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The SU_9 Structure of E_8 and Boson-Fermion Couplings

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Abstract

We construct the E_8 algebra in terms of the tensorial representations of its SU_9 maximal subalgebra. We then construct the supersymmetric gauge-invariant boson-fermion coupling, and decompose it completely into terms exhibiting color SU_3 and family SU_5 symmetries. This work promotes a scheme of E_8 super grand unification that is based on a perfect symmetry between particles and antiparticles, rather than a symmetry between quark-lepton generations and their enigmatic mirror conjugates. Whereas the emergence of a definite chirality for low-energy weak interactions still depends on the yet unresolved problem of symmetry breaking, a picture of weak decays emerges, in which multiple W vector bosons, rather than a single one, are the fundamental weak mediators between multiple charged leptons and associated multiple neutrinos, or between multiple upquarks and their associated multiple downquarks.

1 Introduction

The E_8 algebra is the largest exceptional Lie algebra. It was proposed long time ago as a grand gauge unification symmetry for elementary particles and their strong and electroweak interactions[1], [2], [3] which can lead to intrinsic supersymmetry between fermions, gauge bosons, and possibly Higgs bosons, because its fundamental representation, to which the fermions can belong, is also the adjoint representation, to which the gauge bosons, and the Higgs bosons, would belong. More recently, phenomenological implications of such supersymmetric E_8 unification have been highlighted[4], [5] by Adler stressing the possible role of condensates in supersymmetric Yang-Mills theories.

However, the E_8 algebra leads to an immensely huge system of particles, the spelling out of its detailed structure is extremely extensive. On the other hand, the associated theory in its original form[1], [2] predicts several quark-lepton generations together with their mirror conjugates. This is shown, for example, by considering the decomposition of the fundamental (adjoint) multiplet with respect to $SU_5 \times SU_5$,

$$248 = (1, 24) + (24, 1) + (5^*, 10) + (5, 10^*) + (10, 5) + (10^*, 5^*) \tag{1}$$

If one SU_5 is considered to be the grand unified symmetry of Georgi and Glashow[6] for one generation of leptons and quarks, while the other SU_5 is just a horizontal family symmetry, then we can see that the theory could describe up to five generations of fermions, each generation consisting of $(15 = 5^* + 10)$ Weyl fermions, together with their mirror conjugates $(15^* = 5 + 10^*)$, and other particles. Whereas the mirror particles are supposed to be very heavy, the detailed mechanism of mass generation for ordinary and mirror particles was never completed, thus far, and crucial phenomenological predictions were never confronted with experiment.

Our purpose in this article is to attempt a detailed mathematical exhibition of the boson-fermion gauge coupling terms, and try to grasp some observations from that. However, in this work, we shall present a *new approach to particle assignments*, in which perfect symmetry between particles and antiparticles is promoted, rather than a symmetry between quark-lepton generations and their mirror counterparts. The low-energy chirality (or left-handedness) of weak interactions will remain to be clarified in another development, when the symmetry breaking scenario should be completed.

We shall begin, in the following section, by writing down the E_8 algebra using the tensor calculus of its SU_9 maximal subalgebra. This will be followed by introducing the structures that would assist in writing the gauge-invariant boson-fermion couplings in a decomposition of SU_9, exhibiting color SU_3 and family SU_5.

2 The E_8 Algebra in SU_9 Notation

Let us be begin by introducing the SU_9 generators in the form ${J_a}^b$. Here, the indices $\{a, b, \cdots\}$ would correspond to the fundamental complex representation of SU_9, $(a = 1, \cdots, 9)$. The generators ${J_a}^b$ are traceless, ${J_a}^a = 0$, with the summation over repeated indices is always implied. The SU_9 algebra is defined by the commutation relation

$$\left[{J_a}^b, {J_c}^d\right] = \left({\delta_a}^d {J_c}^b - {\delta_c}^b {J_a}^d\right) \tag{2}$$

The number of SU_9 generators is 80. The number of E_8 generators is 248. The decomposition of the latter with respect to SU_9 is $\mathbf{248} = \mathbf{80} + \mathbf{84} + \mathbf{84}^*$. In order to complete the E_8 algebra, we need to introduce the conjugate generators Q_{abc} and Q^{abc}. These are totally antisymmetric with respect to their indices, hence each giving the multiplicity $(9\times8\times7)/(3\times2) = 84$. As SU_9 tensors, the Q_{abc} and Q^{abc} generators have the following commutation relations with the SU_9 generators ${J_a}^b$,

$$\left[{J_a}^b, Q_{cde}\right] = -\left(\delta_c^b Q_{ade} + {\delta_d}^b Q_{aec} + {\delta_e}^b Q_{acd} - \frac{1}{3}{\delta_a}^b Q_{cde}\right) \tag{3}$$

$$\left[{J_a}^b, Q^{cde}\right] = \left({\delta_a}^c Q^{bde} + {\delta_a}^d Q^{bec} + {\delta_a}^e Q^{bcd} - \frac{1}{3}{\delta_a}^b Q^{cde}\right) \tag{4}$$

Now we introduce the commutators

$$\begin{cases} \left[Q_{abc}, Q_{def}\right] = \frac{1}{3!}\epsilon_{abcdefghx} Q^{ghx} \\ \left[Q^{abc}, Q^{def}\right] = -\frac{1}{3!}\epsilon^{abcdefghx} Q_{ghx} \end{cases} \tag{5}$$

and this closes the algebra:

$$\left[Q_{abc}, Q^{def}\right] = -{\delta_a}^d {\delta_b}^e {J_c}^f + \cdots \tag{6}$$

Here, the meaning of the dots $(\cdots)$ on the right side is that we must antisymmetrize with respect to $\{a, b, c\}$, and with respect to $\{d, e, f\}$, with the total number of terms is 18, all related to the displayed term by proper permutations of indices.

