Confinement of Charge Creation and Annihilation Centers by Nakanishi-Lautrup Field

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(Dated: March 13, 2014)

The electromagnetic field model including Nakanishi-Lautrup (NL) field of quantum electrodynamics (QED) can easily treat creation and annihilation of positive and negative charge pairs, although it is difficult for Maxwell's equations to treat them. However, the model does not directly satisfy the charge conservation equation and permits single charge creation and annihilation. It is shown that the potential energy of NL field for a pair of charge creation and annihilation centers is proportional to their distance. It causes the confinement of charge creation and annihilation centers, which means the charge conservation for this model. The quark confinement might be also explained by the energy of NL field.

KEYWORDS: confinement, Maxwell's equations, charge conservation, Nakanishi-Lautrup field

Maxwell's equations have been used for analyses of electromagnetic field since J. C. Maxwell found the equations in 1865[1]. In early 1930s, E. Fermi proposed modified electromagnetic field model for QED[2–4], where he assumed that 4-D vector potential satisfy d' Alembert equation even in the case except Lorenz gauge condition. Gupta and Bleuler gave subsidiary conditions to Fermi's model in 1950[5, 6]. In 1960s, Nakanishi and Lautrup proposed the auxiliary field called Nakanishi-Lautrup (NL) field[7–10] to describe Lorentz covariant electromagnetic field model for QED. It is now included in the model of QED and Yang-Mills theory[11–14]. Recently, we found that the electromagnetic field model including a Lorentz scalar field, which is equivalent to NL field with Feynman gauge, can easily treat creation and annihilation of positive and negative charge pairs, although it is difficult for Maxwell's equations to treat them[15, 16]. However, the model does not directly satisfy the charge conservation equation and permits single charge creation and annihilation. In this paper, it is shown that the NL field induces the additional field energy and causes the confinement of charge creation and annihilation centers, which means the charge conservation for this model.

Maxwell's equations are given by

$$\mathbf{J} = \nabla \times \mathbf{H} - \varepsilon \frac{\partial \mathbf{E}}{\partial t}, \tag{1}$$

$$\rho = \varepsilon \nabla \mathbf{E}, \tag{2}$$

$$\nabla \times \mathbf{E} + \mu \frac{\partial \mathbf{H}}{\partial t} = 0, \tag{3}$$

$$\nabla \mathbf{H} = 0, \tag{4}$$

where $\mathbf{J}$ and ρ are current and charge density, ε and μ are permittivity and permeability, $\mathbf{E}$ and $\mathbf{H}$ are electric and magnetic field, respectively. Eqs. (1) and (2) directly give the following equation of the charge conservation,

$$\nabla \mathbf{J} + \frac{\partial \rho}{\partial t} = 0. \tag{5}$$

The creation and annihilation of positive and negative charge pairs are ordinarily described by the following equation, which is given by semiconductor physics[17–19],

$$\nabla \mathbf{J}_p + \frac{\partial \rho_p}{\partial t} = -\nabla \mathbf{J}_n - \frac{\partial \rho_n}{\partial t} = G, \tag{6}$$

where ρ_p and ρ_n are positive and negative charge concentration, $\mathbf{J}_p$ and $\mathbf{J}_n$ are positive and negative charge current density, and G is charge creation-annihilation rate. Since Maxwell's equations satisfy the principle of superposition[20], positive and negative charges must individually satisfy Eqs. (1) and (2). Therefore, positive charges satisfy

$$\mathbf{J}_p = \nabla \times \mathbf{H}_p - \varepsilon \frac{\partial \mathbf{E}_p}{\partial t}, \tag{7}$$

and

$$\rho_p = \varepsilon \nabla \mathbf{E}_p, \tag{8}$$

where $\mathbf{E}_p$ and $\mathbf{H}_p$ denote electric and magnetic field induced by positive charges, respectively. Eqs. (7) and (8) directly give

$$\nabla \mathbf{J}_p + \frac{\partial \rho_p}{\partial t} = 0, \tag{9}$$

which contradicts (6) in the case of $G \neq 0$. Since this situation is same for negative charges, it is difficult for Maxwell's equations to treat creation and annihilation of charge pairs.

