The SU_7 Structure of O_{14} and Boson-Fermion Couplings

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Abstract

We construct the O_{14} algebra in terms of the tensorial representations of its SU_7 subalgebra. Subsequently, we construct the gauge-invariant boson-fermion coupling, and decompose it completely into terms exhibiting color SU_3 and family SU_3 symmetries. A picture of the particle spectrum emerges where quarks and leptons, as well as vector bosons, would appear as singlets or triplets with respect to family SU_3. Accordingly, we conjecture the possible existence of a 4th generation of fermions, as well as the imminent existence of other W-like vector bosons, in high-energy collider experiments.

1 Introduction

The O_{14} Lie algebra is a consolidating framework for an SU_7 grand unification[1], [2], [3] model. It is also the maximal internal symmetry that emerges in an 18 dimensional gravidynamic unification[4] of the spacetime and internal symmetries of *four generations* of quarks and leptons. With the advent of high-energy collider experiments, it is very important that we can have deeper understanding of such extended unification theories, with the purpose of deriving some observational implications.

The fundamental multiplet of O_{14} to which the fermionic *Weyl* particles could belong has 64 components. These would contain two **16**'s and two $\mathbf{16}^*$'s of the O_{10} subalgebra. Hence, normally, in the sense of O_{10} or SU_5 grand unification[5], [6], [7], they would be regarded as describing two generations of quarks and leptons together with their *mirror conjugates* (with regard to low-energy weak interactions). However, the decomposition of the 64-component multiplet with respect to the SU_7 subalgebra takes the form:

$$\mathbf{64} = \mathbf{1} + \mathbf{7} + \mathbf{21}^* + \mathbf{35} \tag{1}$$

Here, the **7** is the fundamental SU_7 multiplet, while the **21** is a 2nd rank antisymmetric SU_7 tensor, and the **35** is a 3rd rank antisymmetric SU_7 tensor.

Our purpose in this article is to present a description of the O_{14} unification model in terms of SU_7 and its subsequent decomposition into a *color* SU_3 and a *family* SU_3 subalgebras. The new picture that shall be uncovered pertains to the *multiplicity* of the weak bosons, the understanding of whose phenomenology *may evade the rather enigmatic picture that is associated with mirror generations.*

In the following section, we shall start by formulating the O_{14} algebra in terms of the formalism that pertains to its SU_7 subalgebra. We then proceed to construct the gauge-invariant boson-fermion couplings in a decomposition of SU_7 exhibiting color SU_3 and family SU_3.

2 The O_{14} Algebra in SU_7 Notation

The generators of the O_{14} algebra are 91 in number. They are composed of the SU_7 generators $J_a{}^b$, this being traceless $J_a{}^a = 0$, a U_1 generator J, and conjugate generators Q_{ab} and Q^{ab}. The latter are antisymmetric SU_7 tensors. Notice that the indices $(a, b, c, \cdots)$ will be used to correspond to the fundamental SU_7 multiplet, $a = 1, 2, \ldots, 7$, and a summation over repeated indices is always implied.

We begin by writing the commutators that pertain to the SU_7 subalgebra. We have

$$\left[J_a{}^b, J_c{}^d\right] = \left(\delta_c{}^b J_a{}^d - \delta_a{}^d J_c{}^b\right) \tag{2}$$

Of course, the U_1 generator J would commute with $J_a{}^b$. Now, with regard to the commutation relations of Q_{ab} and Q^{ab} with J and $J_a{}^b$, we first have

$$\begin{cases} [J, Q_{ab}] = \frac{2}{\sqrt{7}} Q_{ab} \\ \left[J, Q^{ab}\right] = -\frac{2}{\sqrt{7}} Q^{ab} \end{cases} \tag{3}$$

And we have

$$\begin{cases} \left[J_a{}^b, Q_{cd}\right] = \delta_c{}^b Q_{ad} - \delta_d{}^b Q_{ac} - \frac{2}{7}\delta_a{}^b Q_{cd} \\ \left[J_a{}^b, Q^{cd}\right] = -\delta_a{}^c Q^{bd} + \delta_a{}^d Q^{bc} + \frac{2}{7}\delta_a{}^b Q^{cd} \end{cases} \tag{4}$$

The O_{14} algebra is completed with the followings:

$$\begin{cases} [Q_{ab}, Q_{cd}] = 0 \\ \left[Q^{ab}, Q^{cd}\right] = 0 \\ \left[Q_{ab}, Q^{cd}\right] = \frac{2}{\sqrt{7}}\left(\delta_a{}^c \delta_c{}^d - \delta_a{}^d \delta_b{}^c\right) J + \left(\delta_a{}^c J_b{}^d - \delta_a{}^d J_b{}^c + \delta_b{}^d J_a{}^c - \delta_b{}^c J_a{}^d\right) \end{cases} \tag{5}$$

Notice that the following quadratic (Casimir) operator commutes with the generators J, $J_a{}^b$, Q_{ab}, and Q^{ab}, and all the Jacobi identities involving these generators are satisfied.

$$J^2 + J_a{}^b J_b{}^a + \frac{1}{2} Q_{ab} Q^{ab} + \frac{1}{2} Q^{ab} Q_{ab} \tag{6}$$

3 The Adjoint Multiplet

The adjoint multiplet of the O_{14} algebra is a set of components that correspond to the generators. For example, let us introduce the module

$$\mathcal{A} = AJ + A_a{}^b J_b{}^a + \frac{1}{2} A_{ab} Q^{ab} + \frac{1}{2} A^{ab} Q_{ab} \tag{7}$$

Here $A_a{}^b$ is traceless, $A_a{}^a = 0$, while A_{ab} and A^{ab} are antisymmetric in their SU_7 indices. Introducing another similar module $\mathcal{B}$, with components $\{B, B_a{}^b, B_{ab}, B^{ab}\}$, we can compute the commutator $[\mathcal{A}, \mathcal{B}]$, to obtain a new adjoint module $\mathcal{F}$. From the O_{14} algebra, we can easily obtain the components of $\mathcal{F}$ as composites of the $\mathcal{A}$ and the $\mathcal{B}$ components:

$$\begin{cases} F = \frac{1}{\sqrt{7}} \left(A^{ab} B_{ab} - A_{ab} B^{ab} \right) \\ F_a{}^b = -\left(A_a{}^c B_c{}^b - A_c{}^b B_a{}^c \right) - \left(A_{ac} B^{bc} - A^{bc} B_{ac} \right) - \frac{1}{7} \delta_a{}^b \left(A^{cd} B_{cd} - A_{cd} B^{cd} \right) \\ F_{ab} = -\frac{2}{\sqrt{7}} \left(A B_{ab} - A_{ab} B \right) + \left(A_a{}^c B_{bc} - A_b{}^c B_{ac} \right) + \left(A_{ac} B_b{}^c - A_{bc} B_a{}^c \right) \\ F^{ab} = \frac{2}{\sqrt{7}} \left(A B^{ab} - A^{ab} B \right) - \left(A_c{}^a B^{bc} - A_c{}^b B^{ac} \right) - \left(A^{ac} B_c{}^b - A^{bc} B_c{}^a \right) \end{cases} \tag{8}$$