That the above algebra is correct with proper signs and factors can be verified by checking all Jacobi identities. The generators are normalized such that the following Casimir operator commutes with all of them.

$$ {J_a}^b {J_b}^a + \frac{1}{3!} Q_{abc} Q^{abc} + \frac{1}{3!} Q^{abc} Q_{abc} \tag{7}$$

3 The Adjoint Multiplet

Corresponding to the generators of the above E_8 algebra we can introduce a multiplet $\mathcal{A} = \{{A_a}^b, A_{abc}, A^{abc}\}$ according to the following module

$$\mathcal{A} = {A_a}^b {J_b}^a + \frac{1}{3!} A_{abc} Q^{abc} + \frac{1}{3!} A^{abc} Q_{abc} \tag{8}$$

Introducing another module for $\mathcal{B} = \{{B_a}^b, B_{abc}, B^{abc}\}$ in the same way, we can compute the commutator $[\mathcal{A}, \mathcal{B}]$. The resulting coefficients of the generators would describe a composite multiplet $\mathcal{F} = \{{F_a}^b, F_{abc}, F^{abc}\}$. The latter components are given by

$$\begin{cases} {F_a}^b = \left({A_a}^c {B_c}^b - {A_c}^b {B_a}^c\right) + \frac{1}{2}\left\{A_{acd} B^{bcd} - A^{bcd} B_{acd} - \frac{1}{9}{\delta_a}^b \left(A_{cde} B^{cde} - A^{cde} B_{cde}\right)\right\} \\ F_{abc} = {A_a}^d B_{bcd} - {A_b}^d B_{acd} + {A_c}^d B_{abd} - A_{bcd} {B_a}^d + A_{acd} {B_b}^d - A_{abd} {B_c}^d + \frac{1}{36}\epsilon_{abcdefghx} A^{def} B^{ghx} \\ F^{abc} = -{A_d}^a B^{bcd} + {A_d}^b B^{acd} - {A_d}^c B^{abd} + A^{bcd} {B_d}^a - A^{acd} {B_d}^b + A^{abd} {B_d}^c - \frac{1}{36}\epsilon^{abcdefghx} A_{def} B_{ghx} \end{cases} \tag{9}$$

The above result can be used, by setting $A = \Omega$, to define the infinitesimal variations of the multiplet $\mathcal{B}$ with respect to E_8 transformations. With a parameter multiplet $\{{\Omega_a}^b, \Omega_{abc}, \Omega^{abc}\}$, we can define the folllowing infinitesimal variations for the $\mathcal{B}$ components:

$$\begin{cases} \delta {B_a}^b = \left({\Omega_a}^c {B_c}^b - {\Omega_c}^b {B_a}^c\right) + \frac{1}{2}\left\{\Omega_{acd} B^{bcd} - \Omega^{bcd} B_{acd} - \frac{1}{9}{\delta_a}^b \left(\Omega_{cde} B^{cde} - \Omega^{cde} B_{cde}\right)\right\} \\ \delta B_{abc} = {\Omega_a}^d B_{bcd} - {\Omega_b}^d B_{acd} + {\Omega_c}^d B_{abd} - \Omega_{bcd} {B_a}^d + \Omega_{acd} {B_b}^d - \Omega_{abd} {B_c}^d + \frac{1}{36}\epsilon_{abcdefghx} \Omega^{def} B^{ghx} \\ \delta B^{abc} = -{\Omega_d}^a B^{bcd} + {\Omega_d}^b B^{acd} - {\Omega_d}^c B^{abd} + \Omega^{bcd} {B_d}^a - \Omega^{acd} {B_d}^b + \Omega^{abd} {B_d}^c - \frac{1}{36}\epsilon^{abcdefghx} \Omega_{def} B_{ghx} \end{cases} \tag{10}$$

The above variations for the components of the $\mathcal{B}$, and with similar variations for an $\mathcal{A}$ multiplet, we can verify the invariance of the following bilinear form:

$$\mathcal{A} \cdot \mathcal{B} = {A_a}^b {B_b}^a + \frac{1}{3!} A_{abc} B^{abc} + \frac{1}{3!} A^{abc} B_{abc} \tag{11}$$

4 The Supersymmetric E_8 Gauge-Invariant Couplings

Let us consider Weyl fermions Ψ, with Dirac conjugates $\bar{\Psi}$, in the adjoint representation of E_8. The coupling of these to vector bosons $\mathcal{V}$, also in the adjoint representation of E_8, comes from the gauge-invariant Lagrangian term

$$\bar{\Psi} i\gamma^{\mu}\nabla_{\mu}\Psi \qquad \nabla_{\mu}\Psi = \partial_{\mu} - i\left[\mathcal{V}_{\mu}, \Psi\right] \tag{12}$$

Hence, we shall be interested in constructing the E_8 invariant coupling

$$\bar{\Psi}\left[\mathcal{V}, \Psi\right] \tag{13}$$

where we shall suppress the gamma matrices and the vector indices.

Introducing SU_9 components $\{\psi_a{}^b, \psi_{abc}, \psi^{abc}\}$, and their corresponding Dirac conjugates $\{\bar{\psi}_a{}^b, \bar{\psi}_{abc}, \bar{\psi}^{abc}\}$, for the fermionic multiplet, and gauge boson components $\{V_a{}^b, V_{abc}, V^{abc}\}$, our strategy for constructing the invariant coupling is by starting with the invariant bilinear

$$\bar{\psi}_a^b F_b{}^a + \frac{1}{3!}\bar{\psi}_{abc}F^{abc} + \frac{1}{3!}\bar{\psi}^{abc}F_{abc} \tag{14}$$

and then building the $\mathcal{F}$ components from the commutator $[\mathcal{V}, \Psi]$, as we have developed in the preceding section. The followings are the resulting terms:

$$\left\{\begin{array}{l} V_a{}^b\bar{\psi}_c{}^a\psi_b{}^c - V_a{}^b\bar{\psi}_b{}^c\psi_c{}^a + \frac{1}{2}V_a{}^b\bar{\psi}^{acd}\psi_{bcd} - \frac{1}{2}V_a{}^b\bar{\psi}_{bcd}\psi^{acd} \\ +\frac{1}{2}V_{abc}\bar{\psi}_d{}^a\psi^{bcd} - \frac{1}{2}V_{abc}\bar{\psi}^{abd}\psi_d{}^c + \frac{1}{216}\epsilon^{abcdefghx}V_{abc}\bar{\psi}_{def}\psi_{ghx} \\ +\frac{1}{2}V^{abc}\bar{\psi}_{abd}\psi_c{}^d - \frac{1}{2}V^{abc}\bar{\psi}_a{}^d\psi_{bcd} - \frac{1}{216}\epsilon_{abcdefghx}V^{abc}\bar{\psi}^{def}\psi^{ghx} \end{array}\right. \tag{15}$$