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In order to solve the above problem, we introduce Nakanishi-Lautrup field B and a gauge parameter α. The Lagrangian density of the electromagnetic field $\mathcal{L}_{EM}$ is given by[10]

$$\mathcal{L}_{EM} = -\frac{1}{4}F^{\nu\lambda}F_{\nu\lambda} + B\partial^\nu A_\nu + \frac{1}{2}\alpha B^2 - \mu J^\nu A_\nu, \quad (10)$$

where J^ν and A^ν denote 4-D current $(c\rho, \mathbf{J})$ and 4-D vector potential $(\psi/c, \mathbf{A})$, respectively, and $F^{\nu\lambda}$ is given by

$$F^{\nu\lambda} = \partial^\nu A^\lambda - \partial^\lambda A^\nu. \quad (11)$$

The above Lagrangian density gives the following equations.

$$\mu J_\nu = \Box A_\nu - \partial_\nu \partial^\lambda A_\lambda - \partial_\nu B, \quad (12)$$

$$\partial^\nu A_\nu + \alpha B = 0, \quad (13)$$

$$\pi^\nu = \frac{\partial \mathcal{L}_{EM}}{\partial(\partial_0 A_\nu)} = (B, -\mathbf{E}/c), \quad (14)$$

where π^ν denotes 4-D canonical momentum density and $\Box$ is d'Alembertian defined by $\Box \equiv \partial_0^2 - \nabla^2$.

Since $\mathbf{E}$ and $\mathbf{H}$ are written by

$$\mathbf{E} = -\nabla\psi - \frac{\partial \mathbf{A}}{\partial t}, \quad (15)$$

$$\mathbf{H} = \frac{1}{\mu}\nabla \times \mathbf{A}, \quad (16)$$

Eqs. (1) and (2) are rewritten by Eqs. (12), (15) and (16) as

$$\mathbf{J} = \nabla \times \mathbf{H} - \varepsilon\frac{\partial \mathbf{E}}{\partial t} + \frac{1}{\mu}\nabla B, \quad (17)$$

$$\rho = \varepsilon \nabla \mathbf{E} - \varepsilon \frac{\partial B}{\partial t}. \quad (18)$$

Then, the charge creation-annihilation rate is given by

$$G = \nabla \mathbf{J} + \frac{\partial \rho}{\partial t} = -\frac{1}{\mu}\Box B. \quad (19)$$

The above relation enable us to treat creation and annihilation of positive and negative charge pairs. It should be noticed that $G = 0$ needs not $B = 0$ but $\Box B = 0$. When we consider about Lorentz covariance, $\mathbf{E}$, $\mathbf{H}$, $\mathbf{A}$, ψ, $\mathbf{J}$ and ρ have same transformation as Maxwell's equations, although B, G, and $\Box$ are not changed by Lorentz transformation. Although Eq. (13) does not satisfy the gauge invariance, if a scalar function Λ satisfies $\Box\Lambda = 0$, $\mathbf{E}$, $\mathbf{H}$, and B are not changed by the transformation of

$$\mathbf{A}' = \mathbf{A} + \nabla\Lambda, \quad (20)$$

$$\psi' = \psi - \frac{\partial \Lambda}{\partial t}. \quad (21)$$

The above model is a natural extension from 3-D to 4-D field for the complex electromagnetic field $\mu\mathbf{H} + i\mathbf{E}/c$. Maxwell's equations, given by Eqs. (1), (2), (3), (4), (15), and (16), can be written by using 3-D complex field as

$$\begin{pmatrix} \mu H_x + iE_x/c \\ \mu H_y + iE_y/c \\ \mu H_z + iE_z/c \end{pmatrix} = \begin{pmatrix} -i\partial_0 & -\partial_z & \partial_y & -\partial_x \\ \partial_z & -i\partial_0 & -\partial_x & -\partial_y \\ -\partial_y & \partial_x & -i\partial_0 & -\partial_z \end{pmatrix} \begin{pmatrix} A_x \\ A_y \\ A_z \\ i\psi/c \end{pmatrix}, \quad (22)$$

$$\mu \begin{pmatrix} J_x \\ J_y \\ J_z \\ ic\rho \end{pmatrix} = \begin{pmatrix} i\partial_0 & -\partial_z & \partial_y \\ \partial_z & i\partial_0 & -\partial_x \\ -\partial_y & \partial_x & i\partial_0 \\ \partial_x & \partial_y & \partial_z \end{pmatrix} \begin{pmatrix} \mu H_x + iE_x/c \\ \mu H_y + iE_y/c \\ \mu H_z + iE_z/c \end{pmatrix}. \quad (23)$$

