the 28-dim D4 Lie Algebra.

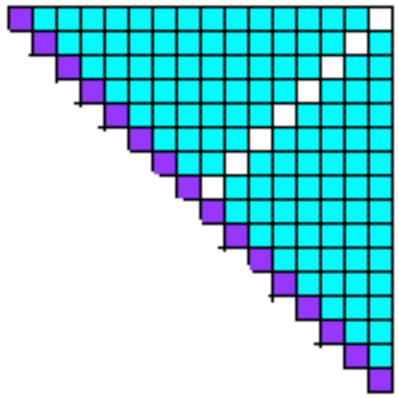
Ramon Llull describe that Octonion structure for the full two copies of D4 in Cl(8)
in terms of his V-wheel with 7+7 = 14 vertices

(image adapted from quisestlullus.narpan.net web site)



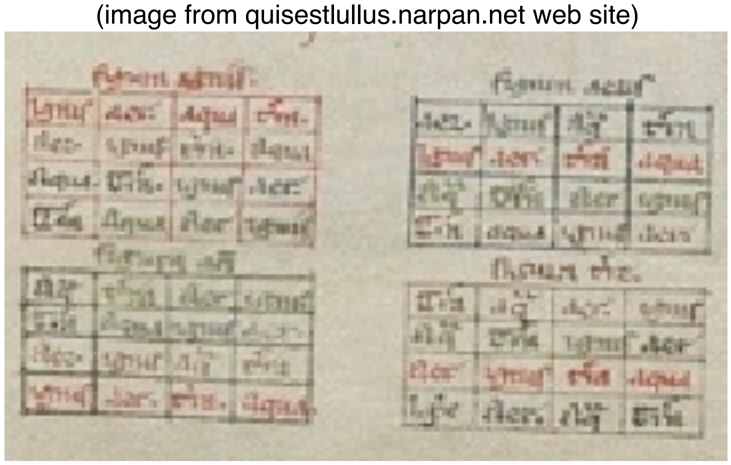
where 21 red lines connect the vertices of one set of 7 vertices of one D4
and 21 black lines connect the vertices of the other 7 vertices of the other D4.
Each heptagon can be used to describe the 480 different Octonion Products.

As to **the 136-dim Symmetric part of 16x16 Real matrices**
the 78-dim Tarot does not contain any of the 8 antidiagonal elements
but contains a single 1-dim U(1) from E6 / D5xU(1)
plus 16 of 64 entries in each of two 64-element triangular blocks



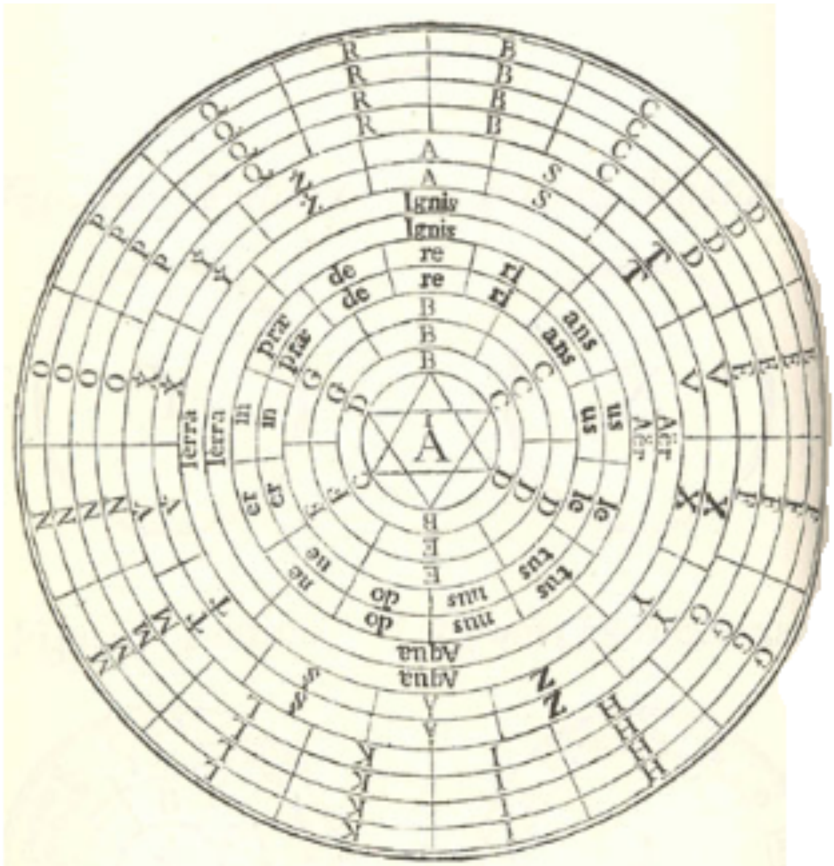
Physically one of the 64-element blocks of the Symmetric Part of Tarot corresponds to
the 8 components (with respect to 8-dim Kaluza-Klein spacetime)
of the 8 First-Generation Fundamental Fermion Particles
and
the other 64-element block corresponds to the components of the Fermion Antiparticles.

Ramon Llull's 64-element Elemental Figure is effectively an 8x8 matrix corresponding to
(image from quisestlullus.narpan.net web site)



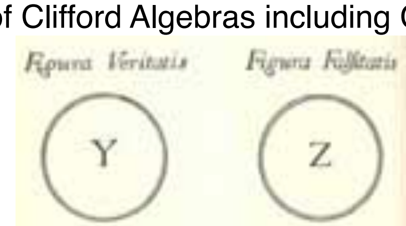
the Fermion Particle and Fermion Antiparticle 64-element Symmetric Tarot blocks
thus
constructing a 64+64 = 128-dimensional D8 half-Spinor space
that when combined with the 120-dimensional D8 of the Llullian A-Wheel produces
120-dim D8 + 128-dim D8 half-Spinor = 248-dim E8 Lie Algebra

As to how all the parts of Llull's model fit together, Lull used a system of revolving concentric circles - his Universal Figure (image from quisestlullus.narpan.net web site)



The outer 4 rings of 16 elements each represent the Elemental Figure Fermion Particles
The next (going in toward the center) 2 rings of 14 elements each represent the D4 and its Lie subalgebras producing the Gauge Groups of Gravity and the Standard Model.
The next 2 rings of 4 elements each represent the 4+4 = 8 dimensions of Spacetime.
The next 2 rings of 13 elements each represent the 26-dimensional String Theory with Strings seen as World-Lines of Fermions, producing a Bohm-type Quantum Theory.
At the center are 3 rings of 6 elements each surrounding a 6-vertex Star of David whose 3x6 + 6 = 24 elements correspond to the vertices of the 24-cell that is the basic element of the Integral Domain Lattice of Integral Quaternions that describe each of the 4-dimensional parts of (4+4)-dimensional Kaluza-Klein Spacetime.

The binary overall structure of Clifford Algebras including Cl(8) is described by Llull



in terms of his True-False Y-wheel and Z-wheel.

In his time around 1300 A.D. Ramon Llull could not use the language of 2000 A.D. math and physics to explain his Tarot-type model to the world. As Anthony Bonner' said in his book Doctor Illuminatus (Princeton 1993):
"... Even ...[Llull's]... disciple, le Myesier, complained ... about "the confusion caused by the meanings of the alphabet of the Ars demonstrativa and its sixteen figures, which confound the mind." ...
Llull ... tr[ied] to persuade the Parisian schoolmen ...[by]... us[ing] the bizarre vocabulary more sparingly, and modify[ing] the Art itself so that it would not look so alarming. ...
Towards the beginning of 1290 in Montpelier, therefore, Llull set about ... beginning a new phase of the Art ...
As a result of the "weakness of human intellect", the number of figures [wa]s reduced and the algebraic notation vanishe[d] ...
Llull's last works were written in December 1315 in Tunis, at which point he disappears from history. ... he must have died sometime between then and March 1316
...
the Dominican inquisitor general of Aragon, Nicholas Eymerich (1320-99), began a campaign against the doctrines of Ramon Llull ...[that]... culminated ... in two events:
The first was the publication in January ... 13676 ... of the Directorium inquisitorum, his notorious manual on inquisitorial methods ... it contained a list of a hundred errors of Ramon Llull ... on February 6, a papal bull was promulgated censuring Llull and condemning twenty of his books ... Llull's followers ... won in 1416 ... the promulgation by the Papal Court ... invalidating the bull of forty years earlier. ... Veneration of Llull was ... permitted withn the Franciscan Order and locally in Majorca ... his feast day was set on 3 July ...
The second condemnation of Llullist doctrines came from ... the Faculty of Theology of the University of Paris. In 1390 ... the Faculty of Theology publish[ed] an edict prohibiting the teaching of Llullist doctrines. ...[It]... cut off ... considerable interest in Llull in Paris ...".