Notice that all indices in the above expression are in the complex fundamental representation of SU_9. Our purpose now is to split these SU_9 indices into three sectors: a first index denoted by 1 corresponding to a particle of charge -1, three indices denoted by $\{i, j, k, \cdots\}$ corresponding to charge $\frac{1}{3}$ quarks (color SU_3), and five indices denoted by $\{r, s, t, \cdots\}$ corresponding to neutral particles (family SU_5). This means that the electric charge operator, in the fundamental representation of SU_9, has diagonal eigenvalues

$$Q = \text{diag}\{-1, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 0, 0, 0, 0, 0\} \tag{16}$$

According to this splitting, we shall define the components of the V's and the ψ's with a somewhat familiar particle names.

First for the vector bosons $V_a{}^b$, we make the following definitions for the split components:

$$\left\{\begin{array}{lll} V_1{}^1 \to -\mathcal{A} + 2\mathcal{Z} & V_1{}^i \to \mathcal{X}^i & V_1{}^r \to \mathcal{W}^r \\ V_i{}^1 \to \mathcal{X}_i & V_i{}^j \to \mathcal{G}_i{}^j + \frac{1}{3}\delta_i{}^j\mathcal{A} - \frac{1}{3}\delta_i{}^j\mathcal{Z} & V_i{}^r \to \mathcal{Y}_i{}^r \\ V_r{}^1 \to \mathcal{W}_r & V_r{}^i \to \mathcal{Y}_r{}^i & V_r{}^s \to \mathcal{H}_r{}^s - \frac{1}{5}\delta_r{}^s\mathcal{Z} \end{array}\right. \tag{17}$$

Here $\mathcal{A}$ is the photon field, $\mathcal{Z}$ is like the Z-boson of electroweak theory, ${\mathcal{G}_i}^j$ is traceless and represents the vector gluons of color SU_3, ${\mathcal{H}_r}^s$ is traceless and represents neutral gauge fields of family SU_5 that mediate between similarly charged particles of different generations, the $\{\mathcal{W}^r, \mathcal{W}_r\}$ are five charged particles and their conjugates like $W^{\pm}$ of electroweak theory, the $\{\mathcal{X}^i, \mathcal{X}_i\}$ are like the charge $\pm\frac{4}{3}$ colored leptoquarks, or quark-antiquark, vector bosons of SU_5 grand unification, and $\{{\mathcal{Y}_i}^r, {\mathcal{Y}_r}^i\}$ are five colored particles that are like the charge $\pm\frac{1}{3}$ leptoquark bosons of SU_5 grand unification.

For the vector bosons V_{abc} we make the assignments:

$$\begin{cases} V_{1ij} \to \epsilon_{ijk}\mathcal{Q}^k \qquad V_{1ir} \to \mathcal{U}_{ir} \qquad V_{1rs} \to \mathcal{E}_{rs} \\ V_{ijk} \to \epsilon_{ijk}\mathcal{F}^* \qquad V_{ijr} \to \epsilon_{ijk}{\mathcal{P}_r}^k \qquad V_{irs} \to \mathcal{D}_{irs} \\ V_{rst} \to \frac{1}{2}\epsilon_{rstuv}\mathcal{N}^{uv} \end{cases} \tag{18}$$

and for their conjugates V^{abc}, we write:

$$\begin{cases} V^{1ij} \to \epsilon^{ijk}\mathcal{Q}_k \qquad V^{1ir} \to \mathcal{U}^{ir} \qquad V^{1rs} \to \mathcal{E}^{rs} \\ V^{ijk} \to \epsilon^{ijk}\mathcal{F} \qquad V^{ijr} \to \epsilon^{ijk}{\mathcal{P}_k}^r \qquad V^{irs} \to \mathcal{D}^{irs} \\ V^{rst} \to \frac{1}{2}\epsilon^{rstuv}\mathcal{N}_{uv} \end{cases} \tag{19}$$

Now moving to the fermionic assignments, we begin with the fermions ${\psi_a}^b$, and make the following assignments:

$$\begin{cases} {\psi_1}^1 \to -a + 2z \qquad {\psi_1}^i \to x^i \qquad {\psi_1}^r \to w^r \\ {\psi_i}^1 \to x_i \qquad {\psi_i}^j \to {g_i}^j + \frac{1}{3}{\delta_i}^j a - \frac{1}{3}{\delta_i}^j z \qquad {\psi_i}^r \to {y_i}^r \\ {\psi_r}^1 \to w_r \qquad {\psi_r}^i \to {y_r}^i \qquad {\psi_r}^s \to {h_r}^s - \frac{1}{5}{\delta_r}^s z \end{cases} \tag{20}$$

Notice that for fermionic particles, we are using small characters that are correspond to their supersymmetric vector boson gauge partners. For example, a is the photino, z is the the zino (corresponding to the $\mathcal{Z}$ boson), ${g_i}^j$ are the gluinos, etc. For the Dirac conjugates of the above, we write

$$\begin{cases} {\bar{\psi}_1}{}^1 \to -\bar{a} + 2\bar{z} \qquad {\bar{\psi}_i}{}^1 \to \bar{x}_i \qquad {\bar{\psi}_r}{}^1 \to w_r \\ {\bar{\psi}_1}{}^i \to \bar{x}^i \qquad {\bar{\psi}_i}{}^j \to {\bar{g}_i}{}^j + \frac{1}{3}{\delta_i}^j \bar{a} - \frac{1}{3}{\delta_i}^j \bar{z} \qquad {\bar{\psi}_r}{}^i \to {\bar{y}_r}{}^i \\ {\bar{\psi}_1}{}^r \to \bar{w}^r \qquad {\bar{\psi}_i}{}^r \to {\bar{y}_i}{}^r \qquad {\bar{\psi}_r}{}^s \to {\bar{h}_r}{}^s - \frac{1}{5}{\delta_r}^s \bar{z} \end{cases} \tag{21}$$

