The Topological Skyrme Model and the Current and the Constituent Quarks in Quantum Chromodynamics

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Abstract

It is well known that a quark manifests its current-character and simultaneously its constituent-character, depending upon what energy scale or length scale one is interested in for a particular phenomenon. Here we show that there is a fundamental conflict between these two concepts, when one looks carefully as to how the electric charge for a quark is defined in these two, current and constituent, structures. This is a crisis for quantum chromodynamics. We show then that the topological Skyrme Model comes to the rescue. We prove in this paper as to how unambiguously and uniquely, it is the Skyrme Model which solves this conundrum in Quantum Chromodynamics.

Keywords: Skyrme Model, current quarks, constituent quarks, electric charge, Standard Model, Quantnum Chromodynamics

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The topological Skyrme Model is an intriguing and interesting model of hadron physics [1,2,3]. In recent years, it is continuing to prove its worth in nuclear physics and particle physics [4]. Another basic aspect of the topological Skyrme Model is its unique prediction of a very heavy scalar Sigma meson particle, which may compete with the putative Higgs particle discovered recently or be close to it [5]. However its most amazing manifestation is the fact that through the anomaly term in the Skyrme Model, a fundamental link is established between the short distance behaviour (exhibited through the quarks) and the long distance behaviour (through the octet mesonic fields). Thus one finds that the anomaly in the Skyrme Model, yields "quarks without quarks" [2]. But what kind link is it and what type of quarks one is talking about - the current quarks or the constituent quarks? We look into this problem in this paper.

Current quarks, which arise in the basic definition of the Quantum Chromodynamics Lagrangean, are taken as being pointlike and having zero or very small masses. The quarks, which arise in the low energy hadron models, having a finite size and having a subsantial amount of mass, are called the constituent quarks [6]. These two concepts coexist without conflict, as the size and the mass, are treated as basic properties, depending upon the energy scale or the size scale relevant in a particular phenomenon. They connect one-to-one with each other as to the above properties. These, as should be, also have the same defining quantum numbers: e.g. the electric charge and the baryon number of say the up-quark, of either kind (the current and the constitent quarks), have the same value of 2/3 and 1/3 respectively.

For the constituent quarks in the group $SU(3)_F$, one has the well known Gell-Mann-Nishijima formula for the electric charge [6] for the the three flavours: u-, d- and s- quarks

$$Q = T_3 + \frac{Y}{2} \tag{1}$$

Here T_3 and Y correspond to the two diagonal generators of the group $SU(3)_F$. Here Y=B+S with B=1/3 is the baryon number and S the strangeness quantum number. Thus for the u-quark, Q(u) = 2/3.

So primarily motivated by the above definition, in their study of the quark model in the large colour limit of $SU(N)_c$ QCD, Witten et. al. [7,8], took the charges of the u- and d-quarks to be fixed as 2/3 and -1/3 respectively. This, the same and the static value of the charges, they took for any arbitrary number of colours. Note that in this picture of Witten, the quarks are necessarily of the current-kind.

However the author showed [9], that the above static current quark charges were wrong ones to use. It was shown that these would mess up the whole structure of the baryons in the corresponding topological Skyrme odel. The reader may see [9] for details. Here it will suffice to state that, it was also demonstraed that in QCD, it being part of the Standard Model, the correct charges to take are the colour dependent ones given below.

In that paper [9] the author had shown that contary to the popular belief against it, the electric charge is actually consistently quantized in the Standard Model. The same generalized for arbitrary number of colours in the Standard Model group $SU(N)_c \otimes SU(2)_L \otimes U(1)_Y$ the electric charge is:

$$Q = I_3 + \frac{1}{2N_c} \tag{2}$$

Here I_3 is the diagonal generator of the weak $SU(2)_L$ group in the Standard Model, and $\frac{1}{N_c}$ = B is the baryon number.

Thus

$$Q(u) = \frac{1}{2}(1 + \frac{1}{N_c}) \tag{3}$$

So for $N_c = 3$ one gets the charge of 2/3 and due to the above colour dependence, for $N_c \to \infty$ it becomes 1/2.