In short:

Ramon Llull expanded the 78-dim Tarot outline structure to the old full 256-dim IFA
including the E8 Lie Algebra and the realistic structure of E8 Physics
but
he was 600 years ahead of rediscovery of his mathematics
and
700 years ahead of the time of detailed experimental confirmation
with the result that
the Paris-based Establishment of his time ignored and attacked his work
even when he tried to dumb it down to their level.

Llull to Cartan-Dirac-Riesz-E8Physics

Llull's description of the D8 Lie Algebra of dimension 120 = 8 (16-1)
remained undeveloped and unappreciated for 600 years
until Killing and Cartan classified Lie Groups.
Roger Penrose in his book "The Road to Reality" (Knopf 2004) said:
"... classification ...[of]... Lie groups ... started with Wilhelm Killing ... whose basic papers appeared in 1888-1890, and was essentially completed in 1894 ... by ... Elie Cartan ...
It turns out that there are four families, known as Am, Bm, Cm, Dm ... of respective dimension m(m+2), m(2m+1), m(2m+1), m(2m-1), called the classical groups ...
and five exceptional groups known as E6, E7, E8, F4, G2,
of respective dimension 78, 133, 248, 52, 14. ...".
The connection of the 248-dim Lie Algebra E8 with the Clifford Algebra Cl(16)
and its 120-dim bivector algebra D8 and 128-dim half-spinor space
only became clear in the 1900s based on the work of Cartan
as further developed by Jovet, Sauter, and Riesz mathematically
and applied by Dirac to physics.
Pertti Lounesto in his article on "History of Clifford Algebras"
in the book "Clifford Numbers and Spinors" by Marcel Riesz (Kluwer 1993) said:
"... E. Cartan 1908 ... identified the Clifford algebras Cl(p,q) as matrix algebras with entries in R [Real Numbers], C [Complex Numbers], H [Quaternions], R+R, H+H
and found a periodicity of 8 ...
Cartan also observed spinor modules of orthogonal Lie algebras in 1913 ...
Jovet 1930 and Sauter 1930 replaced column spinors by square matrices in which only the first column was non-zero -
thus spinor spaces became minimal left ideals in a matrix algebra.
Riesz 1947 used primitive idempotents of Clifford algebras to construct spinor spaces as minimal left ideals in Clifford algebras ...".

Roger Penrose in his book "The Road to Reality" (Knopf 2004) said:
"... One reason that Clifford Algebras are important is for their role in defining spinors. In physics, spinors made their appearance in Dirac's equation for the electron (Dirac 1928), the electron's state being a spinor quantity ...".
In the 1900s, Irving Ezra Segal showed the connection between the Conformal Group and the Dark Energy of Gravity; MacDowell and Mansouri showed how gauging the Conformal Group produces Gravity; and the Dirac Equation was generalized to the Standard Model. During the 2000s Dark Energy was observed by WMAP and Planck and the LHC discovered a Higgs state, confirming the basic Standard Model structure so

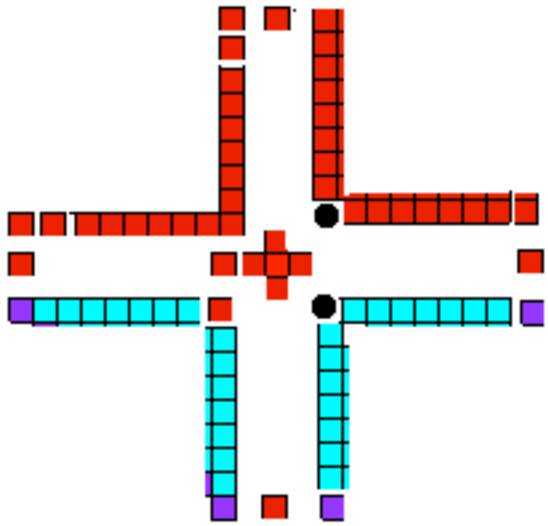
it is only now (the 2000s) that the formal Written Human Culture has caught up with the informal Oral Ancient African Culture and the 700-year old model of Ramon Llull in understanding a realistic Unified Theory of the Laws of Nature.

Appendix1: E6 to D4

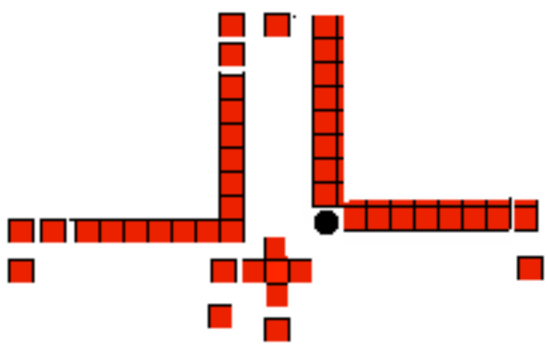
There are two chains from E6 to D4:

The chain E6 to D5 to D4

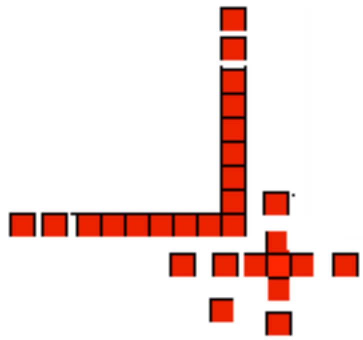
78-dim E6



contains 45-dim D5 by E6 / D5xU(1) rank 2 Type EIII space (CxO)P2 whose related Type V Exceptional Complex Domain is not of Tube Type. In E8 Physics its Shilov Boundary is seen as a bundle with fibre S1xS7 and base space whose own fibration is S1 -> S9 -> CP4. It is used in E8 Physics as a representation space for first-generation Fermion Particles and AntiParticles.

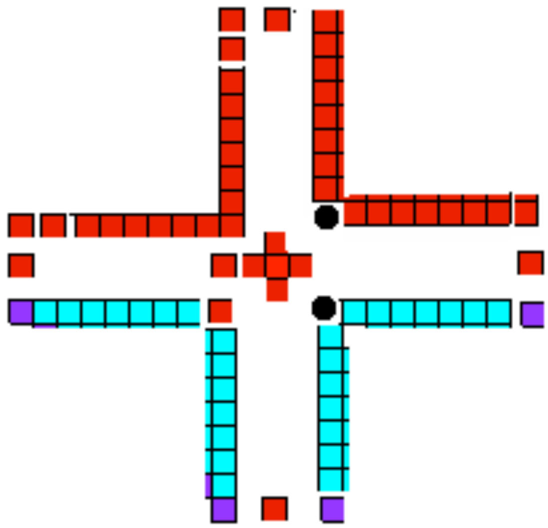


which contains 28-dim D4 by D5 / D4xU(1) Lie sphere rank 2 Type BDI space whose related Type IV(8) Complex Domain is Tube Type with Shilov Boundary RP1 x S7.
It is used in E8 Physics as a representation for 8-dimensional Spacetime.