We can use the above result in order to obtain the O_{14} infinitesimal transformations for the components of the adjoint module $\mathcal{B}$. This is done simply by substituting Ω for A in the above. Hence, with an adjoint parameter multiplet $\{\Omega, \Omega_a{}^b, \Omega_{ab}, \Omega^{ab}\}$, we obtain the following infinitesimal transformations:

$$\begin{cases} \delta B = \frac{1}{\sqrt{7}} \left(\Omega^{ab} B_{ab} - \Omega_{ab} B^{ab} \right) \\ \delta B_a{}^b = -\left(\Omega_a{}^c B_c{}^b - \Omega_c{}^b B_a{}^c \right) - \left(\Omega_{ac} B^{bc} - \Omega^{bc} B_{ac} \right) - \frac{1}{7} \delta_a{}^b \left(\Omega^{cd} B_{cd} - \Omega_{cd} B^{cd} \right) \\ \delta B_{ab} = -\frac{2}{\sqrt{7}} \left(\Omega B_{ab} - \Omega_{ab} B \right) + \left(\Omega_a{}^c B_{bc} - \Omega_b{}^c B_{ac} \right) + \left(\Omega_{ac} B_b{}^c - \Omega_{bc} B_a{}^c \right) \\ \delta B^{ab} = \frac{2}{\sqrt{7}} \left(\Omega B^{ab} - \Omega^{ab} B \right) - \left(\Omega_c{}^a B^{bc} - \Omega_c{}^b B^{ac} \right) - \left(\Omega^{ac} B_c{}^b - \Omega^{bc} B_c{}^a \right) \end{cases} \tag{9}$$

Under the above transformations, defined for any two adjoint modules $\mathcal{A}$ and $\mathcal{B}$, the following bilinear is left invariant:

$$\mathcal{A} \cdot \mathcal{B} = AB + A_a{}^b B_b{}^a + \frac{1}{2} A_{ab} B^{ab} + \frac{1}{2} A^{ab} B_{ab} \tag{10}$$

4 The Fundamental Spinorial Multiplet

The fundamental multiplet of O_{14} has 64 components. This comes from the fact that a spinor in 14 dimensions has $2^7 = 128$ components. However, this can be decomposed into two *chiralities*, each of which would correspond to an irreducible representation of dimension 64. As mentioned earlier, the 64 components of a fundamental O_{14} multiplet would decompose like $\mathbf{64} = \mathbf{1} + \mathbf{7} + \mathbf{21}^* + \mathbf{35}$, with respect to SU_7. Hence, in order to construct a fundamental module, we introduce the operators K, K_a, K^{ab}, and K_{abc}. Here, K^{ab} and K_{abc} are antisymmetric in their SU_7 indices. We now write the commutators of these operators with the generators of O_{14}. First, we have the following commutators with the U_1 generator J:

$$\begin{cases} [J, K] = \frac{\sqrt{7}}{2} K \\ [J, K_a] = -\frac{5}{2\sqrt{7}} K_a \\ \left[J, K^{ab}\right] = \frac{3}{2\sqrt{7}} K^{ab} \\ [J, K_{abc}] = -\frac{1}{2\sqrt{7}} K_{abc} \end{cases} \tag{11}$$

The followings are the commutators with the SU_7 generators ${J_a}^b$:

$$\begin{cases} \left[J_a^b, K\right] = 0 \\ \left[J_a^b, K_c\right] = {\delta_c}^b K_a - \frac{1}{7}{\delta_a}^b K_c \\ \left[J_a^b, K^{cd}\right] = -\left({\delta_a}^c K^{bd} - {\delta_a}^d K^{bc} - \frac{2}{7}{\delta_a}^b K^{cd}\right) \\ \left[J_a^b, K_{cde}\right] = {\delta_c}^b K_{ade} - {\delta_d}^b K_{ace} + {\delta_e}^b K_{acd} - \frac{3}{7}{\delta_a}^b K_{cde} \end{cases} \tag{12}$$

The followings are the commutators with the generators Q_{ab}:

$$\begin{cases} [Q_{ab}, K] = 0 \\ [Q_{ab}, K_c] = K_{abc} \\ \left[Q_{ab}, K^{cd}\right] = -\left({\delta_a}^c {\delta_b}^d - {\delta_a}^d {\delta_b}^c\right) K \\ [Q_{ab}, K_{cde}] = \frac{1}{2}\epsilon_{abcdefg} K^{fg} \end{cases} \tag{13}$$

The followings are the commutators with the generators Q_{ab}:

$$\begin{cases} \left[Q^{ab}, K\right] = -K^{ab} \\ \left[Q^{ab}, K_c\right] = 0 \\ \left[Q^{ab}, K^{cd}\right] = \frac{1}{3!}\epsilon^{abcdefg} K_{efg} \\ \left[Q^{ab}, K_{cde}\right] = \begin{cases} \left({\delta_d}^a {\delta_e}^b - {\delta_d}^b {\delta_e}^a\right) K_c + \left({\delta_c}^b {\delta_e}^a - {\delta_c}^a {\delta_e}^b\right) K_d \\ + \left({\delta_c}^a {\delta_d}^b - {\delta_c}^b {\delta_d}^a\right) K_e \end{cases} \end{cases} \tag{14}$$

We can verify that the Jacobi identities involving any two O_{14} generators with either of K, K_a, K^{ab}, or K_{abc}, are all satisfied.