The model including NL field, given by Eqs. (3), (4), (13), (15), (16), (17), and (18), can be written by using 4-D complex field as

$$\begin{pmatrix} \mu H_x + iE_x/c \\ \mu H_y + iE_y/c \\ \mu H_z + iE_z/c \\ -\alpha B \end{pmatrix} = \begin{pmatrix} -i\partial_0 & -\partial_z & \partial_y & -\partial_x \\ \partial_z & -i\partial_0 & -\partial_x & -\partial_y \\ -\partial_y & \partial_x & -i\partial_0 & -\partial_z \\ \partial_x & \partial_y & \partial_z & -i\partial_0 \end{pmatrix} \begin{pmatrix} A_x \\ A_y \\ A_z \\ i\psi/c \end{pmatrix}, \quad (24)$$

$$\mu \begin{pmatrix} J_x \\ J_y \\ J_z \\ ic\rho \end{pmatrix} = \begin{pmatrix} i\partial_0 & -\partial_z & \partial_y & -\partial_x \\ \partial_z & i\partial_0 & -\partial_x & -\partial_y \\ -\partial_y & \partial_x & i\partial_0 & -\partial_z \\ \partial_x & \partial_y & \partial_z & i\partial_0 \end{pmatrix} \begin{pmatrix} \mu H_x + iE_x/c \\ \mu H_y + iE_y/c \\ \mu H_z + iE_z/c \\ -B \end{pmatrix}. \quad (25)$$

Now we compare the calculation result given by Maxwell's equations and the electromagnetic field model including NL field, using a simple structure. Fig. 1 shows the example structure consisting of a sphere with radius R including large amount of positive and negative moving charges confined in the sphere, which satisfy

$$\mathbf{J}_p + \mathbf{J}_n = \rho_p + \rho_n = \mathbf{E}_p + \mathbf{E}_n = \mathbf{H}_p + \mathbf{H}_n = 0, \quad (26)$$

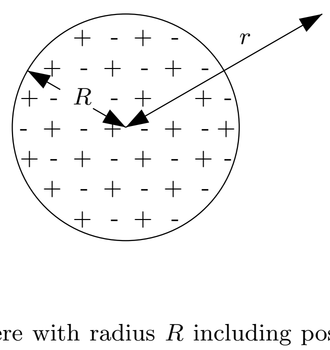

FIG. 1. A sphere with radius R including positive and negative moving charges to be annihilating with time constant τ.

where $\mathbf{E}_n$ and $\mathbf{H}_n$ denote electric and magnetic field induced by negative charges, respectively. Then, the NL fields B_p and B_n induced by annihilation of positive and negative charges also satisfy

$$B_p + B_n = 0. \tag{27}$$

It is assumed that the positive and negative charges $Q_p = 4\pi\rho_p R^3/3$ and $Q_n = 4\pi\rho_n R^3/3$ in the sphere linearly decrease by charge pair annihilation with time as

$$Q_p = -Q_n = Q_0(1 - \frac{t}{\tau}), \tag{28}$$

where Q_0 is the absolute value of positive and negative charge at $t = 0$ and τ is the time constant of charge pair annihilation. By using spherical coordinate system and Gauss's law, the electric field out of the sphere has only radial component as

$$E_p = -E_n = \frac{Q_0(1-\frac{t}{\tau})}{4\pi\varepsilon r^2}, \tag{29}$$

Since this structure has spherical symmetry, the magnetic field does not exist[20]. In the case of Maxwell's equations, the radial component of the current J_p and J_n out of the sphere are needed by Eqs. (1) and (29) as

$$J_p = -J_n = -\varepsilon\frac{\partial E_p}{\partial t} = \frac{Q_0}{4\pi\tau r^2}. \tag{30}$$

The above result does not describe the real condition, because the positive and negative charge currents cannot exist out of the sphere. Maxwell's equations cannot change charge concentration without current because of the charge conservation of Eq. (5). If we consider the NL field B_p and B_n out of the sphere induced by annihilation of positive and negative charges, they are given by analogy of the relation between charge and potential as

$$B_p = -B_n = \frac{\mu Q_0}{4\pi\tau r}. \tag{31}$$

The positive and negative charge current density J_p and J_n out of the sphere are given by Eqs. (17), (29), and (31) as

$$J_p = -J_n = -\varepsilon\frac{\partial E_p}{\partial t} + \frac{1}{\mu}(\nabla B_p)_r = 0. \tag{32}$$

There is no current out of the sphere. The electromagnetic field model including NL field gives the reasonable result.