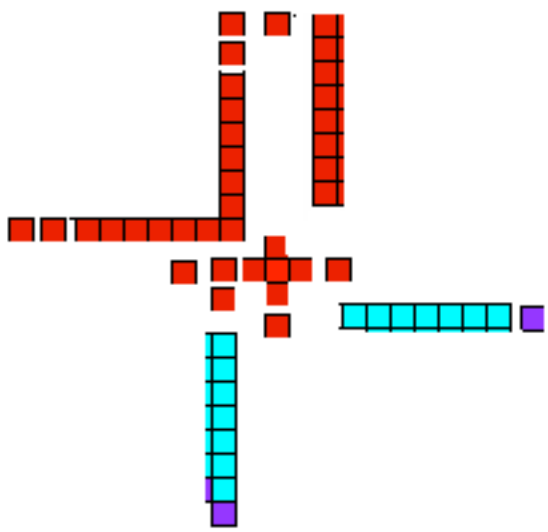


The chain E6 to F4 to B4 to D4

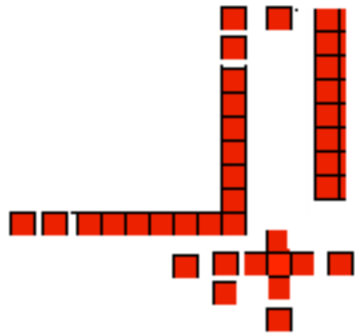
78-dim E6



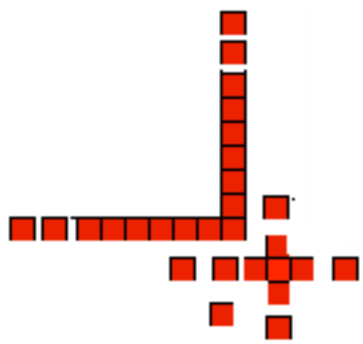
contains 52-dim F4 by E6 / F4 Type EIV rank 2 space that is the set of OP2 in (CxO))2 and is related to the 26-dim traceless part J(3,O)o of the 27-dim Jordan Algebra J(3,O)



which contains 36-dim B4 by F4 / B4 = OP2 = Octonionic Projective Plane

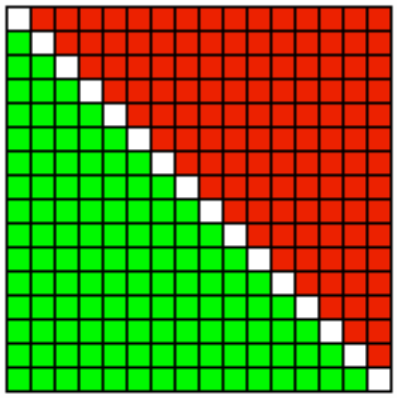


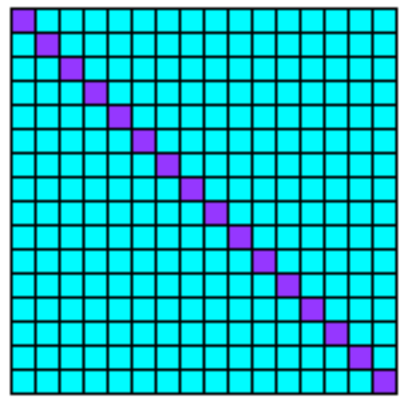
which contains 16+12 = 28-dim D4 by B4 / D4 = OP1= the 8-sphere S8



Appendix2: Some Details of E8 encoding in IFA

IFA Cl(8) = 256-dim 16x16 Real Matrices M(16,R) =
= 120-dim Antisymmetric16x16 + 136-dim Symmetric 16x16





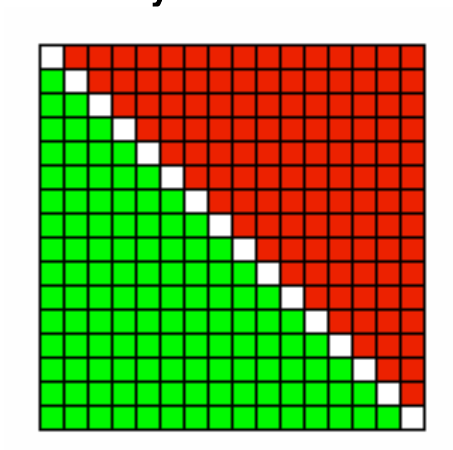
For Antisymmetric 16x16 each red entry above the diagonal is the negative of the corresponding green entry below the diagonal and the 16 diagonal entries are zero so the number of Antisymmetric entries is 120 corresponding to the D8 Lie Algebra.

For Symmetric 16x16 each cyan entry above the diagonal is equal to the corresponding cyan entry below the diagonal and the 16 diagonal entries are non-zero so the number of Symmetric entries is 120+16 = 136.
8 of the 136 Symmetric entries of the IFA Cl(8) 16x16 Matrix do not correspond to E8
but
the other 136-8 = 128 = 64 + 64 correspond to 128-dim half-spinor of D8.

Since **248-dim E8 = 120-dim D8 + 128-dim half-spinor of D8 =**
= 120-dim Antisymmetric part + 128 of 136-dim Symmetric part of M(16,R)

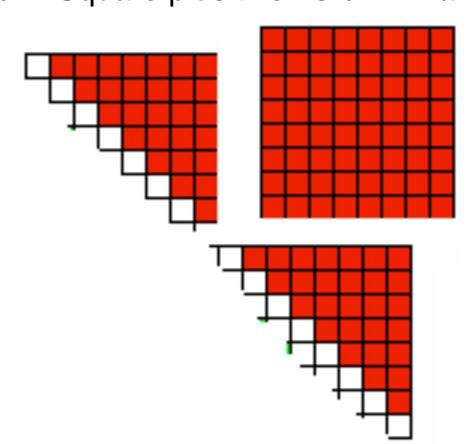
256-dim IFA Cl(8) contains 120+128 = 248-dim E8
and so
encodes the structure of E8 Physics of Gravity and the Standard Model.

Antisymmetric Part:



Due to the diagonal-reflection symmetry of Antisymmetric and Symmetric matrices, only the upper triangular parts of the matrices need to be used in visualization.

The 120-dim Antisymmetric part corresponds to the 120-dim D8 Lie Algebra.
It has 3 components: a 64-dim Square plus two 28-dim Triangles.



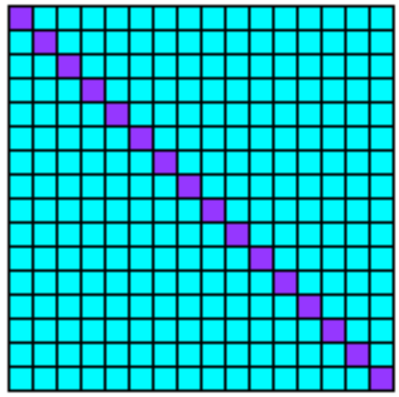
Each 28-dim Triangle corresponds to a 28-dim D4 Lie Algebra
so that
the 64-dim Square corresponds to the Coset Space D8 / D4xD4

The 64-dim Square also corresponds to a U(8) subalgebra of D8
representing relationships between 8-dim Position and 8-dim Momentum
of 8-dim Spacetime that obtains a (4+4)-dim Kaluza-Klein structure.

One of the 28-dim D4 contains a 15-dim A3 = D3 Lie Algebra of the Conformal Group
that produces Gravity by a generalized MacDowell-Mansouri mechanism.

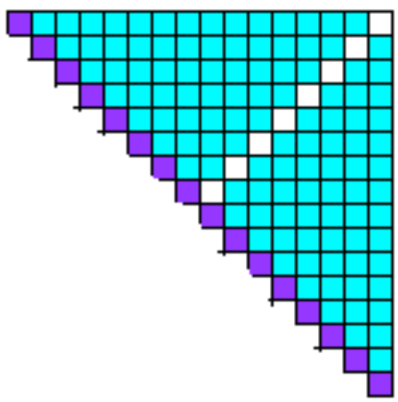
The other 28-dim D4 contains a 15-dim A3 = D3 Lie Algebra with an A3 = SU(3)
subalgebra that in conjunction with 8-dim Kaluza-Klein spacetime containing
4-dim Internal Symmetry Space part CP2 = SU(3) / SU(2)xU(1)
produces the Standard Model gauge groups by the Batakis mechanism.