Now, in order to be able to write Lagrangians for Weyl fermions, we must also introduce the conjugate representation with operators K^*, K^a, K_{ab}, and K^{abc}. This would be needed to represent the Dirac conjugate of the fermion field. Again, K_{ab} and K^{abc} are antisymmetric SU_7 tensors. The commutators with the U_1 generator J are the followings:

$$\begin{cases} [J, K^*] = -\frac{\sqrt{7}}{2} K^* \\ [J, K^a] = \frac{5}{2\sqrt{7}} K^a \\ [J, K_{ab}] = -\frac{3}{2\sqrt{7}} K_{ab} \\ \left[J, K^{abc}\right] = \frac{1}{2\sqrt{7}} K^{abc} \end{cases} \tag{15}$$

The commutators with the SU_7 generators ${J_a}^b$ are:

$$\begin{cases} \left[{J_a}^b, K^*\right] = 0 \\ \left[{J_a}^b, K^c\right] = -{\delta_a}^c K^b + \frac{1}{7}{\delta_a}^b K^c \\ \left[{J_a}^b, K_{cd}\right] = {\delta_c}^b K_{ad} - {\delta_d}^b K_{ac} - \frac{2}{7}{\delta_a}^b K_{cd} \\ \left[{J_a}^b, K^{cde}\right] = -{\delta_a}^c K^{bde} + {\delta_a}^d K^{bce} - {\delta_a}^e K^{bcd} + \frac{3}{7}{\delta_a}^b K^{cde} \end{cases} \tag{16}$$

The commutators with the generators Q_{ab} are:

$$\begin{cases} [Q_{ab}, K^*] = K_{ab} \\ [Q_{ab}, K^c] = 0 \\ [Q_{ab}, K_{cd}] = -\frac{1}{3!}\epsilon_{abcdefg} K^{efg} \\ \left[Q_{ab}, K^{cde}\right] = \begin{cases} \left({\delta_a}^e{\delta_b}^d - {\delta_a}^d{\delta_b}^e\right) K^c + \left({\delta_a}^c{\delta_b}^e - {\delta_a}^e{\delta_b}^c\right) K^d \\ + \left({\delta_a}^d{\delta_b}^c - {\delta_a}^c{\delta_b}^d\right) K^e \end{cases} \end{cases} \tag{17}$$

The commutators with the generators Q^{ab} are:

$$\begin{cases} \left[Q^{ab}, K^*\right] = 0 \\ \left[Q^{ab}, K^c\right] = -K^{abc} \\ \left[Q^{ab}, K_{cd}\right] = \left({\delta_c}^a{\delta_d}^b - {\delta_c}^b{\delta_d}^a\right) K^* \\ \left[Q^{ab}, K^{cde}\right] = -\frac{1}{2}\epsilon^{abcdefg} K_{fg} \end{cases} \tag{18}$$

Again, the Jacobi identities involving any two O_{14} generators with either of K^*, K^a, K_{ab}, or K^{abc}, are all satisfied. On the other hand, with the two conjugate sets of generators, we can construct a quadratic (Casimir) operator of the following form,

$$K^*K + K^aK_a + \frac{1}{2}K_{ab}K^{ab} + \frac{1}{3!}K^{abc}K_{abc} \tag{19}$$

and verify that it commutes with all the O_{14} generators.

We are now ready to introduce the *fundamental* multiplet and its conjugate. This can be done by writing modules of the following forms,

$$\begin{cases} \mathcal{B} = BK^* + B_aK^a + \frac{1}{2}B^{ab}K_{ab} + \frac{1}{3!}B_{abc}K^{abc} \\ \\ \mathcal{B}^* = B^*K + B^aK_a + \frac{1}{2}B_{ab}K^{ab} + \frac{1}{3!}B^{abc}K_{abc} \end{cases} \tag{20}$$

In order to obtain the infinitesimal transformations of the above conjugate multiplets, and the gauge coupling later on, we need to compute the commutator of the following adjoint module $\mathcal{A}$ with any of the above,

$$\mathcal{A} = AJ + A_a{}^bJ_b{}^a + \frac{1}{2}A_{ab}Q^{ab} + \frac{1}{2}A^{ab}Q_{ab} \tag{21}$$

Computing the commutator of the adjoint module $\mathcal{A}$ with the fundamental module $\mathcal{B}$ we obtain a fundamental module $\mathcal{F}$. Using the preceding commutation relations, we obtain for the components of $\mathcal{F}$,

$$\begin{cases} F = -\frac{\sqrt{7}}{2}AB + \frac{1}{2}A_{ab}B^{ab} \\ \\ F_a = \frac{5}{2\sqrt{7}}AB_a - A_a{}^bB_b - \frac{1}{2}A^{bc}B_{abc} \\ \\ F^{ab} = -\frac{3}{2\sqrt{7}}AB^{ab} + A^{ab}B - A_c{}^aB^{bc} + A_c{}^bB^{ac} - \frac{1}{12}\epsilon^{abcdefg}A_{cd}B_{efg} \\ \\ F_{abc} = \begin{pmatrix} \frac{1}{2\sqrt{7}}AB_{abc} - A_{ab}B_c + A_{ac}B_b - A_{bc}B_a \\ \\ -A_a{}^dB_{bcd} + A_b{}^dB_{acd} - A_c{}^dB_{abd} - \frac{1}{4}\epsilon_{abcdefg}A^{de}B^{fg} \end{pmatrix} \end{cases} \tag{22}$$

Also, computing the commutator of the adjoint module $\mathcal{A}$ with the fundamental module $\mathcal{B}^*$ we obtain a fundamental module $\mathcal{F}^*$. Using the preceding commutation relations, we obtain for the components of $\mathcal{F}^*$,

$$\begin{cases} F^* = \frac{\sqrt{7}}{2}AB^* - \frac{1}{2}A^{ab}B_{ab} \\ \\ F^a = -\frac{5}{2\sqrt{7}}AB^a + A_c{}^aB^c + \frac{1}{2}A_{bc}B^{abc} \\ \\ F_{ab} = \frac{3}{2\sqrt{7}}AB_{ab} - A_{ab}B^* + A_a{}^cB_{bc} - A_b{}^cB_{ac} + \frac{1}{12}\epsilon_{abcdefg}A^{cd}B^{efg} \\ \\ F^{abc} = \begin{pmatrix} -\frac{1}{2\sqrt{7}}AB^{abc} + A^{ab}B^c - A^{ac}B^b + A^{bc}B^a \\ \\ A_d{}^aB^{bcd} - A_d{}^bB^{acd} + A_d{}^cB^{abd} + \frac{1}{4}\epsilon^{abcdefg}A_{de}B_{fg} \end{pmatrix} \end{cases} \tag{23}$$