Next we consider about the electromagnetic field energy including NL field[21]. By using Eqs. (1), (2), (14), (17) and (18), $cJ^\nu\pi_\nu$ is written by

$$cJ^\nu\pi_\nu = \mathbf{JE} + c^2\rho B = -\nabla\left(\mathbf{E}\times\mathbf{H} - \frac{1}{\mu}B\mathbf{E}\right) - \frac{\partial}{\partial t}\left(\frac{\varepsilon E^2}{2} + \frac{\mu H^2}{2} + \frac{B^2}{2\mu}\right). \tag{33}$$

Since the above equation is regarded as the continuity equation for energy density, $\mathbf{JE} + c^2\rho B$ is energy annihilation rate, $\mathbf{E}\times\mathbf{H} - B\mathbf{E}/\mu$ is the energy flow vector, and $(\varepsilon E^2 + \mu H^2 + B^2/\mu)/2$ is the energy density. The NL field induces the additional energy density of $B^2/2\mu$.

The NL field permits the existence of charge creation and annihilation centers by Eq. (19). As shown by Eq. (31), the NL field B induced by a point charge creation or annihilation center is given by

$$B = -\frac{\mu\sigma}{4\pi r}, \tag{34}$$

where σ denotes the creating charge per unit time. If the charge creation or annihilation center is isolated, the potential energy of the NL field V_{NL} in a surrounding sphere with radius R is given by

$$V_{NL} = 4\pi\int_0^R \frac{B^2}{2\mu}r^2 dr = \frac{\mu\sigma^2 R}{8\pi}. \tag{35}$$

Since the potential energy is proportional to R, an isolated charge creation or annihilation center cannot stably exist. However, some kinds of pairs of charge creation and annihilation centers can stably exist. Table I shows the force between two centers A and B that create or annihilate positive or negative charges, where the upper 4 cases induce attraction and the others induce repulsion. Only the upper 4 pairs can stably exist, because attractive force reduces the potential energy of NL field. Fig. 2 shows the creation and annihilation centers for positive charges, where d denotes their distance. The total NL field B_{pair} induced by the pair of creation and annihilation centers for positive charges shown in Fig. 2 is given by

$$B_{pair} = -\frac{\mu\sigma}{4\pi}\left(\frac{1}{r} - \frac{1}{\sqrt{r^2 + d^2 - 2rd\cos\theta}}\right), \tag{36}$$

where the charge creation and annihilation rates are assumed to be equal to σ, because the difference between the creation and annihilation rates induces similar potential energy as Eq. (35). If we assume the surrounding

TABLE I. Force between A and B centers with creation and annihilation functions for positive and negative charges.

Charge-A	Function-A	Charge-B	Function-B	Force
positive	creation	positive	annihilation	attraction
positive	creation	negative	creation	attraction
negative	creation	negative	annihilation	attraction
positive	annihilation	negative	annihilation	attraction
positive	creation	positive	creation	repulsion
negative	creation	negative	creation	repulsion
positive	annihilation	positive	annihilation	repulsion
negative	annihilation	negative	annihilation	repulsion
positive	creation	negative	annihilation	repulsion
positive	annihilation	negative	creation	repulsion

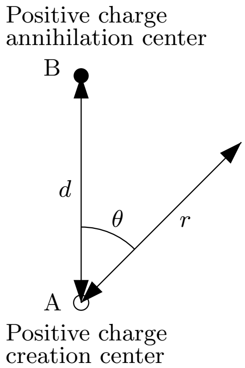

FIG. 2. Positive charge creation and annihilation centers with their distance d.

sphere radius R is enough larger than d ($R \gg d$), the NL field potential energy of the pair V_{NLpair} in the sphere is given by

$$V_{NLpair}(d) = 2\pi \int_0^R \left(\int_0^\pi \frac{B_{pair}^2}{2\mu} \sin\theta d\theta \right) r^2 dr$$
$$= \frac{\mu\sigma^2}{8\pi} \left(\int_0^R 2dr - \int_0^R \int_0^\pi \frac{r\sin\theta}{\sqrt{r^2+d^2-2rd\cos\theta}} d\theta dr \right)$$
$$= \frac{\mu\sigma^2}{8\pi} \left(2R - \int_0^R \frac{d+r-|d-r|}{d} dr \right)$$
$$= \frac{\mu\sigma^2 d}{8\pi}. \quad (37)$$

Therefore the potential energy is proportional to the distance d and the attractive force between charge creation and annihilation centers is constant. It causes the confinement of charge creation-annihilation centers, which means the charge conservation in this model instead of Eq. (5). The above discussion does not depend on the gauge parameter α.