Symmetric Part:

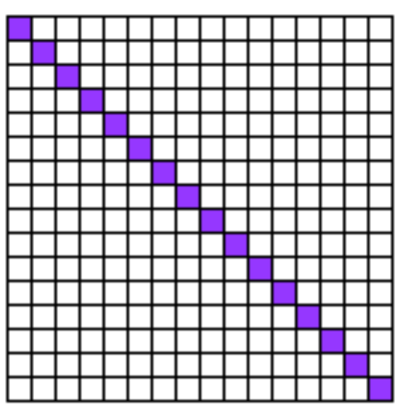


Due to the diagonal-reflection symmetry of Antisymmetric and Symmetric matrices, only the upper triangular parts of the matrices need to be used in visualization.

The 120 + 16 = 136-dim Symmetric part of the Cl(8) matrix algebra M(16,R)



contains 8 anti-diagonal elements that are not contained in the E8 Lie Agebra, but both Cl(8) and E8 contain the 16-element Diagonal whose elements



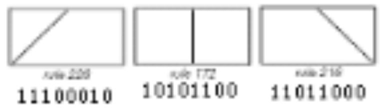
give 16 terms Cl(8) = 16x16 Real Matrix Algebra Primitive Idempotent

$f = (1/2)(1 + e_{1248}) (1/2)(1 + e_{2358}) (1/2)(1 + e_{3468}) (1/2)(1 + e_{4578}) =$
$= (1/16)(1 + e_{1248} + e_{2358} + e_{3468} + e_{4578} + e_{5618} + e_{6728} + e_{7138}$
$- e_{3567} - e_{4671} - e_{5712} - e_{6123} - e_{7234} - e_{1345} - e_{2456} + e_J)$

which in terms of the 256 Elementary Cellular Automata are

- $+ e_{12345678}$

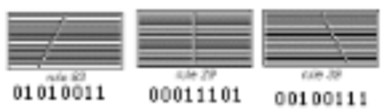
- $+ e_{6728} + e_{3468} + e_{4578}$ to



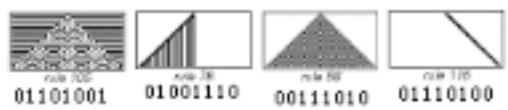
- $+ e_{2358} + e_{1248} + e_{5618} + e_{7138}$

- $+ 1$ to

- $- e_{5712} - e_{1345} - e_{6123}$



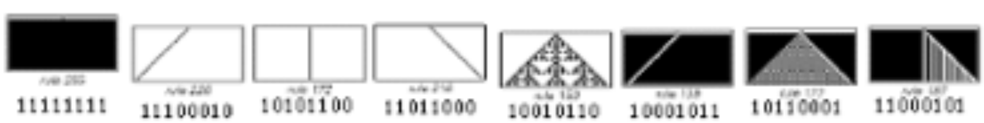
- $- e_{4671} - e_{7234} - e_{2456} - e_{3567}$



which are related to the two 8-dim Cl(8) half-spinors

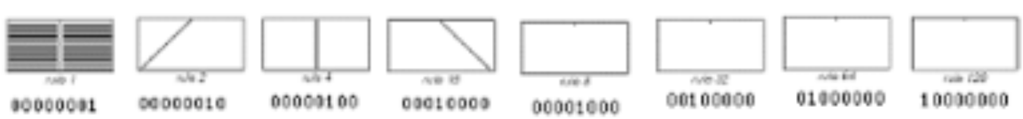
the 8 +half-spinors

the 8 -half-spinors



which in turn are related by Triality to the 8-dim Cl(8) vectors

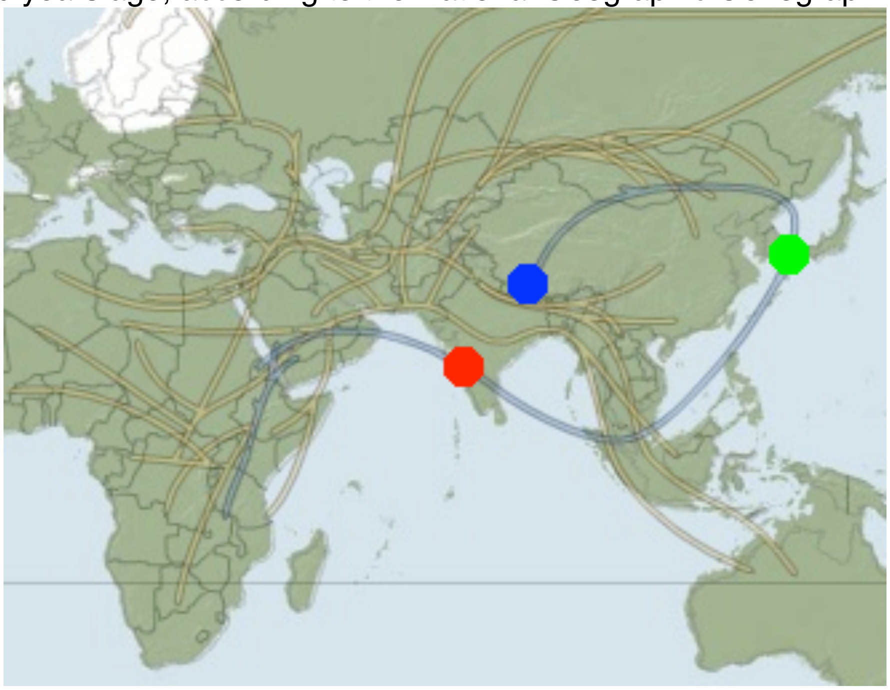
the 8 vectors



When the 56 + 56 off-diagonal elements are added to the 8 + 8 Cl(8) half-spinors you get the 64 + 64 = 128 elements of a half-spinor space of Cl(16) = Cl(8) x Cl(8).

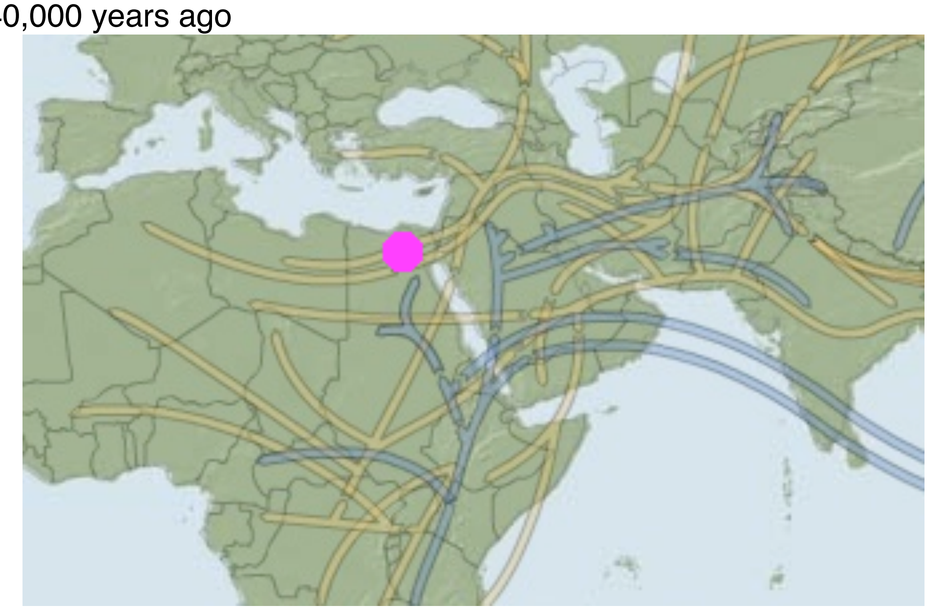
Appendix3 - Comparison of Arabian Sea Africa-India connection with Nile River Africa-Egypt connection

About 50,000 years ago, according to the National Geographic Genographic project,



Y-chromosome DNA indicated that basic human physiology had emerged from Africa to India then to Japan and on to Tibet

By about 40,000 years ago



Y-chromosome DNA population M96 had branched into the Nile River Valley

Around 50,000 years ago
Japanese were separated from their African Homeland by
the East Pacific Ocean, the Sunda Shelf, the Indian Ocean, and the Arabian Sea
which was so great a distance that contact with Africa was so tenuous
that they only retained about 1/2 of their IFA Cultural Heritage as evidenced
by the fact that Shinto Futomani Divination uses 128 elements, 1/2 of the 256 of IFA
Tibetan were separated from their African Homeland by Mountains and Land of China,
the East Pacific Ocean, the Sunda Shelf, the Indian Ocean, and the Arabian Sea
which was so great a distance that contact with Africa was so much more tenuous
that they only retained about 1/4 of their IFA Cultural Heritage as evidenced
by the fact that the I Ching uses 64 elements, 1/4 of the 256 of IFA
Indians were separated from their African Homeland by the Arabian Sea
which was close enough to Africa to maintain regular contact
but far enough that they felt isolated from the very close contact needed
to maintain the details of the oral traditions of IFA, so the Indian priests of IFA chose
to put the IFA Information System into writing and to do so developed Sanskrit
and wrote the Rig Veda.