Substituting Ω for A in the foregoing expressions, we can obtain the infinitesimal O_{14} transformations that pertain to the components of the fundamental $\mathcal{B}$ module and its conjugate $\mathcal{B}^*$. Hence, with an adjoint parameter multiplet $\{\Omega, \Omega_a{}^b, \Omega_{ab}, \Omega^{ab}\}$, we obtain the following infinitesimal transformations for the fundamental $\mathcal{B}$ multiplet:

$$\begin{cases} \delta B = -\frac{\sqrt{7}}{2}\Omega B + \frac{1}{2}\Omega_{ab}B^{ab} \\ \delta B_a = \frac{5}{2\sqrt{7}}\Omega B_a - \Omega_a{}^b B_b - \frac{1}{2}\Omega^{bc}B_{abc} \\ \delta B^{ab} = -\frac{3}{2\sqrt{7}}\Omega B^{ab} + \Omega^{ab}B - \Omega_c{}^a B^{bc} + \Omega_c{}^b B^{ac} - \frac{1}{12}\epsilon^{abcdefg}\Omega_{cd}B_{efg} \\ \delta B_{abc} = \begin{pmatrix} \frac{1}{2\sqrt{7}}\Omega B_{abc} - \Omega_{ab}B_c + \Omega_{ac}B_b - \Omega_{bc}B_a \\ -\Omega_a{}^d B_{bcd} + \Omega_b{}^d B_{acd} - \Omega_c{}^d B_{abd} - \frac{1}{4}\epsilon_{abcdefg}\Omega^{de}B^{fg} \end{pmatrix} \end{cases} \tag{24}$$

For the fundamental $\mathcal{B}^*$ multiplet, we obtain

$$\begin{cases} \delta B^* = \frac{\sqrt{7}}{2}\Omega B^* - \frac{1}{2}\Omega^{ab}B_{ab} \\ \delta B^a = -\frac{5}{2\sqrt{7}}\Omega B^a + \Omega_c{}^a B^c + \frac{1}{2}\Omega_{bc}B^{abc} \\ \delta B_{ab} = \frac{3}{2\sqrt{7}}\Omega B_{ab} - \Omega_{ab}B^* + \Omega_a{}^c B_{bc} - \Omega_b{}^c B_{ac} + \frac{1}{12}\epsilon_{abcdefg}\Omega^{cd}B^{efg} \\ \delta B^{abc} = \begin{pmatrix} -\frac{1}{2\sqrt{7}}\Omega B^{abc} + \Omega^{ab}B^c - \Omega^{ac}B^b + \Omega^{bc}B^a \\ \Omega_d{}^a B^{bcd} - \Omega_d{}^b B^{acd} + \Omega_d{}^c B^{abd} + \frac{1}{4}\epsilon^{abcdefg}\Omega_{de}B_{fg} \end{pmatrix} \end{cases} \tag{25}$$

The above infinitesimal transformations would leave invariant, the following bilinear associated with the fundamental $\mathcal{B}$ and $\mathcal{B}^*$ modules:

$$\mathcal{B}^* \cdot \mathcal{B} = B^*B + B^aB_a + \frac{1}{2}B_{ab}B^{ab} + \frac{1}{3!}B^{abc}B_{abc} \tag{26}$$

5 The O_{14} Gauge-Invariant Fermionic Lagrangian

Let us consider Weyl fermions Ψ that belong to a fundamental O_{14} module (like the above $\mathcal{B}$), with Dirac conjugates $\bar{\Psi}$ that belong to the conjugate module (like $\mathcal{B}^*$). The coupling of these to vector bosons $\mathcal{V}$, that constitute an adjoint O_{14} module, would come from the gauge-invariant Lagrangian,

$$\bar{\Psi}i\gamma^\mu\nabla_\mu\Psi \qquad \nabla_\mu\Psi = \partial_\mu - i\left[\mathcal{V}_\mu, \Psi\right] \tag{27}$$

Hence, we shall construct the fermionic kinetic bilinears and the coupling terms:

$$\bar{\Psi}(i\gamma\cdot\partial)\Psi \qquad \bar{\Psi}\gamma\cdot[\mathcal{V}, \Psi] \tag{28}$$

Hence introducing SU_7 components $\{\psi, \psi_a, \psi^{ab}, \psi_{abc}\}$, and their corresponding Dirac conjugates $\{\bar\psi, \bar\psi^a, \bar\psi_{ab}, \bar\psi^{abc}\}$, for the fermionic multiplet, and gauge boson components $\{V, V_a{}^b, V^{ab}, V^{ab}\}$, our strategy for constructing the invariant coupling is by starting with the invariant bilinear,

$$\bar\psi F + \bar\psi^a F_a + \frac{1}{2}\bar\psi_{ab}F^{ab} + \frac{1}{3!}\bar\psi^{abc}F_{abc} \tag{29}$$

and then building the $\mathcal{F}$ components from the commutator $[\mathcal{V}, \psi]$, as we have detailed in the preceding section. The followings are the resulting coupling terms, suppressing the vector indices that are carried by the components of $\mathcal{V}_\mu$ and the Dirac gamma matrices:

$$\left\{\begin{array}{l} -\frac{\sqrt{7}}{2}V\bar\psi\gamma\psi + \frac{5}{2\sqrt{7}}V\bar\psi^a\gamma\psi_a - \frac{3}{4\sqrt{7}}V\bar\psi_{ab}\gamma\psi^{ab} + \frac{1}{12\sqrt{7}}V\bar\psi^{abc}\gamma\psi_{abc} \\ \\ -V_a{}^b\bar\psi^a\gamma\psi_b + V_a{}^b\bar\psi_{bc}\gamma\psi^{ac} - \frac{1}{2}V_a{}^b\bar\psi^{acd}\gamma\psi_{bcd} \\ \\ +\frac{1}{2}V_{ab}\bar\psi\gamma\psi^{ab} - \frac{1}{2}V_{ab}\bar\psi^{abc}\gamma\psi_c - \frac{1}{24}\epsilon^{abcdefg}V_{ab}\bar\psi_{cd}\gamma\psi_{efg} \\ \\ +\frac{1}{2}V^{ab}\bar\psi_{ab}\gamma\psi - \frac{1}{2}V^{ab}\bar\psi^c\gamma\psi_{abc} - \frac{1}{24}\epsilon_{abcdefg}V^{ab}\bar\psi^{cde}\gamma\psi^{fg} \end{array}\right. \tag{30}$$