For the fermions ψ_{abc}, we make the assignments:

$$\begin{cases} \psi_{1ij} \to \epsilon_{ijk} q^k \qquad \psi_{1ir} \to u_{ir} \qquad \psi_{1rs} \to e_{rs} \\ \psi_{ijk} \to \epsilon_{ijk} f^* \qquad \psi_{ijr} \to \epsilon_{ijk} {p_r}^k \qquad \psi_{irs} \to d_{irs} \\ \psi_{rst} \to \frac{1}{2}\epsilon_{rstuv}\nu^{uv} \end{cases} \tag{22}$$

The Dirac conjugates of the above are

$$\begin{cases} \bar{\psi}^{1ij} \to \epsilon^{ijk} \bar{q}_k \qquad \bar{\psi}^{1ir} \to \bar{u}^{ir} \qquad \bar{\psi}^{1rs} \to \bar{e}^{rs} \\ \bar{\psi}^{ijk} \to \epsilon^{ijk} \bar{f}^* \qquad \bar{\psi}^{ijr} \to \epsilon^{ijk} \bar{p}_k^r \qquad \bar{\psi}^{irs} \to \bar{d}^{irs} \\ \bar{\psi}^{rst} \to \frac{1}{2}\epsilon^{rstuv}\bar{\nu}_{uv} \end{cases} \tag{23}$$

For the fermions ψ^{abc}, we make the assignments:

$$\begin{cases} \psi^{1ij} \to \epsilon^{ijk} q_k \qquad \psi^{1ir} \to u^{ir} \qquad \psi^{1rs} \to e^{rs} \\ \psi^{ijk} \to \epsilon^{ijk} f \qquad \psi^{ijr} \to \epsilon^{ijk} {p_k}^r \qquad \psi^{irs} \to d^{irs} \\ \psi^{rst} \to \frac{1}{2}\epsilon^{rstuv}\nu_{uv} \end{cases} \tag{24}$$

The Dirac conjugates of the above are

$$\begin{cases} \bar{\psi}_{1ij} \to \epsilon_{ijk} \bar{q}^k \qquad \bar{\psi}_{1ir} \to \bar{u}_{ir} \qquad \bar{\psi}_{1rs} \to \bar{e}_{rs} \\ \bar{\psi}_{ijk} \to \epsilon_{ijk} \bar{f} \qquad \bar{\psi}_{ijr} \to \epsilon_{ijk} \bar{p}_r^k \qquad \bar{\psi}_{irs} \to \bar{d}_{irs} \\ \bar{\psi}_{rst} \to \frac{1}{2}\epsilon_{rstuv}\bar{\nu}^{uv} \end{cases} \tag{25}$$

With the above assignments for bosonic and fermionic particles, we can proceed to decompose the various SU_9 invariant terms of the boson-fermion couplings given earlier in (15). However, we must remember to respect the tracelessness and the symmetries of the various particle assignments. For tracelessness, we have

$$\begin{cases} {\mathcal{G}_i}^i = 0 \qquad {\mathcal{H}_r}^r = 0 \\ {g_i}^i = 0 \qquad {h_r}^r = 0 \\ {\bar{g}_i}^i = 0 \qquad {\bar{h}_r}^r = 0 \end{cases} \tag{26}$$

The followings are antisymmetric with respect to their SU_5 family indices (r, s):

$$\begin{cases} \mathcal{E}_{rs} & \mathcal{E}^{rs} \\ \mathcal{N}_{rs} & \mathcal{N}^{rs} \\ \mathcal{D}_{irs} & \mathcal{D}^{irs} \\ e_{rs} & e^{rs} & \bar{e}_{rs} & \bar{e}^{rs} \\ \nu_{rs} & \nu^{rs} & \bar{\nu}_{rs} & \bar{\nu}^{rs} \\ d_{irs} & d^{irs} & \bar{d}_{irs} & \bar{d}^{irs} \end{cases} \tag{27}$$

Computing the split couplings of (15), that are symmetric with respect to color SU_3 and family SU_5, the number of terms involved attains several thousands. However, after symbolic manipulations and simplifications, we are left with 394 terms. In the followings, we shall give these terms according to the kind of bosonic particles involved.

4.1 Couplings to the Photon $\mathcal{A}$

The terms that couple the fermionic particles to the photon $\mathcal{A}$ are:

$$\mathcal{A} \times \begin{pmatrix} -\bar{f}\gamma f + \bar{f}^{*}\gamma f^{*} + \frac{1}{2}\bar{e}_{rs}\gamma e^{rs} - \frac{1}{2}\bar{e}^{rs}\gamma e_{rs} - \bar{w}_{r}\gamma w^{r} + \bar{w}^{r}\gamma w_{r} \\ +\frac{2}{3}\bar{u}_{ir}\gamma u^{ir} - \frac{2}{3}\bar{u}^{ir}\gamma u_{ir} - \frac{1}{6}\bar{d}_{irs}\gamma d^{irs} + \frac{1}{6}\bar{d}^{irs}\gamma d_{irs} \\ +\frac{2}{3}\bar{p}_{i}{}^{r}\gamma p_{r}{}^{i} - \frac{2}{3}\bar{p}_{r}{}^{i}\gamma p_{i}{}^{r} - \frac{1}{3}\bar{q}_{i}\gamma q^{i} + \frac{1}{3}\bar{q}^{i}\gamma q_{i} \\ -\frac{4}{3}\bar{x}_{i}\gamma x^{i} + \frac{4}{3}\bar{x}^{i}\gamma x_{i} - \frac{1}{3}\bar{y}_{i}{}^{r}\gamma y_{r}{}^{i} + \frac{1}{3}\bar{y}_{r}{}^{i}\gamma y_{i}{}^{r} \end{pmatrix} \tag{28}$$

The above displays the photonic couplings of all charged fermionic particles. Remember that we are dealing with Weyl fermions. Electron-like particles of charge -1 are described by e_{rs}, their number being $5 \times 4/2 = 10$, and their antiparticles are e^{rs}. The particle f is also electron-like, with f^* as its antiparticle. We also have 5 electron-like particles, the w^r, with antiparticles w_r. These are actually supersymmetric partners to the bosonic W's.