Here we notice right away, the basic and fundamental conflict in the way the electric charge gets defined in these two basic and successful models of the strong interaction. The current quark charge as given by eqn (2) is fundamentally different from the constituent quark charge given by eqn (1). The two are completely incompatible. Being such a primary quantum number as the electric charge, these can still not be reduced from one to the other. This is a basic puzzle and it just cannot be ignored.

Note that as per the current understanding, the constituent quark is taken as a current quark surrounded by a sea of quark-antiquark pairs and umpteen number of gluons. Gluons are uncharged, and the charge of the quark-antiquark pairs is zero and hence the electric charge of such a constituent quark is that of its core current quark. Thus such a constituent quark entity has the same colour dependent electric charge as the current quark itself. And this is at complete variance with the consituent quark charge structure (eqn. (1)) of the $SU(3)_F$ quark model.

We can also look at at it this way. Given a particular quark, it cannot be simultaneously treated as being of the current and the constituent kind - as we actually do in several models. For example we say that a massless current quark, bound inside the MIT bag, due to its confinement, develops the constituent-quark mass. Given the conflicting structures of the electric charge for them, this should now be completely wrong. So what is the solution to this conundrum?

Below we show that the topological Skyrme Model comes to the rescue. We show below that this fundamental conflict between the current and the constituent pictures of the quark in QCD, requires the presence of the topological Skyrme Model to solve this problem intrinsically.

Now we show what the structure of the electric charge in the Skyrme Model is. Let us start with the Skyrme Lagrangian [1,2,3,4]

$$L_S = \frac{{f_\pi}^2}{4} Tr(L_\mu L^\mu) + \frac{1}{32e^2} Tr[L_\mu, L_\nu]^2 \tag{4}$$

where $L_\mu = U^\dagger \partial_\mu U$. The U field for the three flavour case for example is

$$U(x) = exp[\frac{i\lambda^a \phi^a(x)}{f_\pi}] \tag{5}$$

with ϕ^a the pseudoscalar octet of π, K and η mesons. In the full topological Skyrme this is supplemented with a Wess-Zumino effective action

$$\Gamma_{WZ} = \frac{-i}{240\pi^2} \int_\Sigma d^5x \epsilon^{\mu\nu\alpha\beta\gamma} Tr[L_\mu L_\nu L_\alpha L_\beta L_\gamma] \tag{6}$$

on surface Σ. Let the field U be transformed by the charge operator Q as

$$U(x) \rightarrow e^{i\Lambda Q} U(x) e^{-i\Lambda Q} \tag{7}$$

where all the charges are counted in units of the absolute value of the electronic charge.

Making $\Lambda = \Lambda(x)$ a local transformation the Noether current is [10]

$$J_\mu{}^{em}(x) = j_\mu{}^{em}(x) + j_\mu{}^{WZ}(x) \tag{8}$$

where the first one is the standard Skyrme term and the second is the Wess-Zumino term [1,2,3]

$$j_\mu{}^{WZ}(x) = \frac{N_c}{48\pi^2}\epsilon_{\mu\nu\lambda\sigma} Tr L^\nu L^\lambda L^\sigma (Q + U^\dagger Q U) \tag{9}$$

Next we take the U(1) of electromagnetism as a subgroup of the three flavour SU(3). Its generators can be found by the canonical methods. As the charge operator can be simultaneously diagonalized along with the third component of isospin and hypercharge we write it as

$$Q = \begin{pmatrix} q_1 & 0 & 0 \\ 0 & q_2 & 0 \\ 0 & 0 & q_3 \end{pmatrix}$$

The electric charge of pseudoscalar octet mesons are known. these give

$$q_1 - q_2 = 1, q_2 = q_3 \tag{10}$$

Hence one obtains

$$Q = (q_2 + \frac{1}{3})\mathbf{1}_{3x3} + \frac{1}{2}\lambda_3 + \frac{1}{2\sqrt{3}}\lambda_8 \tag{11}$$