All the indices in the above expression pertain to the fundamental multiplet of SU_7. In the followings, we shall split these indices into three sectors: a first index denoted by 1 and corresponding to a particle of charge -1, a tri-valued index denoted by $(i, j, k, \ldots)$ and corresponding to charge $\frac{1}{3}$ quarks and *color* SU_3, a tri-valued index denoted by $(r, s, t, \ldots)$ and corresponding to neutral particles and *family* SU_3. Hence, the electric charge operator, for the fundamental multiplet of SU_7, takes on the diagonal eigenvalues,

$$Q = \text{diag}\{-1, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 0, 0, 0\} \tag{31}$$

According to the prescribed splitting, we shall introduce the components of Ψ, $\bar\Psi$, and $\mathcal{V}$, with somewhat familiar particle terminology. You will notice that, since we are dealing with Weyl fermions, we shall use the symbol (e) to denote any electron-like particle (charge -1), and (e^*) any positron-like particle. We shall use the symbol (ν) to denote any neutrino-like particle (charge 0), and (ν^*) the antiparticle. We shall use the symbol (u) to denote any upquark-like particle (charge $\frac{2}{3}$), with (u^*) the antiparticle. We shall use the symbol (d) to denote any downquark-like particle (charge $-\frac{1}{3}$), with (d^*) the antiparticle. Quark-like particles will carry an SU_3 color index like $(i, j, k, \cdots)$. Any particle could carry an SU_3 family index like $(r, s, t, \cdots)$.

The singlet ψ and its conjugate $\bar\psi$ will represent an antineutrino:

$$\psi \to \nu^* \qquad \bar\psi \to \bar{\nu^*} \tag{32}$$

For ψ_a, and $\bar\psi^a$, we have

$$\left\{\begin{array}{ll} \psi_1 \to e & \bar\psi^1 \to \bar e \\ \\ \psi_i \to d^*_i & \bar\psi^i \to \bar{d^*}^i \\ \\ \psi_r \to \nu_r & \bar\psi^r \to \bar\nu^r \end{array}\right. \tag{33}$$

Notice that we have introduced 3 neutrinos in association with the electron.

For ψ^{ab}, and $\bar{\psi}_{ab}$, we have

$$\begin{cases} \psi^{1i} \to u^i & \bar{\psi}_{1i} \to \bar{u}_i \\ \psi^{1r} \to (e^*)^r & \bar{\psi}_{1r} \to \bar{e^*}_r \\ \psi^{ij} \to \epsilon^{ijk} u^*_k & \bar{\psi}_{ij} \to \epsilon_{ijk} \bar{u^*}^k \\ \psi^{ir} \to d^{ir} & \bar{\psi}_{ir} \to \bar{d}_{ir} \\ \psi^{rs} \to \epsilon^{rst} \nu^*_t & \bar{\psi}_{rs} \to \epsilon_{rst} \bar{\nu^*}^t \end{cases} \tag{34}$$

For ψ_{abc}, and $\bar{\psi}^{abc}$, we have

$$\begin{cases} \psi_{1ij} \to \epsilon_{ijk} d^k & \bar{\psi}^{1ij} \to \epsilon^{ijk} \bar{d}_k \\ \psi_{1ir} \to u^*_{ir} & \bar{\psi}^{1ir} \to \bar{u^*}^{ir} \\ \psi_{1rs} \to \epsilon_{rst} e^t & \bar{\psi}^{1rs} \to \epsilon^{rst} \bar{e}_t \\ \psi_{ijk} \to \epsilon_{ijk} e^* & \bar{\psi} \to \epsilon^{ijk} \bar{e^*} \\ \psi_{ijr} \to \epsilon_{ijk} u^k_r & \bar{\psi}^{ijr} \to \epsilon^{ijk} \bar{u}^r_k \\ \psi_{irs} \to \epsilon_{rst} (d^*)^t_i & \bar{\psi}^{irs} \to \epsilon^{rst} (\bar{d^*})^i_t \\ \psi_{rst} \to \epsilon_{rst} \nu & \bar{\psi}^{rst} \to \bar{\nu} \end{cases} \tag{35}$$

Now proceeding to the vector bosons, we begin with $V_a{}^b$, which being traceless, is given the following assignments:

$$\begin{cases} V_1{}^1 \to \mathcal{A} + \frac{1}{4}\mathcal{Z} & V_1{}^i \to \mathcal{X}^i & V_1{}^r \to \mathcal{W}^r \\ V_i{}^1 \to \mathcal{X}_i & A_i{}^j \to \mathcal{G}_i{}^j - \frac{1}{3}\delta_i{}^j \mathcal{A} + \frac{1}{4}\delta_i{}^j \mathcal{Z} & V_i{}^r \to \mathcal{Y}_i{}^r \\ V_r{}^1 \to \mathcal{W}_r & V_r{}^i \to \mathcal{Y}_r{}^i & V_r{}^s \to \mathcal{H}_r{}^s - \frac{1}{3}\delta_r{}^s \mathcal{Z} \end{cases} \tag{36}$$

Notice that $\mathcal{G}_i{}^j$ is traceless in color indices, and $\mathcal{H}_r{}^s$ is traceless in family indices. For V_{ab}, we have

$$\begin{cases} V_{1i} \to \mathcal{P}_i & V_{1r} \to \mathcal{E}_r \\ V_{ij} \to \epsilon_{ijk} \mathcal{U}^k & V_{ir} \to \mathcal{D}_{ir} \\ V_{rs} \to \epsilon_{rst} \mathcal{N}^t & \end{cases} \tag{37}$$

For V^{ab}, we have

$$\begin{cases} V^{1i} \to \mathcal{P}^i & V^{1r} \to \mathcal{E}^r \\ V^{ij} \to \epsilon^{ijk}\mathcal{U}_k & V^{ir} \to \mathcal{D}^{ir} \\ V^{rs} \to \epsilon^{rst}\mathcal{N}_t \end{cases} \tag{38}$$

