Question of Planckian "Action" in Gravitational Wave Detection Experiments

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Abstract

The validity of Planck's constant in gravitational wave detection experiments is brought into question in the context of the framework of quantum mechanics. It is shown that in the absence of a *purely* gravitational measurement of Planck's constant one *cannot* at present rule out the possibility that gravitational quanta may be *scaled* by a more diminutive "action." An experiment that could unequivocally test this possibility is suggested.

Keywords: Gravitational wave detection experiments; Planck's constant; Sub-Planckian action; Graviton; Quantum cosmology; Uncertainty principle

1 Introduction

The search for gravitional waves, one of the centerpieces of general relativity, has been a work in progress for over five decades. Two main forms of detectors are currently in use worldwide. The first, pionered by Weber [1] in the 1960's, is based on the expectation that a passing gravitational wave will induce a mechanical oscillation in a cryogenically cooled cylindrical bar whose resonance can then be amplified and recorded. The second method, using lasers, is designed to measure spacetime geometry variations between mirrors suspended in vacuum using interferometry in a Michelson configuration. Despite the ever increasing sensitivity of these detectors the *direct* detection of gravitational waves in a laboratory setting remains elusive. It is therefore pertinent to inquire at this point in the long history of these experiments whether the failure to detect gravitational waves directly is simply a question of detector sensitivity, or if it is symptomatic of yet to be discovered physics.

It should be observed that if we confine ourselves to pursuing this question exclusively from a quantum mechanical perspective we are inevitably struck by the fact that the graviton is envisaged in analogy with the photon as being *scaled* by

Planck's constant, whose role is generally accepted as fact without questions concerning the possible limit of its validity being asked. It is important to point out that no *purely* gravitational measurement of Planck's constant exists to support such a premise [2]. This realization makes it appear almost unavoidable that consideration be given to the possibility that gravitons, unlike photons, may not be *scaled* by Planck's constant. It should be emphasized from the outset that any discussion of this possibility has as its foundation the irrefutable fact that nature has made available *two* immutable *elementary* "actions" in the context of the framework of quantum mechanics. Namely, Planck's familiar constant, h, which has been shown experimentally to play an indispensable role in the microphysical realm, and a second, more diminutive, *sub-Planckian* "action" formed from two of the basic constants of quantum mechanics, namely (e^2/c) — the ratio of the square of the elementary charge to the velocity of light, which has the value 7.6957×10^{-37} J s (henceforth denoted by the symbol j for simplicity of presentation). The purpose of this paper is to present a summary of results obtained by a heuristic application of this *sub-Planckian* "action" to a spin-2 boson, which renders transparent the possibility that gravitational waves may be subject to a more diminutive quantum of "action" than Planck's.

2 Sub-Planckian quantum mechanical interpretation of gravitational quanta

It is reasonable to expect, from the linear approximation of general relativity's field equations, that this *sub-Planckian* quantized field will be a plane wave, with wave vector k_μ and helicity ± 2, as consisting of quanta. That is, *gravitons* with energy-momentum vector $p^\mu = (j/2\pi)k^\mu$ and spin component in the direction of motion $\pm 2(j/2\pi)$. The fact that there exist only two physical states of polarization for a gravitational wave of given momentum corresponds to the well known result of relativistic quantum theory that a particle of zero mass must have its spin along the direction of motion (positive helicity) or opposite to the direction of motion (negative helicity). For a large assembly of gravitons, all of which have four-momenta

$$p^\mu = (j/2\pi)k^\mu \ , \tag{1}$$

the energy-momentum tensor is given by

$$T_{\mu\nu} = \frac{(j/2\pi)k_\mu k_\nu}{\omega} N \tag{2}$$

where N is the number of gravitons per unit volume. If we now compare this with the *helicity* amplitude for a gravitational plane wave, described by the energy-momentum tensor

$$\langle t_{\mu\nu} \rangle = \frac{k_\mu k_\nu}{16\pi G} \left(|e_+|^2 + |e_-|^2 \right) \ , \tag{3}$$

we can determine the number density of gravitons with helicity ± 2 in a plane wave [5]. We then have

$$N_{\pm} = \frac{w}{16\pi(j/2\pi)G}\, |e_{\pm}|^2 \tag{4}$$

which describes, at the simplest level, what would be interpreted as a gravitational plane wave consisting of quanta (gravitons). Implicit in this description of gravitational radiation is the possibility of subjecting this *sub-Planckian* "action" to experimental scrutiny vis-a-vis Planck's constant independently of the conception of any particular model.