We have 5 up quarks u^{ir} (antiparticles u^{ir}), and 10 down quarks d^{irs} (antiparticles d_{irs}). Also $p_r{}^i$ are 5 up quarks (antiparticles $p_i{}^r$), and q^i is a single down quark (antiparticle q_i). The x^i is a charge $\frac{4}{3}$ fermion (antiparticle x_i) and $y_r{}^i$ are 5 charge $\frac{1}{3}$ fermions (antiparticle $y_i{}^r$). The x's and the y's are supersymmetric partners to quark-antiquark and leptoquark bosons of grand unified theories.

4.2 Couplings to the $\mathcal{Z}$ Boson

Here we have the couplings of the fermionic particles to the $\mathcal{Z}$ boson.

$$\mathcal{Z} \times \left(\begin{array}{l} \bar f \gamma f - \bar f^* \gamma f^* - \frac{4}{5} \bar e_{rs} \gamma e^{rs} + \frac{4}{5} \bar e^{rs} \gamma e_{rs} + \frac{11}{5} \bar w_r \gamma w^r - \frac{11}{5} \bar w^r \gamma w_r \\ -\frac{22}{15} \bar u_{i,r} \gamma u^{ir} + \frac{22}{15} \bar u^{i,r} \gamma u_{ir} + \frac{11}{30} \bar d_{irs} \gamma d^{irs} - \frac{11}{30} \bar d^{irs} \gamma d_{irs} \\ -\frac{13}{15} \bar p_i{}^r \gamma p_r{}^i + \frac{13}{15} \bar p_r{}^i \gamma p_i{}^r + \frac{4}{3} \bar q_i \gamma q^i - \frac{4}{3} \bar q^i \gamma q_i \\ +\frac{7}{3} \bar x_i \gamma x^i - \frac{7}{3} \bar x^i \gamma x_i + \frac{2}{15} \bar y_i{}^r \gamma y_r{}^i - \frac{2}{15} \bar y_r{}^i \gamma y_i{}^r \\ -\frac{3}{10} \bar \nu_{rs} \gamma \nu^{rs} + \frac{3}{10} \bar \nu^{rs} \gamma \nu_{rs} \end{array} \right) \tag{29}$$

The above couplings involve both the charged and the neutral particles. Notice that we have 10 neutrino-like fermions ν_{rs}, with antiparticles ν^{rs}.

4.3 Couplings to the $\mathcal{W}$ Bosons

Here we have the couplings of the fermionic particles to the $\mathcal{W}$ bosons:

$$\mathcal{W}_r \times \left(\begin{array}{l} -\frac{1}{4} \epsilon^{rstuv} \bar e_{st} \gamma \nu_{uv} + \frac{1}{4} \epsilon^{rstuv} \bar \nu_{st} \gamma e_{uv} \\ +\bar a \gamma w^r - \bar w^r \gamma a - \frac{11}{5} \bar z \gamma w^r + \frac{11}{5} \bar w^r \gamma z \\ +\bar u_{is} \gamma d^{irs} - \bar d^{irs} \gamma u_{is} - \bar q^i \gamma p_i{}^r + \bar p_i^r \gamma q^i \\ -\bar w^s \gamma h_s{}^r + \bar h_s{}^r \gamma w^s - \bar x^i \gamma y_i{}^r + \bar y_i{}^r \gamma x^i \end{array} \right) + \text{conj.} \tag{30}$$

In the above, we have fermionic couplings to 5 charged bosons, while the couplings to their charge conjugates are suppressed. Notice that we have here five analogs of the charges W's of electroweak theory, however, they transform various electrons to various neutrinos in a rather subtle way, and likewise, for their mediation between up quarks and down quarks, between photinos and winos, between zinos and winos, etc. We shall return to a discussion of the kind of associated phenomenolgy to be expected in high-energy hadronic collisions.

4.4 Couplings to the $\mathcal{G}$ Gluons

In the followings, we have the couplings of the fermionic particles to the $\mathcal{G}$ gluons of color SU_3. Of course, only the colored fermions, quarks and otherwise, are the ones that participate in these couplings:

$$\mathcal{G}_i{}^j \times \left(\begin{array}{l} -\bar{u}_{jr}\gamma u^{ir} + \bar{u}^{ir}\gamma u_{jr} - \frac{1}{2}\bar{d}_{jrs}\gamma d^{irs} + \frac{1}{2}\bar{d}^{irs}\gamma d_{jrs} \\ -\bar{p}_j{}^r\gamma p_r{}^i + \bar{p}_r{}^i\gamma p_j{}^r - \bar{q}_j\gamma q^i + \bar{q}^i\gamma q_j \\ -\bar{x}_j\gamma x^i + \bar{x}^i\gamma x_j - \bar{y}_j{}^r\gamma y_r{}^i + \bar{y}_r{}^i\gamma y_j{}^r \\ -\bar{g}_j{}^k\gamma g_k{}^i + \bar{g}_k{}^i\gamma g_j{}^k \end{array} \right) \tag{31}$$

Notice that besides the up and down quarks of types $(u_{ir}, d_{irs}, p_i{}^r, q_i)$, and the fermionic partners $(x_i, y_i{}^r)$ of grand unified quark-antiquark and leptoquark bosons, we also have the gluinos g_i^k.