The quark confinement has been energetically studied[22–24] since Gell-Mann and Zweig introduced quarks in hadron's model in 1960s[25, 26]. Although duality model was proposed by Nambu, t'Hooft, and Mandelstam in 1970s[27–30], the theoretical explanation of the confinement has not succeeded yet. Since the potential dependence on the distance between creation and annihilation centers is same as the linear potential of quarks based on the spinning stick model for Regge trajectories[23], the quark confinement might be explained by the energy of NL field.

In conclusion, the electromagnetic field model including NL field can easily treat creation and annihilation of positive and negative charge pairs. The NL field gives the additional field energy, which is proportional to the distance between charge creation and annihilation centers. It causes the confinement of charge creation and annihilation centers, which means the charge conservation in this electromagnetic field model. The quark confinement might be also explained by the energy of NL field.

The author would like to thank Professor Kei-Ichi Kondo of Chiba University for his helpful suggestion about NL field.

[1] J. C. Maxwell, Phil. Trans. R. Soc. Lond. **155**, 459 (1865).
[2] E. Fermi., Rev. Mod. Phys **4**, 87 (1932).
[3] S. S. Schweber, *An Introduction to Relativistic Quantum Field Theory* (Harper and Row, 1962).
[4] F. Mandl and G. Shaw, *Quantum Field Theory 2nd ed.* (Wiley, 2010).
[5] S. N. Gupta, Proc. Phys. Soc. **63**, 681 (1950).
[6] K. Bleuler, Helv. Phys. Acta **23**, 567 (1950).
[7] N. Nakanishi, Progr. Theor. Phys. **35**, 1111 (1966).
[8] N. Nakanishi, Progr. Theor. Phys. Suppl. **51**, 1 (1972).
[9] B. Lautrup, K. Danske Vidensk, Selk. Mat.-fis. Medd. **35**, no. 11 (1967).
[10] N. Nakanishi, *Ba-No-Ryoshiron (Quantum Field Theory)* (Baifukan, in Japanese, 1975).
[11] C. N. Yang and R. L. Mills, Phys. Rev. **96**, 191 (1954).
[12] I. J. R. Aitchison, *An Informal Introduction to Gauge Field Theories* (Cambridge University Press, 1982).
[13] T. Kugo, *Gauge-Ba-No-Ryoshiron (Quantum Theory of Gauge field)* (Baifukan, in Japanese, 1989).
[14] K. Kondo, *Gauge-Ba-No-Ryosiron-Nyumon (Introduction to Quantum Theory of Gauge field)* (Science-Sha, in Japanese, 2013).
[15] H. Mutoh, J. Institute of Image Information and Television Engineers **67**, J89 (2013), https://www.jstage.jst.go.jp/article/itej/67/3/67_J89/_pdf.
[16] H. Mutoh, "Proceedings of the 2013 international image sensor workshop," (2013), https://www.imagesensors.org/Past%20Workshops/2013%20Workshop/2013%20Papers/07-01_052-Mutoh_paper.pdf.
[17] S. M. Sze, *Physics of Semiconductor Devices* (Wiley, New York, 1981).
[18] A. S. Grove, *Physics and Technology of Semiconductor Devices* (Wiley, New York, 1967).
[19] S. Selberherr, *Analysis and Simulation of Semiconductor Devices* (Springer, Wien, 1984).
[20] R. P. Feynman, R. B. Leighton, and M. L. Sands, *Feynman Lectures on Physics, 2* (Addison-Wesley, 1965).
[21] K. J. van Vlaenderen and A. Waser, Hadronic Journal **24**, 609 (2001).
[22] K. G. Wilson, Phys. Rev. **D10**, 2445 (1974).
[23] G. S. Bali, Phys. Rept. **343**, 1 (2001), hep-ph/0001312.
[24] R. Alkofer and J. Greensite, (2006), hep-ph/0610365.
[25] M. Gell-Mann, Physics Letters **8**, 214 (1964).
[26] G. Zweig, CERN Report , No. 8182/TH.401 (1964).
[27] Y. Nambu, Phys. Rev. **D10**, 4262 (1974).
[28] G. 't Hooft, Nucl. Phys. **B79**, 276 (1974).
[29] S. Mandelstam, Phys. Rev. **D11**, 3026 (1975).
[30] A. M. Polyakov, JETP Lett. **20**, 194 (1974).