About 10,000 years later (around 40,000 years ago)
the Builders of the Great Pyramid had migrated throughout the length of the Nile,
along which substantially contiguous settlements enabled them to maintain
enough contact to maintain the details of the oral traditions of IFA so that
when they built the earliest of the pyramids, the Great Pyramid,
they did not deface it with any writing.
However,
the Great Pyramid was a huge engineering project requiring coordinated work

(image from Wikipedia)

by large numbers of people, so an engineering language developed among
the builders, first based upon hand signals (see Stan Tenen's www.meru.org)
and then translated into a written alphabetical language, Hebrew.
After the Great Pyramid complex had been completed,
the Tower of Babel breakdown occurred, the cooperative community fragmented,
skills became diluted so that later pyramids were not up to Great Pyramid standards,
and a less-sophisticated heiroglyphic writing evolved and was used on later structures.

Appendix4 - Poster

The poster on the following page was produced for the NSBP/NSHP 2010
Joint Annual Conference of the
Nartional Society of Black Physicists
and the
National Society of Hispanic Physicists
to be held 10-14 February 2010.

The conference was cancelled
so
the poster is put here so that it might be seen by anyone interested.

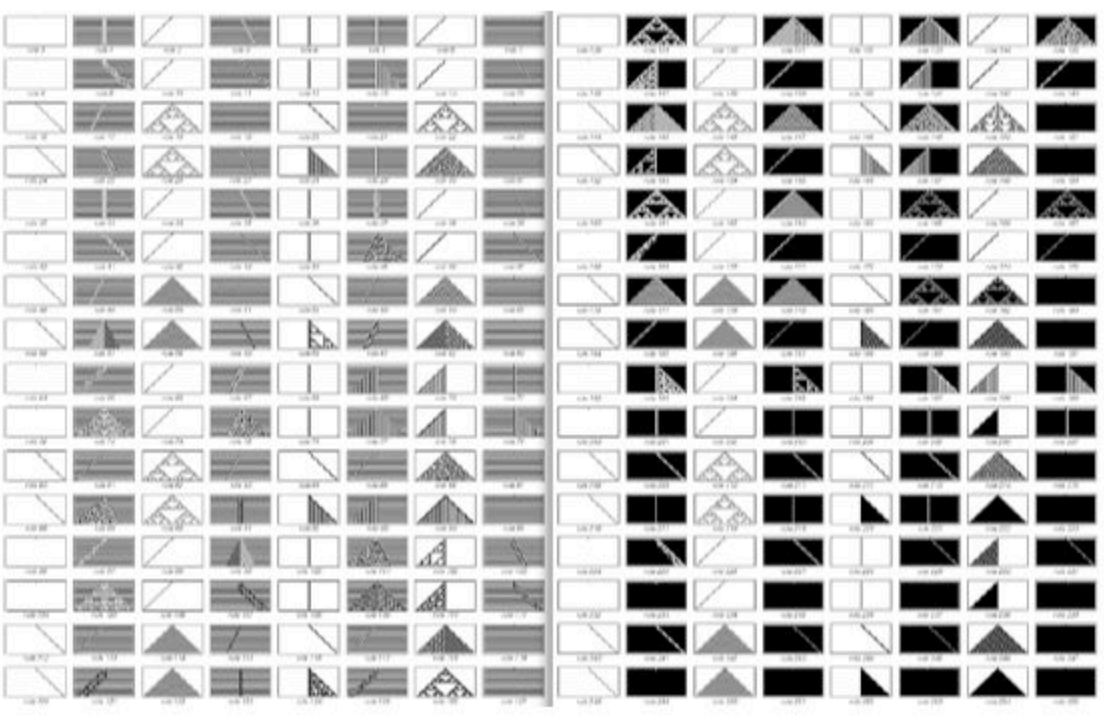
African Origins

Africans developed IFA Oracle divination based
on the square of 16 = 16x16 = 256 = 2^8
corresponding to the vertices of an 8-dimensional hypercube
and to the binary 2-choice Clifford algebra Cl(8)
and so to related ones such as Cl(8)xCl(8) = Cl(16).

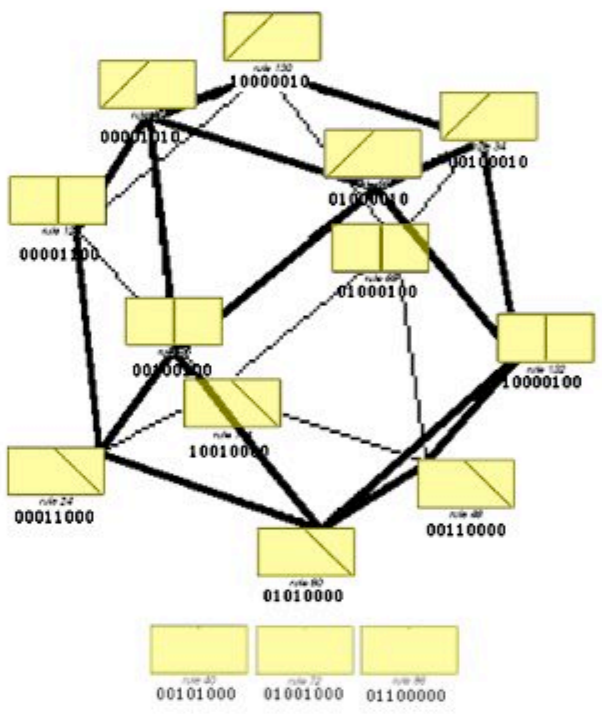
Since the number of sub-hypercubes in an 8-dimensional
hypercube is 6,561 = 81x81 = 3^8, the IFA Oracle has
N=8 ternary 3-structure as well as binary 2-structure:

```
N        2^N                 3^N
0                   1
1                 2   3
2               4 =2x2  9 = 3x3
3             8           27
4          16 = 4x4         81 = 9x9
5        32                   243
6      64 = 8x8                 729 = 27x27
7    128                          2187
8   256 = 16x16                     6561 = 81x81
```

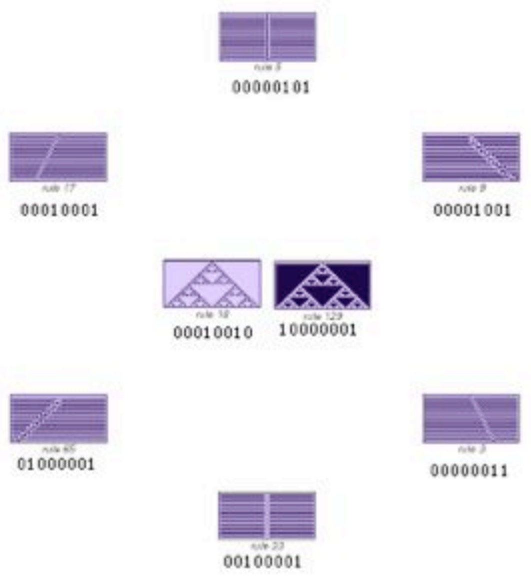
As ancient African games such as Owari show,
binary 2-structure corresponds to static states
and ternary 3-structure corresponds to dynamic states.
Mathematically, using binary 2-choice static states to define
dynamics on 3 ternary neighbor states
produces the 256 Elementary Cellular Automata:



15 of the elements represent
Conformal Spin(2,4) of Gravity:

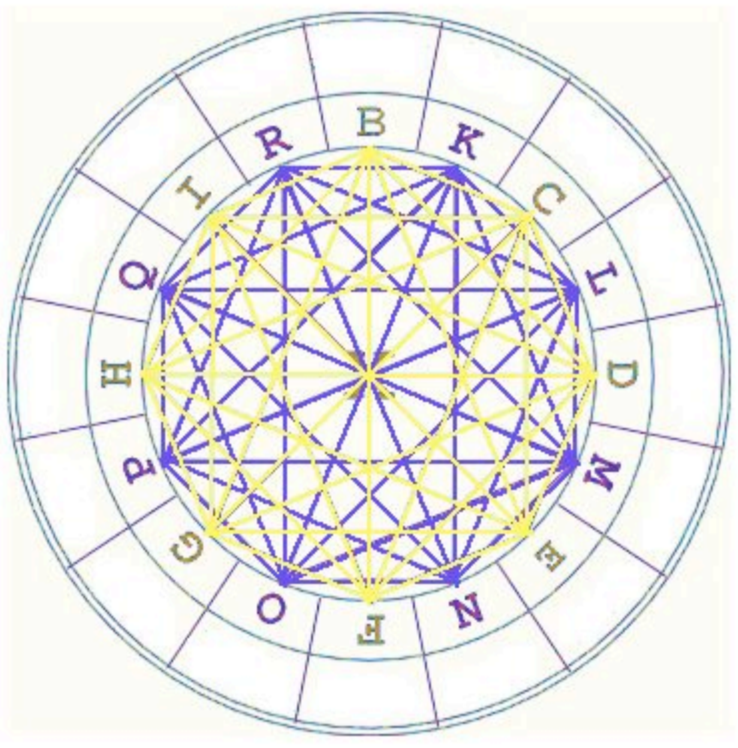


8 of the elements represent
the SU(3) Color Force:

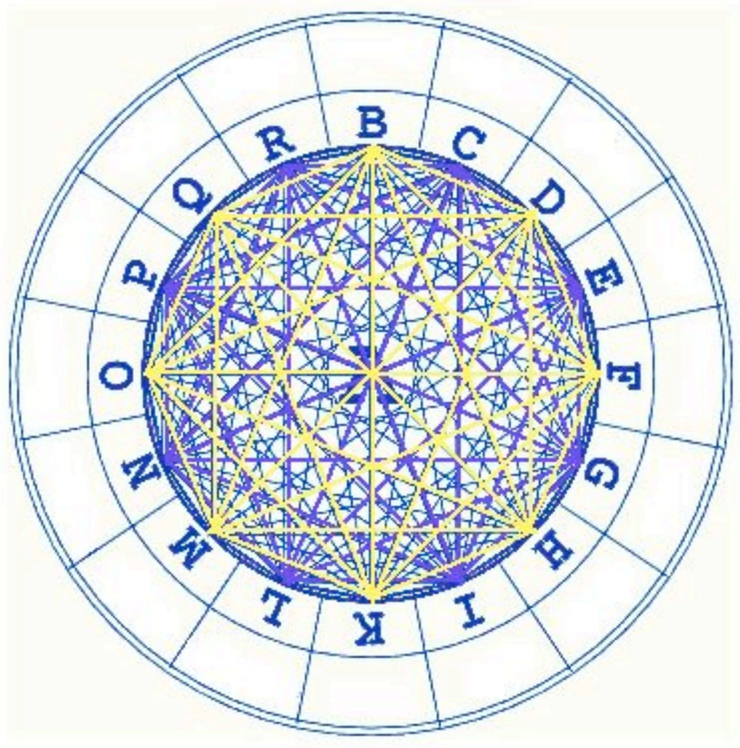


Hispanic Development

Ramon Llull (1232-1316) of Mallorca studied the 16 possibilities of the Ilm al Raml,
which are derived from 16 of the 16x16 = 256-element African IFA divination system,
and found a structure that he summarized
in Wheel Diagrams with 16 vertices connected to each other by lines
and in Cubic 4x4x4 = square 8x8 = 64-element Elemental Figures
(images adapted from lullianarts.net web site):



The 28 gold lines (gold 8-HyperCube points) represent Gravity
and
the 28 purple lines (purple 8-HyperCube points) represent the
Standard Model.

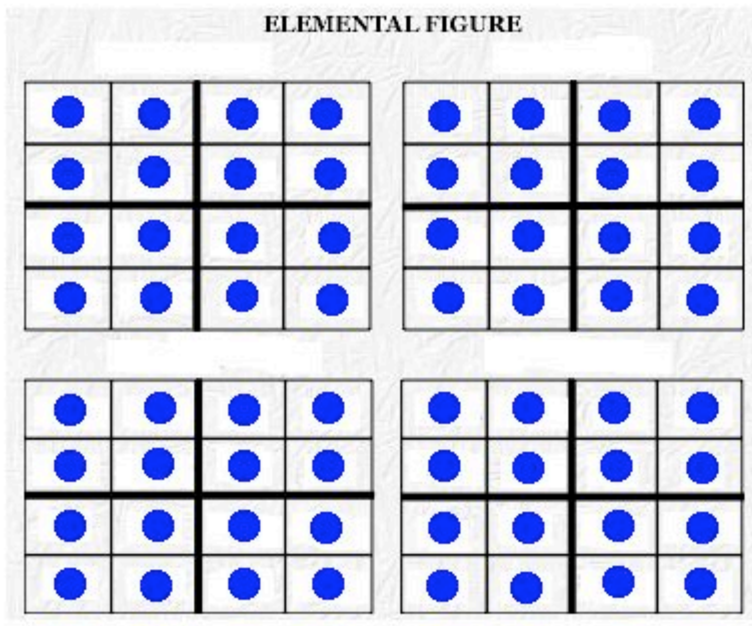


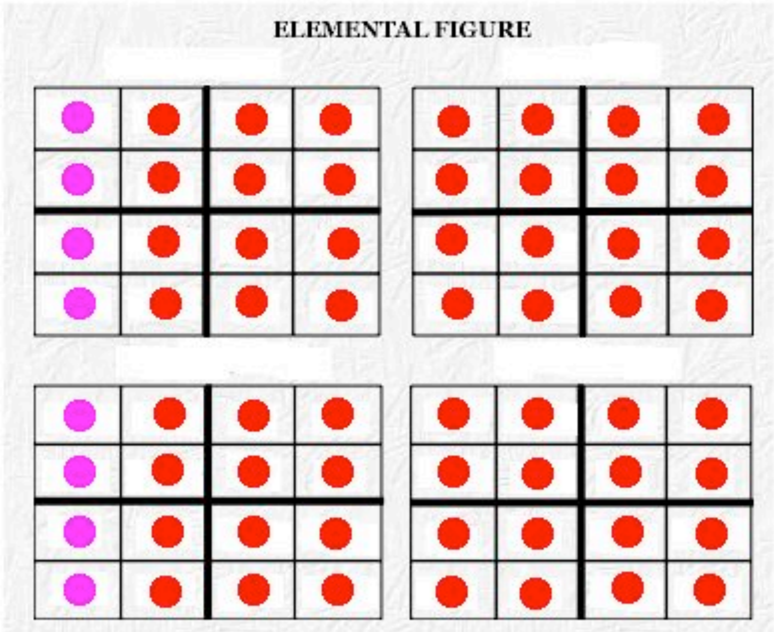
Adding in 64 blue lines (blue 8-HyperCube points without white dots)
gives 120 lines that represent the Spin(16) BiVector Lie Algebra
of the Clifford Algebra Cl(16) = Cl(8) x Cl(8):