4.5 Couplings to the $\mathcal{H}$ Bosons

In the followings, we have the couplings of the fermionic particles to the $\mathcal{H}$ bosons. The latter are the gauge particles of family (or horizontal) SU_5, being neutral particles that transform fermions, charged or neutral, colored or colorless, into particles of the same type, but of different mass.

$$\mathcal{H}_r{}^s \left(\begin{array}{l} -\bar{e}_{st}\gamma e^{rt} + \bar{e}^{rt}\gamma e_{st} - \bar{\nu}_{st}\gamma\nu^{rt} + \bar{\nu}^{rt}\gamma\nu_{st} - \bar{w}_s\gamma w^r + \bar{w}^r\gamma w_s \\ -\bar{u}_{is}\gamma u^{ir} + \bar{u}^{ir}\gamma u_{is} - \bar{d}_{ist}\gamma d^{irt} + \bar{d}^{irt}\gamma d_{ist} \\ +\bar{p}_i{}^r\gamma p_s{}^i - \bar{p}_s{}^i\gamma p_i{}^r + \bar{y}_i{}^r\gamma y_s{}^i - \bar{y}_s{}^i\gamma y_i{}^r \\ -\bar{h}_s{}^t\gamma h_t{}^r + \bar{h}_t{}^r\gamma h_s{}^t \end{array} \right) \tag{32}$$

Of course, only the fermions which carry family SU_5 indices do participate in the above couplings. Notice the existence of the neutral fermions $h_r{}^s$. These 24 fermions are the supersymmetric partners of the neutral SU_5 gauge bosons $\mathcal{H}_r{}^s$ themselves.

4.6 Couplings to the $\mathcal{X}$ Bosons

Here we give the couplings of the fermionic particles to the $\mathcal{X}$ bosons, the socalled leptoquarks in grand unified theories, being carriers of charges $\pm\frac{4}{3}$.

$$\mathcal{X}^i \times \left(\begin{array}{l} -\frac{1}{2}\bar{d}_{irs}\gamma e^{rs} + \frac{1}{2}\bar{e}^{rs}\gamma d_{irs} - \bar{f}\gamma q_i + \bar{q}_i\gamma f^* \\ +\epsilon_{ijk}\bar{u}^{jr}\gamma p_r{}^k + \epsilon_{ijk}\bar{p}_r{}^j\gamma u^{kr} + \bar{w}_r\gamma y_i{}^r - \bar{y}_i{}^r\gamma w_r \\ -\frac{4}{3}\bar{a}\gamma x_i + \frac{4}{3}\bar{x}_i\gamma a + \frac{7}{3}\bar{z}\gamma x_i - \frac{7}{3}\bar{x}_i\gamma z \\ +\bar{x}_j\gamma g_i{}^j - \bar{g}_i{}^j\gamma x_j \end{array} \right) + \text{conj.} \tag{33}$$

In the above, we have suppressed the conjugate couplings involving $\mathcal{X}_i$. Notice that the neutrinos ν_{rs} do not participate in the above couplings.

4.7 Couplings to the $\mathcal{Y}$ Bosons

Here we give the couplings of the fermionic particles to the $\mathcal{Y}$ bosons, also leptoquarks in grand unified theories, being carriers of charges $\pm\frac{1}{3}$.

$$\mathcal{Y}_r{}^i \times \left(\begin{array}{l} -\frac{1}{3}\bar{a}\gamma y_i{}^r + \frac{1}{3}\bar{y}_i{}^r\gamma a - \bar{f}\gamma p_i{}^r + \bar{p}_i{}^r\gamma f^* \\ +\frac{2}{15}\bar{z}\gamma y_i{}^r - \frac{2}{15}\bar{y}_i{}^r\gamma z - \bar{u}_{is}\gamma e^{rs} + \bar{e}^{rs}\gamma u_{is} \\ -\frac{1}{4}\epsilon^{rstuv}\bar{d}_{ist}\gamma\nu_{uv} + \frac{1}{4}\epsilon^{rstuv}\bar{\nu}_{st}\gamma d_{iuv} - \epsilon_{ijk}\bar{q}^j\gamma u^{kr} - \epsilon_{ijk}\bar{u}^{jr}\gamma q^k \\ -\bar{x}_i\gamma w^r + \bar{w}^r\gamma x_i - \epsilon_{ijk}\bar{d}^{jrs}\gamma p_s{}^k - \epsilon_{ijk}\bar{p}_s{}^j\gamma d^{krs} \\ -\bar{g}_i{}^j\gamma y_j{}^r + \bar{y}_j{}^r\gamma g_i{}^j + \bar{h}_s{}^r\gamma y_i{}^s - \bar{y}_i{}^s\gamma h_s{}^r \end{array} \right) + \text{conj.} \qquad (34)$$

In the above, we have suppressed the conjugate couplings involving $\mathcal{Y}_i{}^r$.

4.8 Couplings to the $\mathcal{Q}$ Bosons

Here we give the couplings of the fermionic particles to the $\mathcal{Q}$ bosons. The latter are colored charge $\pm\frac{1}{3}$ vector bosons.

$$\mathcal{Q}_i \times \left(\begin{array}{l} \frac{1}{2}\bar{\nu}_{rs}\gamma d^{irs} - \frac{1}{2}\bar{d}^{irs}\gamma\nu_{rs} + \bar{f}\gamma x^i - \bar{x}^i\gamma f^* \\ +\epsilon^{ijk}\bar{u}_{jr}\gamma y_k{}^r + \epsilon^{ijk}\bar{y}_j{}^r\gamma u_{kr} - \bar{w}^r\gamma p_r{}^i + \bar{p}_r{}^i\gamma w^r \\ \frac{1}{3}\bar{a}\gamma q^i - \frac{1}{3}\bar{q}^i\gamma a - \frac{4}{3}\bar{z}\gamma q^i + \frac{4}{3}\bar{q}^i\gamma z \\ -\bar{q}^j\gamma g_j{}^i + \bar{g}_j{}^i\gamma q^j \end{array} \right) + \text{conj.} \qquad (35)$$

In the above, we have suppressed the conjugate couplings involving $\mathcal{Q}^i$. Notice that the electron-like particles e_{rs} do not participate in the above couplings.