Before writing out the boson-fermion coupling terms, let us check the fermion kinetic bilinears, and the boson bilinears, in terms of the foregoing assignments. For the following O_{14} invariant fermion kinetic bilinears, written in terms SU_7 invariant forms,

$$\bar{\psi}(i\gamma\cdot\partial)\psi + \frac{1}{2}\bar{\psi}_{ab}(i\gamma\cdot\partial)\psi^{ab} + \frac{1}{3!}\bar{\psi}^{abc}(i\gamma\cdot\partial)\psi_{abc} \tag{39}$$

we obtain, after splitting in terms of color SU_3 and family SU_3 invariant forms,

$$\begin{cases} \bar{\nu}(i\gamma\cdot\partial)\nu + \bar{\nu^*}(i\gamma\cdot\partial)\nu^* + \bar{\nu}^r(i\gamma\cdot\partial)\nu_r + \bar{\nu^*}^r(i\gamma\cdot\partial)\nu_r^* \\ +\bar{e}(i\gamma\cdot\partial) + \bar{e^*}(i\gamma\cdot\partial)e^* + \bar{e}_r(i\gamma\cdot\partial)e^r + \bar{e^*}_r(i\gamma\cdot\partial)(e^*)^r \\ +\bar{u}_i(i\gamma\cdot\partial)u^i + \bar{u^*}(i\gamma\cdot\partial)u_i^* + \bar{u}_i^r(i\gamma\cdot\partial)u_r^i + \bar{u^*}^{ir}(i\gamma\cdot\partial)u_{ir}^* \\ +\bar{d}_i(i\gamma\cdot\partial)d^i + \bar{d^*}^i(i\gamma\cdot\partial)d_i^* + \bar{d}_{ir}(i\gamma\cdot\partial)d^{ir} + \bar{d^*}^i_r(i\gamma\cdot\partial)(d^*)_i^r \end{cases} \tag{40}$$

Notice how, with respect to family SU_3 whose fundamental index is r, the fermions of a certain charge type (neutrino-like, electron-like, upquark-like, or downquark-like) clearly demonstrate similar groupings into *singlets* and *triplets*.

Now with regard to the following O_{14} invariant boson bilinears, written in terms of SU_7 invariant forms,

$$V^2 + V_a{}^b V_b{}^a + \frac{1}{2}V_{ab}V^{ab} \tag{41}$$

we obtain, after splitting with respect to the foregoing color SU_3 and family SU_3 symmetries,

$$\begin{cases} V^2 + \frac{4}{3}\mathcal{A}^2 + \frac{7}{12}\mathcal{Z}^2 + \mathcal{G}_i{}^j\mathcal{G}_j{}^i + \mathcal{H}_r^s\mathcal{H}_s^r \\ +2\mathcal{W}_r\mathcal{W}^r + 2\mathcal{X}_i\mathcal{X}^i + 2\mathcal{Y}_i{}^r\mathcal{Y}_r{}^i \\ +\mathcal{E}_r\mathcal{E}^r + \mathcal{N}_r\mathcal{N}^r + \mathcal{P}_i\mathcal{P}^i \\ +\mathcal{U}_i\mathcal{U}^i + \mathcal{D}_{ir}\mathcal{D}^{ir} \end{cases} \tag{42}$$

In order that the above terms take a canonical form, we must have a *rescaling* of some fields. However, we shall not do that in the followings, but one must remember to do the rescaling in practical applications.

Now the splitting of the O_{14} boson-fermion coupling terms (30) can be done, using the preceding assignments. The resulting terms are over 500 in number. However, after simplifications and various arrangements they reduce to 175 terms. In the following subsections, we give these terms that concern the couplings to specific vector bosons.

5.1 Couplings to the V Boson

Here, we give the couplings of the fundamental fermionic particles to the V vector boson that is associated with the U_1 symmetry in O_{14}, which commutes with SU_7.

$$V \times \frac{1}{2\sqrt{7}} \begin{pmatrix} 5\bar{e}\gamma e + \bar{e^*}\gamma e^* + \bar{e}_r\gamma e^r - 3\bar{e^*}_r\gamma(e^*)^r \\ +\bar{\nu}\gamma\nu - 7\bar{\nu^*}\gamma\nu^* + 5\bar{\nu}^r\gamma\nu_r - 3\bar{\nu^*}^r\gamma\nu^*_r \\ -3\bar{u}_i\gamma u^i - 3\bar{u^*}^i\gamma u^*_i + \bar{u}^r_i\gamma u^i_r + \bar{u^*}^{ir}\gamma u^*_{ir} \\ +\bar{d}_i\gamma d^i + 5\bar{d^*}^i\gamma d^*_i - 3\bar{d}_{ir}\gamma d^{ir} + \bar{d^*}^i_r\gamma(d^*)^r_i \end{pmatrix} \tag{43}$$

Notice that all fermions would participate in the above couplings.

5.2 Couplings to the Photon $\mathcal{A}$

Here, we give the couplings of the fundamental fermionic particles to the photon $\mathcal{A}$:

$$\mathcal{A} \times \begin{pmatrix} -\bar{e}\gamma e + \bar{e^*}\gamma e^* - \bar{e}_r\gamma e^r + \bar{e^*}_r\gamma(e^*)^r \\ +\frac{2}{3}\bar{u}_i\gamma u^i - \frac{2}{3}\bar{u^*}^i\gamma u^*_i + \frac{2}{3}\bar{u}^r_i\gamma u^i_r - \frac{2}{3}\bar{u^*}^{ir}\gamma u^*_{ir} \\ -\frac{1}{3}\bar{d}_i\gamma d^i + \frac{1}{3}\bar{d^*}^i\gamma d^*_i - \frac{1}{3}\bar{d}_{ir}\gamma d^{ir} + \frac{1}{3}\bar{d^*}^i_r\gamma(d^*)^r_i \end{pmatrix} \tag{44}$$

Of course, only the charged particles would participate in the above.

5.3 Couplings to the $\mathcal{Z}$ Boson

Here we give the couplings of the fundamental fermionic particles to the $\mathcal{Z}$ vector boson:

$$\mathcal{Z} \times \frac{1}{4} \begin{pmatrix} 4\bar{\nu}\gamma\nu + \frac{4}{3}\bar{\nu}^r\gamma\nu_r - \frac{8}{3}\bar{\nu^*}^r\gamma\nu^*_r \\ -\bar{e}\gamma e - 3\bar{e^*}\gamma e^* + \frac{5}{3}\bar{e}_r\gamma e^r - \frac{1}{3}\bar{e^*}_r\gamma(e^*)^r \\ +2\bar{u}_i\gamma u^i + 2\bar{u^*}^i\gamma u^*_i - \frac{2}{3}\bar{u}^r_i\gamma u^i_r - \frac{2}{3}\bar{u^*}^{ir}\gamma u^*_{ir} \\ -3\bar{d}_i\gamma d^i - \bar{d^*}^i\gamma d^*_i - \frac{1}{3}\bar{d}_{ir}\gamma d^{ir} + \frac{5}{3}\bar{d^*}^i_r\gamma(d^*)^r_i \end{pmatrix} \tag{45}$$

Notice that the antineutrino particle ν^*, which is singlet with respect to SU_7, does not participate in the above couplings.