In the standard way we use $U = A(t)U_c(\mathbf{x})A(t)^{-1}$ where A is the collective coordinate. We obtain the B=1 electric charge from the Skyrme term in terms of the left-handed generators L_α only as [10]

$$Q^{em} = \frac{1}{2}(L_3 - (A^\dagger\lambda_3 A)_8 \frac{N_c B(U_c)}{\sqrt{3}}) + \frac{1}{2\sqrt{3}}(L_8 - (A^\dagger\lambda_8 A)_8 \frac{N_c B(U_c)}{\sqrt{3}}) \tag{12}$$

The Wess-Zumino term contributes

$$Q^{WZ} = N_c B(U_c)(q_2 + \frac{1}{3} + \frac{1}{2\sqrt{3}}(A^\dagger\lambda_3 A)_8 + \frac{1}{6}(A^\dagger\lambda_3 A)_8) \tag{13}$$

Hence the total electric charge is [10,11]

$$Q = I_3 + \frac{1}{2}Y + (q_2 + \frac{1}{3})N_c B(U_c) \tag{14}$$

Using the above expression, Balachandran et. al. [11, p. 210], state that, "The last term vanishes once we take the down quark charge q_2 to be -1/3, and we are left with the Gell-Mann Nishijima formula" (eqn. (1) above). They say so, only because they have ignored the colour dependence of the hypercharge in the Skyrme Model. This is wrong as one just can not ignore the colour dependence of the hypercharge in the second term in eqn. (14) above, while at the same time, colour is fully included in the last term. This insconsistency is avoided by taking for B=1 the hypercharge to be Y = $\frac{N_c}{3}$ [10,12]. Actually it is well-known that this is the proper hypercharge to use in the Skyrme Model [1,2,3].

And only for N_c=3 does the hypercharge in the Skyrme Model reduce to the one in the $SU(3)_F$ model of Gell-Mann. This fact too is well known. For example, in Ref. [13] the proper represenatations for say, the proton and the neutron, arise for $N_F = 3$ only with N_c=3. One gets extra strangeness and extra hypercharge for these states fo N_c 3 and these vanish only for N_c=3.

Now if we take q_2 (which is the colour-dependent charge of the d-quark) as arising from eqn. (2) and equal to -1/3, then the second term (which is also colour-dependent) cancels. Hence the correct Gell-Mann Nishijima expression for the electric charge of the constituent quarks in $SU(3)_F$ arise only when we take N_c=3 in the above expression for hypercharge. As this is occuring due to the topological mechanisms of the Skyrme Model, we may justifiably conclude, that the colour dependence of the electric charge of the QCD quark is eaten up by the Skyrme Model and which thereafter, due to the colour dependence arisisng from within itself, produces the correct electric charge of the Gell-Mann-Nishijima formula for the constituent quaks for the $SU(3)_F$ group, but only when N_c=3.

Thus, what the above suggests is a phenomenological composite onion-like picture of the baryons. At short distance, we have the current quarks with their colour dependent electric charges as given in eqn (2). Outside, it is surroundided by a baryonic structure arising from the topological Skyrme Model with its full electric charge contribution as given in eqn. (14). And as per the above discussion, this together produecs the correct constituent quark electric charge structure.

In summary, we find that the defintion of the electric charges, for the current quark and the constituent quarks in QCD, are fundamentally incompatible. This leads to a basic conflict between the two concepts in the quark model - a crisis for the quark model. We have shown that the topological Skyrme Model comes to the rescue. An onion-like model of baryons, with the inner part being made up of the current quarks, and the outer part made up of the the structure arising from the topological Skyrme Model. The inner part colour dependence of the electric charge of the quarks then, uniquely, gets cancelled by the outer Skyrme Model part with its own colour dependence. The stucture of the rest of the electric charge still has colour dependence in the hypercharge. Only for N_c=3 does it reduce to the proper Gell-Mann Nishijima expression for the electric charge for the constituent quarks, Note that this new picture of the connection between the current and the constituent quark is at variance with the conventional one popularly held today - that of the constituent quark as a current quark surrounded by a sea of quark-antiquark and a bunch of gluons. This also explains how and in what manner, does the Skyrme Model actually yields "quarks without quarks" [2]. We would like to emphasize that this means that the topological Skyrme Model is as fundamantal to an understanding of the baryons, as Quantum Chromodynamics is.

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