4.9 Couplings to the $\mathcal{U}$ Bosons

Here we give the couplings of the fermionic particles to the $\mathcal{U}$ bosons. The latter are 5 vector bosons of charges $\pm\frac{2}{3}$, being supersymmetric partners to the quarks u_{ir}.

$$\mathcal{U}^{ir}\times\left(\begin{array}{l}\frac{2}{3}\bar{a}\gamma u_{ir}-\frac{2}{3}\bar{u}_{ir}\gamma a-\frac{22}{15}\bar{z}\gamma u_{ir}+\frac{22}{15}\bar{u}_{ir}\gamma z\\ +\bar{d}_{irs}\gamma w^{s}-\bar{w}^{s}\gamma d_{irs}-\bar{e}_{rs}\gamma {y_i}^{s}+\bar{y}_i{}^{s}\gamma e_{rs}\\ +\bar{u}_{is}\gamma {h_r}^{s}-\bar{h}_r{}^{s}\gamma u_{is}+\bar{u}_{jr}\gamma {g_i}^{j}-\bar{g}_i{}^{j}\gamma u_{jr}\\ +\bar{\nu}_{rs}\gamma {p_i}^{s}-\bar{p}_i{}^{s}\gamma \nu_{rs}-\epsilon_{ijk}\bar{q}^{j}\gamma {y_r}^{k}-\epsilon_{ijk}\bar{y}_r{}^{j}\gamma q^{k}\\ +\epsilon_{ijk}\bar{x}^{j}\gamma {p_r}^{k}+\epsilon_{ijk}\bar{p}_r{}^{j}\gamma x^{k}-\frac{1}{4}\epsilon_{ijk}\epsilon_{rstuv}\bar{d}^{jst}\gamma d^{kuv}\end{array}\right)+\text{conj.}\qquad(36)$$

In the above we suppress the conjugate terms involving $\mathcal{U}_{ir}$.

4.10 Couplings to the $\mathcal{E}$ Bosons

Here we give the couplings of the fermionic particles to the $\mathcal{E}$ vector bosons. The latter are 10 charge ± 1 particles that are supersymmetric partners to the electron-like particles e_{rs}.

$$\mathcal{E}_{rs}\times\left(\begin{array}{l}-\frac{1}{2}\bar{a}\gamma e^{rs}+\frac{1}{2}\bar{e}^{rs}\gamma a+\frac{1}{2}\bar{f}\gamma\nu^{rs}-\frac{1}{2}\bar{\nu}^{rs}\gamma f^{*}\\ +\frac{4}{5}\bar{z}\gamma e^{rs}-\frac{4}{5}\bar{e}^{rs}\gamma z+\frac{1}{4}\epsilon^{rstuv}\bar{d}_{itu}\gamma {p_v}^{i}-\frac{1}{4}\epsilon^{rstuv}\bar{p}_t{}^{i}\gamma d_{iuv}\\ +\frac{1}{4}\epsilon^{rstuv}\bar{w}_t\gamma\nu_{uv}-\frac{1}{4}\epsilon^{rstuv}\bar{\nu}_{tu}\gamma w_v+\frac{1}{2}\bar{x}_i\gamma d^{irs}-\frac{1}{2}\bar{d}^{irs}\gamma x_i\\ -\bar{e}^{rt}\gamma {h_t}^{s}-\bar{h}_t{}^{r}\gamma e^{st}+\bar{y}_i{}^{r}\gamma u^{is}+\bar{u}^{ir}\gamma {y_i}^{s}\end{array}\right)+\text{conj.}\qquad(37)$$

In the above we suppress the conjugate terms involving $\mathcal{E}^{rs}$.

4.11 Couplings to the $\mathcal{F}$ Bosons

Here we give the couplings of the fermionic particles to the $\mathcal{F}$ vector bosons. The latter are charge ± 1 particles that are supersymmetric partners to the (f, f^*) fermions.

$$\mathcal{F}\times\left(\begin{array}{l}-\bar{a}\gamma f^{*}+\bar{f}\gamma a-\bar{f}\gamma z+\bar{z}\gamma f^{*}\\ -\bar{x}_i\gamma q^{i}+\bar{q}^{i}\gamma x_i-\frac{1}{2}\bar{\nu}\gamma e^{rs}+\frac{1}{2}\bar{e}^{rs}\gamma\nu_{rs}\\ +\bar{p}_r{}^{i}\gamma {y_i}^{r}-\bar{y}_i{}^{r}\gamma {p_r}^{i}\end{array}\right)+\text{conj.}\qquad(38)$$

In the above, we suppress the conjugate terms involving $\mathcal{F}^*$.

4.12 Couplings to the $\mathcal{P}$ Bosons

Here we give the couplings of the fermionic particles to the $\mathcal{P}$ vector bosons. The latter are charge $\pm\frac{2}{3}$ particles, being supersymmetric partners to $(p_i{}^r, p_r{}^i)$ fermions.

$$\mathcal{P}_i{}^r \times \begin{pmatrix} -\frac{2}{3}\bar{a}\gamma p_r{}^i + \frac{2}{3}\bar{p}_r{}^i\gamma a + \bar{f}\gamma y_r{}^i - \bar{y}_r{}^i\gamma f^* \\ +\frac{13}{15}\bar{z}\gamma p_r{}^i - \frac{13}{15}\bar{p}_r{}^i\gamma z - \epsilon^{ijk}\bar{d}_{jrs}\gamma y_k{}^s - \epsilon^{ijk}\bar{y}_j{}^s\gamma d_{krs} \\ -\epsilon^{ijk}\bar{u}_{jr}\gamma x_k - \epsilon^{ijk}\bar{x}_j\gamma u_{kr} - \bar{w}_r\gamma q^i + \bar{q}^i\gamma w_r \\ +\bar{\nu}_{rs}\gamma u^{is} - \bar{u}^{is}\gamma \nu_{rs} + \frac{1}{4}\epsilon_{rstuv}\bar{d}^{ist}\gamma e^{uv} - \frac{1}{4}\epsilon_{rstuv}\bar{e}^{st}\gamma d^{iuv} \\ +\bar{g}_j{}^i\gamma p_r{}^j - \bar{p}_r{}^j\gamma g_j{}^i - \bar{h}_r{}^s\gamma p_s{}^i + \bar{p}_s{}^i\gamma h_r{}^s \end{pmatrix} + \text{conj.} \tag{39}$$

In the above we suppress the conjugate terms involving $\mathcal{P}_r{}^i$.