5.4 Couplings to the $\mathcal{G}$ Bosons

Here we give the couplings of the fundamental fermionic particles to the $\mathcal{G}$ vector bosons of color SU_3:

$$\mathcal{G}_i{}^j \times \begin{pmatrix} \bar{u}_j\gamma u^i - \bar{u^*}^i\gamma u^*_j - \bar{u}^r_j\gamma u^i_r - \bar{u^*}^{ir}\gamma u^*_{jr} \\ +\bar{d}_j\gamma d^i - \bar{d^*}^i\gamma d^*_j + \bar{d}_{jr}\gamma d^{ir} - \bar{d^*}^i_r\gamma (d^*)^r_j \end{pmatrix} \tag{46}$$

Notice that only the colored quarks and antiquarks would participate in the above couplings (the manifest SU_3 color indices are i and j). These group themselves as singlets and triplets with respect the SU_3 family (with manifest family index r)

5.5 Couplings to the $\mathcal{H}$ Bosons

Here we give the couplings of the fundamental fermionic particles to the $\mathcal{H}$ vector bosons of the family (or horizontal) SU_3 symmetry:

$$\mathcal{H}_r{}^s \times \begin{pmatrix} \bar{e}_s\gamma e^r + \bar{e^*}_s\gamma (e^*)^r - \bar{\nu}^r\gamma\nu_s - \bar{\nu^*}^r\gamma\nu^*_s \\ -\bar{u}^r_i\gamma u^i_s - \bar{u^*}^{ir}\gamma u^*_{is} + \bar{d}_{is}\gamma d^{ir} + \bar{d^*}^i_s\gamma (d^*)^r_i \end{pmatrix} \tag{47}$$

In the above, only the fermions that are triplets with respect to family SU_3 would participate in the couplings (the manifest indices of family symmetry are r and s, while the manifest quark color SU_3 index is i).

5.6 Couplings to the $\mathcal{W}$ Bosons

Here, we give the couplings of the fundamental fermionic particles to the $(\mathcal{W}^r, \mathcal{W}_r)$ vector bosons. The latter are conjugate *triplets* with respect to the family SU_3 that are charge ± 1 particles, like the $W^\pm$ of electroweak theory.

$$\mathcal{W}^r \times \begin{pmatrix} -\bar{e}\gamma\nu_r - \bar{e}_r\gamma\nu \\ -\bar{d}_i\gamma u^i_r - \bar{d}_{ir}\gamma u^i \\ +\epsilon_{rst}\bar{u^*}^{is}\gamma (d^*)^t_i - \epsilon_{rst}\bar{\nu^*}^s\gamma (e^*)^t \end{pmatrix} + \text{conj.} \tag{48}$$

In the above, we have suppressed the conjugate terms involving $\mathcal{W}_r$. Notice how the *triplet* of $\mathcal{W}$'s exchanges a singlet charged lepton with a triplet of associated neutrinos, and a singlet neutrino with a triplet of charged leptons. Likewise a singlet upquark is exchanged with a triplet of downquarks, and a triplet of upquarks with a singlet downquark. Notice, as well, how triplets of antineutrinos are exchanged with triplets of charged antileptons, via *family alternation*, using the family SU_3 epsilon symbol. The latter mechanism also involves triplets of anti-upquarks and anti-downquarks.

5.7 Couplings to the $\mathcal{X}$ Bosons

Here we give the couplings of the fundamental fermionic particles to the $(\mathcal{X}^i, \mathcal{X}_i)$ vector bosons. The latter are charge $\pm\frac{4}{3}$ colored bosons, often termed *leptoquarks* in grand unified theories.

$$\mathcal{X}^i \times \begin{pmatrix} -\bar{e}\gamma d_i^* - \bar{d}_i \gamma e^* \\ \\ +\bar{d}_{ir}\gamma (e^*)^r - \bar{e}_r \gamma (d^*)_i^r \\ \\ -\epsilon_{ijk}\bar{u^*}^j \gamma u^k - \epsilon_{ijk}\bar{u^*}^{jr}\gamma u_r^k \end{pmatrix} + \text{conj.} \tag{49}$$

In the above, we have suppressed the conjugate terms with $\mathcal{X}_i$. Notice that the neutrinos do not participate in the above couplings, simply because there are no charge $\pm\frac{4}{3}$ quarks to couple with.

5.8 Couplings to the $\mathcal{Y}$ Bosons

Here we give the couplings of the fundamental fermionic particles to the $(\mathcal{Y}_i^r, \mathcal{Y}_r^i)$ vector bosons. These are charge $\pm\frac{1}{3}$ colored particles, being *triplets* with respect to family SU_3. They are also called leptoquarks in grand unified theories, and in some theories would be responsible for proton decay, since they exchange quarks with antiquarks.

$$\mathcal{Y}_i^r \times \begin{pmatrix} -\bar{e^*}\gamma u_r^i + \bar{e^*}_r \gamma u^i - \epsilon_{rst}\bar{u^*}^{is}\gamma e^t \\ \\ -\bar{d^*}^i \gamma \nu_r - \bar{d^*}_r^i \gamma \nu - \epsilon_{rst}\bar{\nu^*}^s \gamma d^{it} \\ \\ -\epsilon^{ijk}\bar{d}_j \gamma u_{kr}^* - \epsilon^{ijk}\bar{d}_{jr}\gamma u_k^* - \epsilon^{ijk}\epsilon_{rst}\bar{u}_j^s \gamma (d^*)_k^t \end{pmatrix} + \text{conj.} \tag{50}$$

In the above, we suppress the conjugate terms with $\mathcal{Y}_r^i$. Notice how the $\mathcal{Y}$ triplets of vector bosons would exchange singlet leptons with triplet quarks and vice versa. Likewise, triplets of quarks are exchanged with singlets of antiquarks and vice versa. Also, via family alternation, using the epsilon of family SU_3, triplets of leptons are exchanged with triplets of quarks, and triplets of quarks are exchanged with triplets of antiquarks.

5.9 Couplings to the $\mathcal{E}$ Bosons

Here we give the couplings of the fundamental fermionic particles with the $(\mathcal{E}_r, \mathcal{E}^r)$ vector bosons. The latter are charge ± 1 conjugate *triplets* of family SU_3.

$$\mathcal{E}_r \times \begin{pmatrix} \bar{\nu^*}\gamma (e^*)^r + \bar{\nu^*}^r \gamma e^* + \epsilon^{rst}\bar{e}_s \gamma \nu_t \\ \\ +\bar{u^*}^i \gamma (d^*)_i^r + \bar{u^*}^{ir}\gamma d_i^* + \epsilon^{rst}\bar{d}_{is}\gamma u_t^i \end{pmatrix} + \text{conj.} \tag{51}$$

In the above, we suppress the conjugate terms involving $\mathcal{E}^r$. Notice that the SU_7 singlet antineutrino ν^* does participate in the above couplings, being exchanged with a triplet of charged antileptons. Observe, as well, how other singlets are exchanged with triplets, and vice versa, and how triplets are exchanged with triplets via SU_3 family alternation.