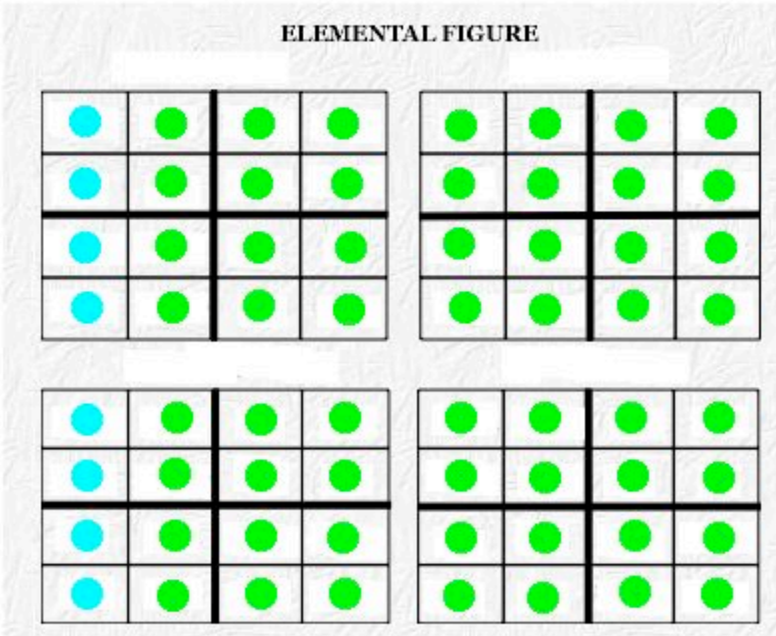
The 64 blue lines, and the 64 pure blue 8-HyperCube points,
represent 8-dimensional Kaluza-Klein Vector SpaceTime
and can be shown as a 64-element Elemental Figure.

By Triality Automorphisms,
the 8+56 = 64-element magenta and red 8-HyperCube points
that represent 8 Fermion Particles
can also be shown as a 64-element Elemental Figure
and
the 8+56 = 64-element cyan and green 8-HyperCube points
that represent 8 Fermion AntiParticles
can also be shown as a 64-element Elemental Figure.

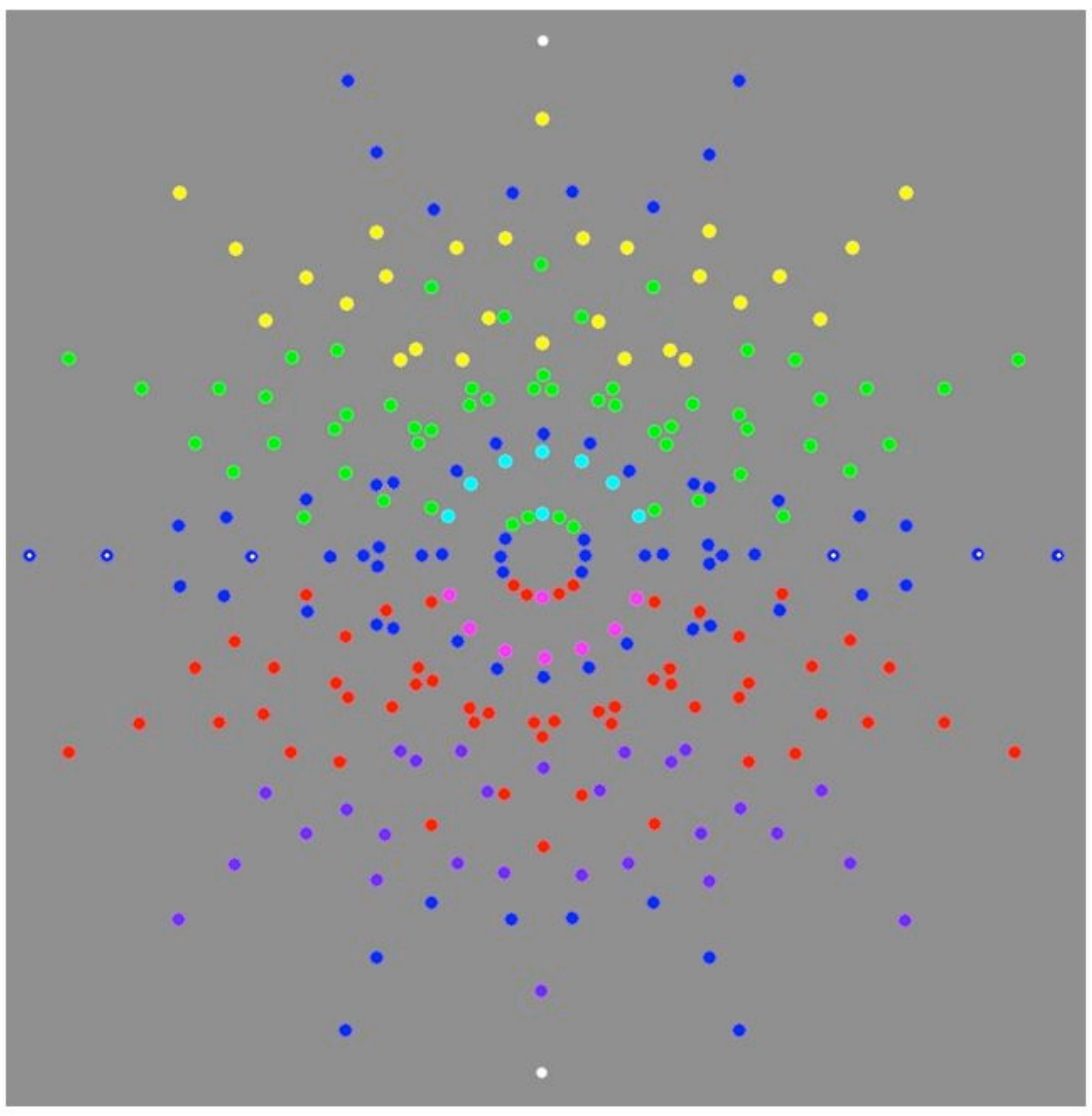
The 64 red-magenta Fermion Particle elements
plus the 64 green-cyan Fermion AntiParticle elements
form the 128 elements of one Half-Spinor representation of Spin(16).