4.13 Couplings to the $\mathcal{D}$ Bosons

Here we give the couplings of the fermionic particles to the $\mathcal{D}$ vector bosons. The latter are 10 charge $\pm\frac{1}{3}$ particles that are supersymmetric partners to the (d_{irs}, d^{irs}) quarks.

$$\mathcal{D}^{irs} \times \begin{pmatrix} -\frac{1}{6}\bar{a}\gamma d_{irs} + \frac{1}{6}\bar{d}_{irs}\gamma a + \frac{11}{30}\bar{z}\gamma d_{irs} - \frac{11}{30}\bar{d}_{irs}\gamma z \\ +\bar{d}_{irt}\gamma h_s{}^t + \bar{h}_r{}^t\gamma d_{ist} + \frac{1}{2}\bar{d}_{jrs}\gamma g_i{}^j - \frac{1}{2}\bar{g}_i{}^j\gamma d_{jrs} \\ +\frac{1}{2}\bar{e}_{rs}\gamma x_i - \frac{1}{2}\bar{x}_i\gamma e_{rs} + \frac{1}{2}\bar{q}_i\gamma \nu_{rs} - \frac{1}{2}\bar{\nu}_{rs}\gamma q_i \\ +\bar{u}_{ir}\gamma w_s + \bar{w}_r\gamma u_{is} - \frac{1}{4}\epsilon_{rstuv}\bar{e}^{tu}\gamma p_i{}^v - \frac{1}{4}\epsilon_{rstuv}\bar{p}_i{}^t\gamma e^{uv} \\ +\epsilon_{ijk}\bar{p}_r{}^i\gamma y_s{}^k - \epsilon_{ijk}\bar{y}_r{}^j\gamma p_s{}^k + \frac{1}{4}\epsilon_{rstuv}\bar{\nu}^{tu}\gamma y_i{}^v - \frac{1}{4}\epsilon_{rstuv}\bar{y}_i{}^t\gamma \nu^{uv} \\ -\frac{1}{4}\epsilon_{ijk}\epsilon_{rstuv}\bar{d}^{jtu}\gamma u^{kv} - \frac{1}{4}\epsilon_{ijk}\epsilon_{rstuv}\bar{u}^{jt}\gamma d^{kuv} \end{pmatrix} + \text{conj.} \tag{40}$$

In the above we suppress the conjugate terms involving $\mathcal{D}_{irs}$.

4.14 Couplings to the $\mathcal{N}$ Bosons

We give here the couplings of the fermionic particles to the $\mathcal{N}$ vector bosons. The latter are neutral supersymmetric partners to the neutrinos (ν_{rs}, ν^{rs}).

$$\mathcal{N}^{rs} \times \left(\begin{array}{l} -\frac{1}{2}\bar{f}\gamma e_{rs} + \frac{1}{2}\bar{e}_{rs}\gamma f^* - \frac{3}{10}\bar{z}\gamma\nu_{rs} + \frac{3}{10}\bar{\nu}_{rs}\gamma z \\ -\frac{1}{2}\bar{d}_{irs}\gamma q^i + \frac{1}{2}\bar{q}^i\gamma d_{irs} + \bar{u}_{ir}\gamma p_s{}^i + \bar{p}_r{}^i\gamma u_{is} \\ +\bar{\nu}_{rt}\gamma h_s{}^t + \bar{h}_r{}^t\gamma\nu_{st} - \frac{1}{4}\epsilon_{rstuv}\bar{d}^{itu}\gamma y_i{}^v + \frac{1}{4}\epsilon_{rstuv}\bar{y}_i{}^t\gamma d^{iuv} \\ -\frac{1}{4}\epsilon_{rstuv}\bar{e}^{tu}\gamma w^v + \frac{1}{4}\epsilon_{rstuv}\bar{w}^t\gamma e^{uv} \end{array} \right) + \text{conj.} \tag{41}$$

In the above we suppress the conjugate terms involving $\mathcal{N}_{rs}$.

5 Discussion

We have presented a new scheme of supersymmetric E_8 gauge unification, where a symmetry between particles and antiparticles is promoted, rather than that between quark-lepton generations and their mirror counterparts. In this scheme, we seem to be able to evade the problem associated with the prediction of mirror generations, any evidence of which has never been found.

One of the interesting points associated with the exhibited coupling terms, that are invariant with respect to color SU_3 and family SU_5, which descend from the SU_9 subalgebra of E_8, regards the participation of several W-like particles (for instance the $\mathcal{W}_r$ and the $\mathcal{W}^r$ bosons, with r being an SU_5 family index) in the decays of leptons (the electron-like particles, for instance, represented here by e_{rs}, and their associated neutrinos ν_{rs}, and some others) and quarks (upquarks and downquarks, represented here by u_{ir} and d^{irs}, respectively, and some others). We observe that such decays would involve more than one W, and for a specific lepton, for instance, more than a single neutrino. In such a complex scheme, it would be more significant to talk about multiple neutrinos associated with multiple charged leptons, rather than a single neutrino associated with a single charged lepton (also multiple downquarks associated with multiple upquarks, etc.). The mediating vector boson would always be a multiple object. High-energy hadronic collisions, with the production of heavier and heavier particles, should be able to reveal whether such a scenario would have any foundation.

We hope to return to several other aspects of this E_8 unification, and phenomenology, in other articles.

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