5.10 Couplings to the $\mathcal{N}$ Bosons

Here we give the couplings of the fundamental fermionic particles to the $(\mathcal{N}_r, \mathcal{N}^r)$ vector bosons. The latter are neutral particles that are conjugate *triplets* with respect to family SU_3.

$$\mathcal{N}_r \times \begin{pmatrix} -\bar{e}\gamma e^r + \bar{e^*}\gamma(e^*)^r - \bar{\nu}^r\gamma\nu + \bar{\nu^*}^r\gamma\nu^* \\ \\ -\bar{d}_i\gamma d^{ir} - \bar{d^*}^i\gamma(d^*)^r_i - \bar{u^*}^{ir}\gamma u^*_i - \bar{u}^r_i\gamma u^i \end{pmatrix} + \text{conj.} \tag{52}$$

In the above, we suppress conjugate terms that involve $\mathcal{N}^r$. Notice that the SU_7 singlet antineutrino ν^* does participate in the above couplings. It is being exchanged with a triplet of antineutrinos. Notice also the underlying exchange between singlets and triplets for all other leptons and quarks.

5.11 Couplings to the U Bosons

Here we give the couplings of the fundamental fermionic particles to the $(\mathcal{U}^i, \mathcal{U}_i)$ vector bosons. The latter are charge $\pm\frac{2}{3}$ colored particles.

$$\mathcal{U}^i \times \begin{pmatrix} -\bar{u}_i\gamma\nu - \bar{u}^r_i\gamma\nu_r + \bar{\nu^*}\gamma u^*_i - \bar{\nu^*}^r\gamma u^*_{ir} \\ \\ -\bar{d}_i\gamma e - \bar{d}_{ir}\gamma e^r - \bar{e^*}\gamma d^*_i + \bar{e^*}_r\gamma(d^*)^r_i \end{pmatrix} + \text{conj.} \tag{53}$$

In the above, we suppress conjugate terms that involve $\mathcal{U}_i$. Notice that the SU_7 singlet antineutrino ν^* does participate in the above couplings. It is being exchanged with the antiquark u^*_i.

5.12 Couplings to the $\mathcal{D}$ Bosons

Here we give the couplings of the fundamental fermionic particles to the $(\mathcal{D}^{ir}, \mathcal{D}_{ir})$ vector bosons. The latter are charge $\pm\frac{1}{3}$ colored particles that are *triplets* with respect to family SU_3.

$$\mathcal{D}^{ir} \times \begin{pmatrix} -\bar{e}\gamma u^*_{ir} - \bar{e}_r\gamma u^*_i + \epsilon_{rst}\bar{u}^s_i\gamma(e^*)^t \\ \\ -\bar{d}_i\gamma\nu^*_r + \bar{d}_{ir}\gamma\nu^* - \epsilon_{rst}\bar{\nu}^s\gamma(d^*)^t_i \\ \\ +\epsilon_{ijk}\bar{d^*}^j\gamma u^k_r + \epsilon_{ijk}\bar{d^*}^j_r\gamma u^k + \epsilon_{ijk}\epsilon_{rst}\bar{u^*}^{js}\gamma d^{kt} \end{pmatrix} + \text{conj.} \tag{54}$$

In the above, we suppress conjugate terms that involve $\mathcal{D}_{ir}$. Notice that the SU_7 singlet antineutrino ν^* does participate in the above couplings. It is being exchanged with the quark d_{ir}.

5.13 Couplings to the $\mathcal{P}$ Bosons

Here we give the couplings of the fundamental fermionic particles to the $(\mathcal{P}_i, \mathcal{P}^i)$ vector bosons. The latter are charge $\pm\frac{2}{3}$ colored particles.

$$\mathcal{P}_i \times \begin{pmatrix} -\bar{u^*}^i\gamma\nu - \bar{u^*}^{ir}\gamma\nu_r - \bar{\nu^*}^r\gamma u_r^i + \bar{\nu^*}\gamma u^i \\ \\ +\epsilon^{ijk}\bar{d}_j\gamma d_k^* + \epsilon^{ijk}\bar{d}_{jr}\gamma(d^*)_k^r \end{pmatrix} + \text{conj.} \tag{55}$$

In the above, we suppress conjugate terms that involve $\mathcal{P}^i$. Notice that the SU_7 singlet antineutrino ν^* does participate in the above couplings. It is being exchanged with the singlet quark u^i.

6 Discussion

In spite of the underlying extensive algebraic formalism, we have been able to write out the detailed expressions for the boson-fermion couplings that exhibit particle content with *color* SU_3 and *family* SU_3 symmetries, and show that the O_{14} unification model can be handled in practice. This effort can be compared to the enormously much more extensive counterpart associated with the development of the supersymmetric E_8 unification model[8], with color SU_3 and family SU_5 symmetries. In another article, we shall detail the algebraic structure of the 18-dimensional gravidynamic model[4] and its relationship to the foregoing O_{14} model.

It is interesting that by presenting the SU_7 structure of the the present quark-lepton unification model in color SU_3 and family SU_3 decomposition, we have noted that the fermionic particles, as well as the bosonic particles, do appear either as *singlets* or as *triplets* with respect to family SU_3. The important question, in this regard, is *whether the observed three generations of quarks and leptons would correspond to this a triplet structure*, and whether a heavier 4th generation, the singlet quarks and leptons in our model, does exist. Whether one can predict the masses of these heavier fermions is a problem that awaits the completion of *a truly predictive effective theory of symmetry breaking*, perhaps different from the deficient Higgs mechanism that seems to lack any real predictive power.

We think that the most important implication of a theory, like the one treated in this article, that could have relevance to high-energy collider experiments is *the possible observation of a family of $W^\pm$-like particles* (a triplet according to this theory). Of course, a host of other vector bosons are part and parcel of such a grand unification theory, but the observation of the extra W's should be the nearest and *most imminent possibility*. We hope to return to other phenomelogical aspects as soon as experimental signs start showing up. Perhaps, even earlier, when our truly predictive theoretical setup, in connection with the effective action of quantum field theory, starts running.

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