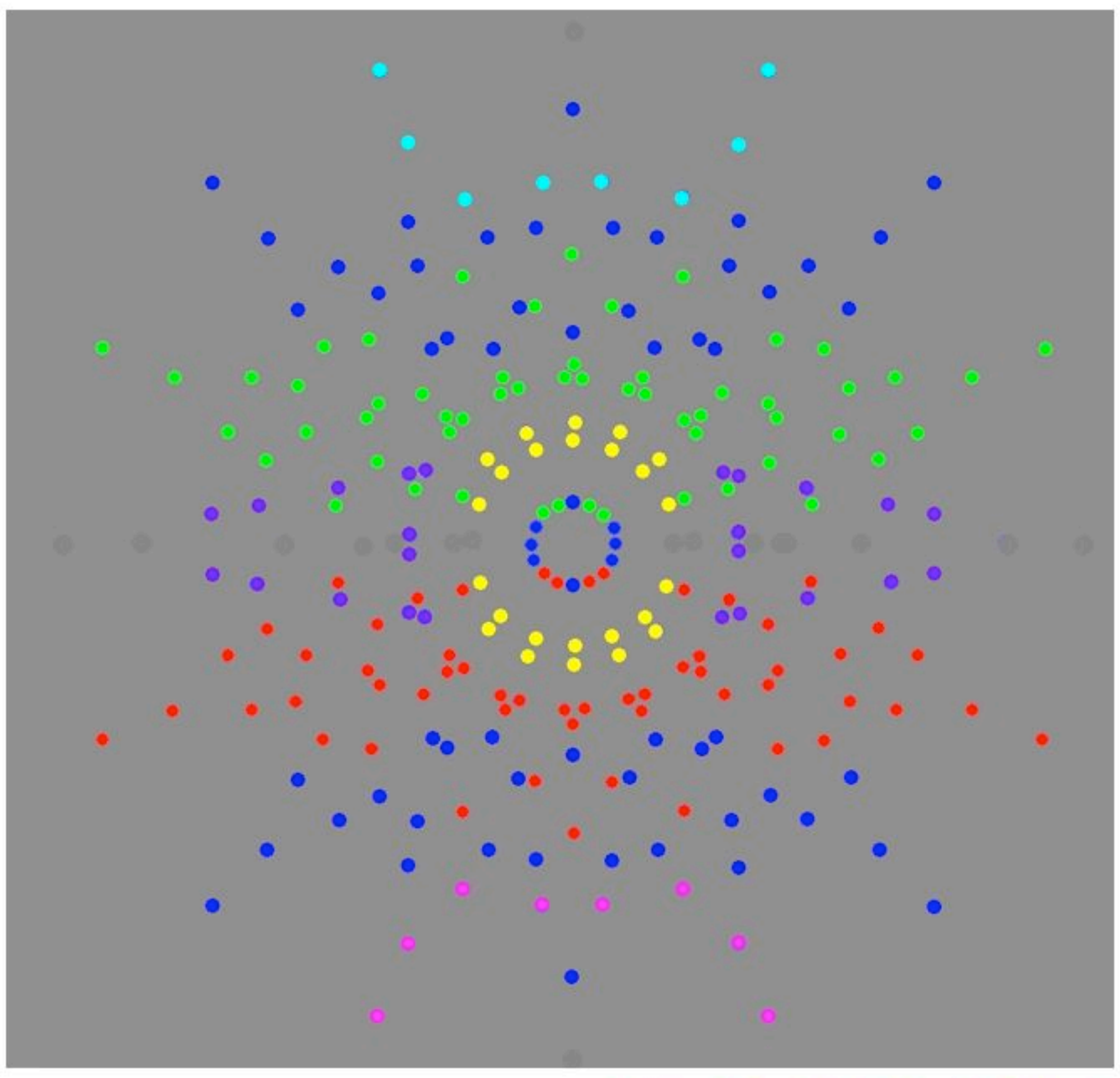




The 120 elements of the Spin(16) BiVector Lie Algebra
plus the 128 elements of a Half-Spinor representation of Spin(16),
both of which live in the Clifford Algebra Cl(16) = Cl(8) x Cl(8),
combine to form the 120+128 = 248-dimensional E8 Lie Algebra:



8-HyperCube = 256 vertices = 1 + 8 + 28 + 56 + (8+48+8+(3+3)) + 56 + 28 + 8 + 1



E8 Lie Algebra = 120+128 = 248 vertices = 8 + 28 + 56 + (28+8+28) + 56 + 28 + 8

E8 Physics

The resulting E8 Physics Model has:

EPR structure similar to that of Joy Christian;

E8 structure modified from that of Garrett Lisi;

Cl(16) = Cl(8)xCl(8) Clifford Algebra structure anticipated by Ramon Llull;

Higgs mechanism produced by formation of M4 x CP2 spacetime as shown by work of Meinhard Mayer;

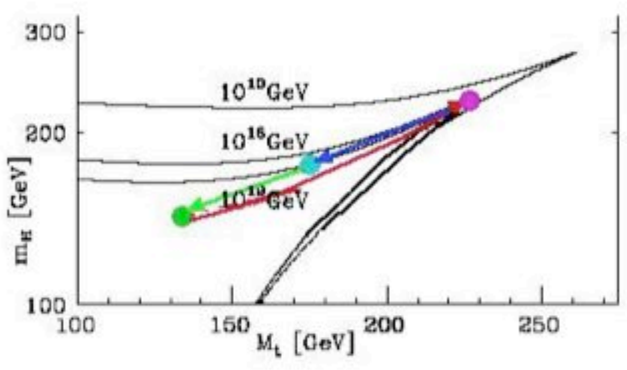
Standard Model Gauge Groups produced therein as shown by work of N. A. Batakis;

Conformal Gravity produced as in the MacDowell-Mansouri mechanism;

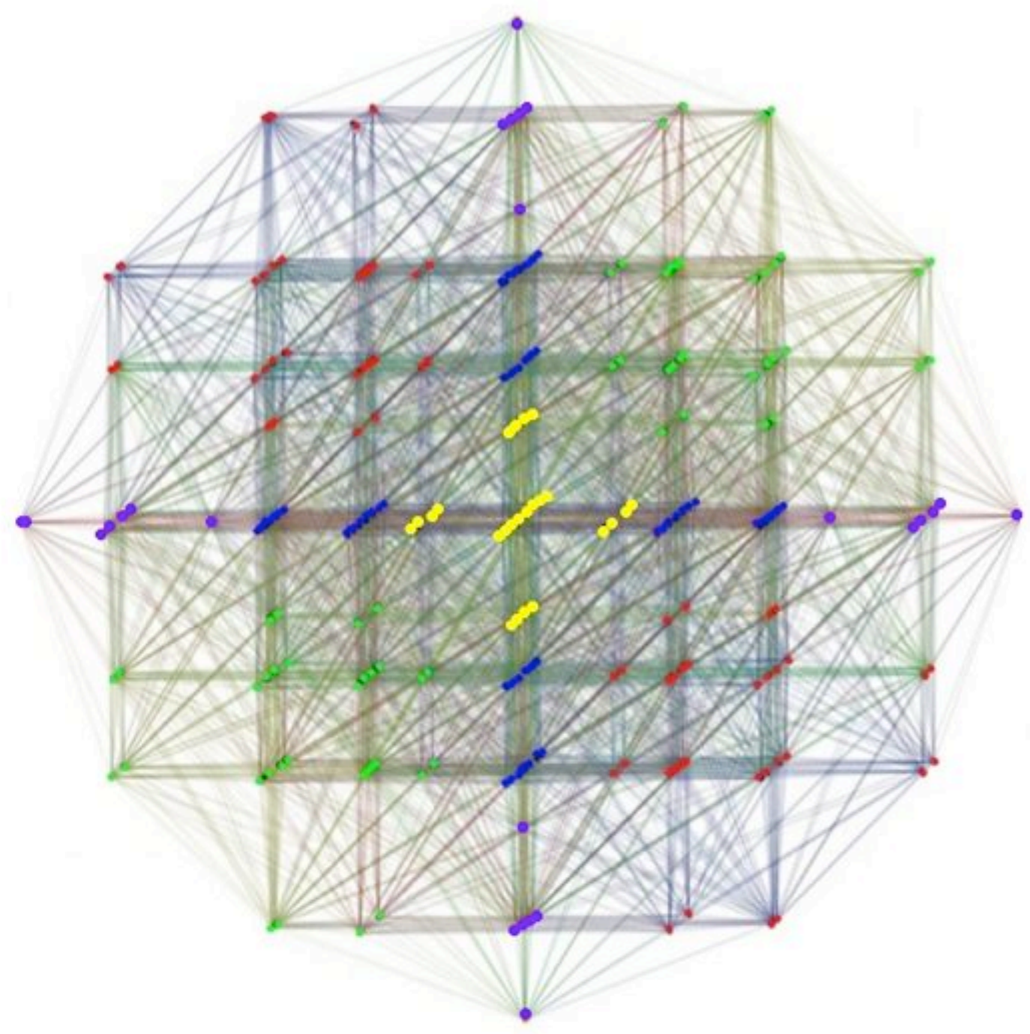
Dark Energy : Dark Matter : Ordinary Matter ratio 75 : 21 : 4
produced by conformal structures similar to those of Irving Ezra Segal;

Force Strength and Particle Mass calculations done using the Work of Hua Luogeng,
particularly work on the Geometry of Complex Domains;

T-quark composite Higgs model based on the work of Yamawaki et al,
resulting in a 3-state T-quark - Higgs system;



and
Algebraic Quantum Field Theory (AQFT) constructed
from a Clifford Real-Periodicity-8 hyperfinite II1 von Neumann algebra factor.



E8 Root Vectors = 112+128 = 240 = 8 + 28 + 56 + (24+8+24) + 56 + 28 + 8