3 Possible experimental test

To have a sensitive unequivocal test one must be able to differentiate between these two elementary "actions." Clearly, the most direct way is to measure the *vibrational* displacement induced in a resonant detector by a passing gravitational wave. To be more specific, let us assume, using Planck's constant, that a graviton of *angular* frequency ω has an energy

$$E = \hbar\omega \ . \tag{5}$$

We can then profit from the fact that the *vibrational* energy induced in a *resonant* detector, by a gravitational wave, can be converted to the fractional change in *vibrational* displacement by making use of the relation between amplitude x_0, energy E and the total mass M for a harmonic oscillator, in the familiar form

$$E = \frac{1}{2} M\omega^2 x_0^2 \ . \tag{6}$$

If we now take as an example Weber's seminal experiment, which used as an antenna a 1400 kg cylindrical aluminum bar that had a natural *resonance* frequency ν_0 of 1660 Hz, we can readily compute the *vibrational* displacement, x, caused by a *single* graviton of *angular* frequency $\omega = 2\pi\nu_0$, and energy $\hbar\omega$. Combining Eqs. (5) and (6) and then substituting these values, we obtain

$$\begin{aligned} x &= (2\hbar/M\omega)^{1/2} \\ &\approx 3.8 \times 10^{-21} m \end{aligned} \tag{7}$$

which may be compared with the *vibrational* displacement caused by a *single* graviton of the same *angular* frequency ω, carrying an energy $(j/2\pi)\omega$, in the analogous form

$$\begin{aligned} x &= [2(j/2\pi)/M\omega]^{1/2} \\ &\approx 1.3 \times 10^{-22} m \ . \end{aligned} \tag{8}$$

Unfortunately, such extraordinarily small displacements could not be measured with the technology available in Weber's day. Indeed, even today neither of these two conflicting results can be ruled out since at present there are no resonant-mass antennas or laser interferometers in operation that have the required sensitivity.

Since Weber's pioneering work in the 1960s numerous projects have been undertaken in an effort to enhance detector sensitivity. One of the more innovative of these efforts has been the development of the Schenberg *spherical* resonant-mass telescope in Brazil [6], which has the advantage of being omnidirectional. When fully operational it will provide information regarding a wave's amplitude, polarization, and direction of source. The detector program, which we shall presently exploit, uses an 1150 kg spherical resonant-mass made of a copper-aluminum alloy, and has a resonance frequency ν_0 of 3200 Hz. The *vibrational* displacement caused by a *single* graviton of *angular* frequency ω can be computed on the basis of Planck's constant by direct substitution in Eq. (7). We thus obtain

$$x \approx 3.0 \times 10^{-21} m \tag{9}$$

which is clearly in conflict with the *sub-Planckian* result obtaining from Eq. (8) for a graviton of the same *angular* frequency

$$x \approx 1.0 \times 10^{-22} m \ . \tag{10}$$

It is anticipated that at the standard quantum limit of sensitivity the Schenberg will be able to resolve a displacement of around 3×10^{-21} meters, which is sufficient to determine if gravitational quanta are scaled by Planck's constant. As for the result of Eq.(10), deriving from the *sub-Planckian* quantum of "action," verification is contingent on the Schenberg surpassing the standard quantum limit of sensitivity by *squeezing* the signal, which should result in a ten-fold increase in sensitivity.

4 Summary

The validity of Planck's constant in gravitational wave detection experiments was brought into question in the context of the framework of quantum mechanics. It was shown that in the absence of a *purely* gravitational measurement of Planck's constant one *cannot* at present rule out the possibility that gravitational quanta may be *scaled* by the more diminutive of nature's *two* elementary "actions," quantitatively given by e^2/c. An experiment that could unequivocally test this possibility was suggested.

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References

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Appendix

The recognition of e^2/c as an elementary *sub-Planckian* quantum of "action" (denoted by the symbol j) inevitably leads to quantum uncertainty, as formulated by Heisenberg, in the analogous form (Messina, 2012)

$$(\Delta x)(\Delta p) \approx j \tag{1}$$

where, as usual, x is the uncertainty of position, and p the uncertainty in momentum. Its implication for the *temporal* events that make up the big bang can be simply illustrated in terms of the *sub-Planckian* unit of time, T_0, analogous to the Planck time $T_p = (\hbar G/c^5)^{1/2}$ in the form

$$\begin{aligned} T_0 &= \left[(j/2\pi)G/c^5\right]^{1/2} \\ &= 1.837 \times 10^{-45} s \end{aligned} \tag{2}$$

where $(j/2\pi)$ is the reduced *sub-Planckian* "action" constant, G is the Newtonian gravitational constant, and c is the velocity of light. Unfortunately, because of the *sub-Planckian* uncertainty principle, Eq.(1), we are prevented from speculating on times shorter than 10^{-44} seconds after the big bang, which is an order of magnitude prior to the Planck era (10^{-43} seconds). The disparity in this temporal sequence of events is, needless to say, cosmologically significant since it implies that a *sub-Planckian* era *preceded* the Planck era in the nascent universe, which should be discernible from its gravitational signature.